Three loop soft gluon corrections to di-lepton and Higgs productions at the LHC

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RECAPP

Harish-Chandra Research Institute, Allahabad

- Introduction
- Scale ambiguity
- Sudakov Resummation of soft gluons at N^3LO
- Drell-Yan and Higgs productions
- Conclusions

In collaboration with

Willy van Neerven, Jack Smith

Snap shot of the talk

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  2) Factorisation scale

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- Soft gluons dominate in some kinematic regions that are accessible at hadron colliders.
- Sudakov resummation of soft gluons can be used to predict for Drell-Yan and Higgs total cross sections and their rapidity distributions at N^3LO and beyond.

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- "BUT"

- We have not found Higgs Boson yet!. The Higgs boson is the last particle to be discovered in SM.

Higgs Mass

[Summer 2004, LEPEWWG]

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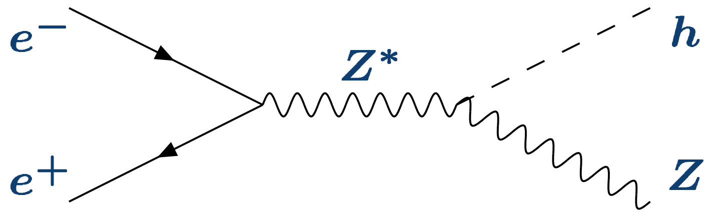
Direct:



$$m_h > 114.4^{+69}_{-45}\ GeV$$

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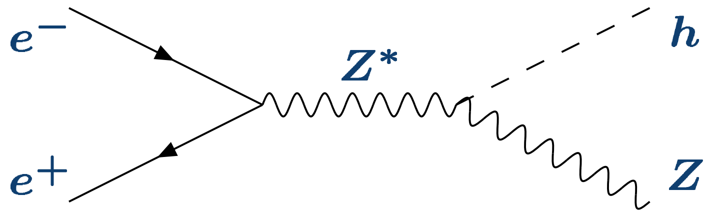

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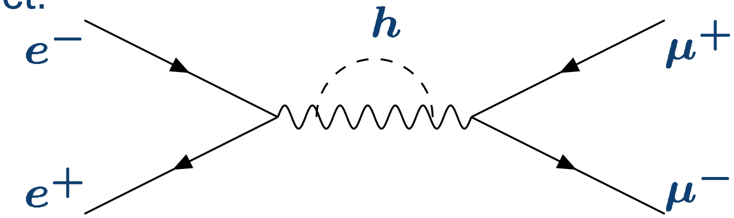

$$m_h < 260\ GeV\ (\ 95\%\ CL)$$

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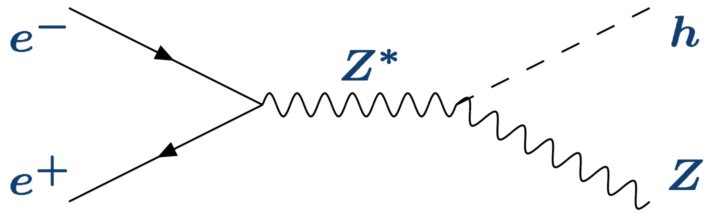

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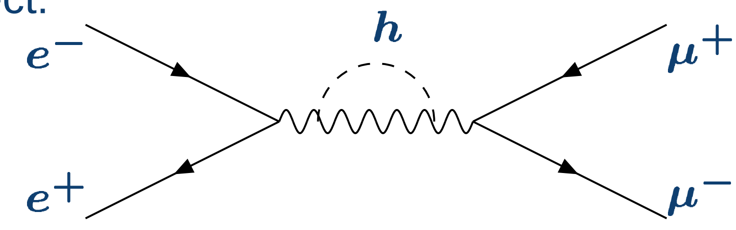

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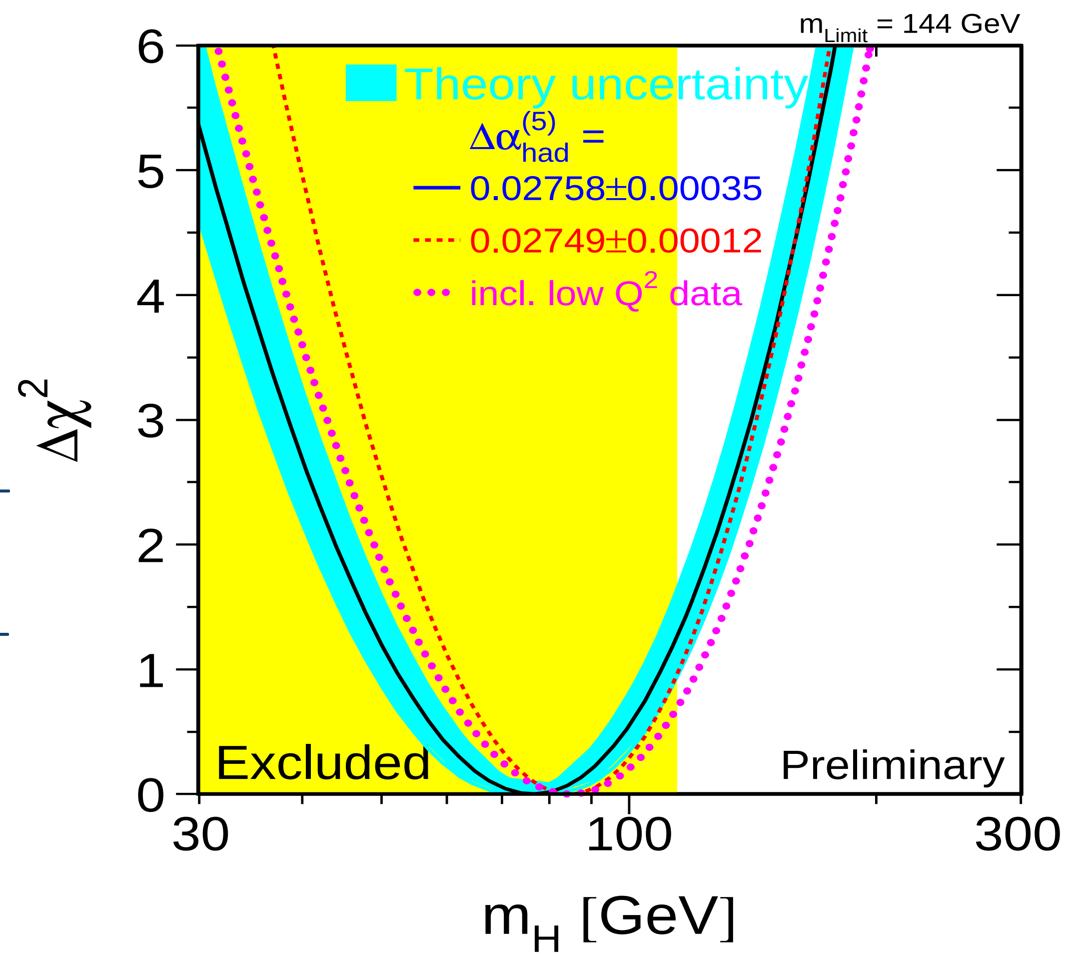
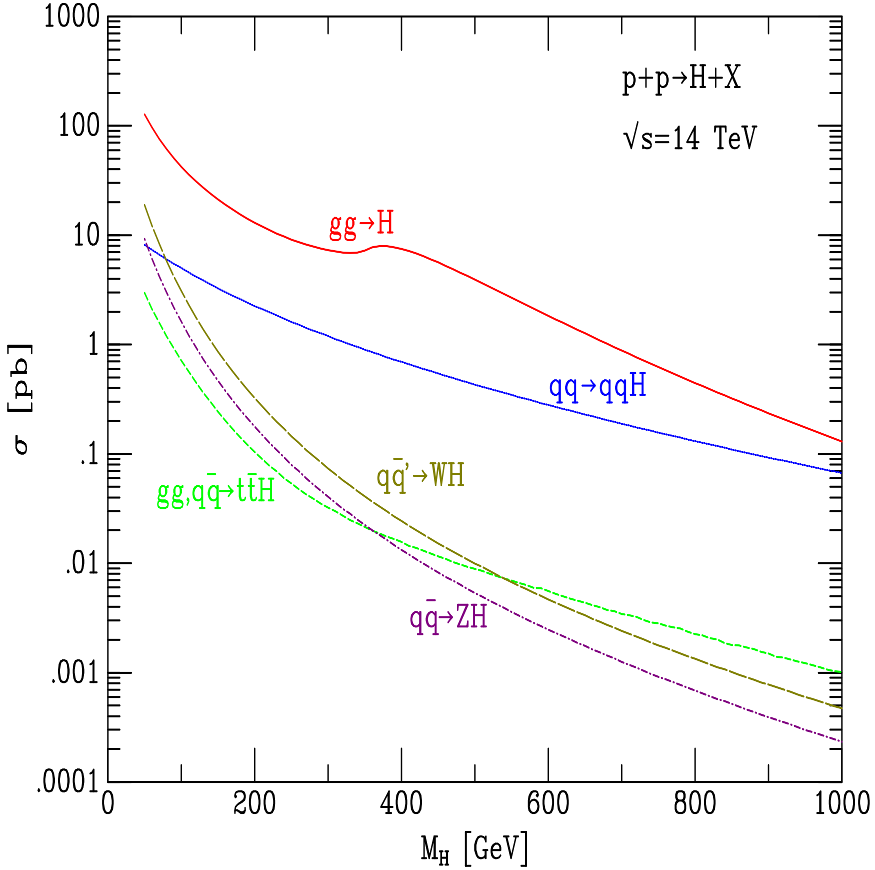


- Recent $D0$ analysis increases top mass to $170.9 \pm 1.1(stat) \pm 1.5(sys)\ GeV$.
- Higgs mass is $76^{+33}_{-24}\ GeV$ at the mininum.

Various Contributions at LHC



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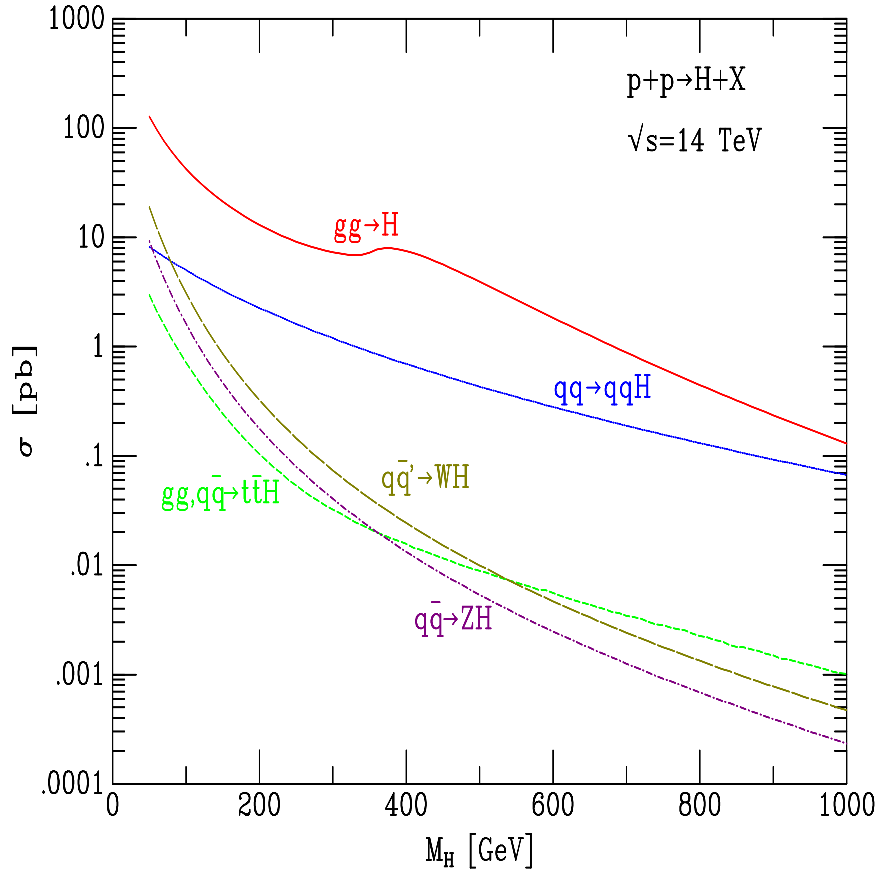

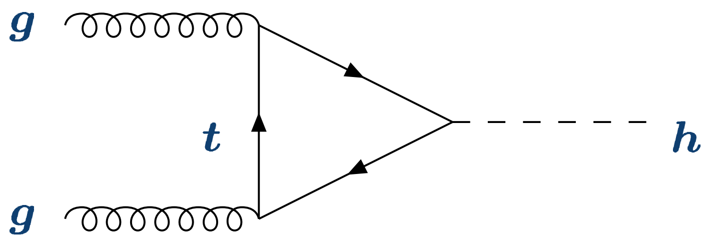

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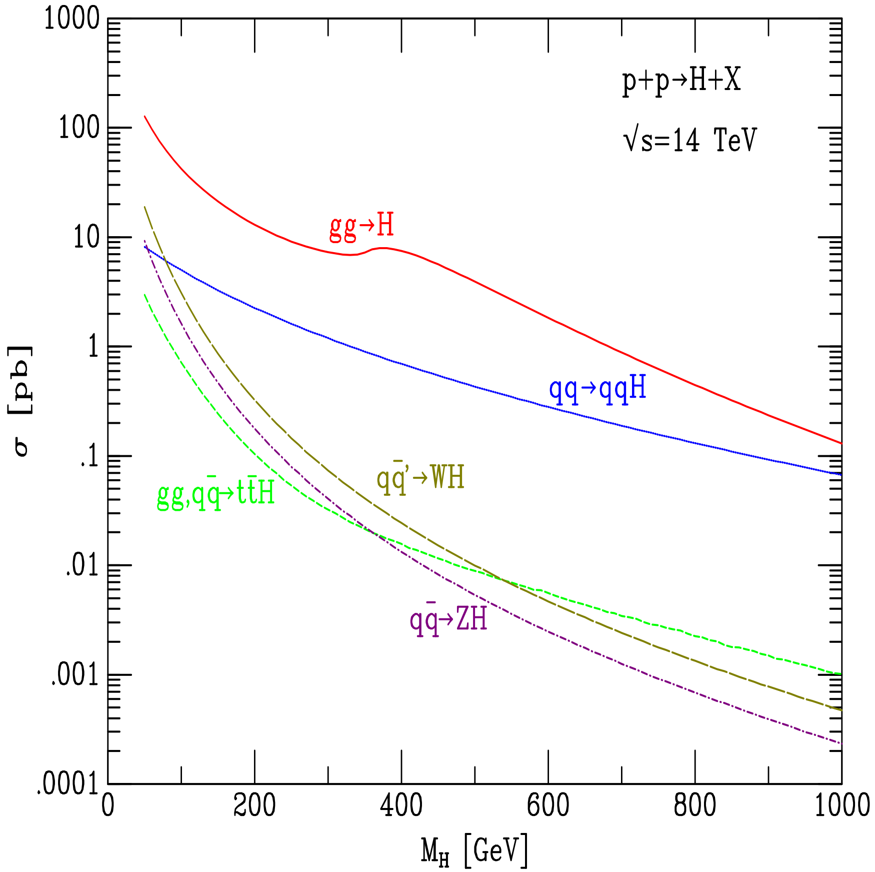

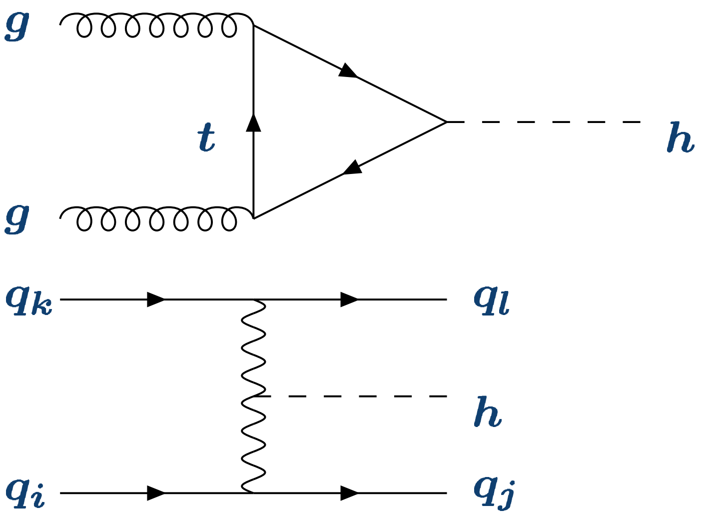

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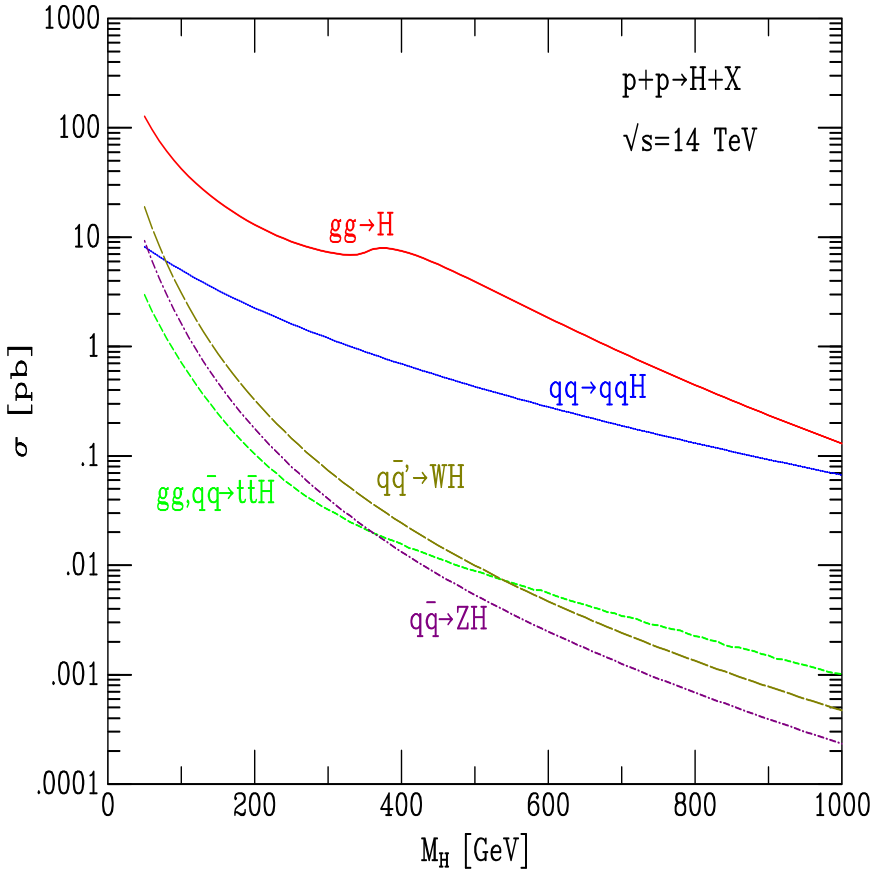

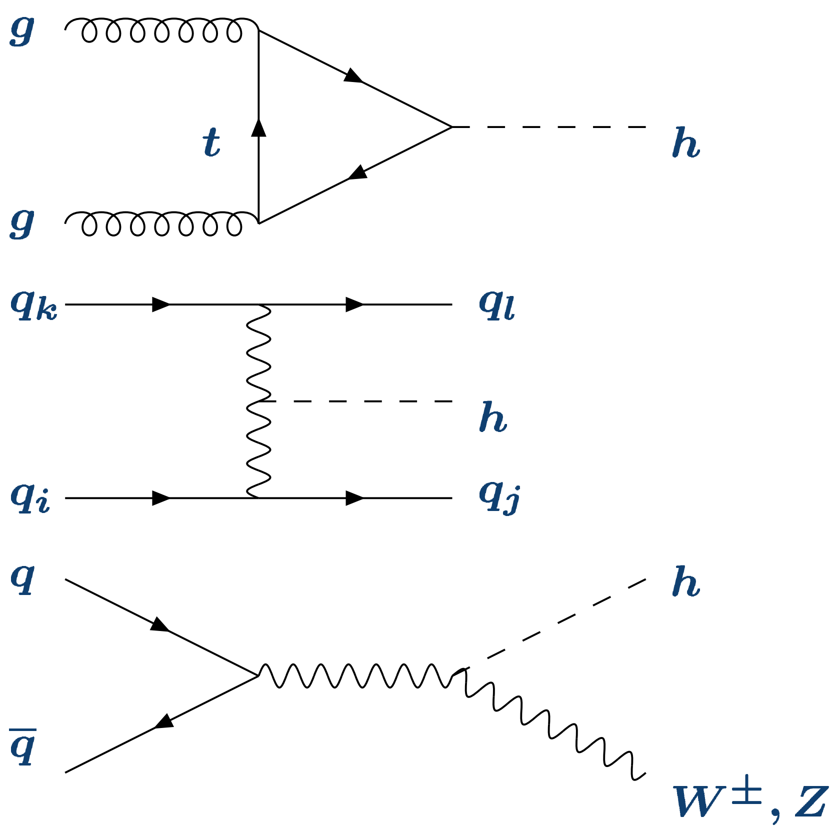

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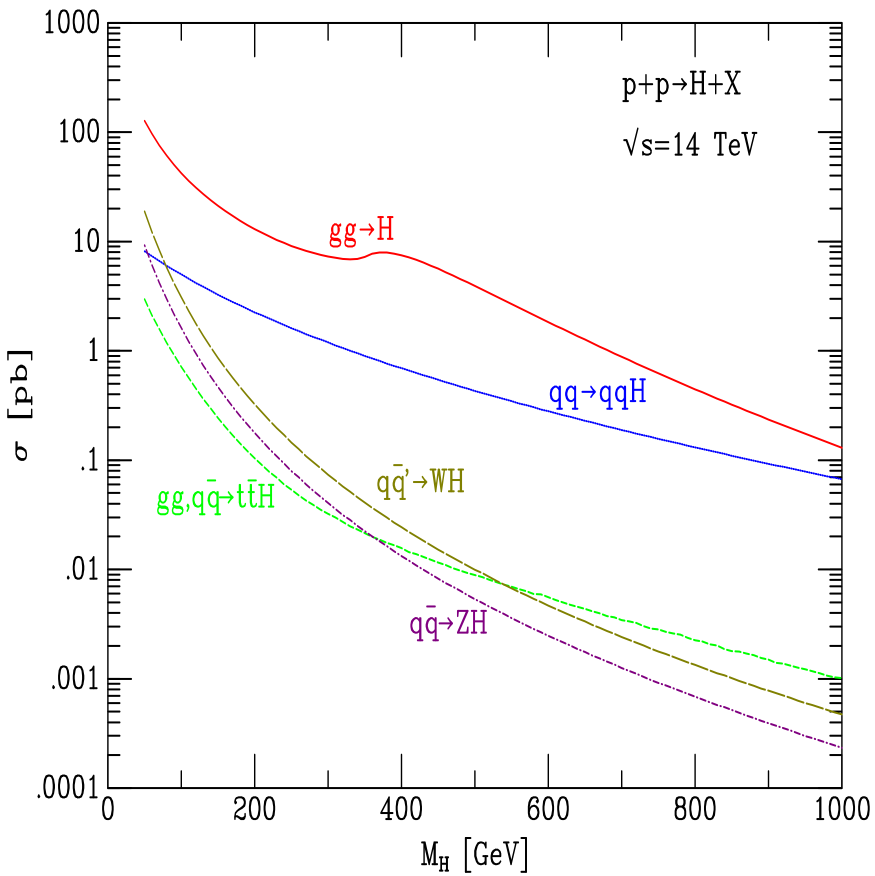

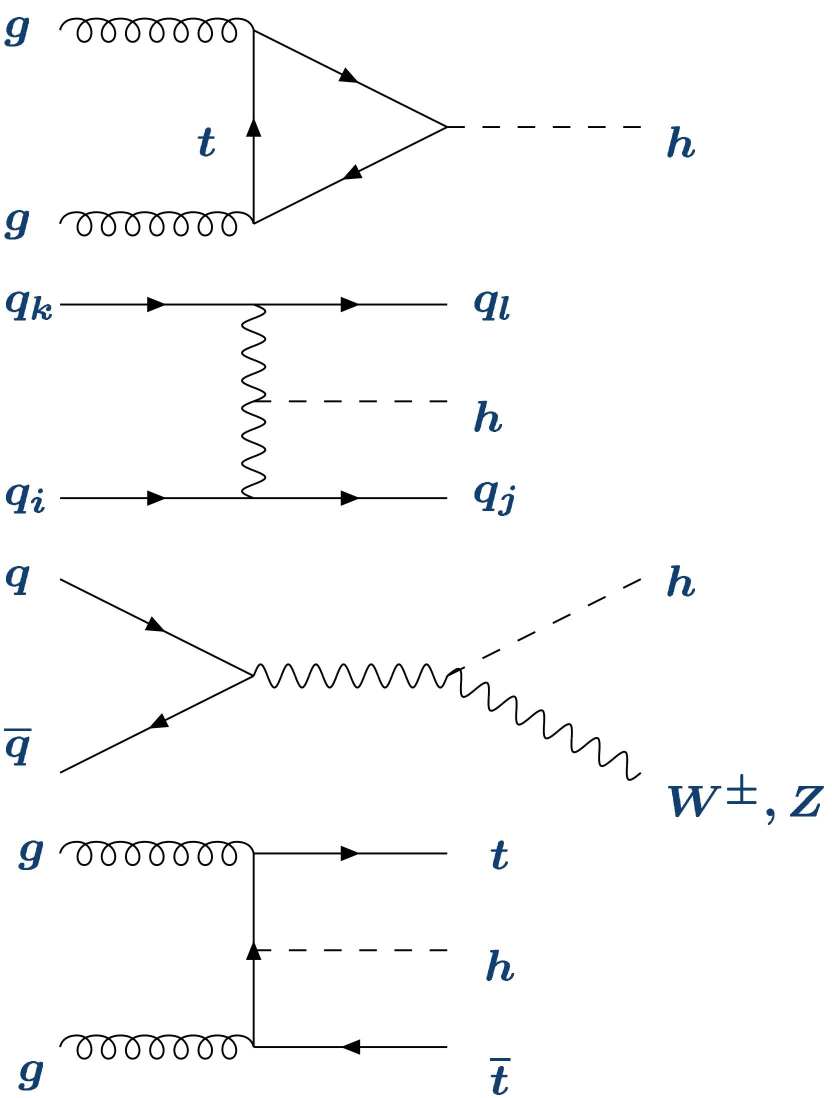

Factorisation Theorem (QCD improved Parton Model)

Collins, Soper, Sterman

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

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- The perturbatively calculable partonic cross section:

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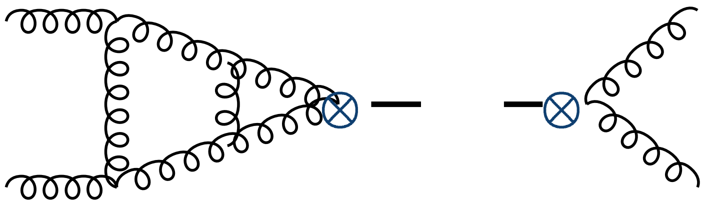
- The Renormalisation group invariance:

$$\frac{d}{d\mu}\sigma^{P_1 P_2}(\tau, m_h^2) = 0, \qquad \mu = \mu_F, \mu_R$$

Higgs Prodution at NNLO (Two Loop level)

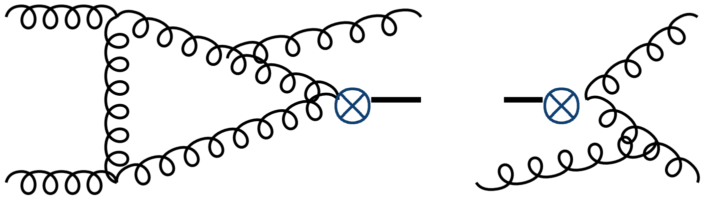
Harlander, Kilgore/ Anastasiou, Melnikov/ V.Ravindran, W. van Neerven, J. Smith,'04

Double Virtual:



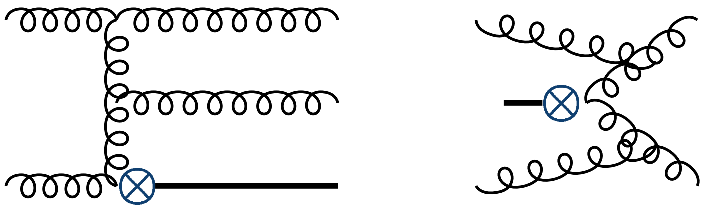
+ 148 terms;

Real Virtual:



+ 635 terms;

Double Real:



+ 594 terms.

In addition:

$$q + g \rightarrow h + X(q, \overline{q}, g) \qquad q_i + q_j(\overline{q}_j) \rightarrow h + X(q, \overline{q}, g)$$

For one loop results, see Djouadi et al, Dawson

Higgs production at LHC and Scale dependence

Harlander, Kilgore/ Anastasiou, Melnikov/ V.Ravindran, W. van Neerven, J.Smith

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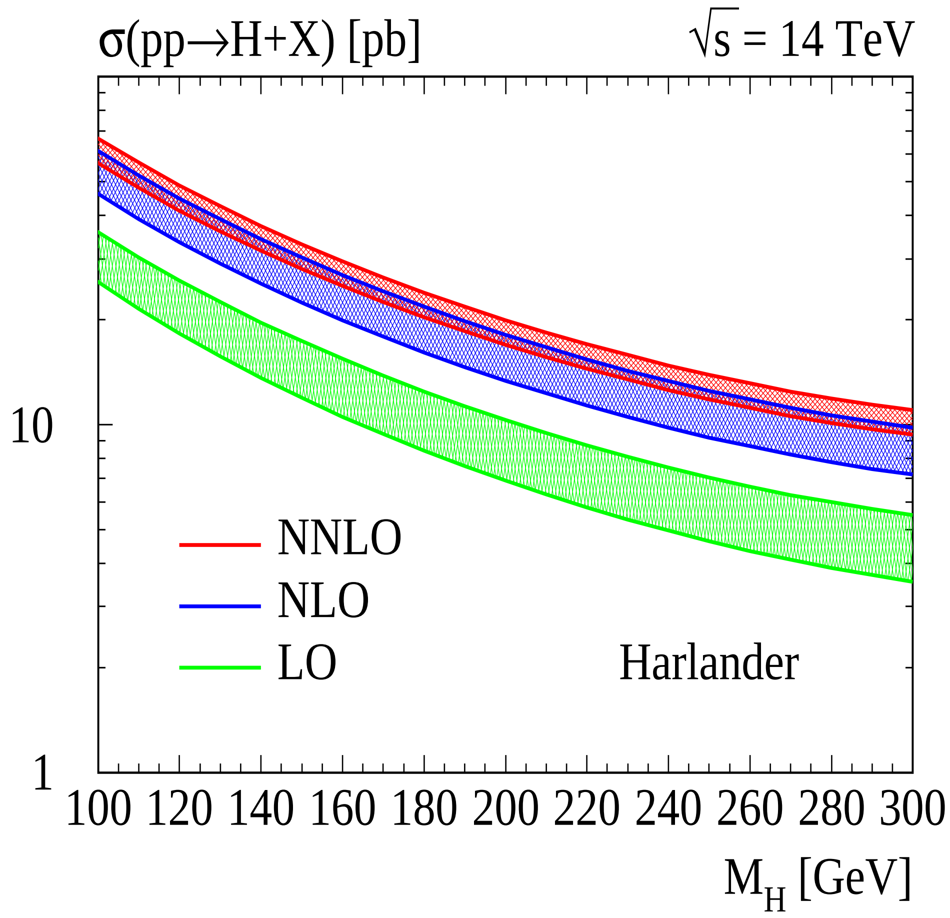

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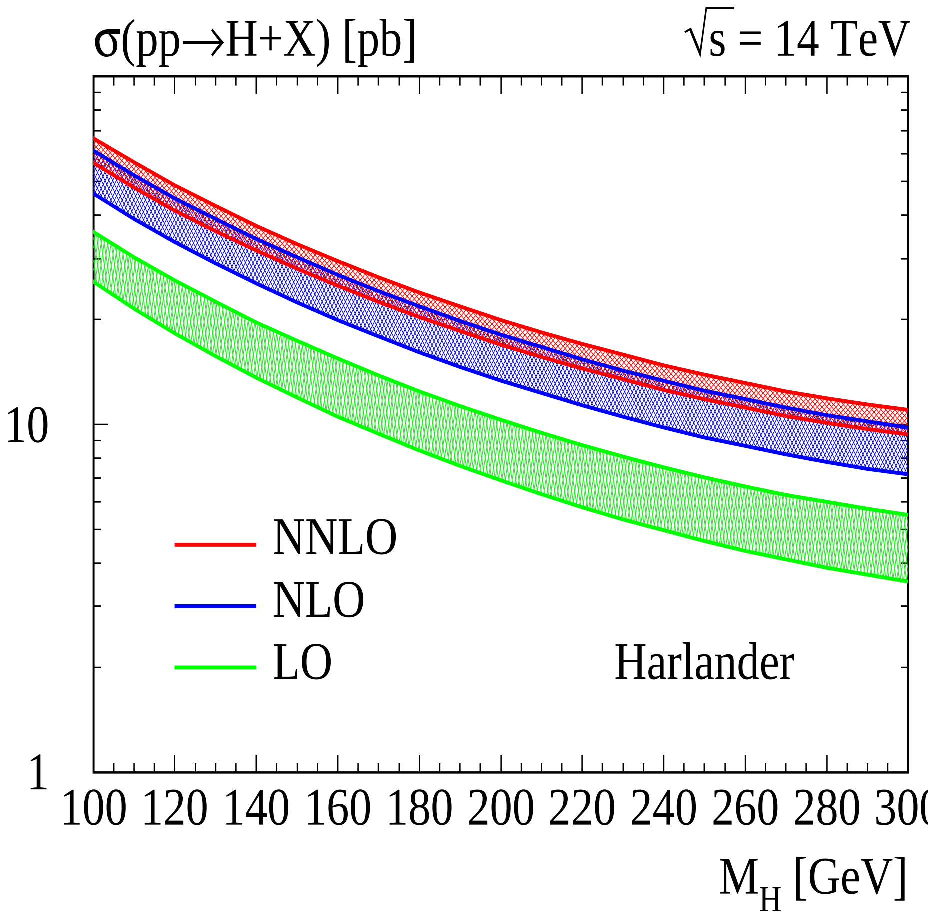

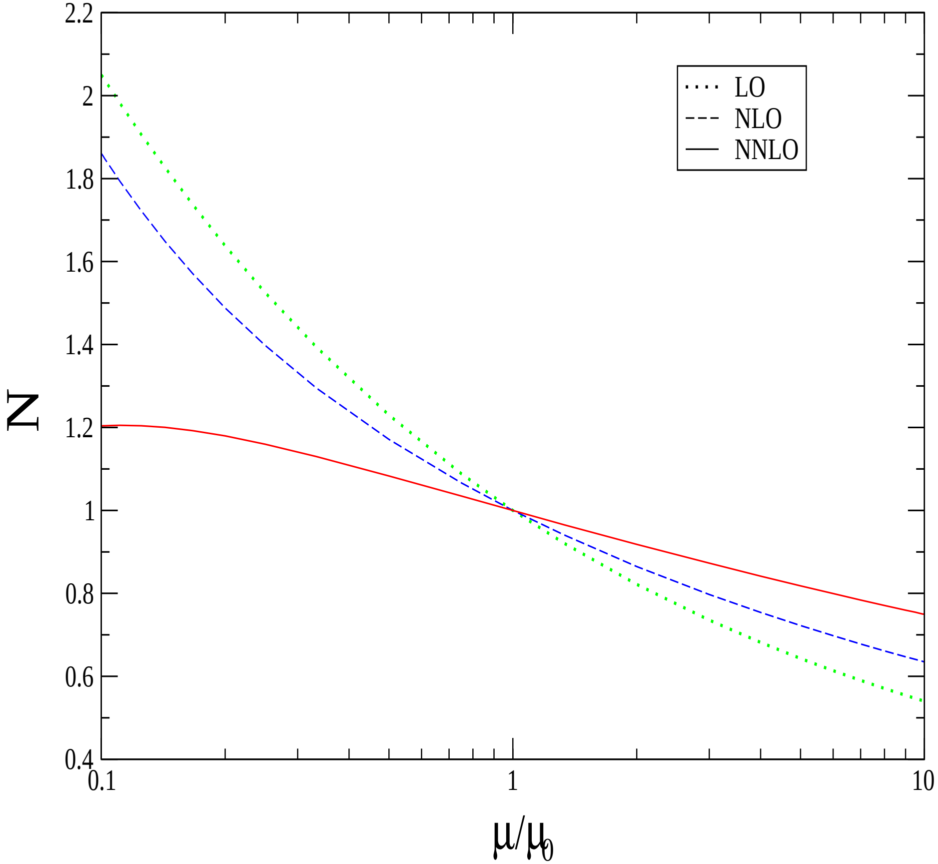

- See Hinchcliff,... for LO and see Dawson, Djouadi et.al for NLO (with finite top mass), NNLO is done in the large top limit $N = \sigma(\mu_R = \mu_F = \mu)/\sigma(\mu_0)$.

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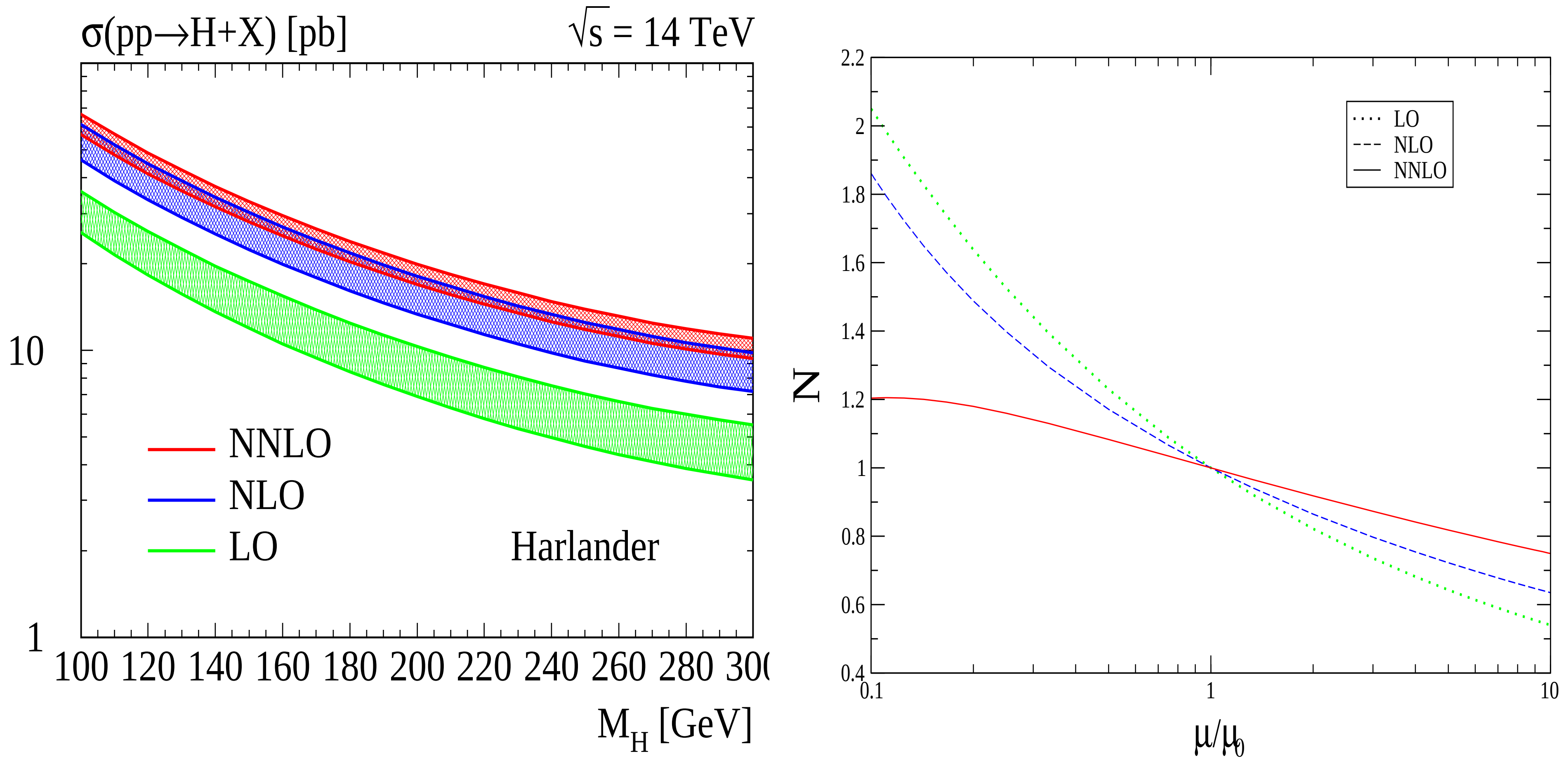

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- Is it the end?

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Catani et al, Harlander and Kilgore

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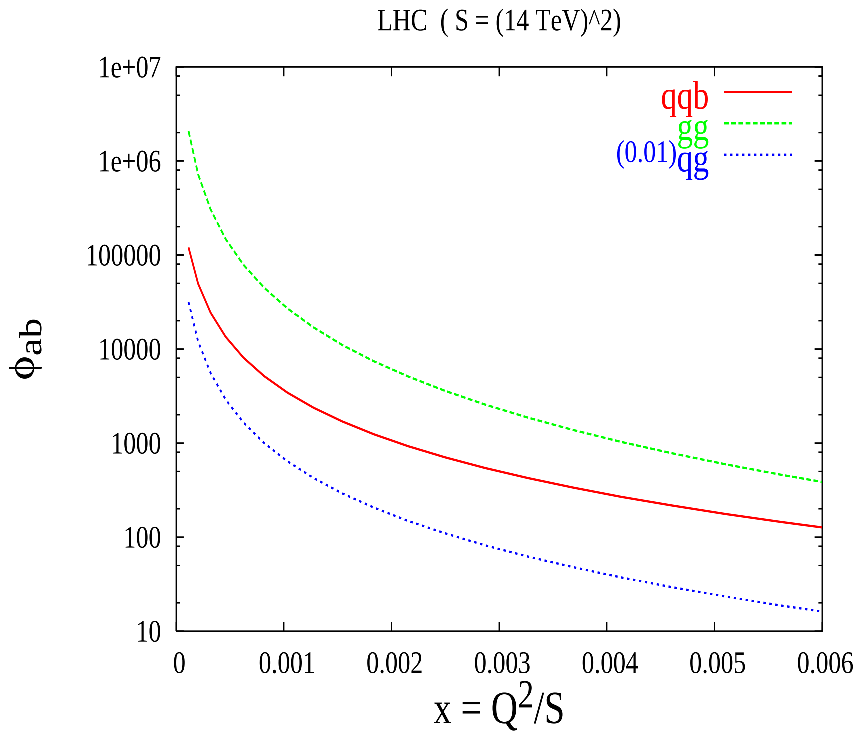

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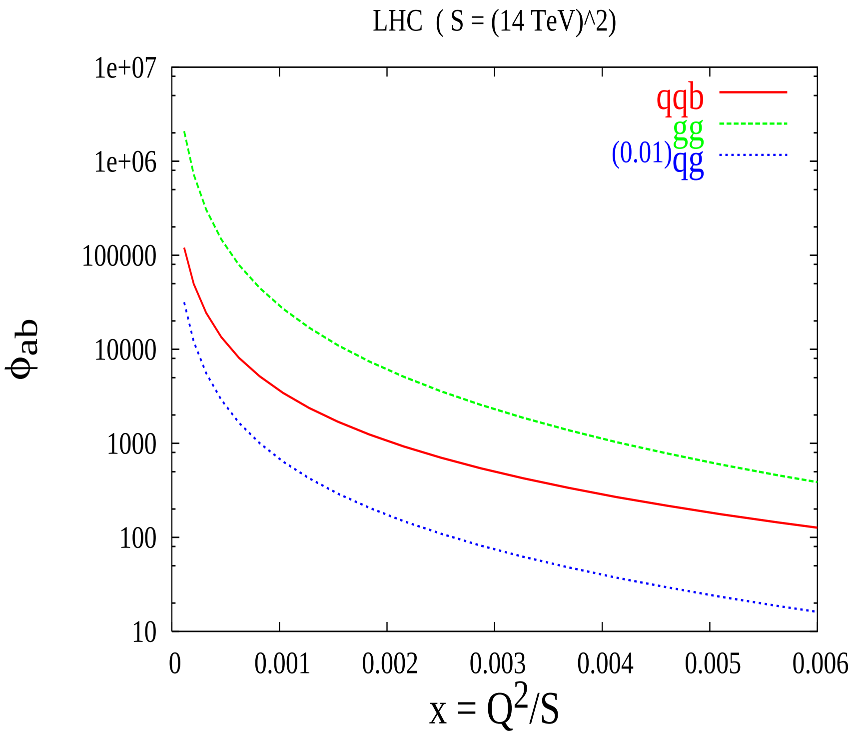

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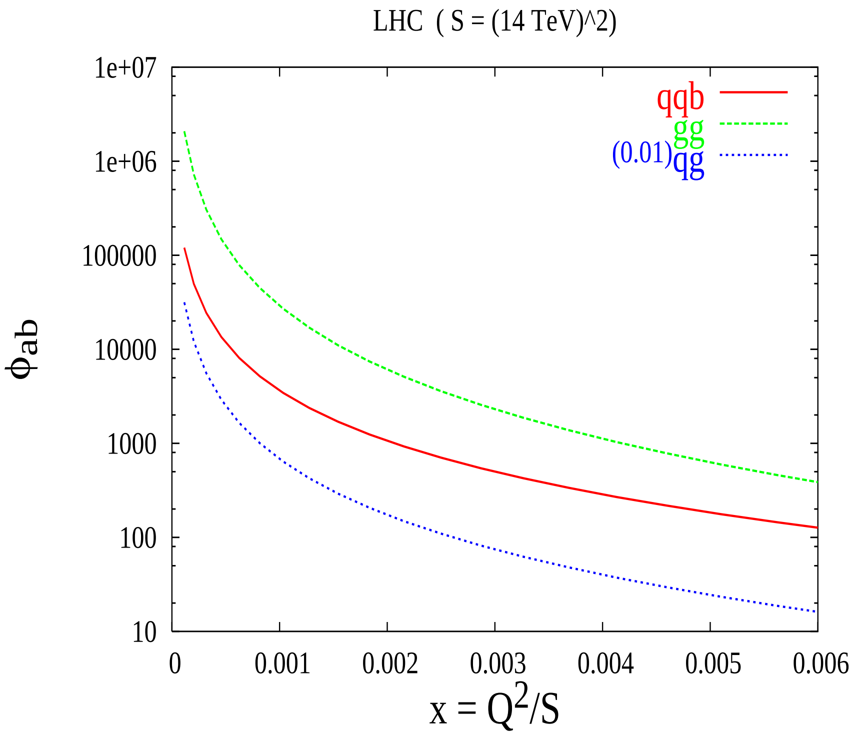

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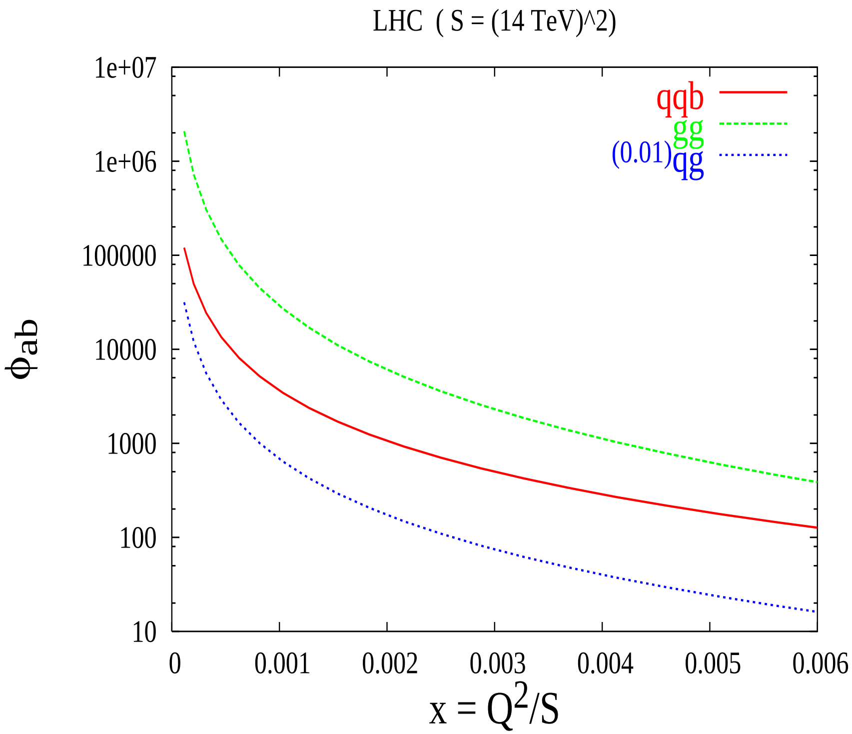

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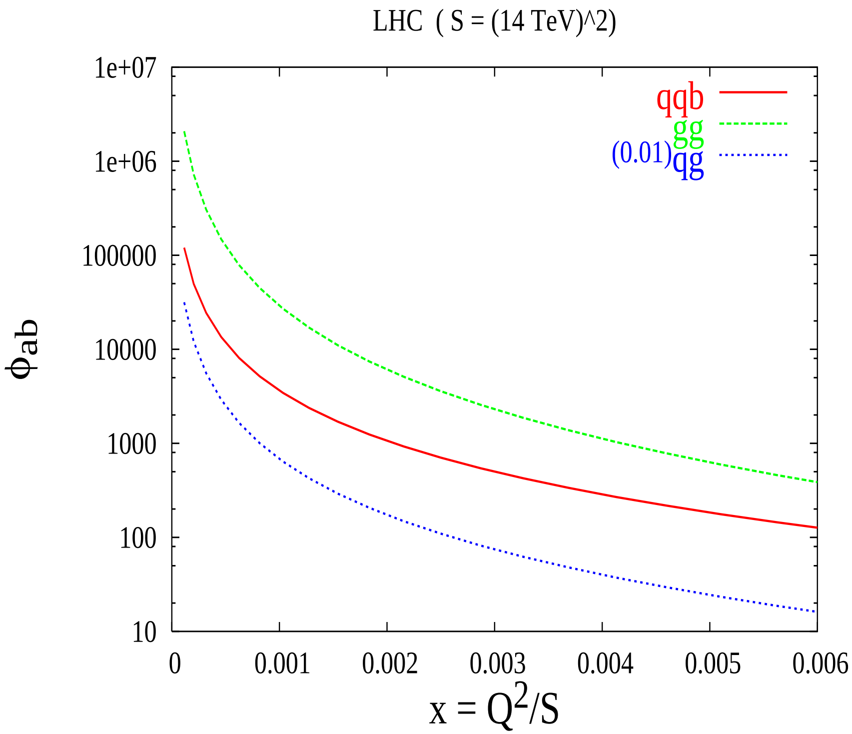

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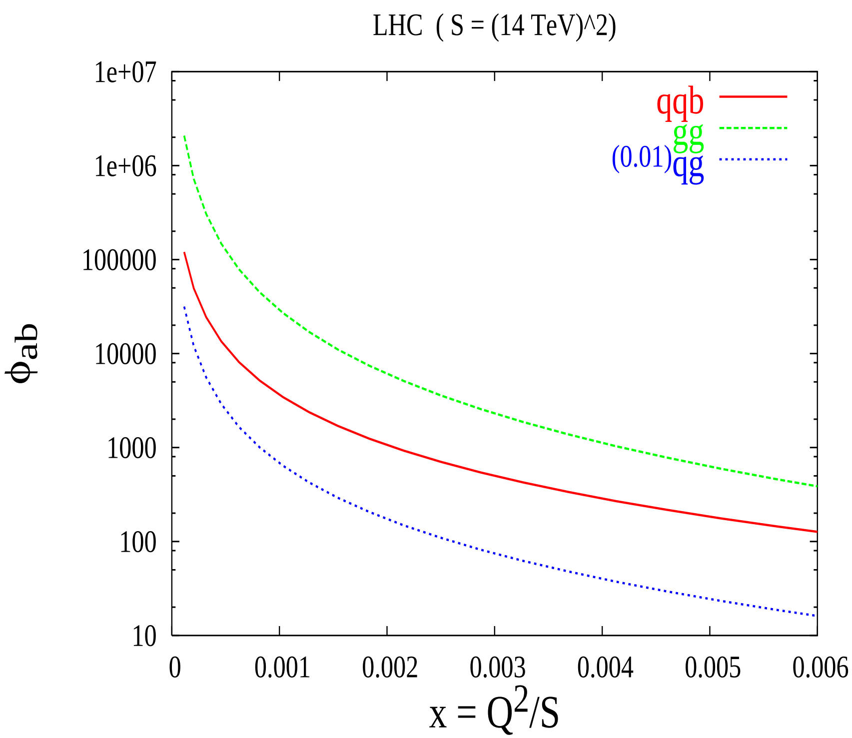

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- Expand the partonic cross section around $x = \tau$.

Soft part

Catani et al, Harlander and Kilgore

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- Expand the partonic cross section around $x = \tau$ or $z = \frac{x}{\tau} = 1$.

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- $\mathcal{C}^{(0)}$:

$$\mathcal{C}^{(0)} = \mathcal{C}_0^{(0)}\, \delta(1-z) + \sum_{k=0}^{\infty} \mathcal{C}_k^{(0)} \left(\frac{\log^k(1-z)}{(1-z)} \right)_+$$

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Soft part

Catani et al, Harlander and Kilgore

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  OR

  Extract from "Form factors and DGLAP kernels" using

  1) Factorisation theorem 2) Renormalisation Group Invariance

  3) Higgs and Drell-Yan NNLO results

Soft plus Virtual at N^3LO and beyond

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V.Ravindran

Using "factorisation" of Virtual, Soft and Collinear:

$$\Delta^{sv}_{I,P}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi^I_P(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)\right)\Big|_{\varepsilon=0} \qquad I=q,g \qquad n=4+\varepsilon$$

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$$\hat a_s = \frac{\hat g_s^2}{16\pi^2} \qquad m=\frac{1}{2} \quad \text{for} \quad \text{DIS}, \quad m=1 \quad \text{for} \quad \text{DY, Higgs}$$

Sudakov Resummation for Form factors

$Vogt, Vermaseren, Moch, V. Ravindran$

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Formal solution upto 4 loops:

$$\hat{\mathcal{L}}_F^{I,(1)} = \frac{1}{\varepsilon^2}\left(-2A_1^I\right) + \frac{1}{\varepsilon}\left(G_1^I(\varepsilon)\right)$$

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- Every order in $\hat{a}_s$, all the poles except the lowest one can be predicted from the previous order results using A and β function.

New observation for single pole in ε

$V.Ravindran, J.\ Smith, W.\ van\ Neerven$

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We conjuctured the form of the single pole: $G_i^I = 2(B_i^I - \gamma_i^I) + f_i^I + \cdots$ to all orders with $f_i^g = (C_A/C_F)\, f_i^q$ for all i

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This completes the understanding of all the poles of the form factors.

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Observable $\boldsymbol{\Delta^I(\alpha_s, Q^2)}$ are finite:

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The remaining poles after UV Operator Renormalisation(Z_{α_s} and Z^I) and Mass factorisation:

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- The structure of soft part should be "similar" to the Form Factors.
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RG invariance of Φ^I implies:

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) = -\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) = -\overline{A}^I(a_s(\mu_R^2))\delta(1-z)$$

Soft gluon Resummation

V.Ravindran

- Threshold resummation formula in z space for DY,Higgs and DIS:

$$
\begin{aligned}
\Phi^{I}_{P}(\hat{a}_s, q^2, \mu^2, z, \varepsilon) \quad = \quad & \left(\frac{m}{1-z} \left\{ \int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I \left(a_s(\lambda^2)\right) \right.\right. \\
& \left.\left. + \overline{G}^{I}_{P} \left(a_s \left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right) \right\} \right)_+ \\
& + \delta(1-z) \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2 \delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\phi}^{I,(i)}_{P}(\varepsilon) \\
& + \left(\frac{m}{1-z}\right)_+ \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \overline{K}^{\,I,(i)}(\varepsilon)
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$$+\delta(1-z) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2\delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \hat{\phi}_P^{I,(i)}(\varepsilon)$$

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- The threshold exponents D^I_i for DY and B^I_i for DIS are related to $\overline{G}^I_P(\varepsilon = 0)$.
- $\overline{G}^I_P(\varepsilon = 0)$ upto three loop gives D^I_i and B^I_i for $i = 1, 2, 3$
- Expansion of $\mathcal{C}e^{\left(2\Phi^I_P\right)}$ leads to soft part of the cross section.
- Fixed order N^3LO soft plus virtual cross sections can be computed(except $\delta(1-z)$)

Higgs and Drell-Yan productions beyond $NNLO$

Universal soft function: $V.Ravindran$

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$$\Phi^g(\hat{a}_s, q^2, z, \mu^2, \varepsilon) = \frac{C_A}{C_F} \Phi^q(\hat{a}_s, q^2, z, \mu^2, \varepsilon)$$

- N^3LO threshold corrections to Higgs production:

$$\sigma(g + g \to Higgs)$$

- N^3LO threshold corrections to Drell-Yan production:

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Soft plus Virtual part at N^3LO for Higgs Production

Moch, Vogt and V.Ravindran

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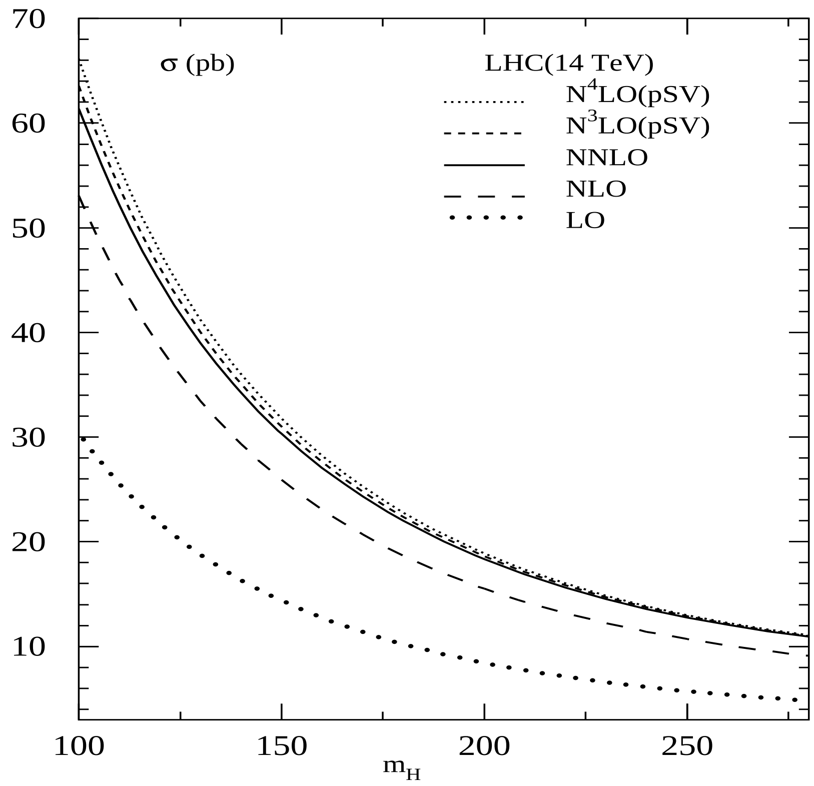

Gluon flux is largest at LHC

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Scale variation at N^3LO for Higgs production

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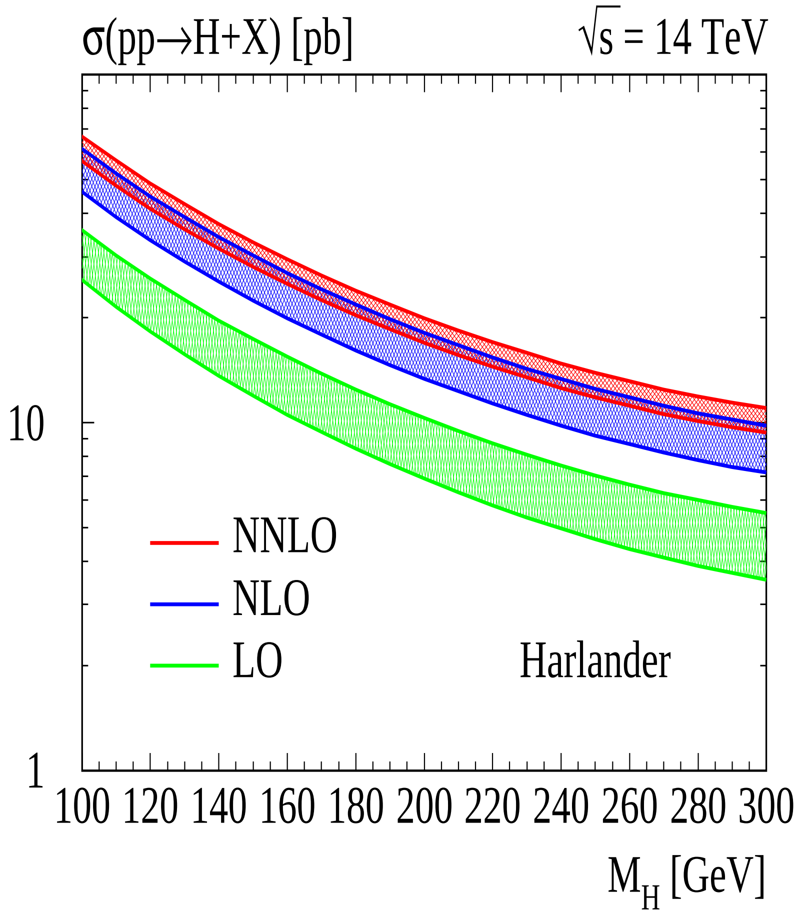

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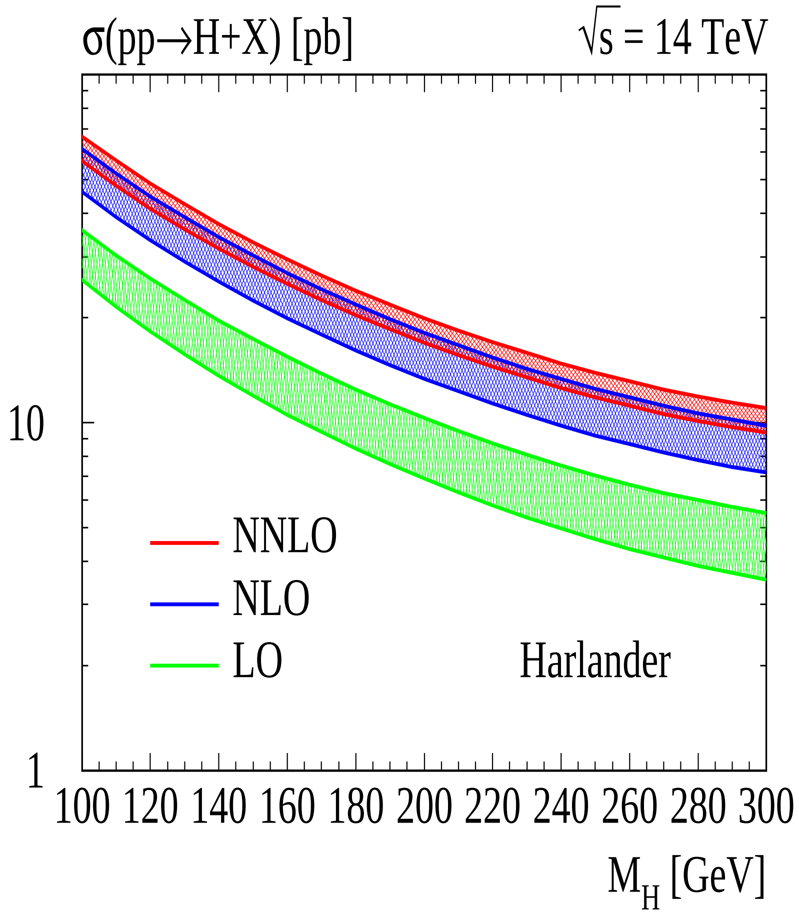

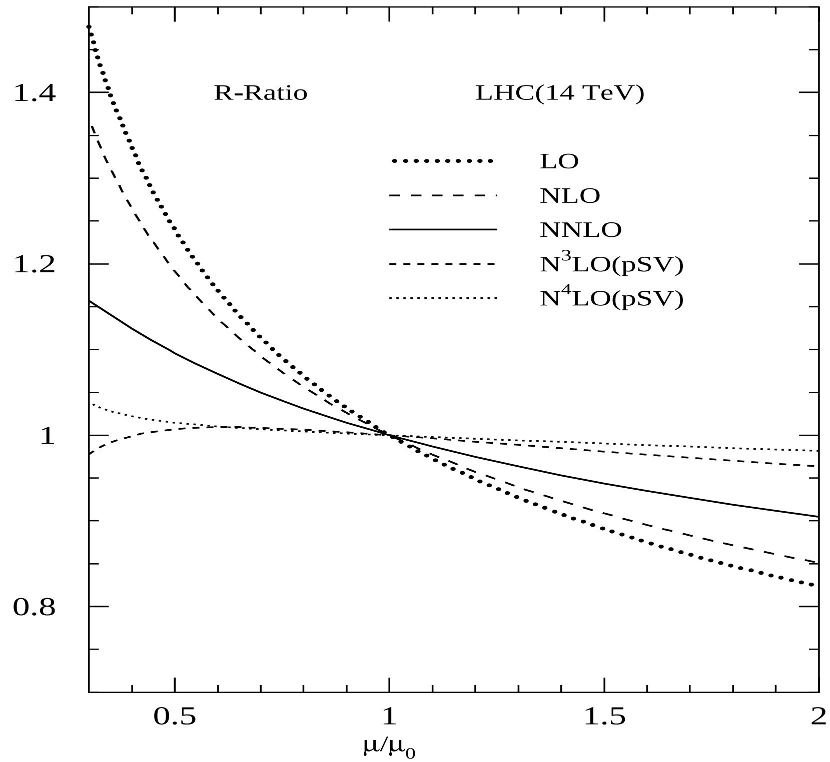

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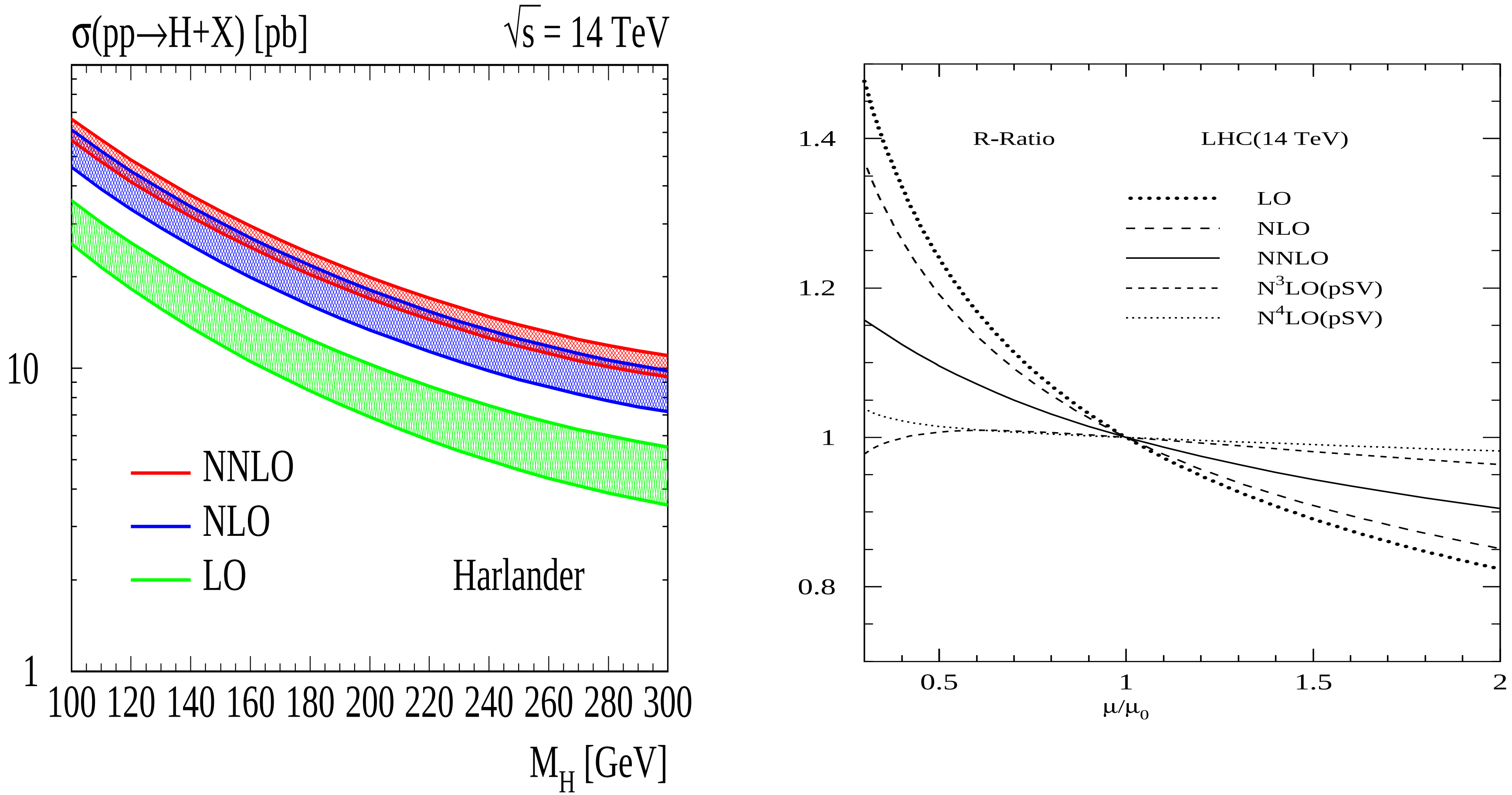

- Scale uncertainity improves a lot
- Perturbative QCD works at LHC

Rapidity distributions

V. Ravindran, J. Smith and W. van Neerven

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$$\frac{d\sigma^I}{dY} = \sigma^I_{\rm Born}(x_1^0, x_2^0, q^2) W^I(x_1^0, x_2^0, q^2) \ , \qquad I = q, b, g \quad ,$$

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$$\times \int_0^1 dz_1 \int_0^1 dz_2\, \delta(x_1^0 - x_1 z_1)\, \delta(x_2^0 - x_2 z_2)\, \Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2,$$

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$$\Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) = \Delta^{\rm hard}_{I,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) + \delta_{a\overline{b}}\Delta^{\rm sv}_{d,I}(z_1, z_2, q^2, \mu_F^2, \mu_R^2),$$

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$$\Delta^{sv}_{d,I}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) : \qquad Z^I(\mu_F^2), \quad \mathcal{F}^I(Q^2), \quad \Gamma(\mu_F^2), \Phi^I(Q^2)$$

Rapidity of Higgs production and its Scale dependence at NNLO

Anastasiou, Melnikov, Petriello

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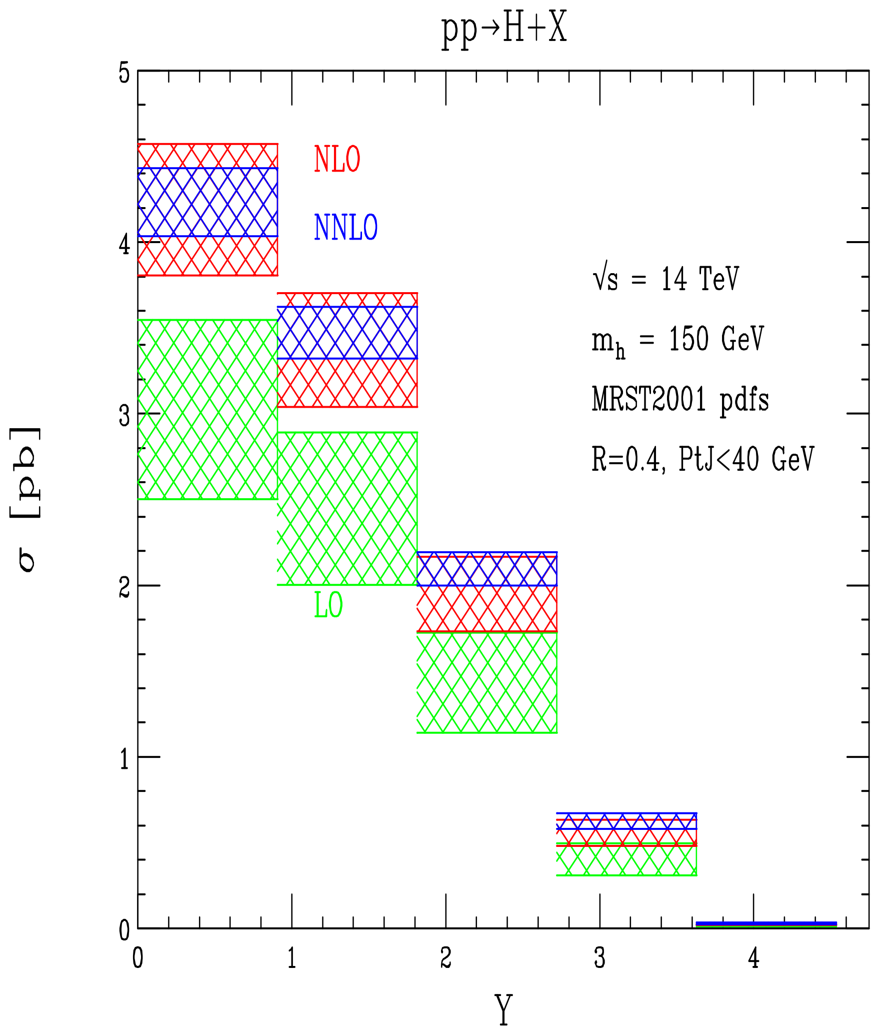

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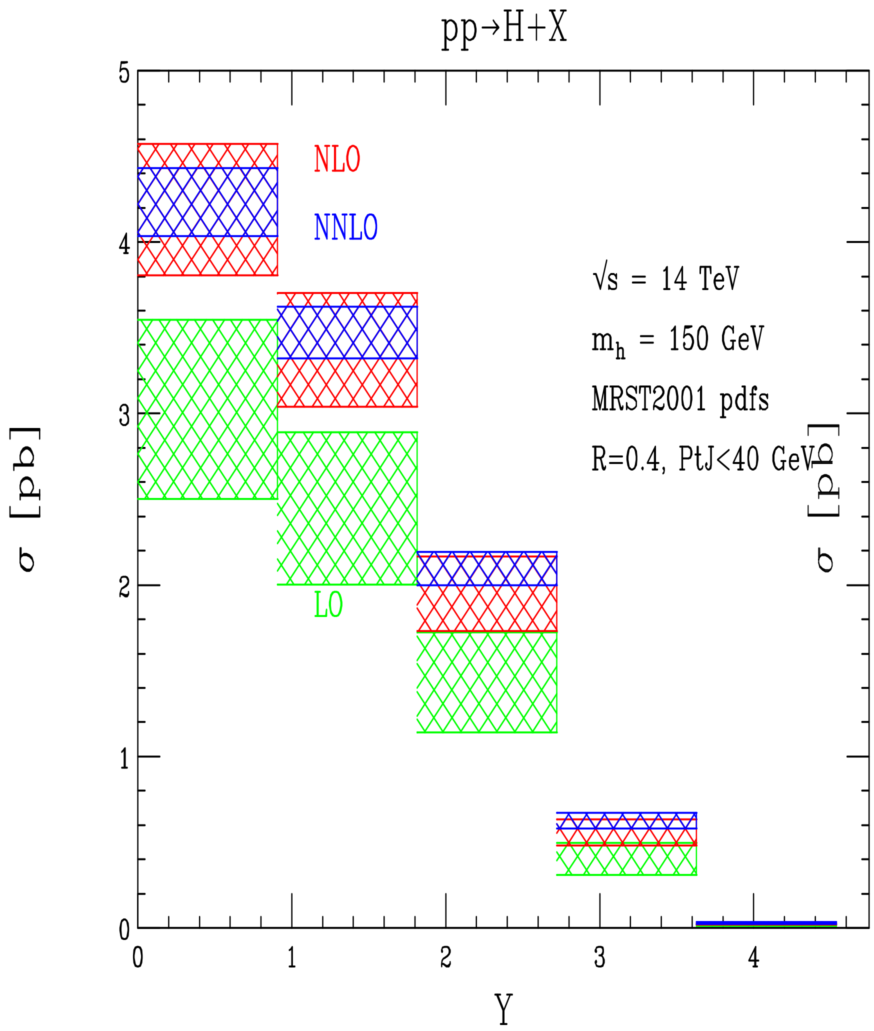

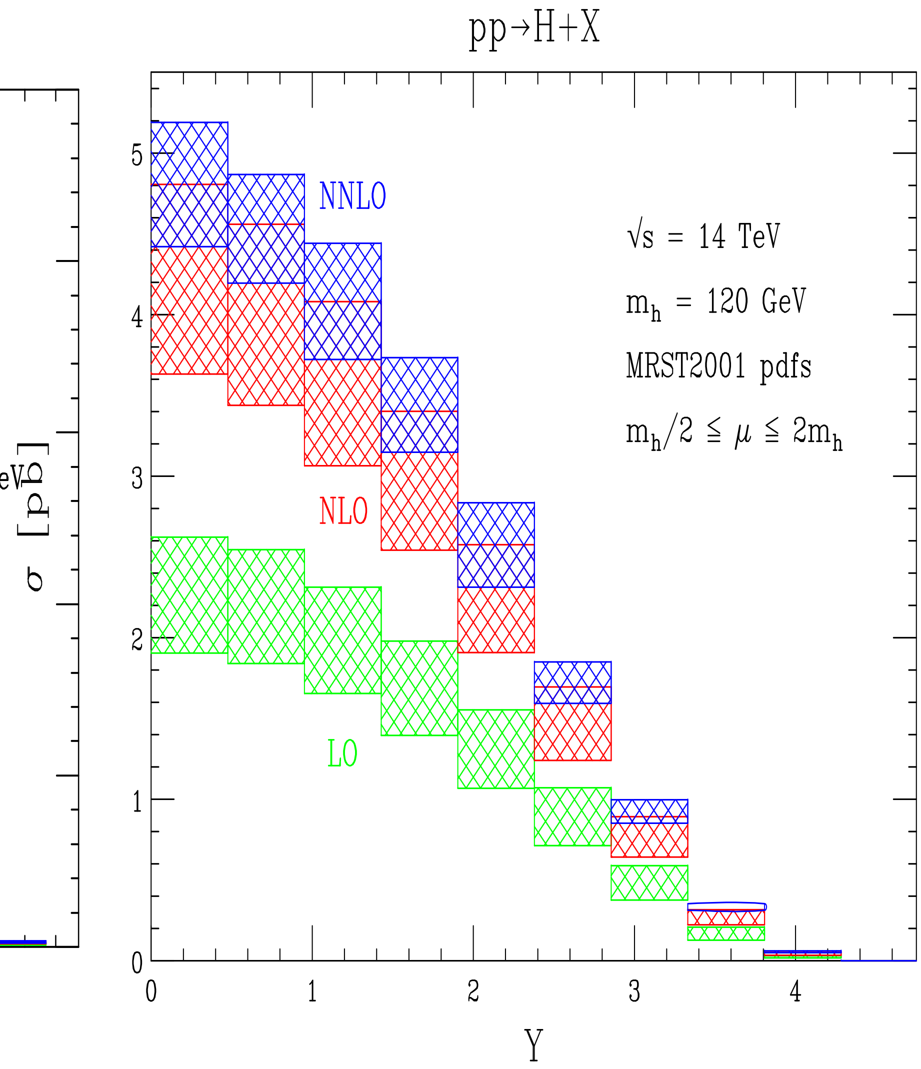

- NNLO exact in the large top limit reduces the scale uncertainity significantly

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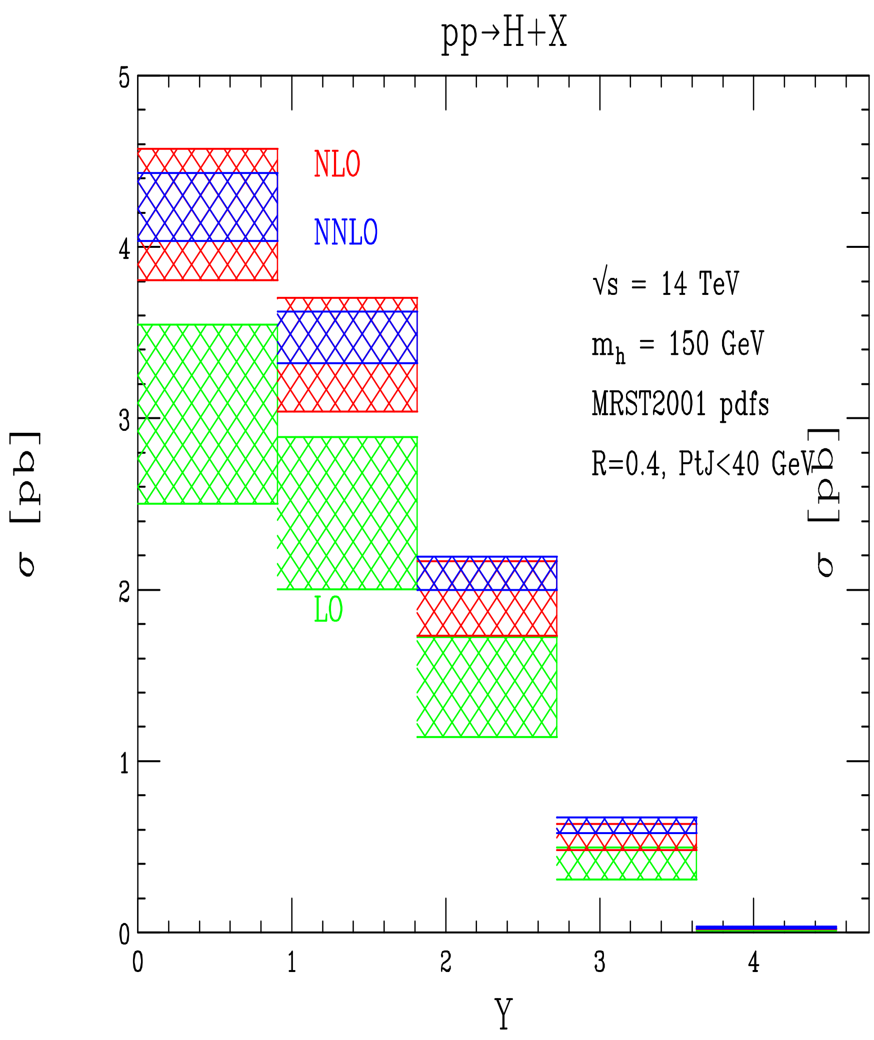

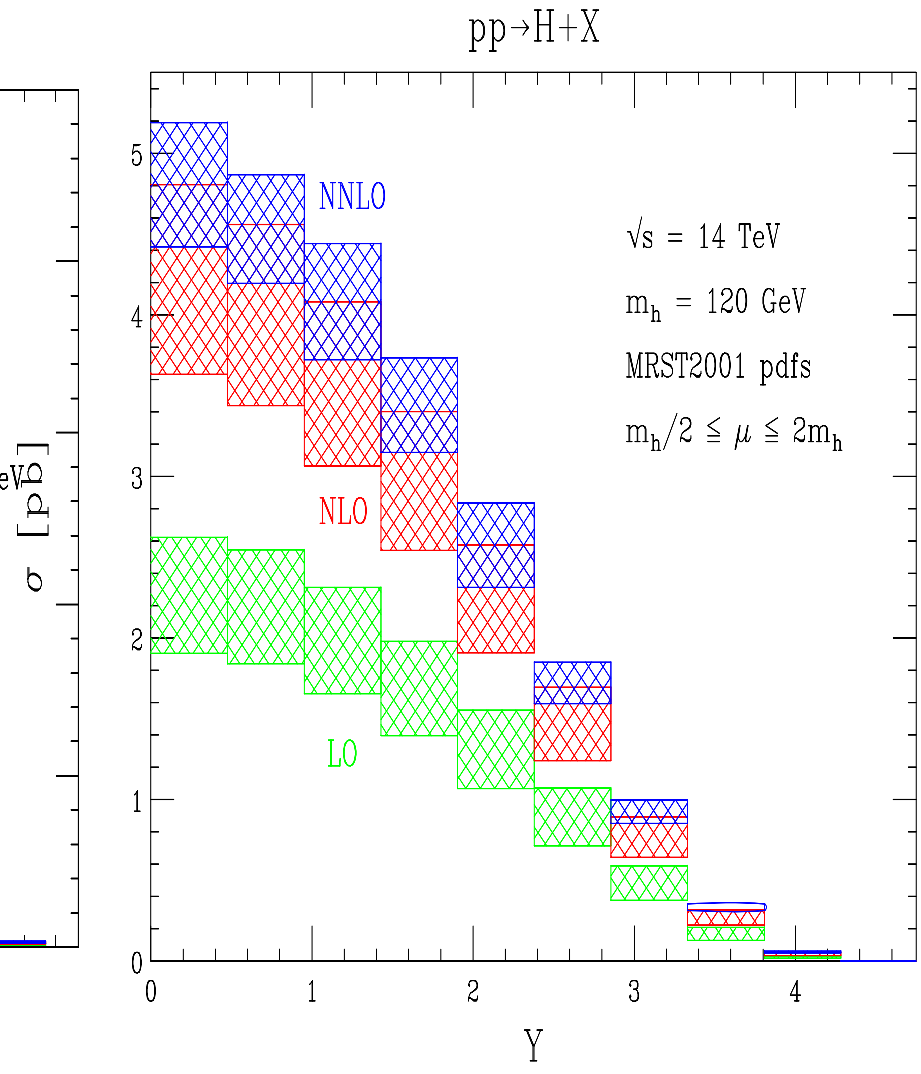

- NNLO exact in the large top limit reduces the scale uncertainity significantly
- One of the most difficult computations in QCD. Is it the end?

Rapidity of Drell-Yan and its Scale dependence at NNLO

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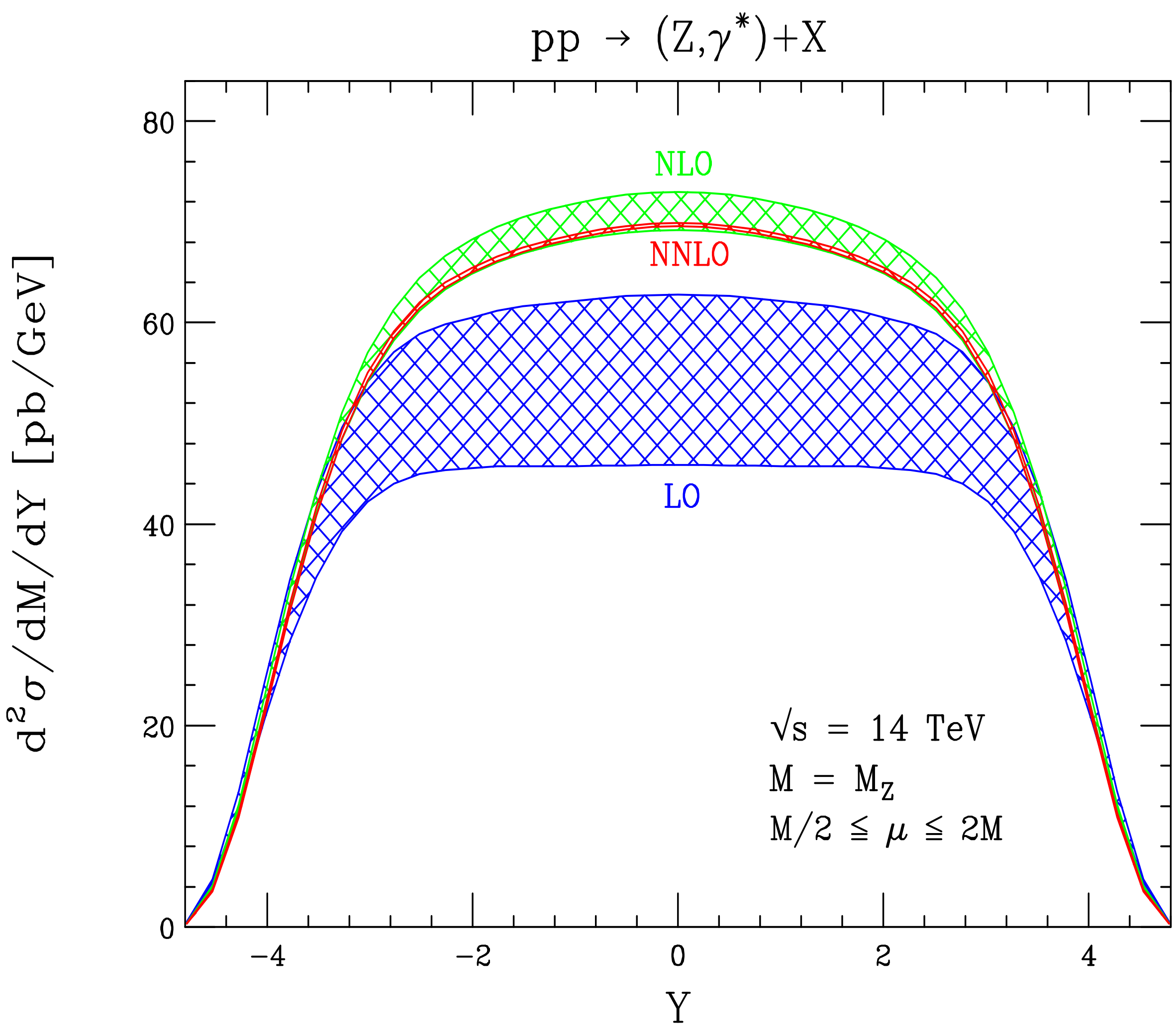

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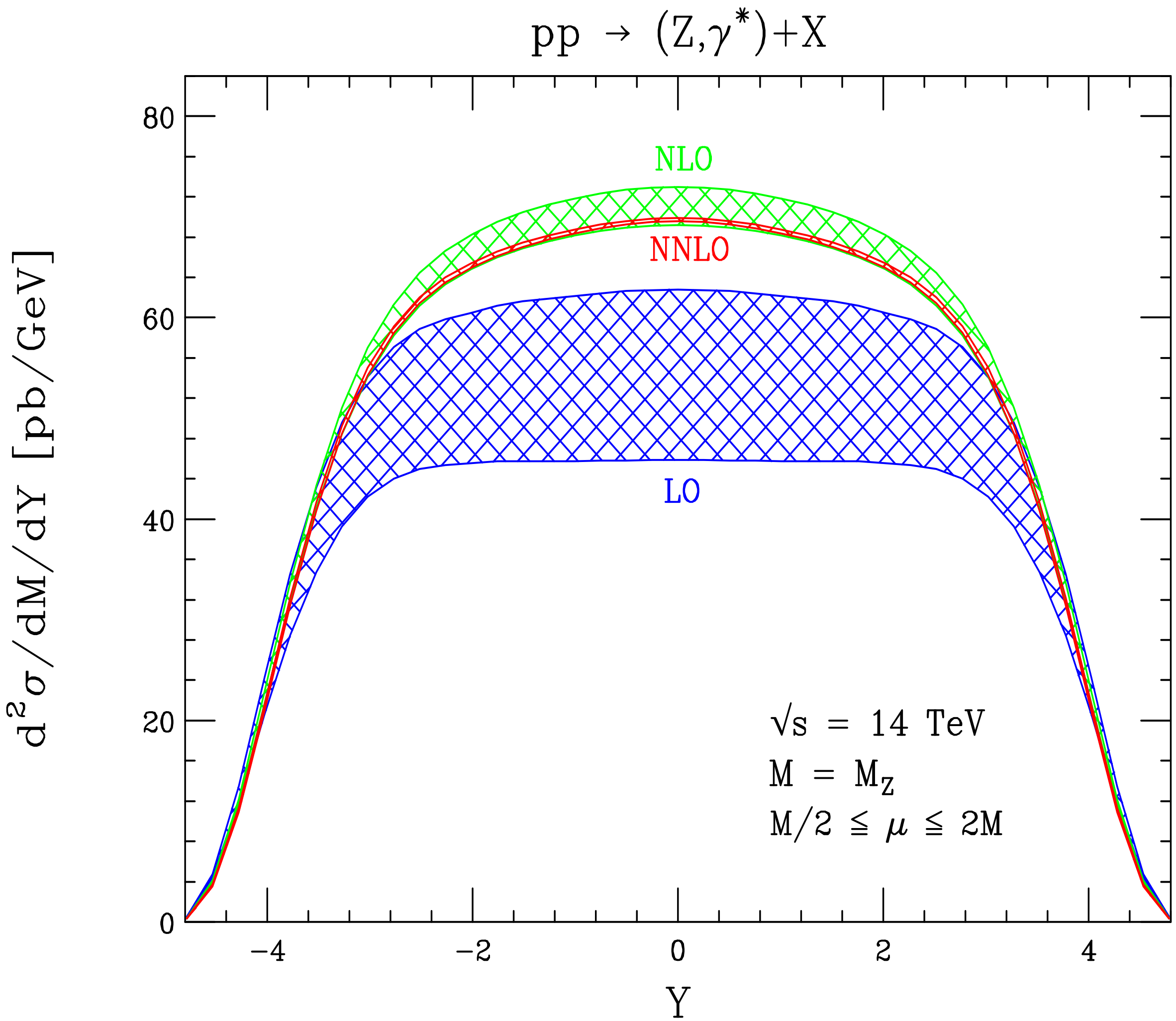

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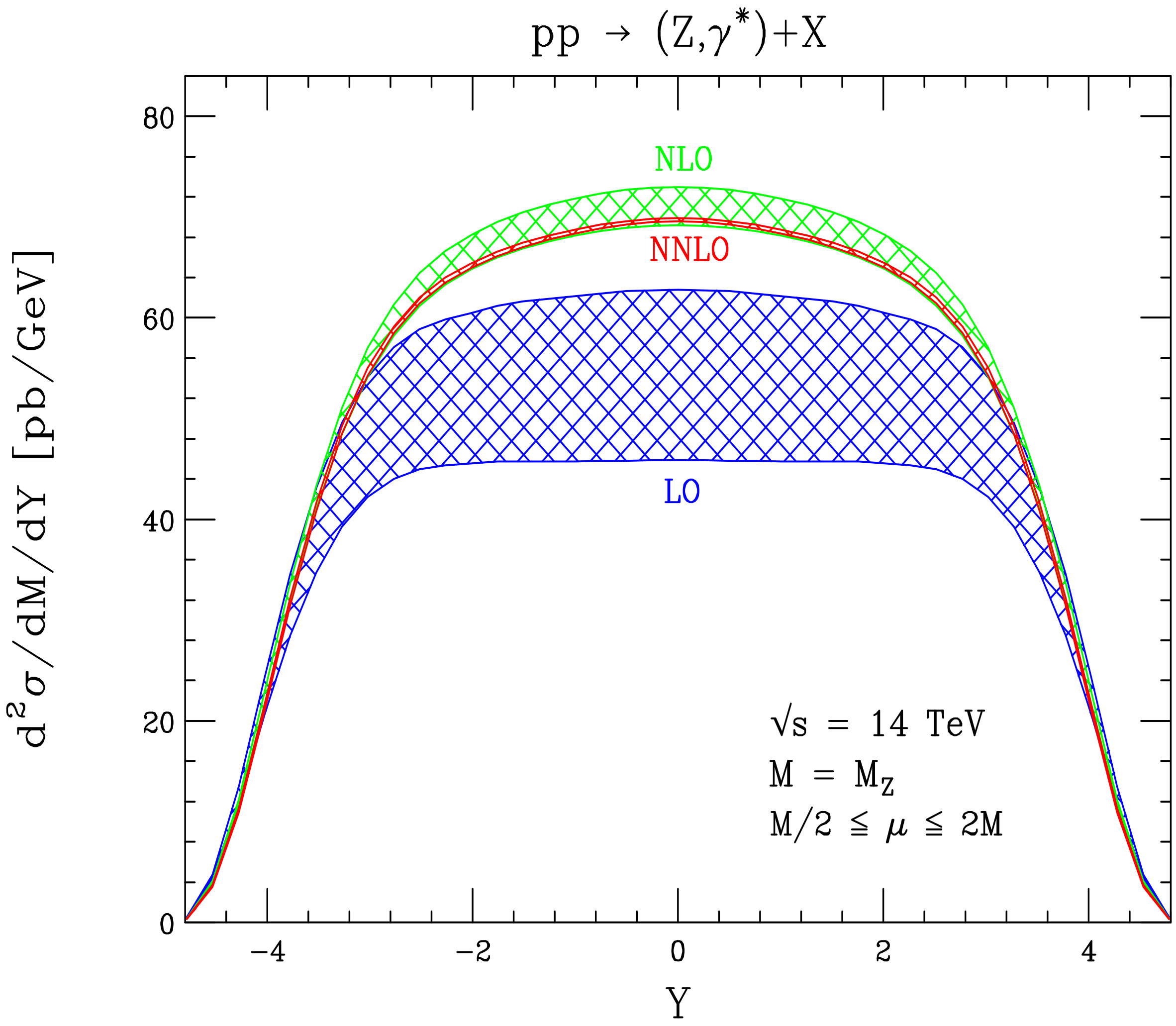

- NNLO exact reduces the scale uncertainity significantly
- Also "most difficult" computation in QCD
  What is next?

Three loop (N^3LO) Soft distribution for rapidity

V.Ravindran, J. Smith and W. van Neerven

Using RGE and Factorisation:

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Using RGE and Factorisation:

$$\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) \quad = \quad \Phi_{d,finite}^I + \Phi_{d,singular}^I$$

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$$\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) = \Phi^I_{d,finite} + \Phi^I_{d,singular}$$

where

$$\Phi^I_{d,finite} = \frac{1}{2}\delta(1-z_2)\left(\frac{1}{1-z_1}\left\{\int_{\mu_R^2}^{q^2(1-z_1)} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right)\right.\right.$$

$$\left.\left. +\overline{G}_d^I\left(a_s\left(q^2(1-z_1)\right), \varepsilon\right)\right\}\right)_+$$

$$+q^2\frac{d}{dq^2}\left[\left(\frac{1}{4(1-z_1)(1-z_2)}\left\{\int_{\mu_R^2}^{q^2(1-z_1)(1-z_2)} \frac{d\lambda^2}{\lambda^2} A^I\left(a_s(\lambda^2)\right)\right.\right.\right.$$

$$\left.\left.\left. +\overline{G}_d^I\left(a_s\left(q^2(1-z_1)(1-z_2)\right), \varepsilon\right)\right\}\right)_+\right]$$

$$+z_1 \leftrightarrow z_2$$

N^3LO_{pSV} results for Drell-Yan rapidity

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$$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$

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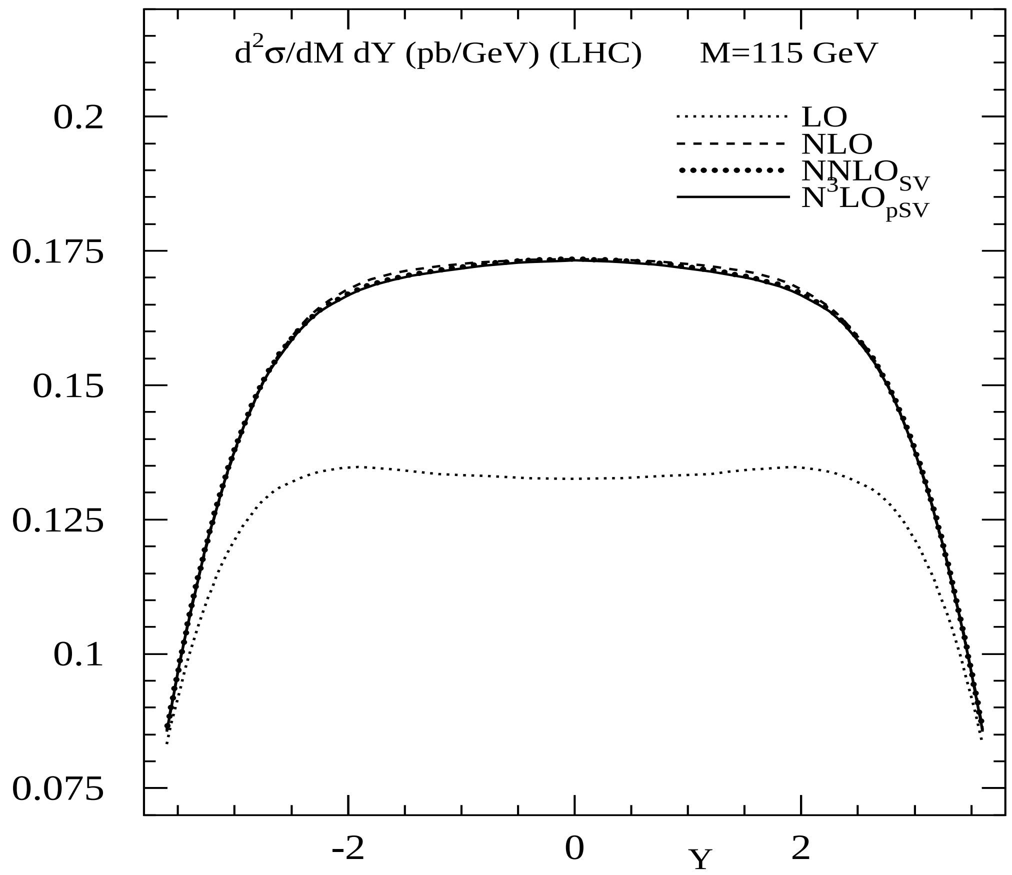

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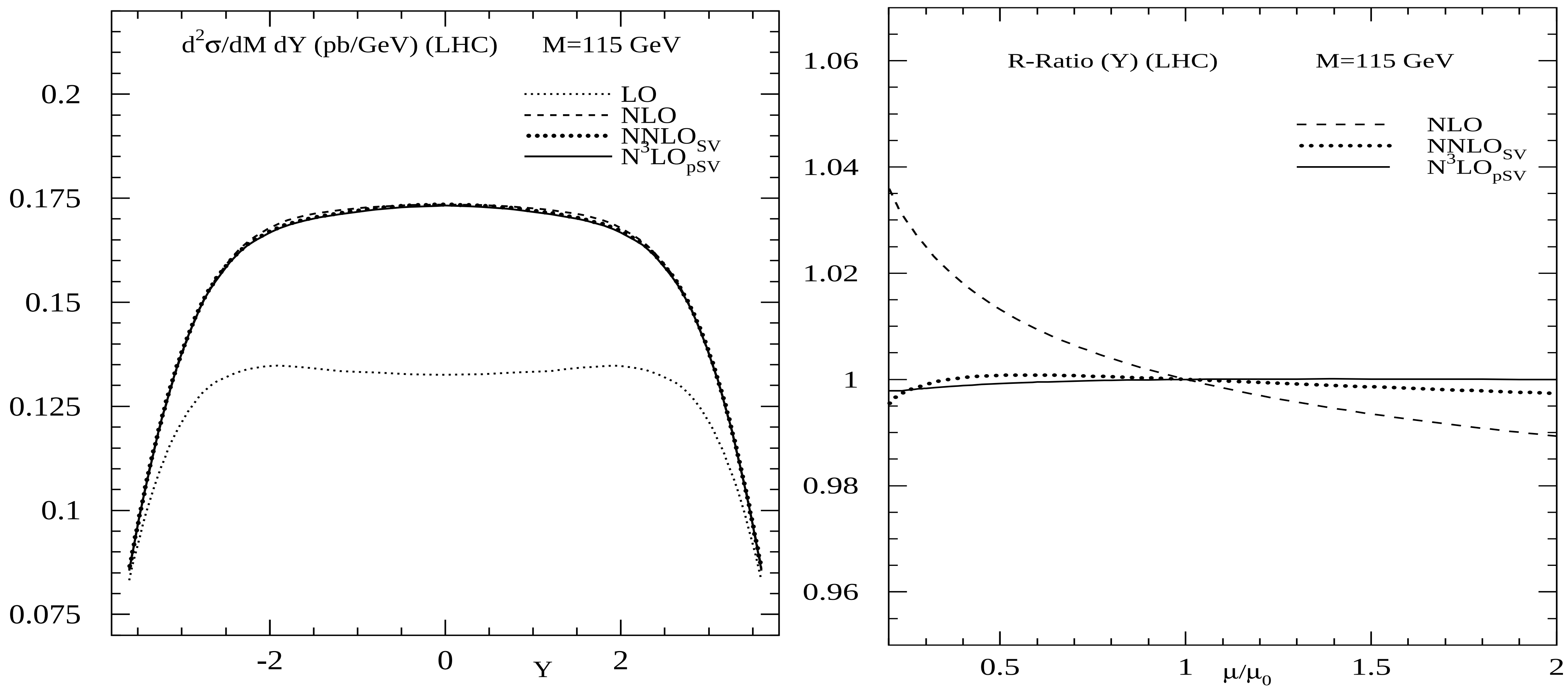

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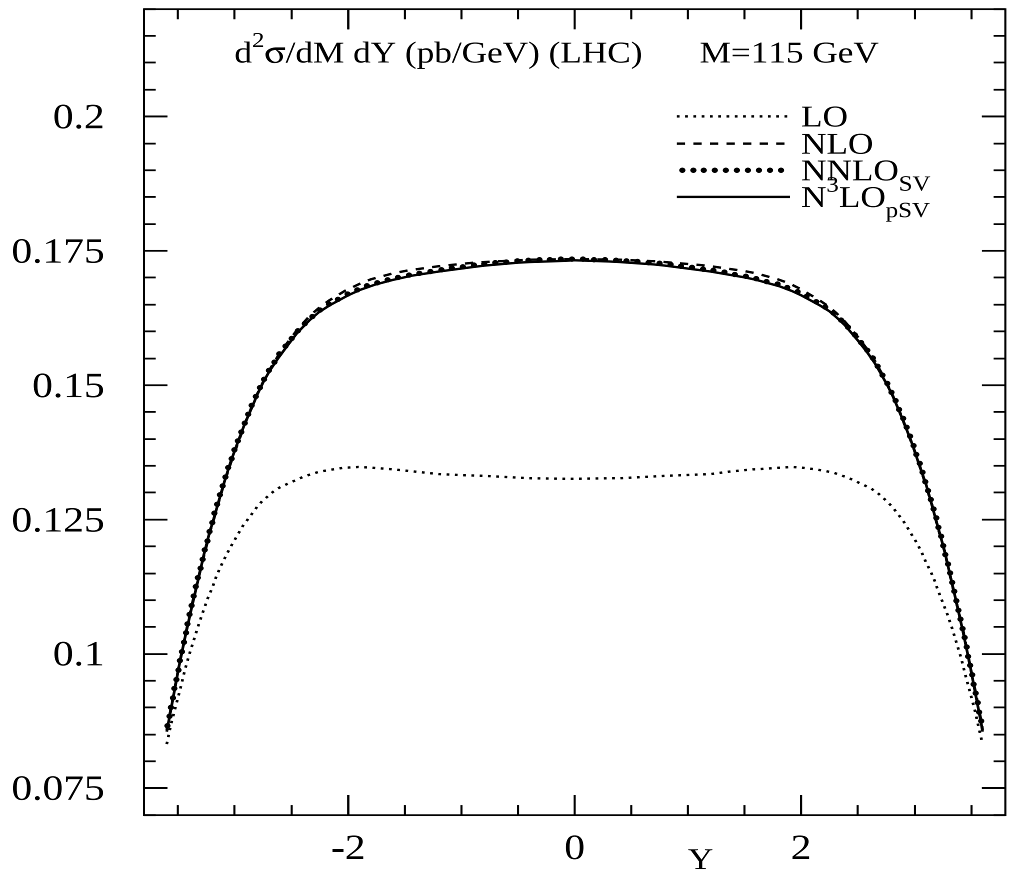

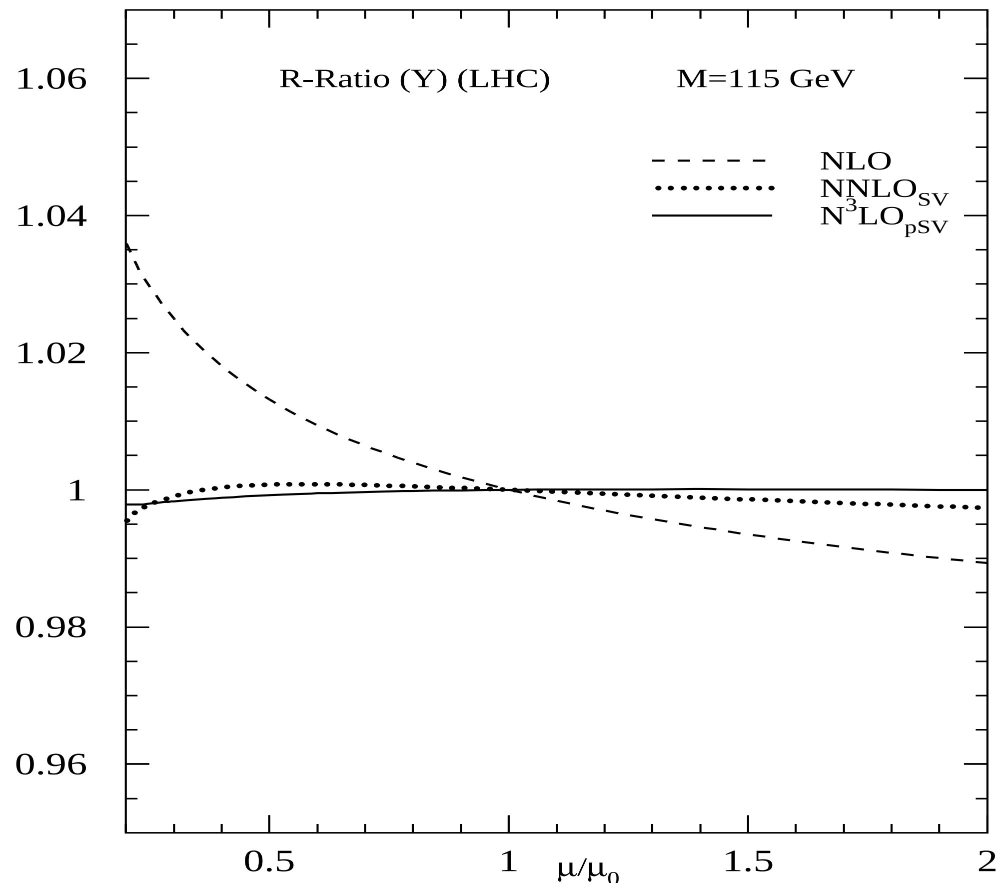

- Compared against Dixon, Anastasiou, Melnikov, Petriello NNLO results for Drell-Yan, $\boldsymbol{Higgs, Z, W^{\pm}}$ productions.

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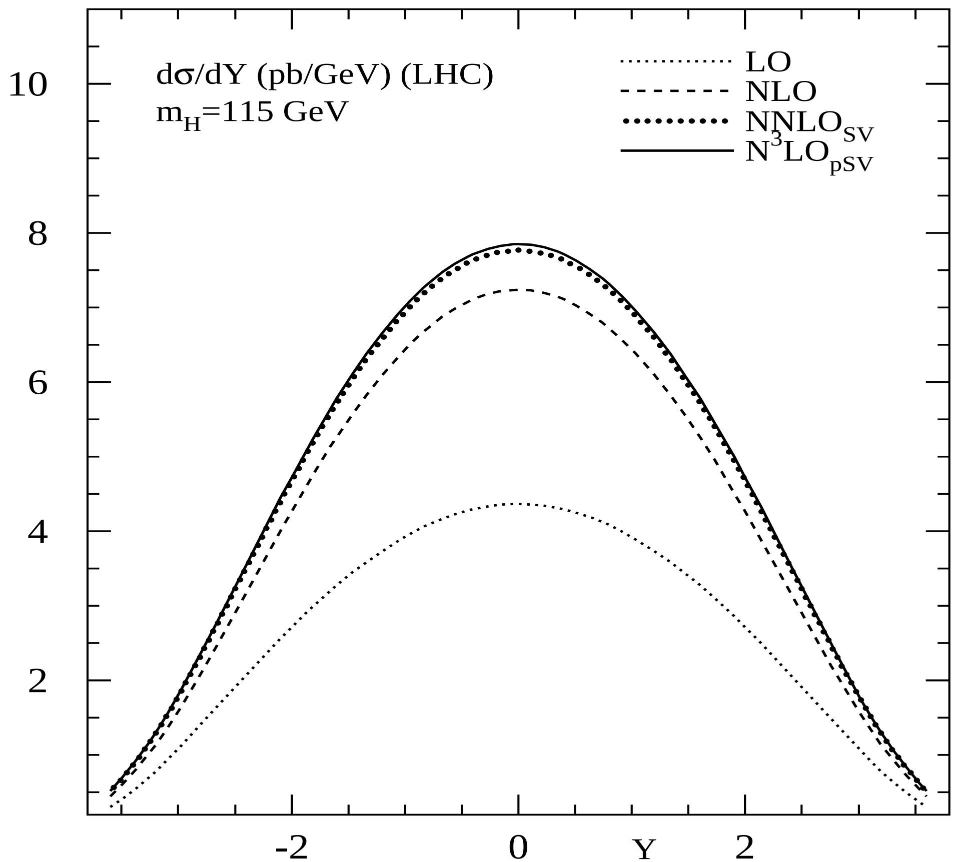

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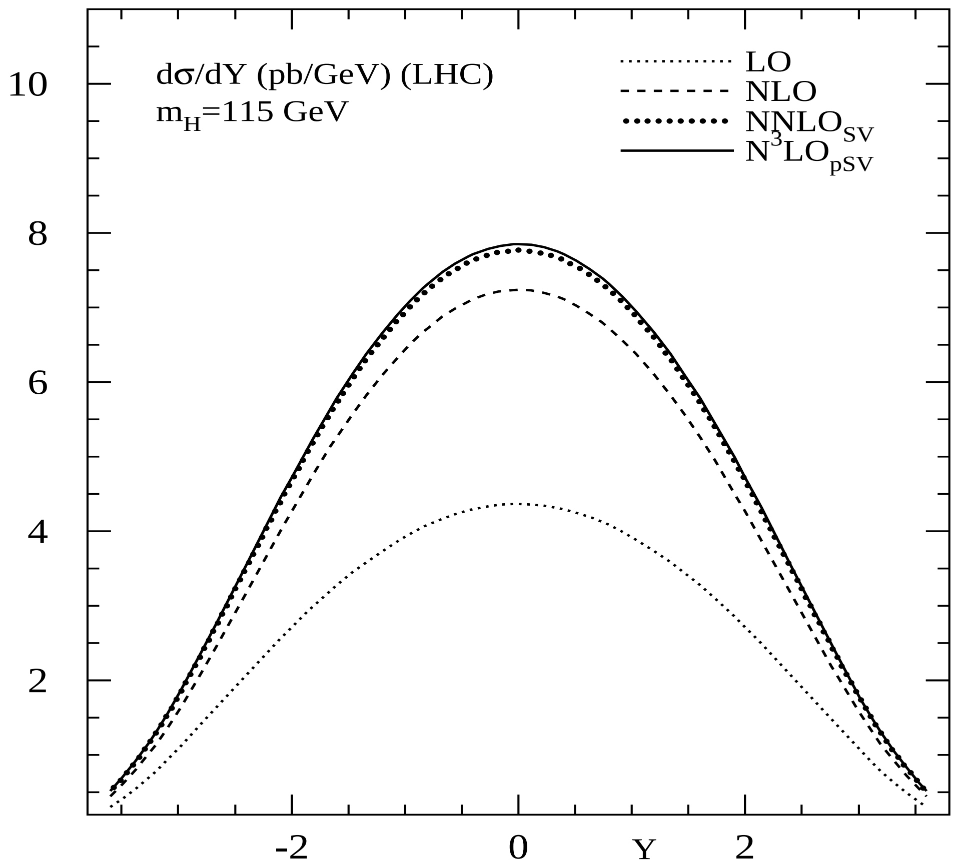

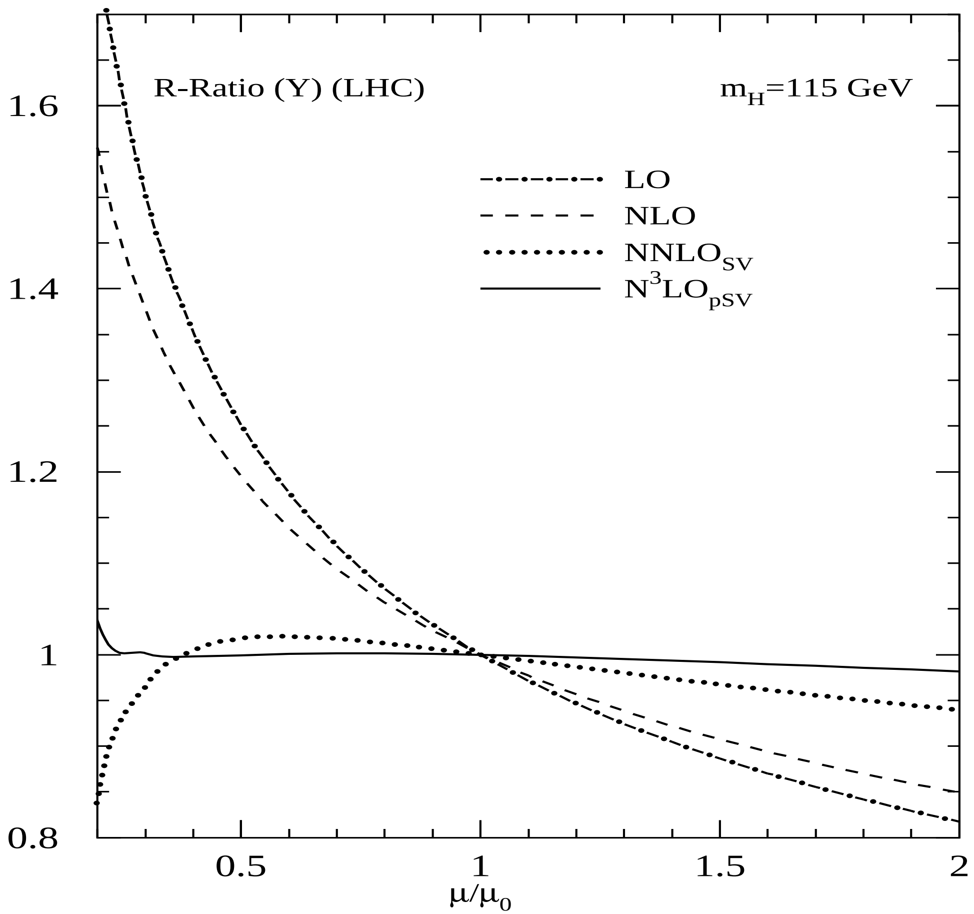

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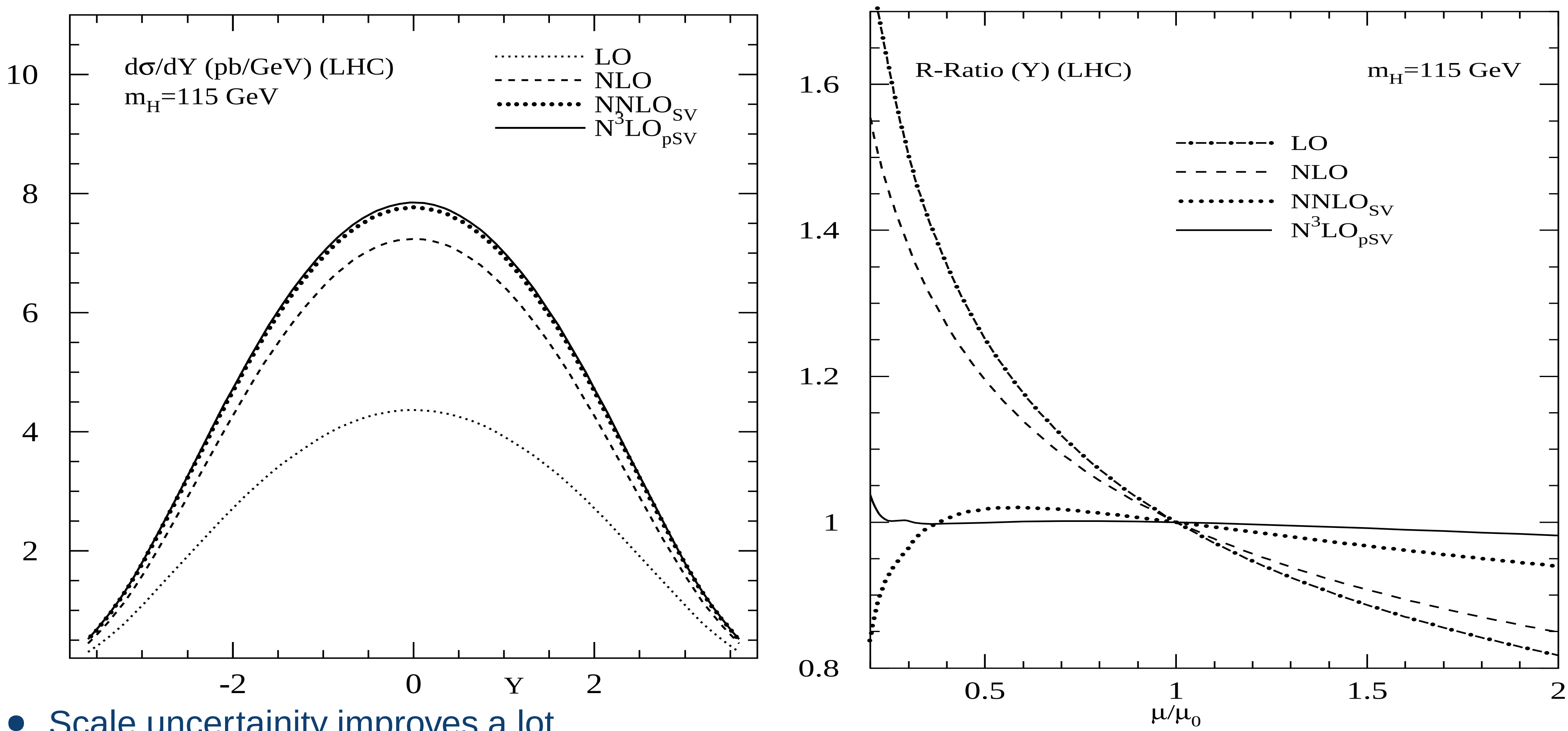

- Scale uncertainity improves a lot

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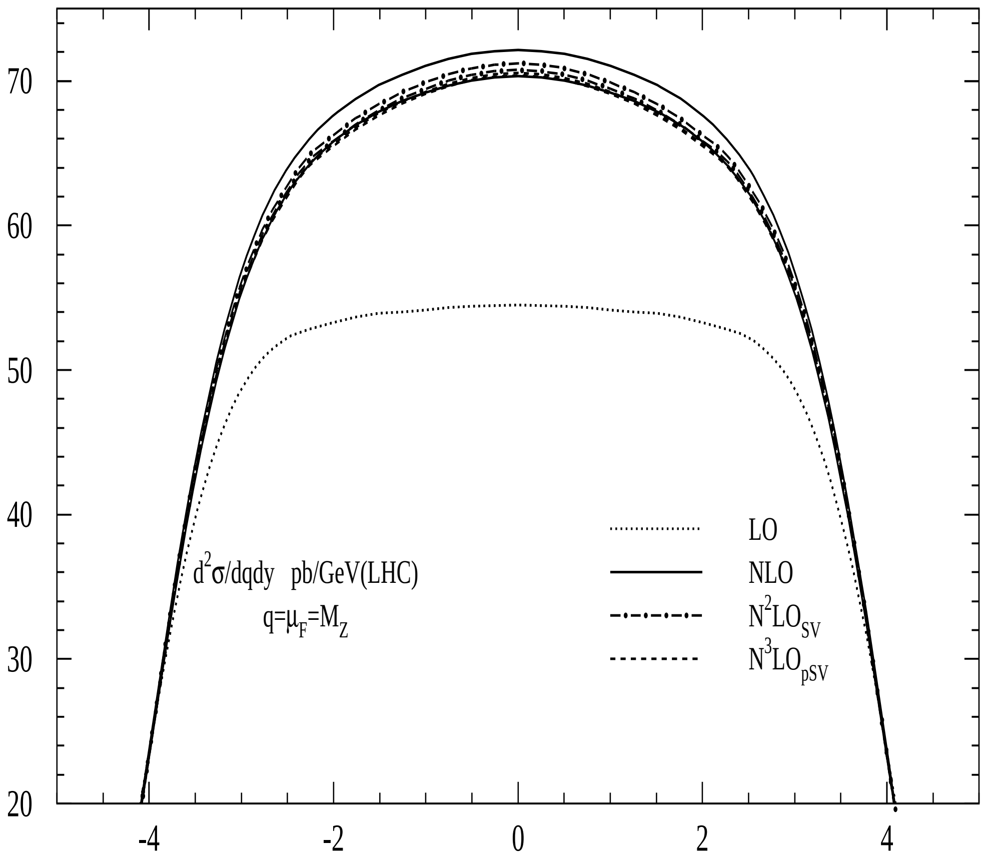

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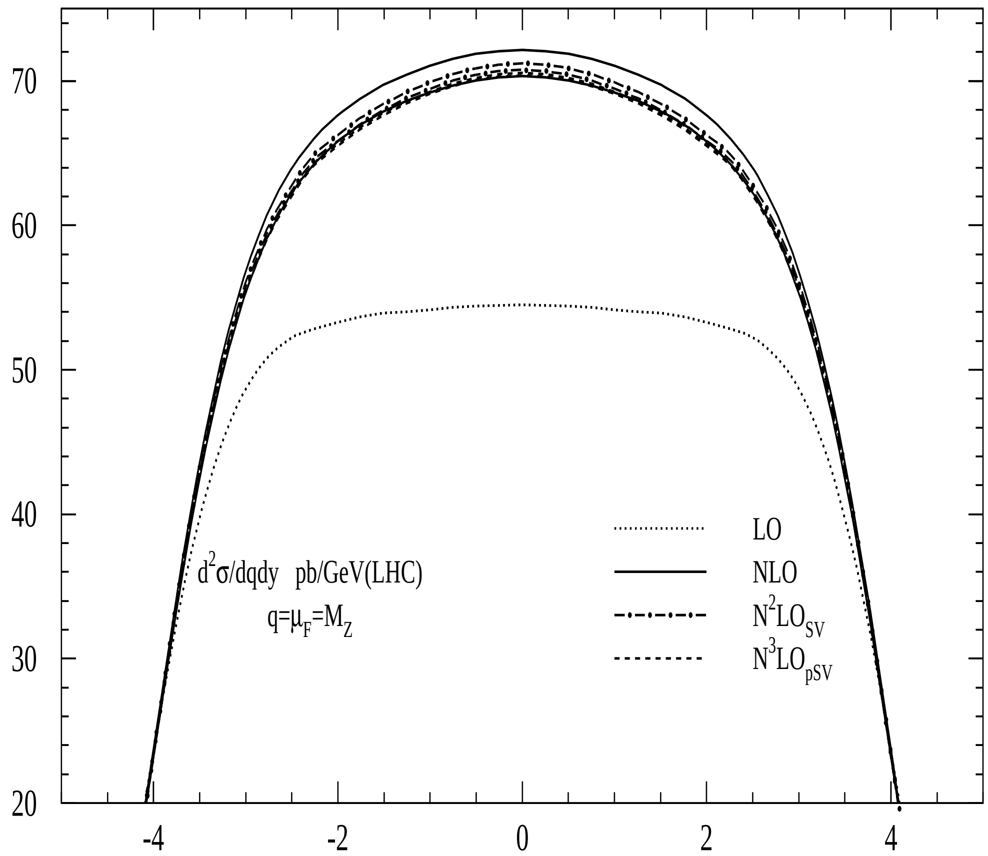

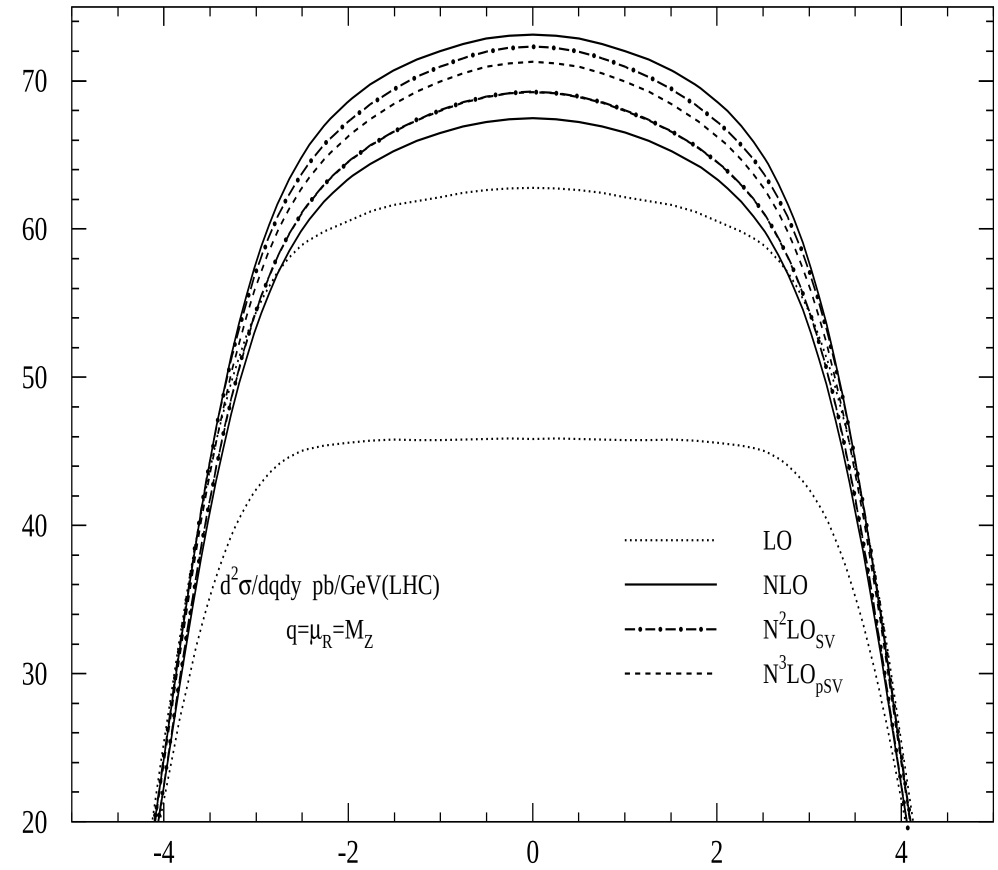

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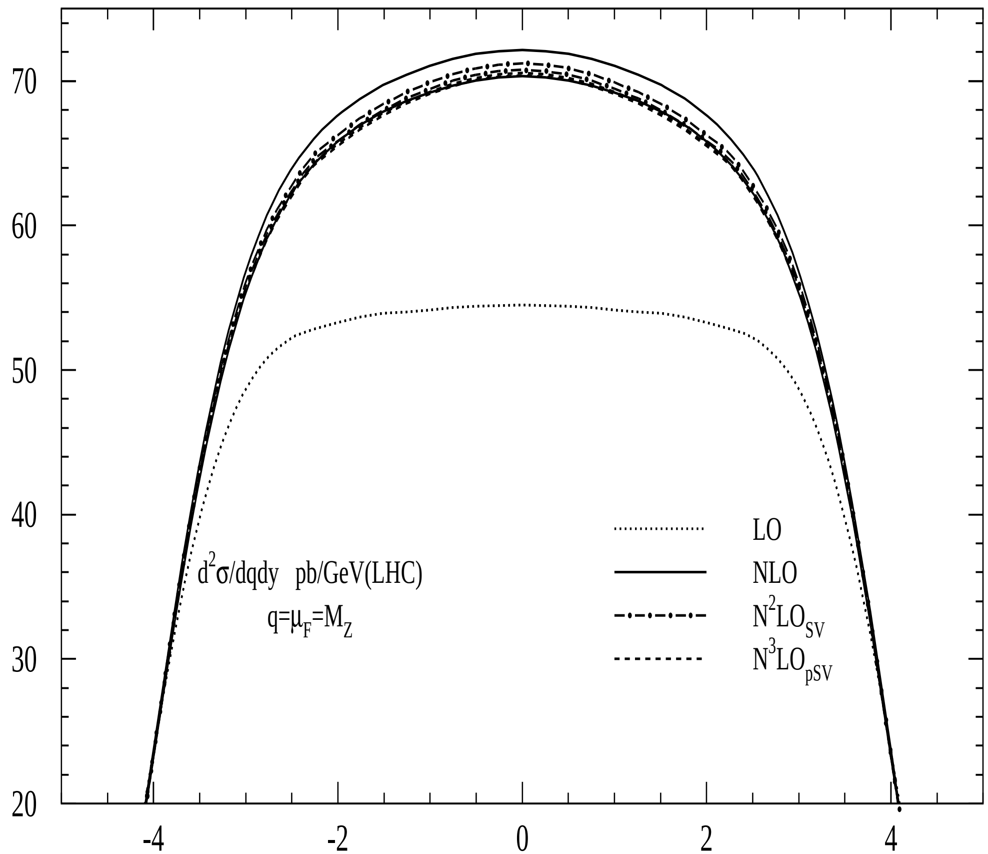

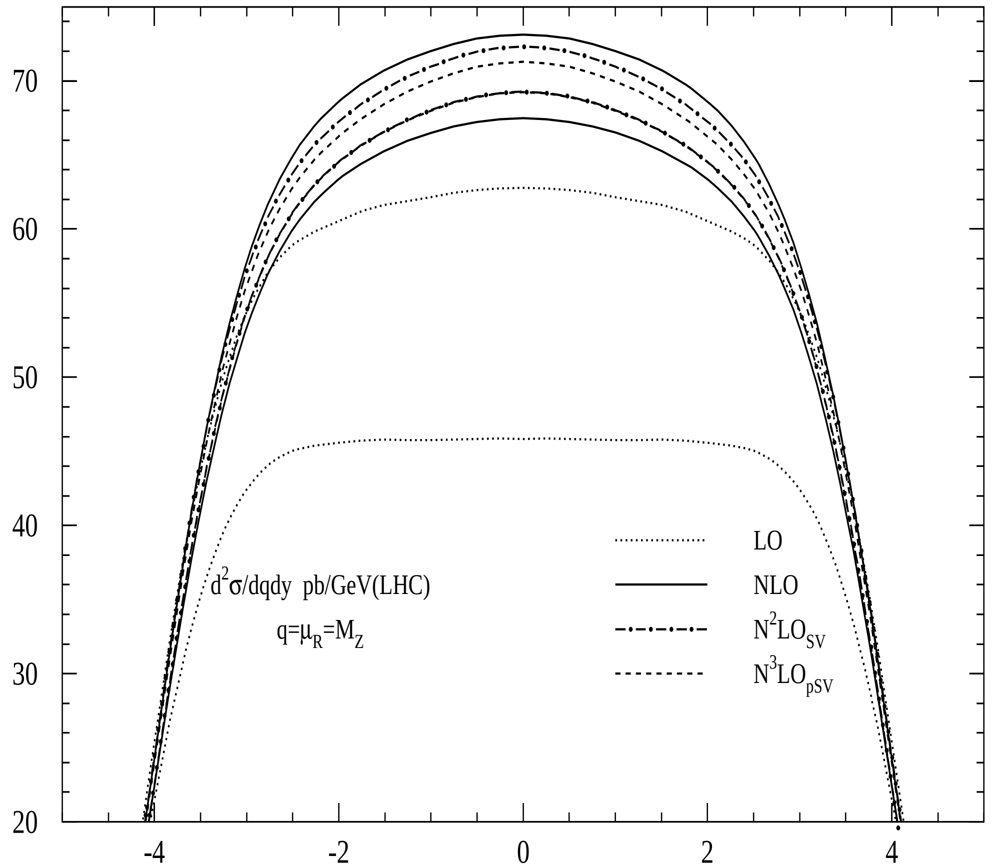

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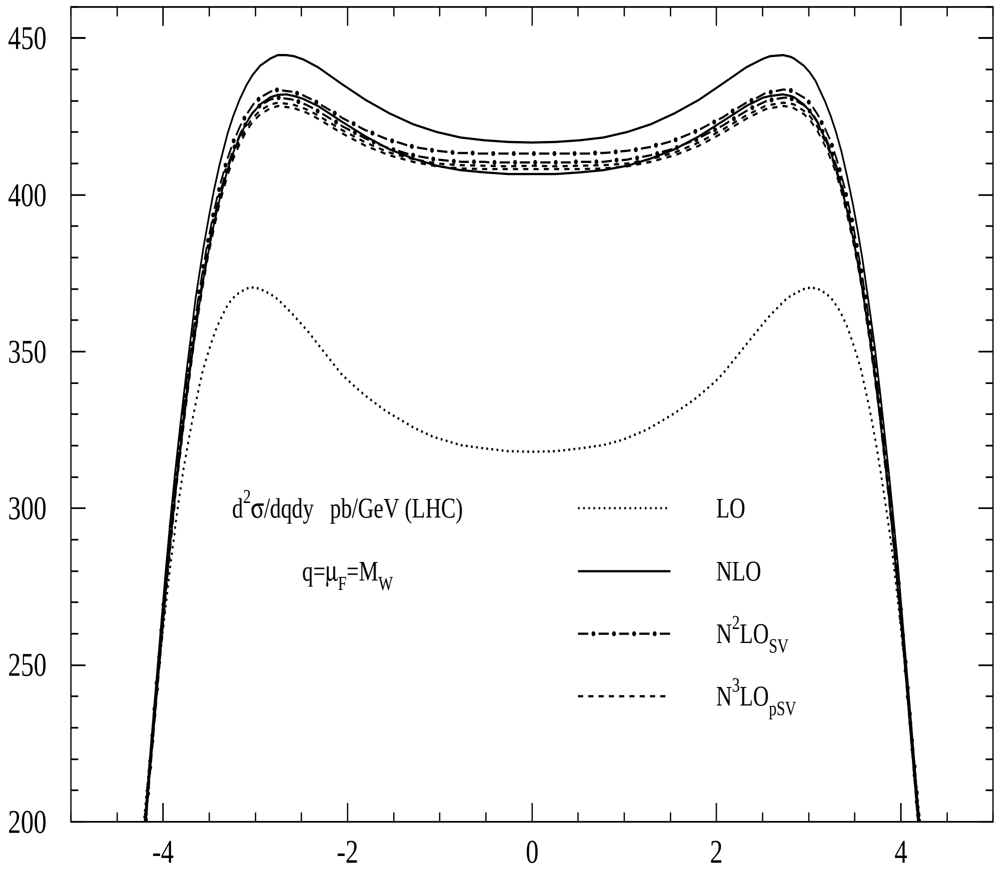

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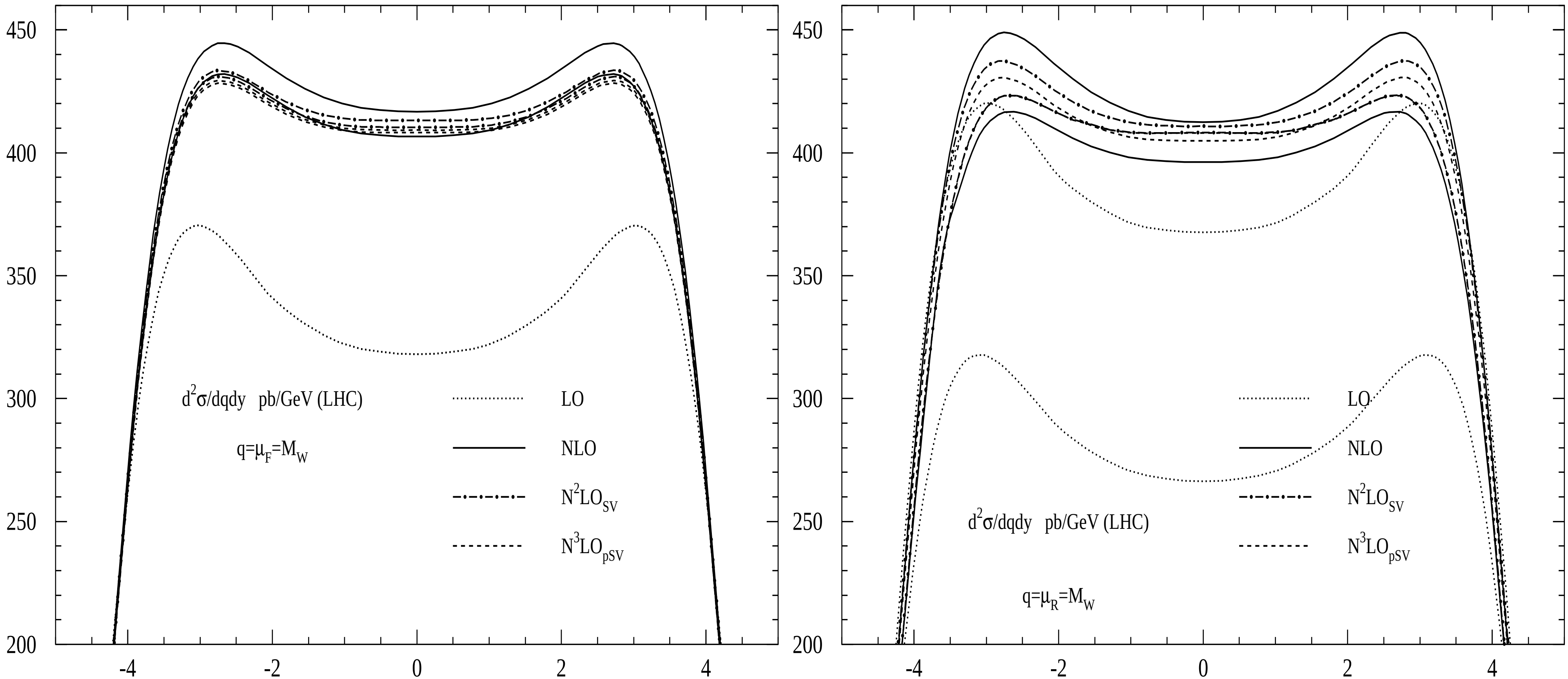

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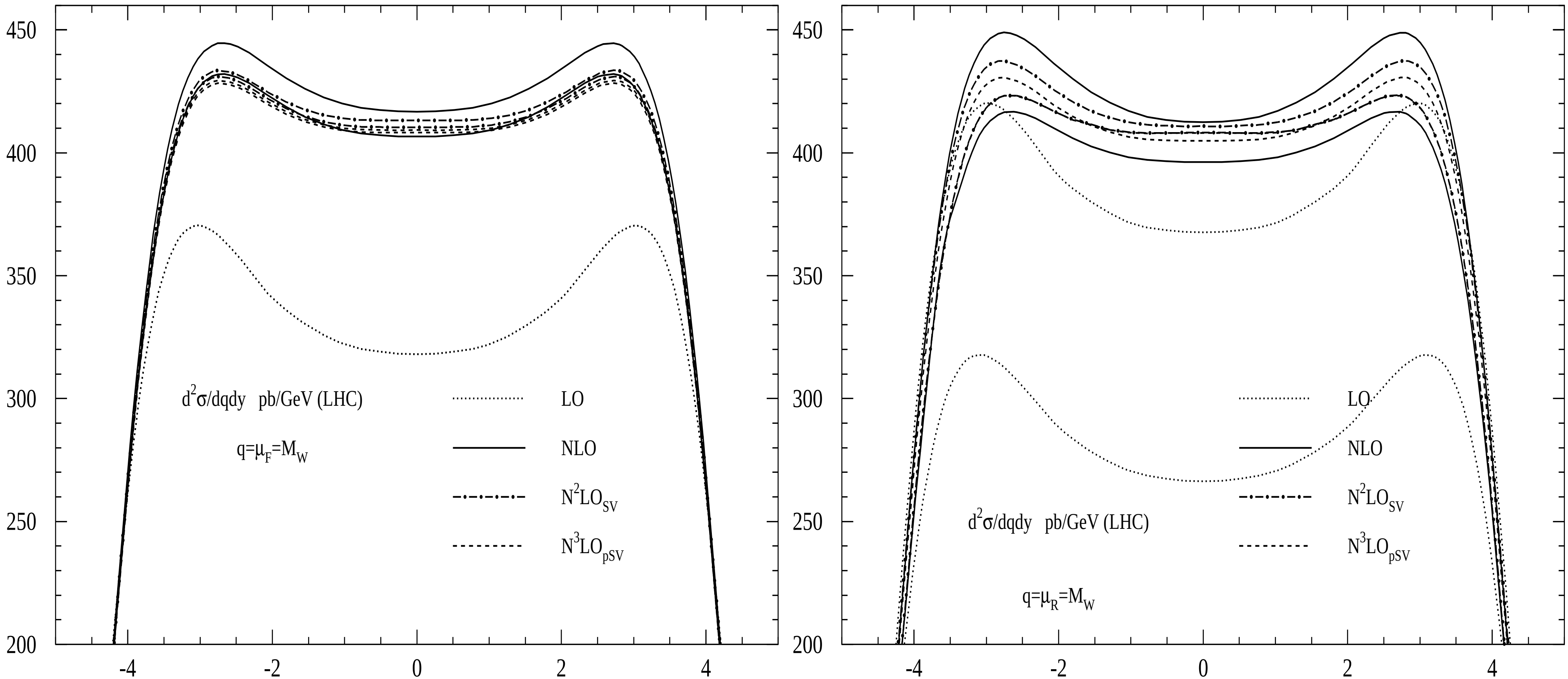
$V.Ravindran, J.Smith$

$$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$



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Thank You