Cosmological Constant, brane tension and large hierarchy in a generalised Randall-Sundrum scenario

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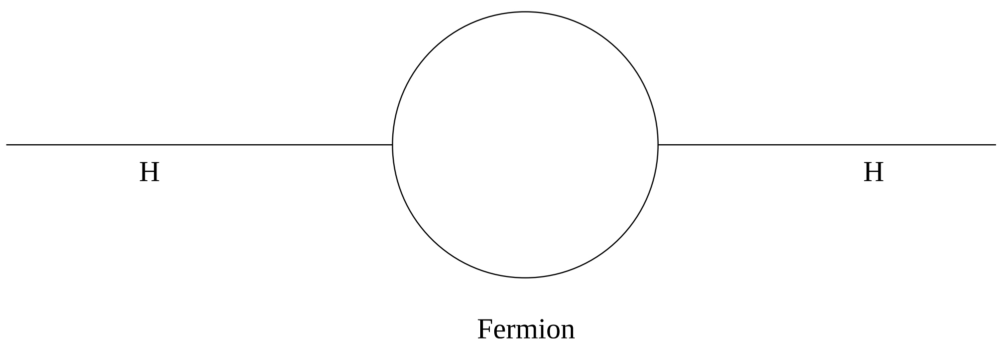
Some Questions

Some of the intriguing questions about our physical universe, which remain unanswered, are:

- Why does it appear to have $(3+1)$ spacetime dimensions? Are there additional unobserved dimensions?
- Why is the ratio of the electroweak scale/Higgs mass (m) to the Planck mass (m_0) so tiny ($\simeq 10^{-16}$)? This gives rise to the *fine tuning* or *naturalness* or *gauge hierarchy problem*.
- Why is the observed value of the cosmological constant Ω extremely small ($\Omega \simeq 10^{-124}$) (in Planck units)? This gives rise to the *cosmological fine tuning problem*

Background

- Standard model has been extremely successful in explaining physical phenomena upto scale close to Tev
- It encounters the fine tuning problem in connection with the large quadratic correction to the mass of the only scalar in the theory —- 'Higgs'



This is further related to the gauge hierarchy problem which refers to the vast disparity between the weak and Planck scale.

$$\delta m_H^2 \sim \Lambda^2$$

where Λ is the cutoff scale i.e.Planck scale or GUT scale

To keep m_H within Tev, one needs extreme fine tuning to cancel the very large loop correction $\rightarrow$ unnatural !

Resolution through Supersmmetry

Supersymmetry removes this unnatural feature at the expense of bringing in the following problems :

- It incorporates a large number of (so far undetected) superpartners in the theory
- Absence of the superpartners implies that the supersymmetry, even if it is true, is a broken symmetry at the present energy scale
- Breakdown of supersymmetry in turn generates large cosmological constant which is not consistent with the present observed small value

- Moreover, what will happen if the signature of supersymmetry is not found at low scale?
- It may be there at high scale, as required in String theory, but that will not resolve the hierarchy problem
- If we do not want to subscribe to exotic ideas like landscape or anthropic principle in favour of fine tuning or unnaturalness – we will have to look for some alternative paths

Extra dimensional Theories – Braneworld

In the brane world models it was shown that :

- Questions (i) and (ii) may be related i.e if the spacetime dimension exceeds four, the hierarchy problem can be solved.
- ADD model resolves the hierarchy problem at the expense of bringing in new length hierarchy.
- Randall-Sundrum two-brane warped geometry model is particularly successful in resloving the fine tuning problem without bringing in any arbitrary intermediate scale between the Planck and the Tev scale.

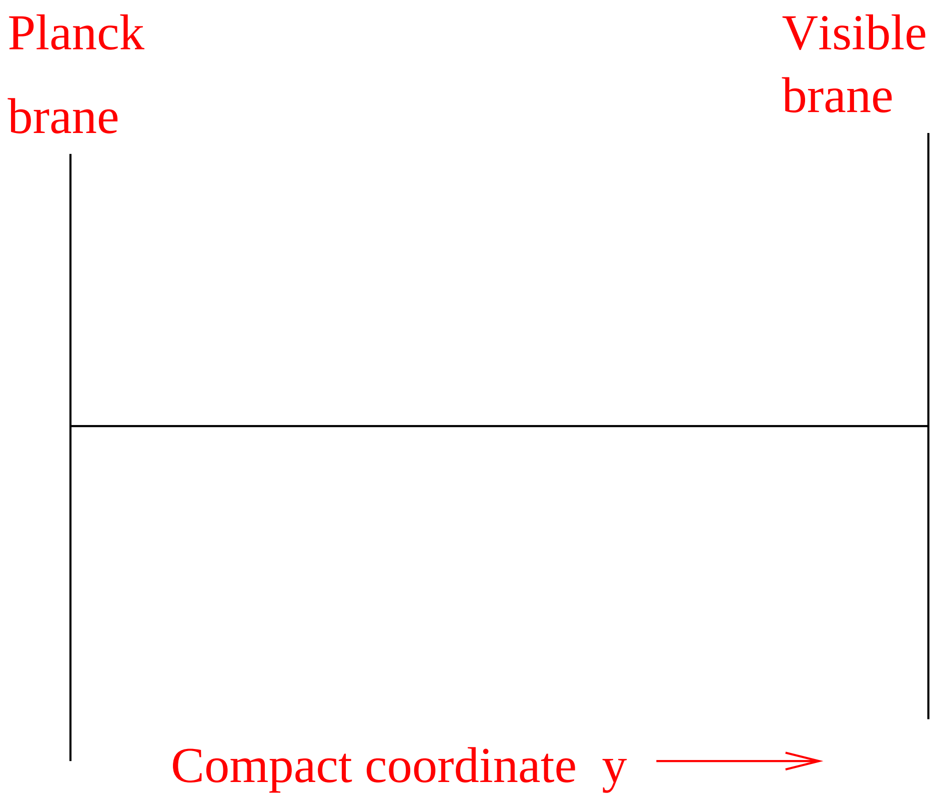
Randall Sundrum Model

Considering one extra spatial dimension Randall and Sundrum proposed a 5 dimensional warped geometric model in an anti-deSitter bulk spacetime

The Einstein action in 5 dimensional ADS space:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_5} \left[\mathcal{R}_d - \Lambda\right]$$

Compactify the extra coordinate $y = r\phi$ on a S_1/Z_2 orbifold with two 3-branes placed at the two orbifold fixed points $\phi = 0, \pi$, where r is the radius of S_1



The Z_2 orbifolded coordinate $y = r\phi$ with $0 \leq \phi \leq \pi$ and r is the radius of the S_1

Action ($M_{Pl(5)} \equiv M$):

$$S = S_{Gravity} + S_{vis} + S_{hid}$$

where,
$$S_{Gravity} = \int d^4x\, r\, d\phi \sqrt{-G}\, [2M^3 R - \underbrace{\Lambda}_{5-dim}]$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}}\, [L_{vis} - V_{vis}]$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}}\, [L_{hid} - V_{hid}]$$

Metric ansatz:

$$ds^2 = e^{-A}\,\eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\phi^2 \;\leftarrow\; \text{extra dim}$$

Computing the warp factor $A(y)$

Warp factor and the brane tensions are found by solving the 5 dimensional Einstein's equation with orbifolded boundary conditions

$$A = 2kr\phi$$

$$V_{hid} = -V_{vis} = 24M^3 k \qquad \left[k^2 = \frac{-\Lambda}{24M^3}\right]$$

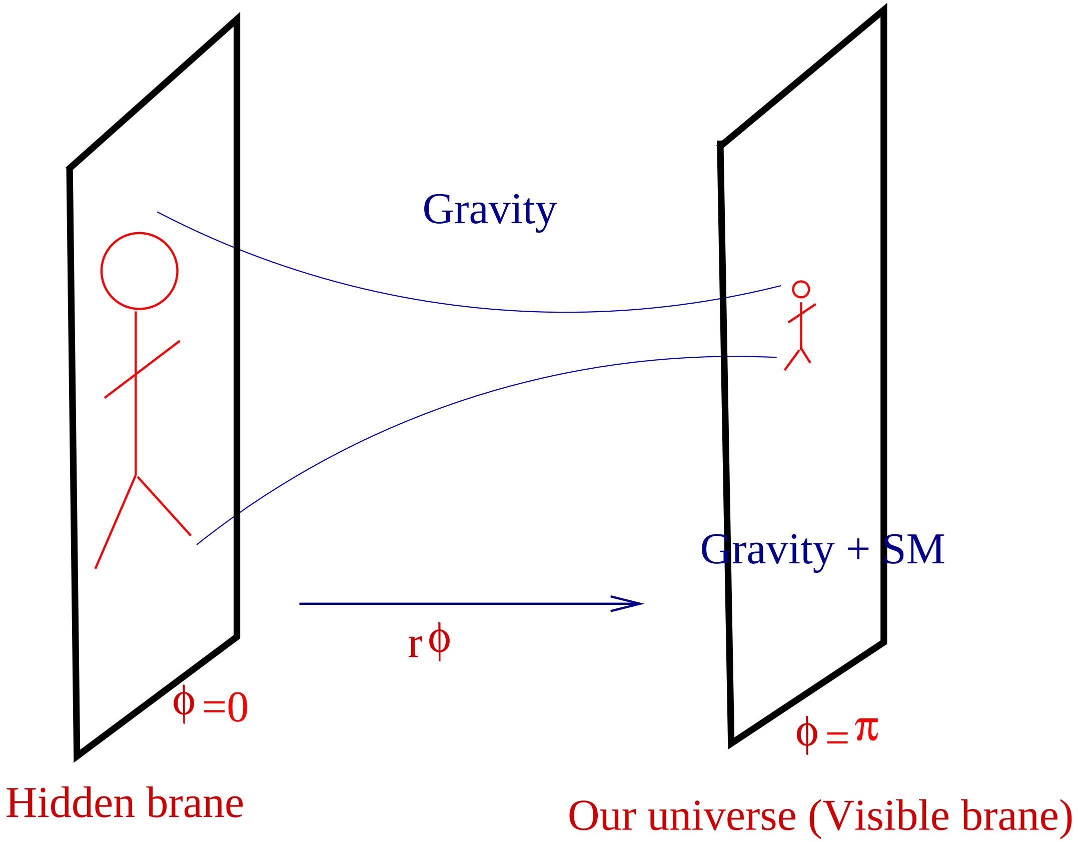
Warping

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \approx (10^{-16})^2$$

$$\Rightarrow kr = \frac{16}{\pi}\ln(10) = 11.6279\ldots \quad \leftarrow \text{RS value}$$

with
$k \sim M_P$ and $r \sim l_P$

So hierarchy problem is resolved without introducing any new scale


Gravity
Gravity + SM
r ϕ
ϕ =0
ϕ = π
Hidden brane
Our universe (Visible brane)

Outcome

A large hierarchy emerges naturally from a small conformal factor

Some features of RS model

- Somewhat unatisfactory but inevitable feature of having a negative tension visible brane to describe our Universe. Such negative tension branes are known to have stability problem.

- The effctive visible 3-brane being flat has zero cosmological constant which is not consistent with its presently observed small value.

- Problem of stabilizing the modulus.

Our work

- We now extend such warped geometric model with non-zero cosmological constant on the visible 3-brane and look for possible occurence of positive tension Tev brane when a large hierarchy exists between the two branes.
- One of the inspiration of looking for positive tension branes emerge from string inspired braneworld models where we have positive tension D-branes to construct low energy phenomenology
- The implications of having a small cosmological constant are examined with special emphasis.

Earlier work

In the original RS scenario, it was proposed that the visible 3-brane being flat has zero cosmological constant.

$$ds^2 = e^{-2kry}\eta_{\mu\nu}dx^\mu dx^\nu + r^2 dy^2$$

But such model was generalized to Ricci flat spaces :

$$R_{\mu\nu} = 0$$

and the warp factor turned out to be the same as obtained by RS

See Chamblin, Hawking, Real : Phys.Rev.D, 61,065007 (2000)

Generalization to ADS and DS spaces

In our work we demonstrate that the condition of zero cosmological constant can be relaxed and a more general warp factor can be obtained.
We start with the metric :

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r^2 dy^2 \ .$$

and evaluate the function $A(y)$ which extremises the action:

$$S = \int d^5x \sqrt{-G}(M^3\mathcal{R} - \Lambda) + \int d^4x \sqrt{-g_i}\mathcal{V}_i$$

where Λ is the bulk cosmological constant, $\mathcal{R}$ is the bulk (5-dimensional) Ricci scalar and $\mathcal{V}_i$ is the tension of the i^{th} brane

The resulting Einstein equations are:

$$^4G_{\mu\nu} - g_{\mu\nu} e^{-2A}[-6A'^2 + 3A''] = -\frac{\Lambda}{2M^3} g_{\mu\nu} e^{-2A}$$

$$-\frac{1}{2} e^{2A}\ {}^4R + 6A'^2 \ = \ -\frac{\Lambda}{2M^3}$$

with the boundary conditions

$$[A'(y)]_i = \frac{\epsilon_i}{12M^3} \mathcal{V}_i \ ,$$

where $\epsilon_{hid} = -\epsilon_{vis} = 1$.

Here ${}^4G_{\mu\nu}$ and ${}^4R$ are the four dimensional Einstein tensor and Ricci scalar respectively.

Rearranging terms we find that one side of the equation contains $A(y)$ and its derivatives and depends on the extra coordinate y alone, while the other side depends on the brane coordinates x^μ alone.

Thus each side is equal to an arbitrary constant, Ω (say).

Thus, we get, :

$$^4G_{\mu\nu} = -\Omega g_{\mu\nu} \qquad ,$$

$$e^{-2A} \left[-6A'^2 + 3A'' - \frac{\Lambda}{2M^3}\right] = \Omega \; .$$

Ω is the four dimensional cosmological constant on the $3 - brane$.

On simplification we obtain,

$$6A'^2 = -\frac{\Lambda}{2M^3} + 2\Omega e^{2A}$$

$$3A'' = \Omega e^{2A} \; .$$

The above corresponds to a constant curvature brane spacetime, as opposed to a Ricci flat spacetime. For example, for $\Omega > 0$ and $\Omega < 0$, $g_{\mu\nu}$ may correspond to dS-Schwarzschild and AdS-Schwarzschild spacetimes respectively.

For Ads bulk i.e. $\Lambda < 0$, we first consider the induced cosmological constant Ω on the visible brane to be negative.

Negative Ω – ADS case

Define the parameter $\omega^2 \equiv -\Omega/3k^2 \geq 0$, The solution for the

warp factor,

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c\,1} + ky\right)$$

Note that the above solution is an exact solution for the warp factor in presence of Ω.

The RS solution $A = ky$ is recovered in the limit $\omega \rightarrow 0$.

The brane tensions are:

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_1^2}e^{2kr\pi} - 1}{\frac{\omega^2}{c_1^2}e^{2kr\pi} + 1}\right]$$

$$\mathcal{V}_{hid} = 12M^3k\left[\frac{1 - \frac{\omega^2}{c_1^2}}{1 + \frac{\omega^2}{c_1^2}}\right]$$

Normalizing the warp factor to unity at the orbifold fixed point $y = 0$, we get:

$$c_1 = 1 + \sqrt{1 - \omega^2}\ .$$

To solve the hierarchy problem, we equate the warp factor at $y = r\pi$ to the ratio of the Higgs to the Planck mass:

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + kr\pi\right) = 10^{-n}\ .$$

We call n as warp factor index.

At this point, we keep n arbitrary, although eventually we will assume it to be $\simeq 16$ to achieve the desired warping from Planck to Tev scale. Defining $kr\pi \equiv x$, the above

equation simplifies to:

$$10^{-n} = \frac{1}{2}\left[c_1 e^{-x} + \frac{\omega^2}{c_1} e^{x}\right] \ ,$$

This further simplifies to,

$$e^{-x} = \frac{10^{-n}}{2}\left[1 \pm \sqrt{1 - \omega^2 10^{2n}}\right]$$

Outcome

Real solutions for e^{-x} exists if $\omega^2 \leq 10^{-2n}$. In other words, for ADS 3-brane there is an an upper bound on the magnitude of the induced cosmological constant which is determined by the extent of desired warping (i.e. the warp factor index) from the hidden brane to the visible brane.

In general for very small ω, $c_1 \simeq 2$, which we will assume from now on. We now set the brane cosmological constant

$\omega^2 \equiv 10^{-N}$, N is termed as the cosmological constant index. Therefore the upper bound on the magnitude of cosmological constant or lower bound on N is

$$N_{min} = 2n$$

Thus, for $n = 16$, it follows that the brane cosmological constant cannot exceed 10^{-32} (in Planck units).

Also solving the previous equ.

$$10^{-N} = 4\left(10^{-n}e^{-x} - e^{-2x}\right) ,$$

$$e^{-x} = \frac{10^{-n}}{2}\left[1 \pm \sqrt{1 - 10^{-(N-2n)}}\right] .$$

From above, it can be seen that for $N \to \infty$ ($\omega^2 \to 0$), the RS case is obtained.

For $N > N_{min}$, there are *two* values of x which give rise to the required warping.
For $N - 2n \gg 1$, these two solutions are:

$$x_1 \simeq n \ln 10 + \frac{1}{4} 10^{-(N-2n)}$$

and

$$x_2 \simeq (N - n) \ln 10 + \ln 4 \ .$$

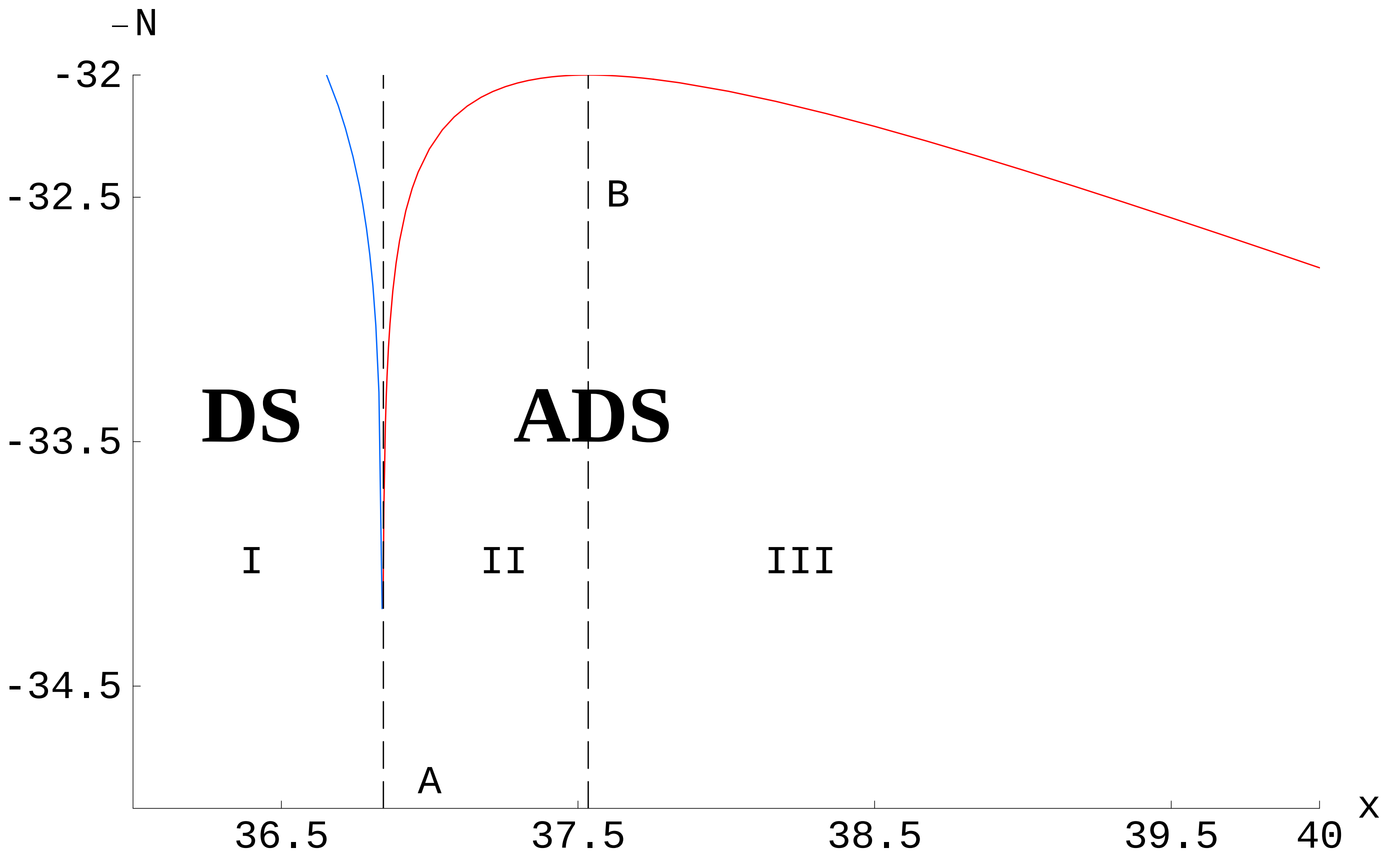
Thus, for $n = 16$ and $N = 124$, we get:

$$x_1 \simeq 36.84 + 10^{-93}\ , \quad x_2 = 250.07\ ,$$

or,

$$kr_1 \simeq 11.73 + 10^{-93}\ , \quad kr_2 = 79.60\ .$$

- The first corresponds to the RS value plus a minute correction,
- The second, corresponds to a larger value of the modulus r.
- The hierarchy problem however is solved for two different values for the modulus when the cosmological constant is negative.



−N
-32
-32.5
-33.5
-34.5
DS
ADS
B
A
I
II
III
x
36.5
37.5
38.5
39.5
40

The brane tension for the visible brane is,

$$\mathcal{V}_{vis} = (12M^3k)\,\frac{1 - 10^{N-2n}\left[1 \pm \sqrt{1 - 10^{-(N-2n)}}\right]}{1 + 10^{N-2n}\left[1 \pm \sqrt{1 - 10^{-(N-2n)}}\right]}\,.$$

Observe that $\mathcal{V}_{vis} = 0$ when $N = N_{min} = 2n$. Further, it is easy to show that $\mathcal{V}_{vis} < 0$ for $x = x_1$, while $\mathcal{V}_{vis} > 0$ for $x = x_2$. Thus the second solution for x i.e $kr\pi$ is associated with a positive tension brane which also produces the desired large hierarchy.

When $N - 2n \gg 1$ The two tensions are approximately given as:

$$\begin{aligned} \mathcal{V}_{vis-1} &\simeq -(12M^3k) \\ \mathcal{V}_{vis-2} &\simeq \frac{1}{3}(12M^3k) \ . \end{aligned}$$

Note that a small negative cosmological constant suffices to render the tension positive, provided the distance between the branes is somewhat larger than the value predicted by RS.

The hidden brane tension is given by:

$$\mathcal{V}_{hid} = (12M^3k)\,\frac{4 - 10^{-N}}{4 + 10^{-N}}\ ,$$

which is always positive.

Positive Ω – DS case

Consider$\Omega > 0$

The warp factor is now given by:

$$e^{-A} = \omega \sinh\left(\ln \frac{c_2}{\omega} - ky\right) \ ,$$

where now $\omega^2 \equiv \Omega/3k^2$ and $c_2 = 1 + \sqrt{1+\omega^2}$.

Equating the above to $m/m_0 = 10^{-n}$,

$$10^{-n} = \frac{1}{2}\left[c_2 e^{-x} - \frac{\omega^2}{c_2} e^{x}\right] \ ,$$

and

$$e^{-x} \;\; = \;\; \frac{10^{-n}}{c_2}\left[1 + \sqrt{1 + \omega^2 10^{2n}}\right]$$

In this case, there are no bounds on ω^2, i.e. the (positive) cosmological constant can be of arbitrary magnitude.

Also, there is a single solution of x, whose precise value will depend on ω^2 and n. This is described in the region I in FIG.1 and it is easy to observe that a small and positive value of the cosmological constant which corresponds to the observed value $\sim 10^{-124}$ in Planckian unit indicates a value for x i.e $kr\pi$ very very close to the RS value 36.84. However for the entire range of $\Omega \geq 0$ the Tev brane tension continues to be negative as in RS case.

This can be seen from the expressions for the brane tensions:

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_2^2}e^{2kr\pi} + 1}{\frac{\omega^2}{c_2^2}e^{2kr\pi} - 1}\right]$$

$$\mathcal{V}_{hid} = 12M^3k\left[\frac{1 + \frac{\omega^2}{c_2^2}}{1 - \frac{\omega^2}{c_2^2}}\right]$$

Recall,

$$c_2 = 1 + \sqrt{1 + \omega^2} > \omega$$

Therefore, $c_2^2 > \omega^2$, and hence $\mathcal{V}_{hid}$ is always positive.

Also the condition of positivity of the warp factor 10^{-n} requires $\frac{\omega^2}{c_2^2} e^{2kr\pi} < 1$ which makes $\mathcal{V}_{vis}$ negative for the entire range of positive values of Ω.

Conclusions

Here by generalizing the RS model with a non-vanishing cosmological constant on the visible brane we show that

- Issue of smallness of cosmological constant, smallness of the factor in gauge hierarchy and brane tensions are intimately related in a generalized Randall-Sundrum (RS) type of warped geometry model .

- Exact solution for the warp factors are determined for both DS and ADS cases.

- Region of positive cosmological constant on the visible 3-brane (de-Sitter) strictly implies negative brane tension

- However visible brane with negative cosmological constant (anti de-Sitter) admits of both positive and negative brane tension.

- For both the cases the desired warping from Planck to Tev scale can be achieved as a proper resolution of the gauge hierarchy problem.

- The magnitude of the negative induced cosmological constant on the 3-brane has an upper bound $\sim 10^{-32}$ in Planck unit.

- For a very tiny but negative value of the induced cosmological constant the hierarchy problem can be resolved for two different values of the modulus, one of which corresponds to a positive tension Tev brane alongwith the positive tension Planck brane.

- In the other region namely $\Omega > 0$ the Tev brane tension turns out to be necessarily negative . The modulus value corresponding to the observed value of the cosmological constant lies very close to the RS value.

Thus in a generalized warp braneworld model,
the fine tuning problem in connection with the Higgs mass requires that the cosmological constant Ω (whether positive or negative) on the Tev brane must be tuned to a very very small value.
In other words:

The fine tuning problem in connection with the Higgs mass and the cosmological fine tuning problem are intimately related and one implies the other!