Photons, hydrodynamics & the QCD Equation-of-State

Hot and Dense Matter in the RHIC-LHC Era

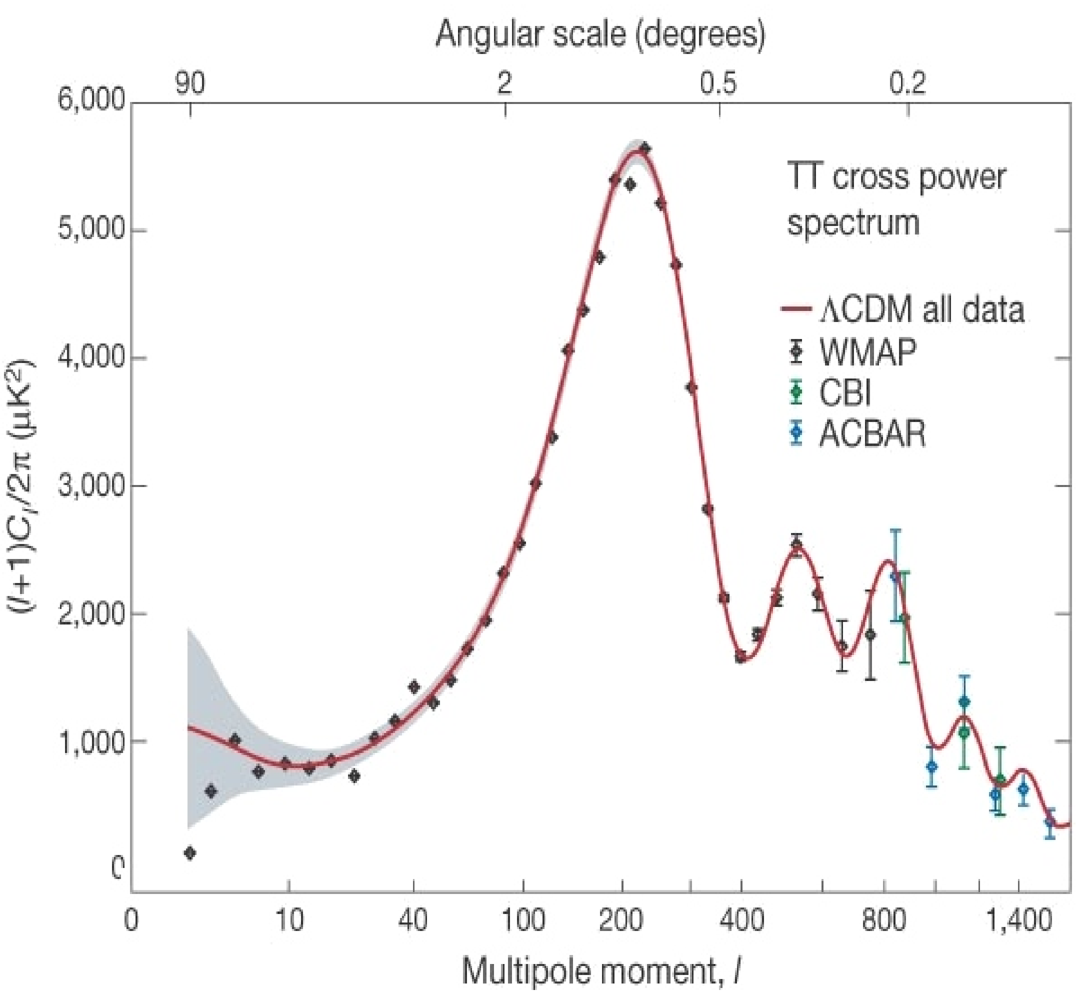
Mumbai, Feb. 12nd -14th, 2007

David d'Enterria (CERN)

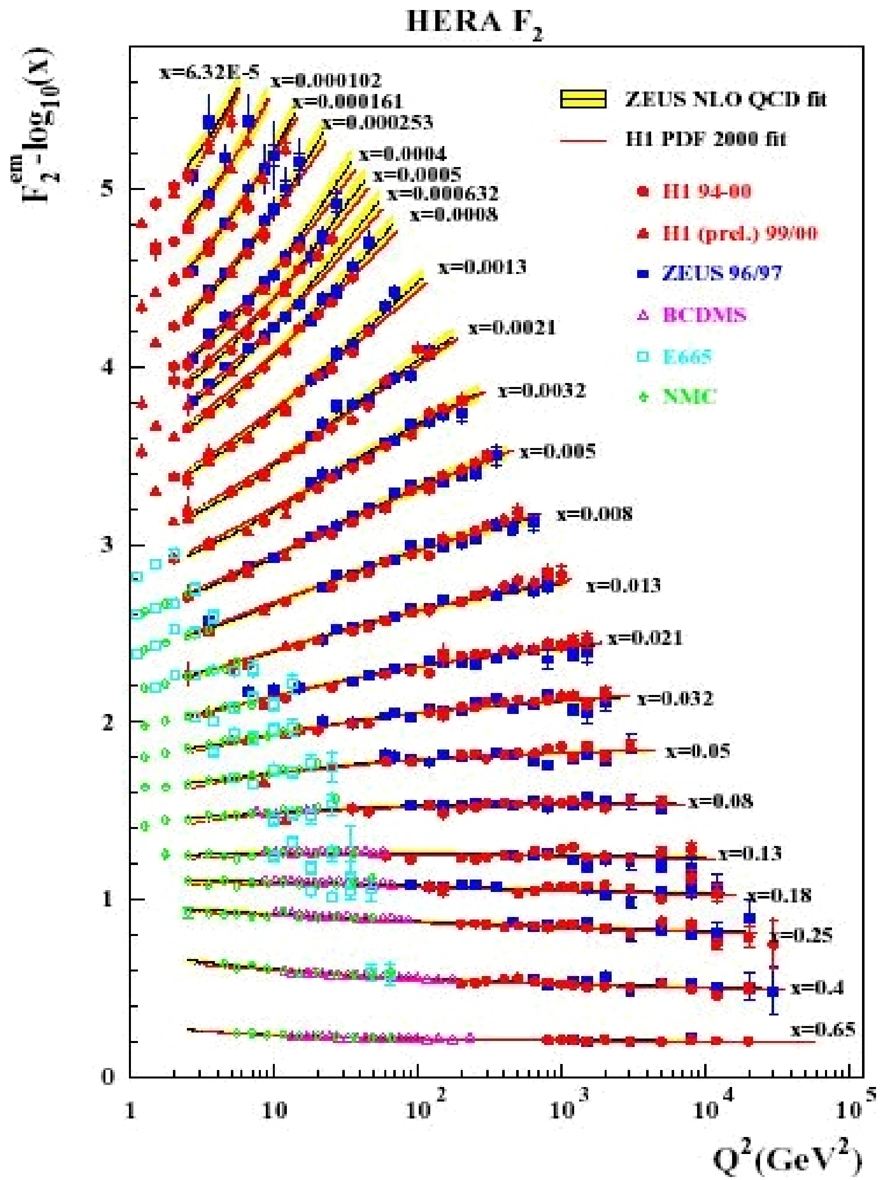
(*) Work with D. Peressounko (& F. Arleo): EPJ-C 46, 451(2006); arXiv:0707.2356; arXiv:0707.2357

Physics text-book plots: HERA, WMAP, ...

- CMB temperature fluctuations:



- DIS scaling violations:



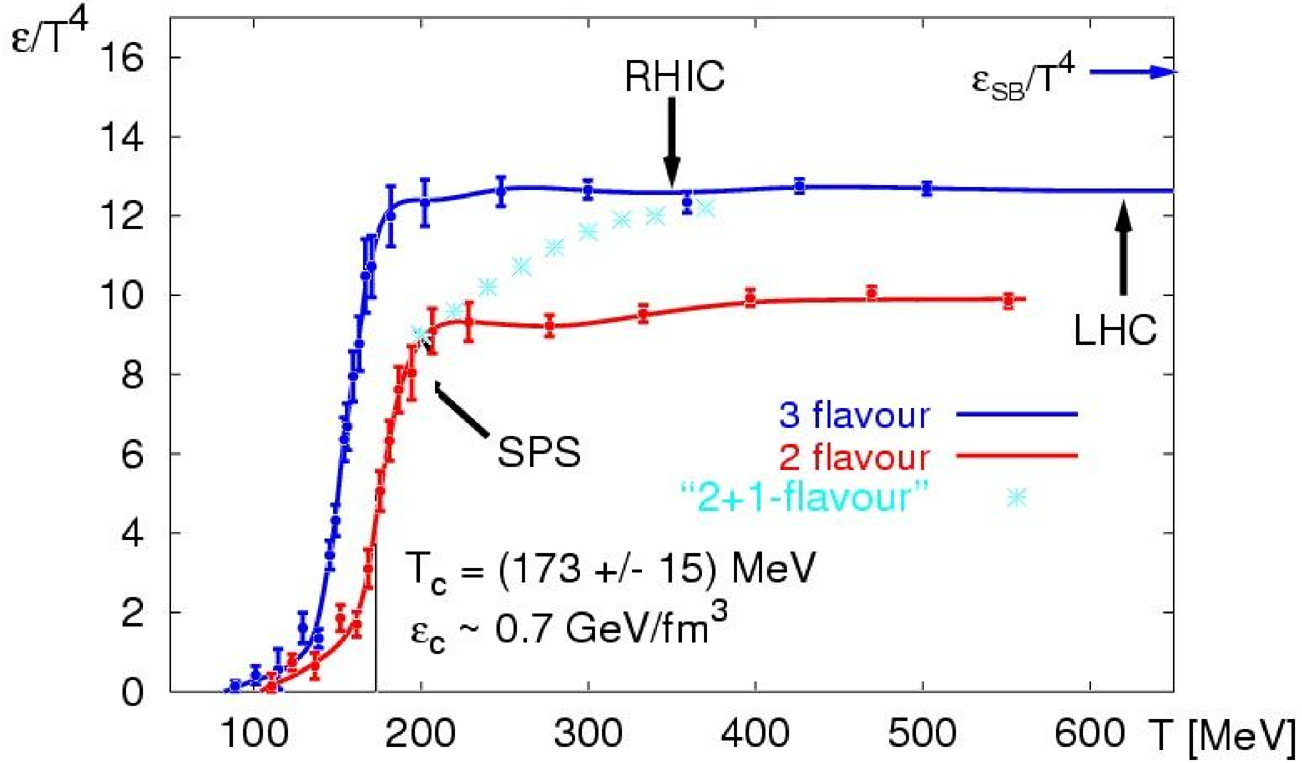
Text-book   plot in high-energy heavy-ion physics ?

Text-book plot in high-energy heavy-ion physics

- Lattice Equation-of-State of QCD matter:

  (i) rise of degs. of freedom at T_c (deconfinement)

  (ii) plateau at high ε (QGP)



[Karsch & Laermann, hep-lat/0305025]

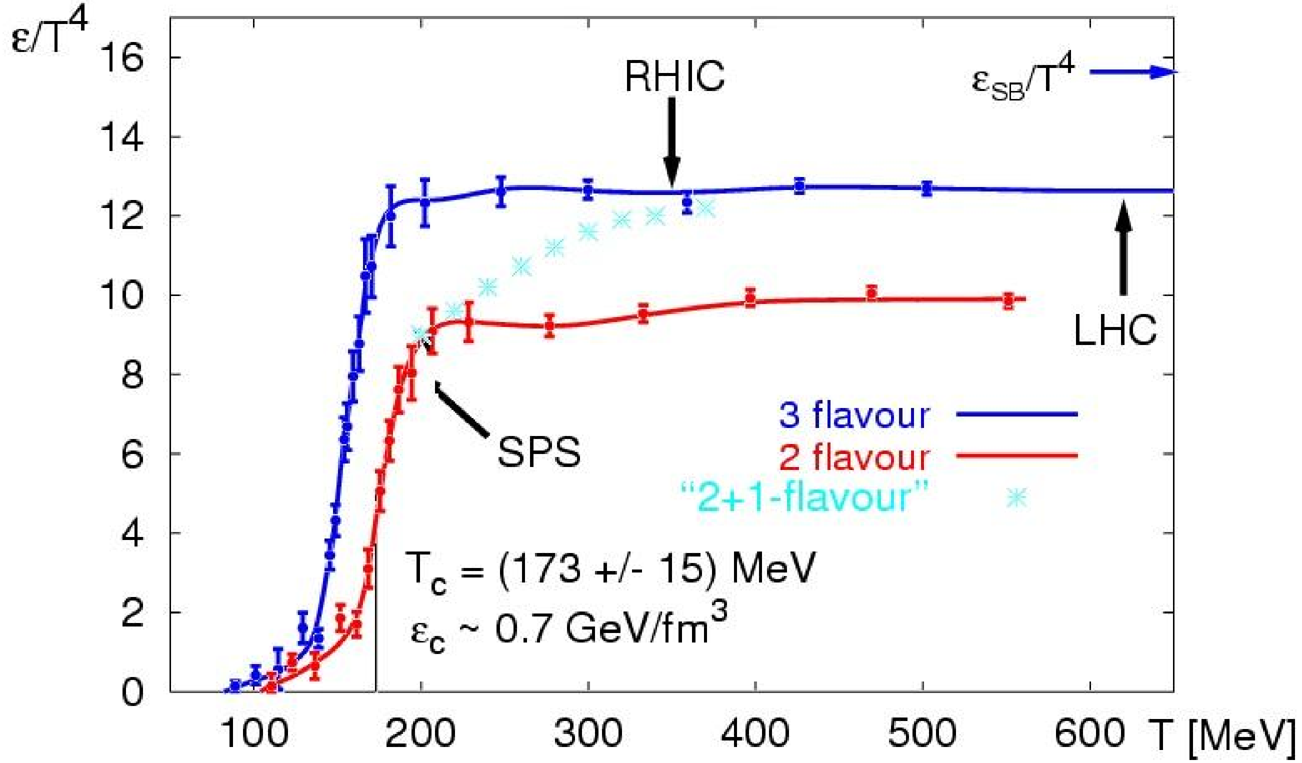
Q: Any chance to reproduce this theoretical plot with experimental data ?

Text-book plot in high-energy heavy-ion physics

- Lattice Equation-of-State of QCD matter:

  (i) rise of degs. of freedom at T_c (deconfinement)

  (ii) plateau at high ε (QGP)



[Karsch & Laermann, hep-lat/0305025]

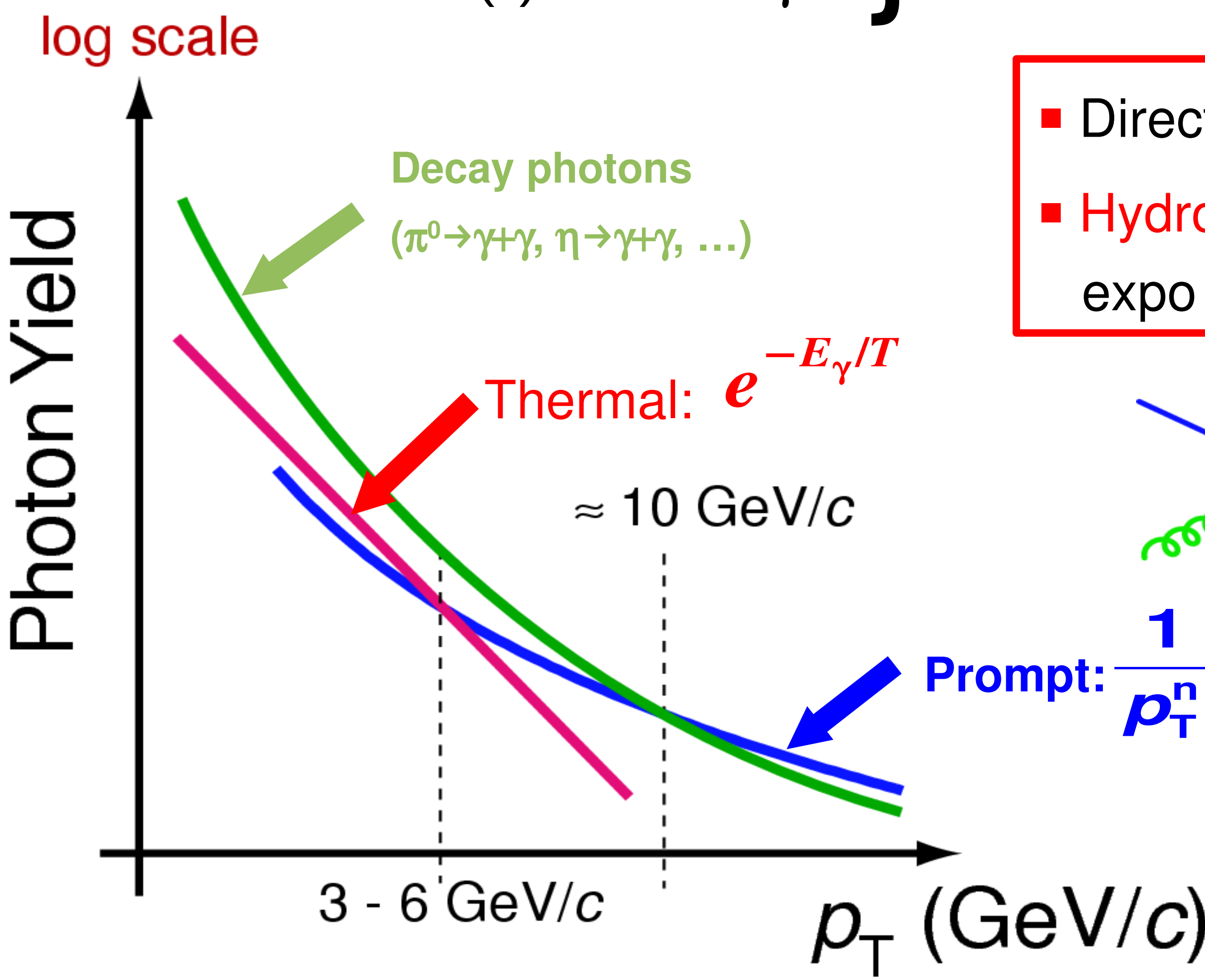
A: Measure thermal photons (slope = T) & hadron multiplicities (~entropy).

Plot: $s/T^3 = f(T)$ for various centralities/energies

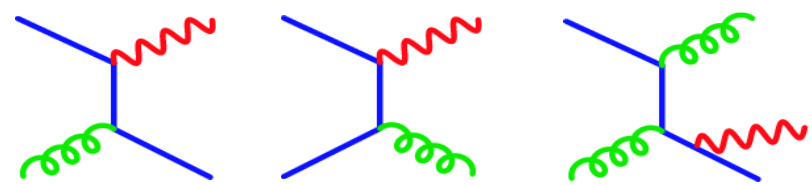
Photon production in A-A collisions

- Three sources: (i) Hadron decay γ → <u>Background</u>

  (ii) Prompt γ
  (ii) Thermal γ } → Direct photon <u>signal</u>



- Direct measure of the medium T
- Hydro model needed to relate expo slope to temperature.



- Initial parton-parton colls.
- Measurable in p-p.
- Computable in pQCD.

Overview

1. Hydrodynamics & thermal photon production:
   - Relativistic fluid-dynamics: longitud. boost-invariant, cylindrical symm.
   - EoS: QGP (ideal gas) + HRG + 1st order phase transition.
   - Initial & final conditions fixed by soft hadron data: $s_0(\tau_0)$, μ_B; Cooper-Frye at T_{fo}
   - Photon rates parametrizations: QGP, HRG
   - RHIC data vs. hydro+pQCD

2. QCD EoS constraints from RHIC data:
   - Thermal γ exponential slopes $\rightarrow$ initial Temperature
   - hadron $dN_{ch}/d\eta$ $\rightarrow$ entropy

3. Thermal photon predictions for LHC.

Hydrodynamical evolution

- Relativistic hydro eqs. (local conservation of energy-momentum & any charge)

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu N_i^\mu &= 0, \quad i = B, S, \ldots \end{aligned}$$

$T^{\mu\nu}$ is energy-momentum tensor (10 independ. vars.)
N_i^μ is charge 4-current (4 independ. vars.)

(5 equations with 14 unknowns)

- (Usual) approximations:

- Ideal (non-viscous) fluid:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} \\ N^\mu &= n u^\mu \end{aligned}$$

} (6 unknowns) ε, P, n, u^μ

- Cylindrical symmetry in transverse direction.
- Lorentz boost invariant in longitudinal dir. (Bjorken scaling)

} $\rightarrow$ "2D+1": (z,r,t)

$$u^\mu = \gamma_r \left(\frac{t}{\tau}, v_r, 0, \frac{z}{\tau} \right) = \gamma_r (\cosh\eta, v_r, 0, \sinh\eta)$$

Hydro results independent of rapidity (valid around y = 0)

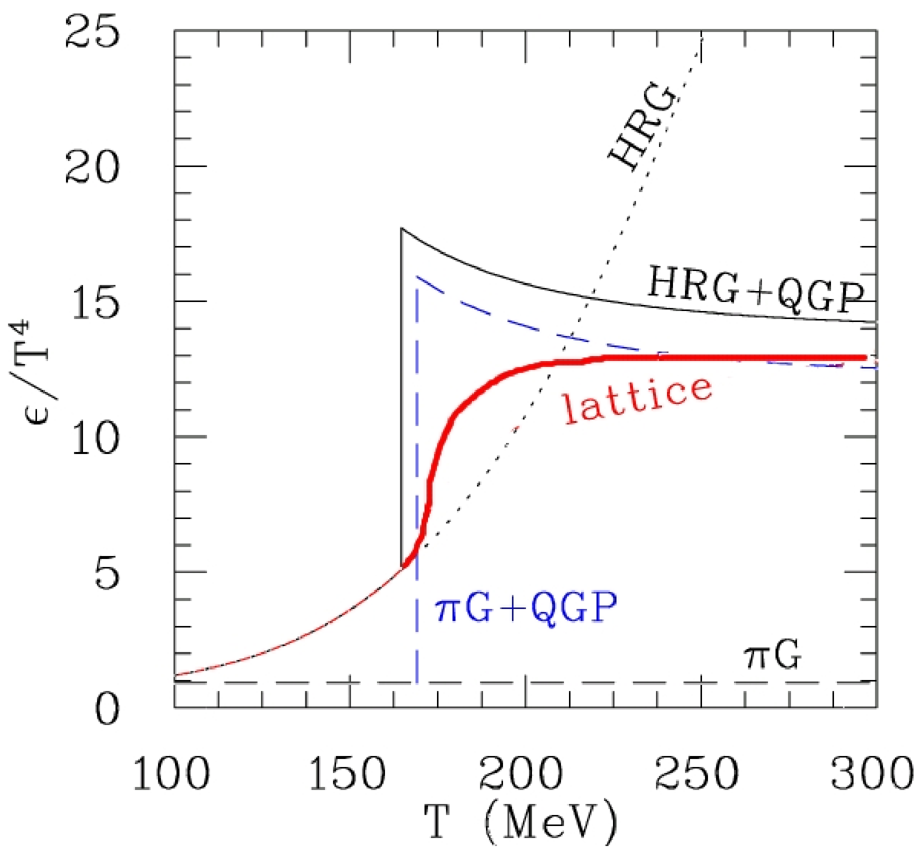
- Equation-of-State (relation between thermodyn. vars. of system):

$P = P(\varepsilon, n)$ closes the system of eqs. (given initial conditions)

- [Numerical solution via MacCormack method]

Hydrodynamics: EoS

- Equation-of-State:



- **QGP**:
  - Lattice parametrization, or
  - Ideal (massless) parton gas + MIT bag

$$\epsilon_{QGP} = \frac{3g_{QGP}}{\pi^2}T^4 + B$$

$$P_{QGP} = \frac{1}{3}(\epsilon_{QGP} - 4B)$$

[2+1 flavors, 16 gluons]

g_{QGP} = 42.5

B = 0.38 GeV/fm^3

- **HRG**: Hadron resonance gas including ~400 hadrons & resonances up to m~2 GeV/c^2. Chem. equilibrium (hadron ratios) fixed at T_C

$$P(T,\mu) = \sum_i g_i \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}^2}{3E}\frac{1}{e^{(E-\mu_i)/T} \pm 1}$$

$$= \sum_i \frac{g_i}{2\pi^2}T^2 m_i^2 \sum_{n=1}^{\infty}\frac{(\mp 1)^{n+1}}{n^2} e^{n\frac{\mu_i}{T}} K_2\left(n\frac{m_i}{T}\right)$$

- Maxwell construction for 1st order phase transition at T_c = 165 MeV:

$$P_{QGP}(T_c) = P_{HRG}(T_c) \Rightarrow T_c = \left(\frac{\pi^2}{g_{QGP} - g_\pi}\right)^{\frac{1}{4}} B^{\frac{1}{4}}$$

, *LH* ~ 1.2 GeV/fm^3

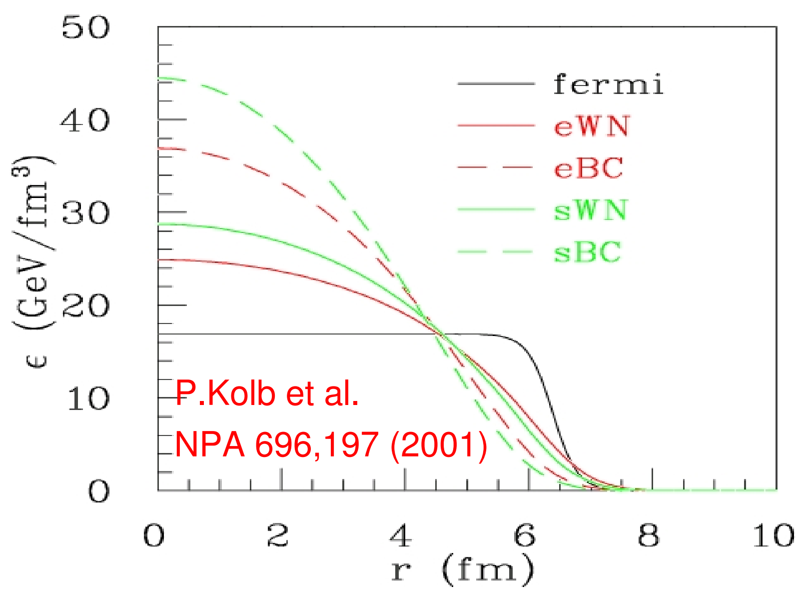
Hydrodynamics: initial conditions (RHIC)

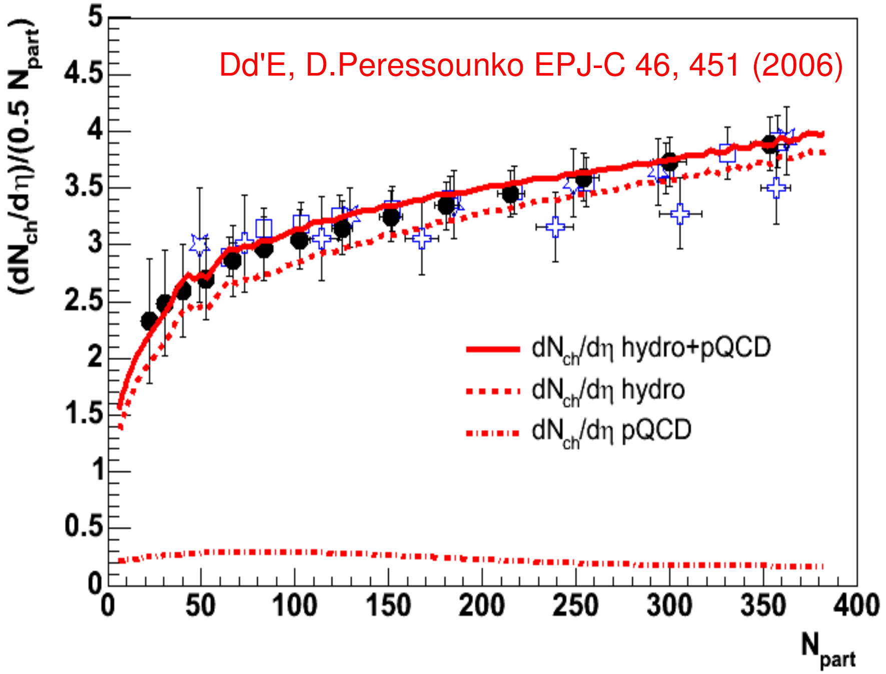
- Initial conditions for head-on (b=0 fm) Au-Au @ 200 GeV:

$\tau_0 = 2R/\gamma = 0.15$ fm/c	(transit time Au-Au: time-scale for secondary parton-parton colls.)
$s_0 \sim 500$ fm^{-3}	(consistent w/ $dN_{ch}/d\eta$). $\varepsilon_0 \propto s_0^{4/3}$ = 220 GeV/fm^3 (source center)
$\mu_B = 25$ MeV	(consistent w/ exp. hadron ratios at y=0)

- Centrality-dependence: Kolb-Heinz-Huovinen-Eskola-Tuominen (Glauber) prescription:

$$s_0(b) = s_0(b=0) \times [0.75\, N_{part}(b) + 0.25\, N_{coll}(b)]$$



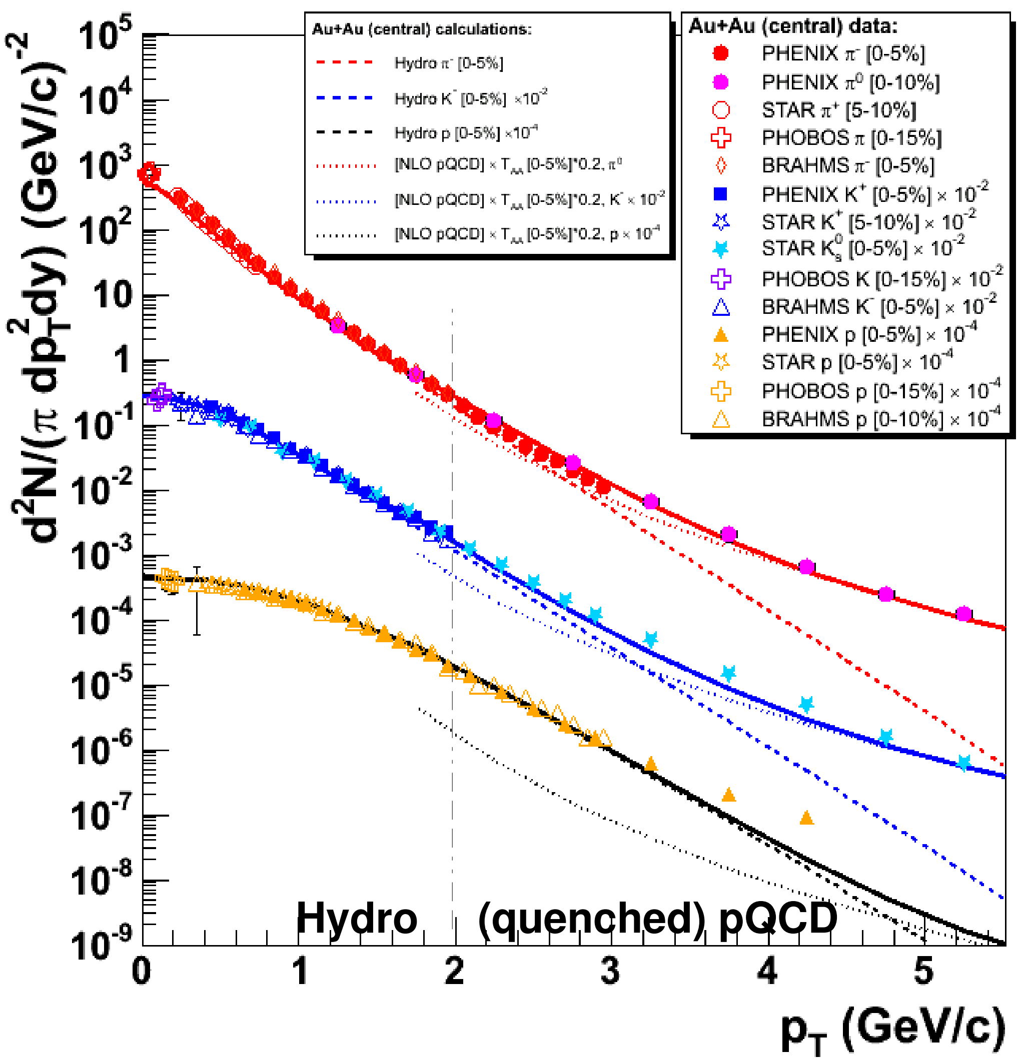


- Chemical freeze-out at: T_{chem}=155 MeV
- End of evolution: Cooper-Frye freeze-out prescription at T_{fo} = 120 MeV

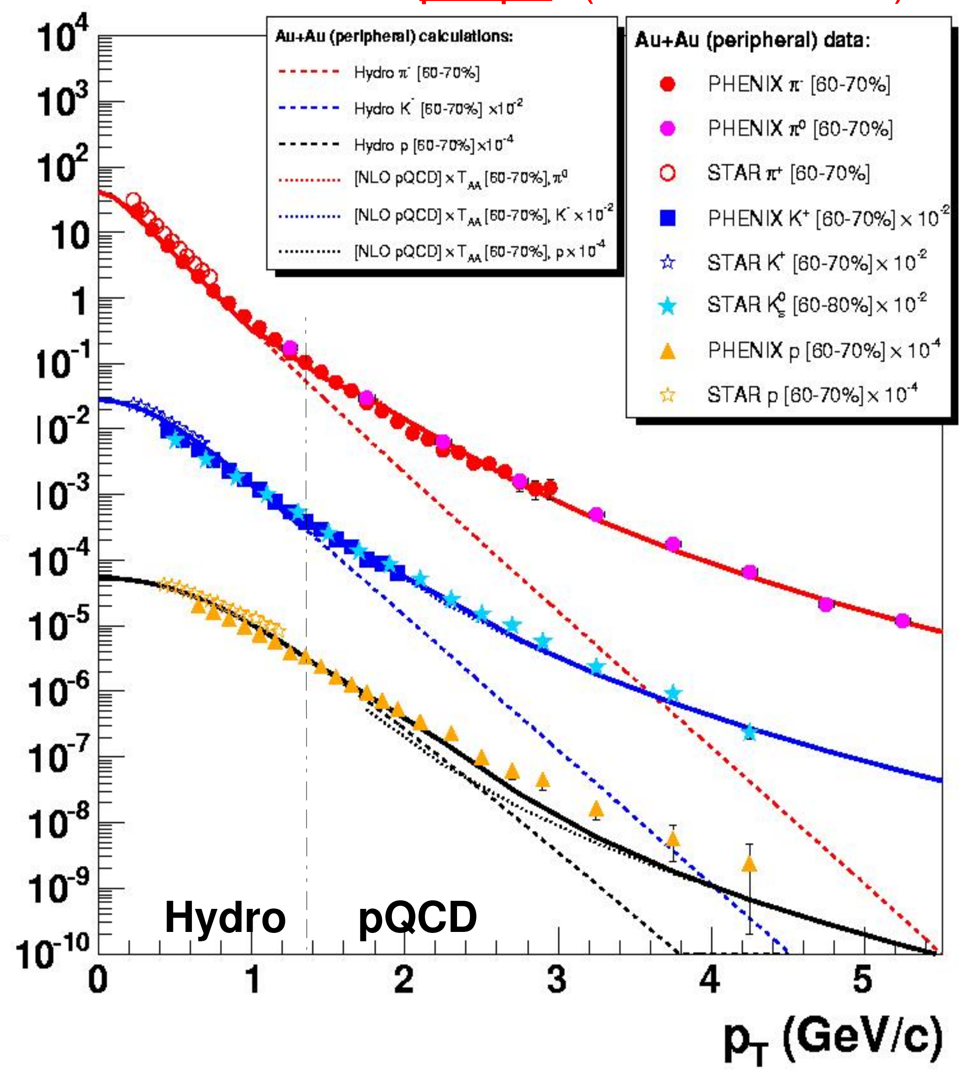
Hadron spectra (RHIC): Hydro+pQCD vs Au-Au data

- $\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$ spectra very well reproduced by hydro + (quenched) NLO pQCD:

Au+Au 0-10% central (<b> = 3.2 fm)



Au+Au 60-70% periph. (<b> = 11.2 fm)

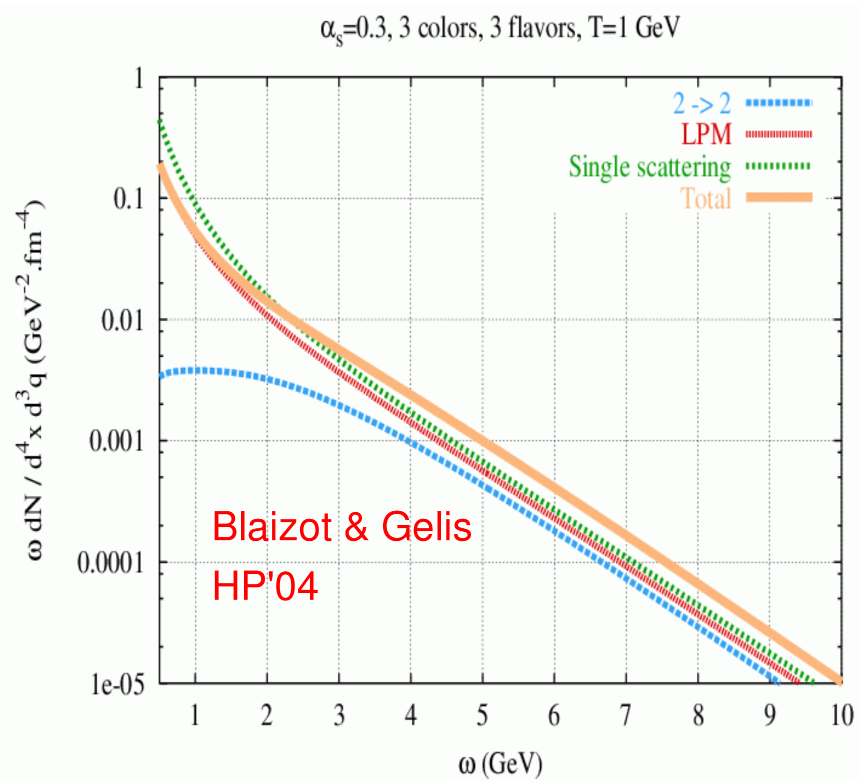


- "Quenched" pQCD: [NLO (W. Vogelsang), CTEQ6 PDFs, KKP FFs] x R_{AA} = 0.2(0.7)

Hydrodynamics: direct photon production

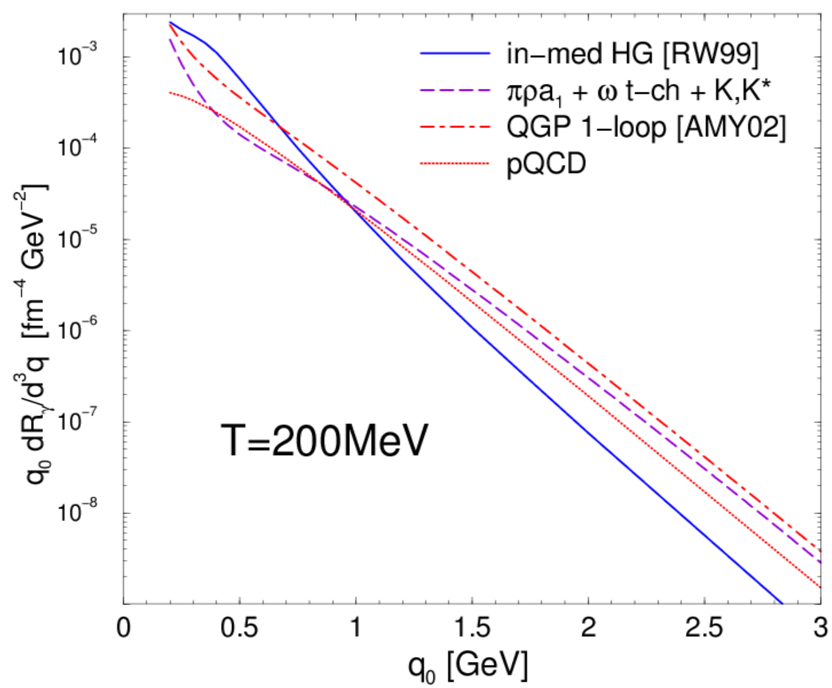
- QGP thermal photon rates by AMY:
  Complete Leading-Log+ LPM suppression

Arnold, Moore, Yaffe JHEP 0112 (2001) 009



- HRG thermal photon rates by
  Turbide et al.: Most recent parametrization, includes channels not accounted for before

Turbide, Rapp, Gale, PRC 69, 014903 (2004)



- Latt. T-dependent α_s parametrization:

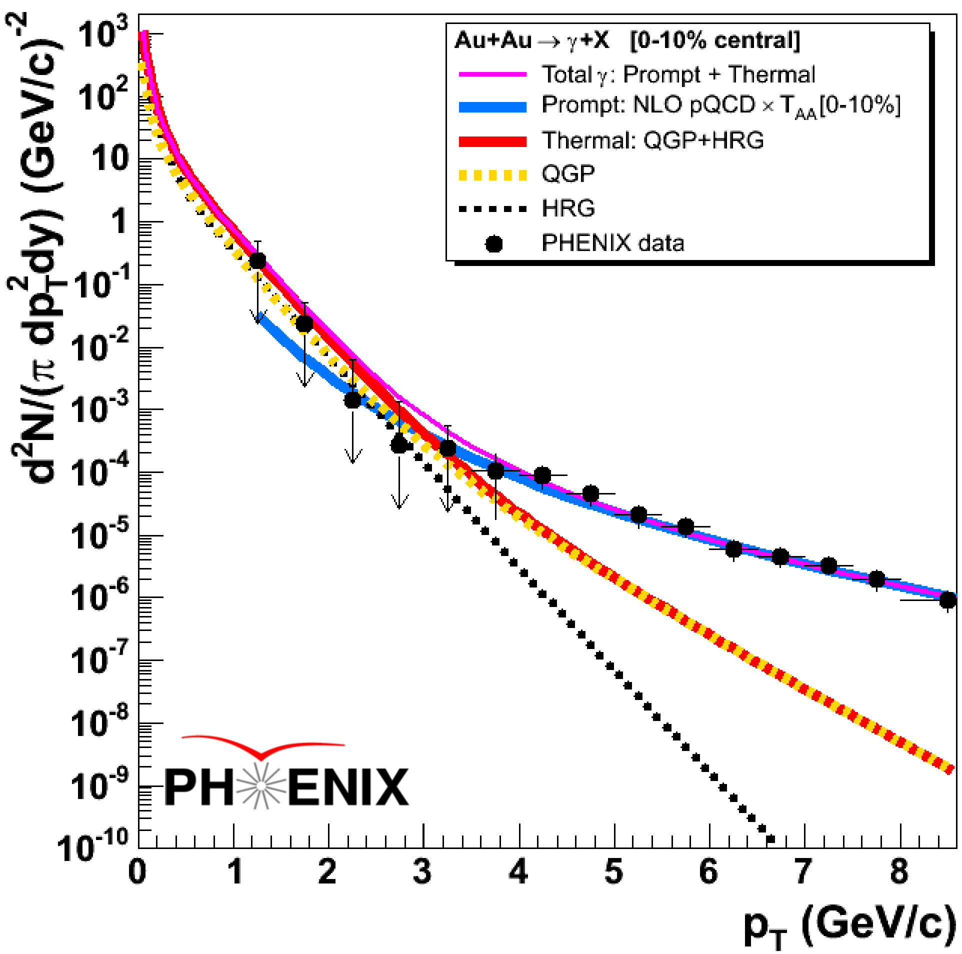
Kaczmarek, Karsch, Zantow, Petercky, PRD 70, 074505 (2004)

$$\alpha_s(T) = 2.095/\{\frac{11}{2\pi}\ln\left(Q/\Lambda_{\overline{MS}}\right) + \frac{51}{22\pi}\ln\left[2\ln(Q/\Lambda_{\overline{MS}})\right]\} \text{ with } Q = 2\pi T$$

Photon spectra (RHIC): Hydro+pQCD vs Au-Au data

- Photon spectra very well reproduced by hydro + N_{coll}-scaled NLO pQCD:

Au+Au 0-10% central (<b> = 3.2 fm)



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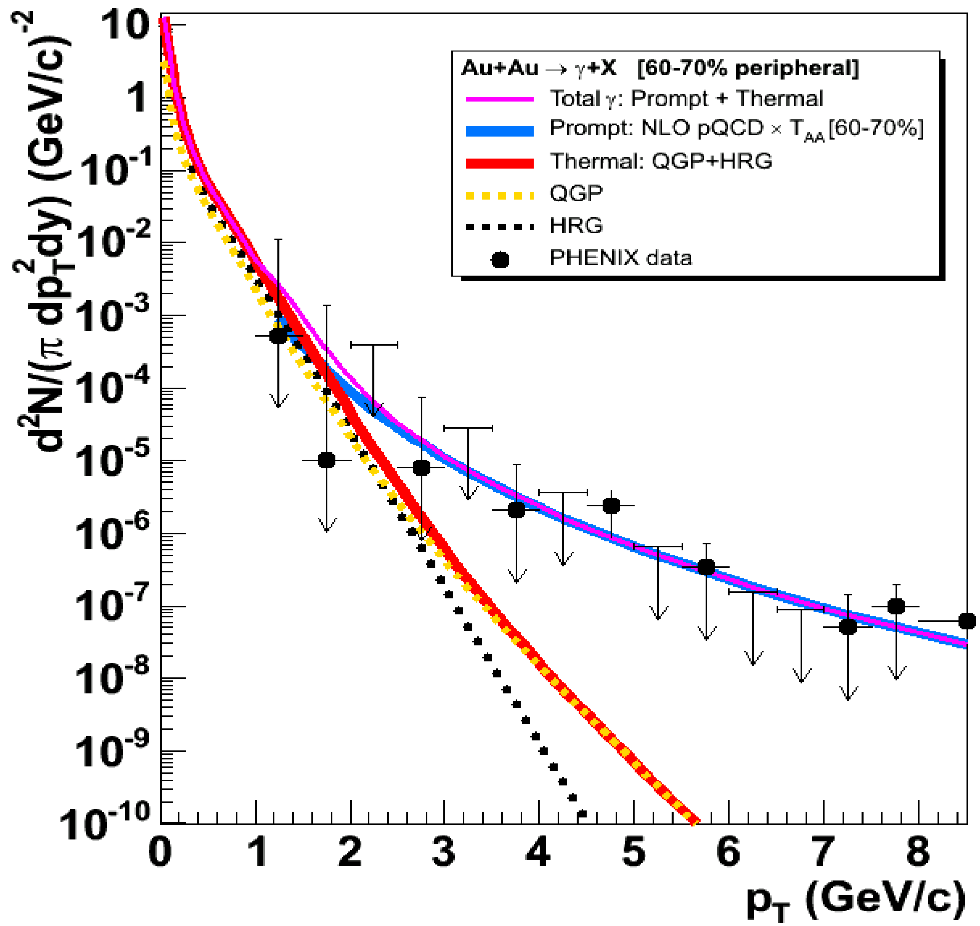
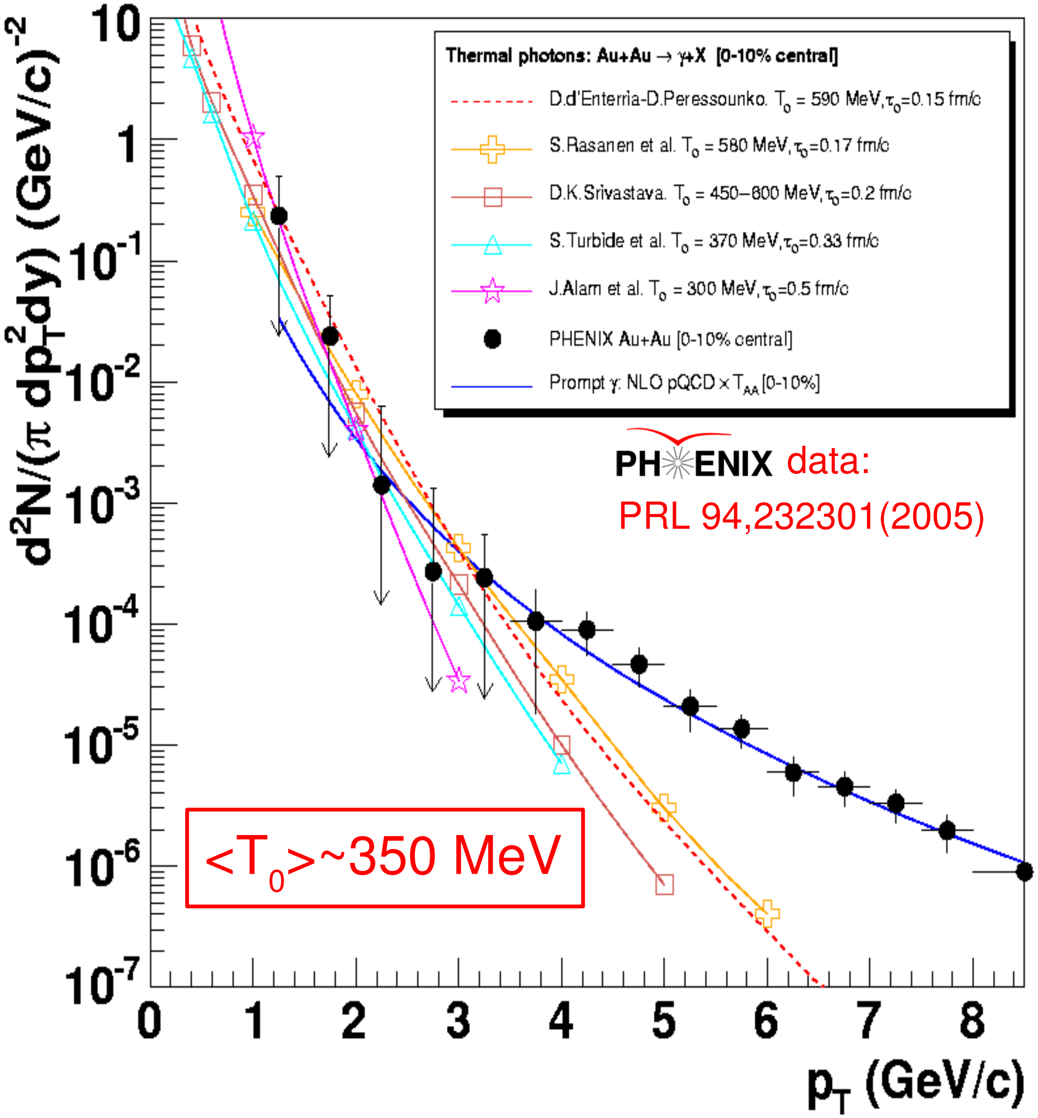
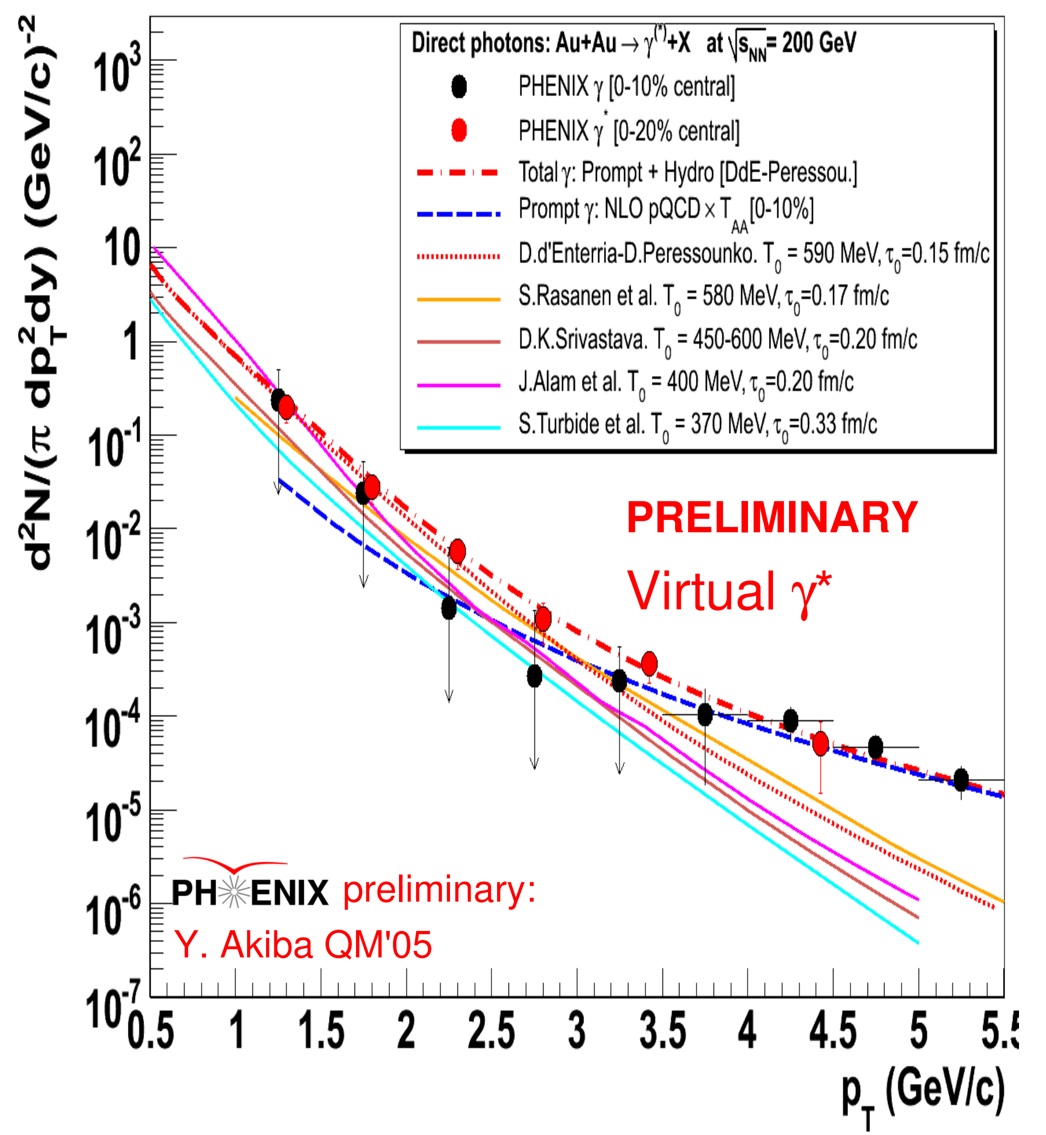


- Hydro consistent with upper limits in the "thermal signal" region p_T = 1 – 4 GeV/c

Photon spectra in the thermal region

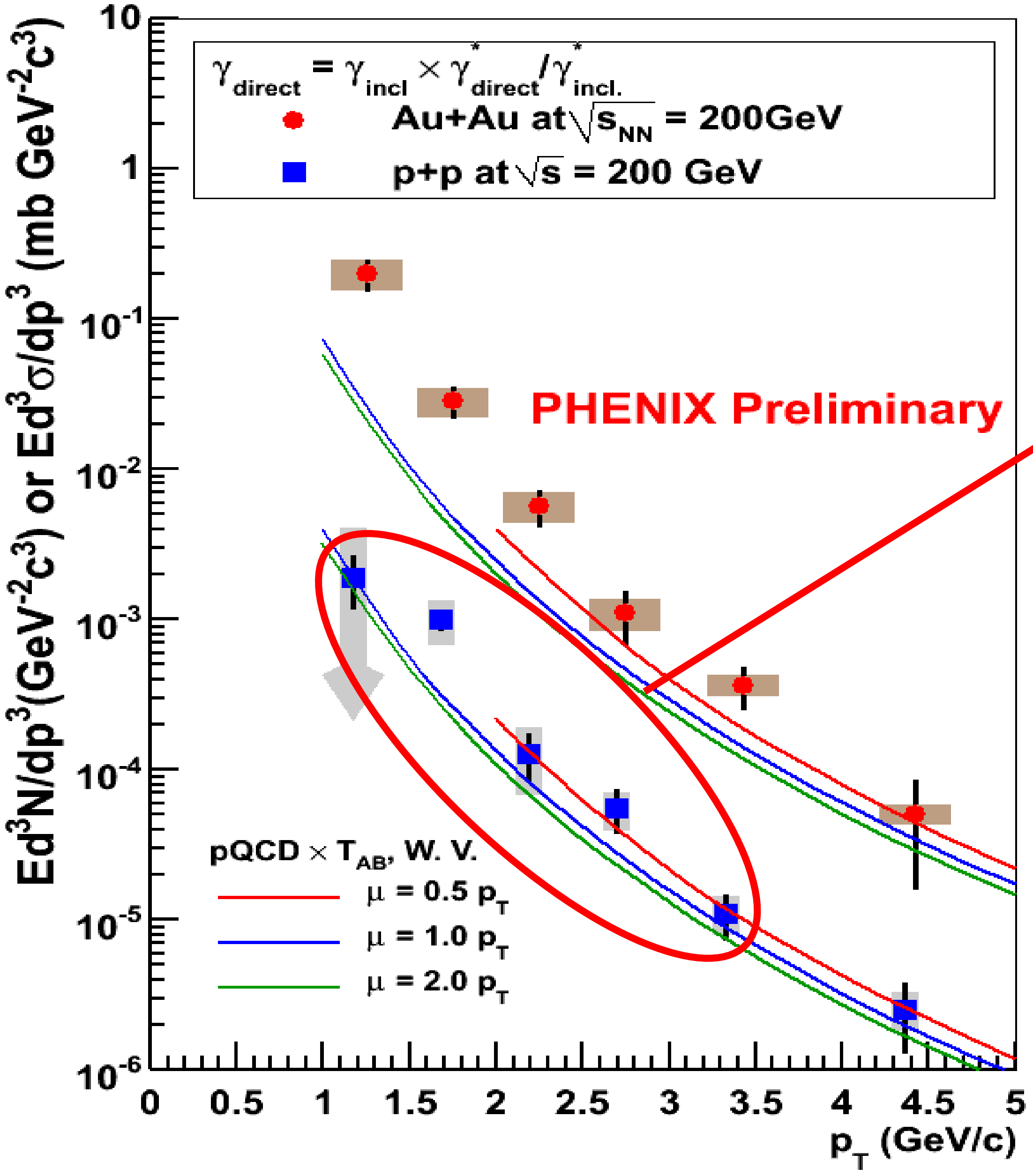
- Current upper limits in p_T = 1 – 4 GeV/c consistent w/ thermal γ component:





Caveat: Prompt-γ reference used is NOT real p+p data but NLO pQCD

Photon spectra in the thermal region



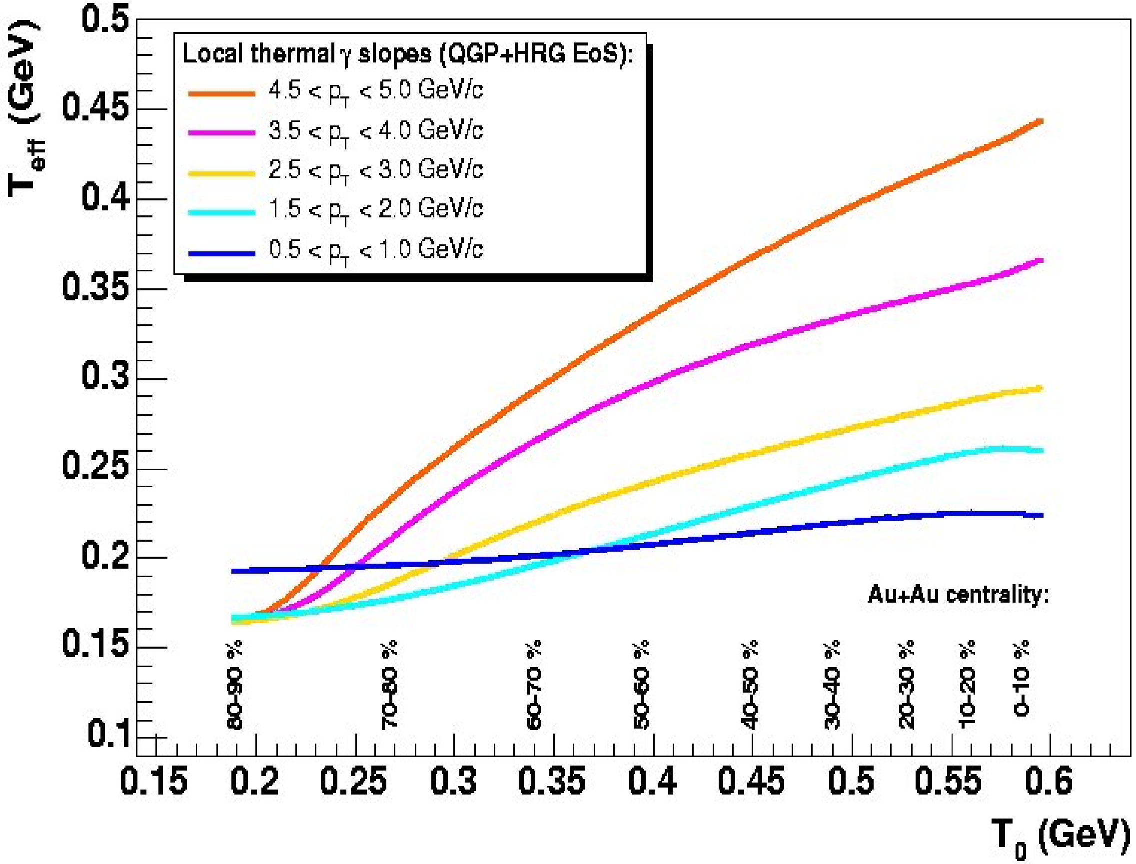
- QM'08 new results:

- New p-p virtual γ^* consistent with NLO pQCD

- Au-Au virtual γ^* excess above N_{coll}-scaled p-p spectra.

PHENIX Preliminary:
T. Dahms, QM'08
Y. Akiba QM'05

Thermal photon slopes vs. initial temperature (hydro)



Increasing Au-Au centrality (T_0) ➡

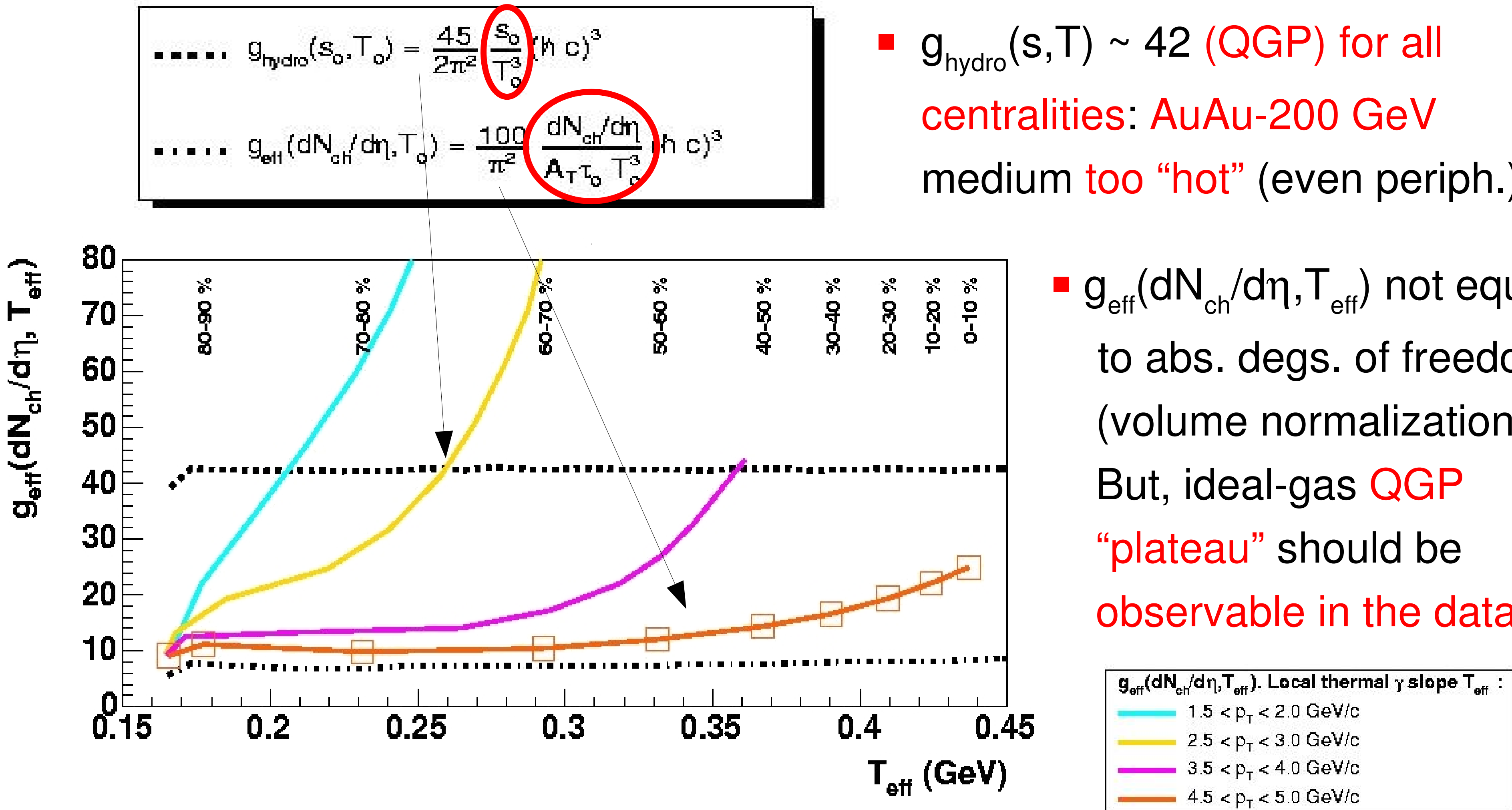
- Exponential fit of thermal photon spectrum in various p_T ranges & centralities:

  - inverse slopes proportional to initial (max.) temperature.

  - the higher the p_T range where the slope is measured (= fitted), the closer is the apparent T to the initial QGP T_0 ~ 500 MeV

QGP EoS from thermal γ & hadron multiplicities (hydro)

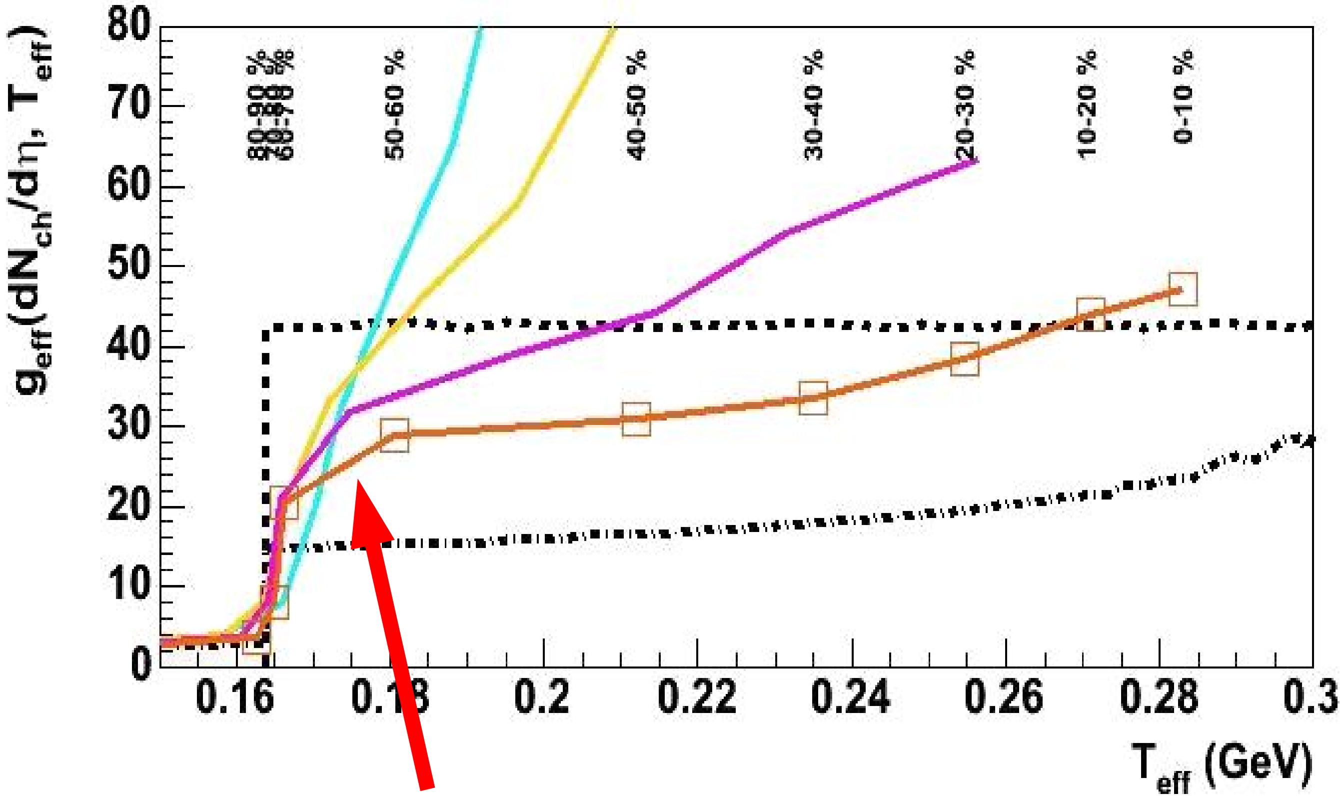
- Correlate thermal-γ slopes (T) & hadron multiplicities ($dN_{ch}/d\eta \propto$ entropy, isentropic expansion) for various centralities. $s \approx 3.6 \cdot \frac{dN}{dV} \approx \frac{4.3}{\langle A_\perp \rangle \cdot \tau_0} \cdot \frac{dN_{ch}}{d\eta}$
- Effective # of degrees of freedom, g(s,T), for various centralities:



- $g_{hydro}(s,T) \sim 42$ (QGP) for all centralities: AuAu-200 GeV medium too "hot" (even periph.)
- $g_{eff}(dN_{ch}/d\eta,T_{eff})$ not equal to abs. degs. of freedom (volume normalization). But, ideal-gas QGP "plateau" should be observable in the data.

QGP EoS from A-A data at lower sqrt(s)

- AuAu @ 200 GeV produces too hot medium (QGP for all centralities): not sensitive to any centrality-dependent (strong) change due to phase transition.
- Try lighter nuclei &/or lower √s. Preliminary hydro calcs. for AuAu @ 62 GeV



D.d'E. & D.Peressounko
(in preparation)

- Apparent phase transition change in $g_{eff}(dN_{ch}/d\eta,T_{eff})$ for centrality 50-60%
- Even better: try more central collisions for lighter/lower-√s: AuAu,pp @ 30-40 GeV

LHC predictions

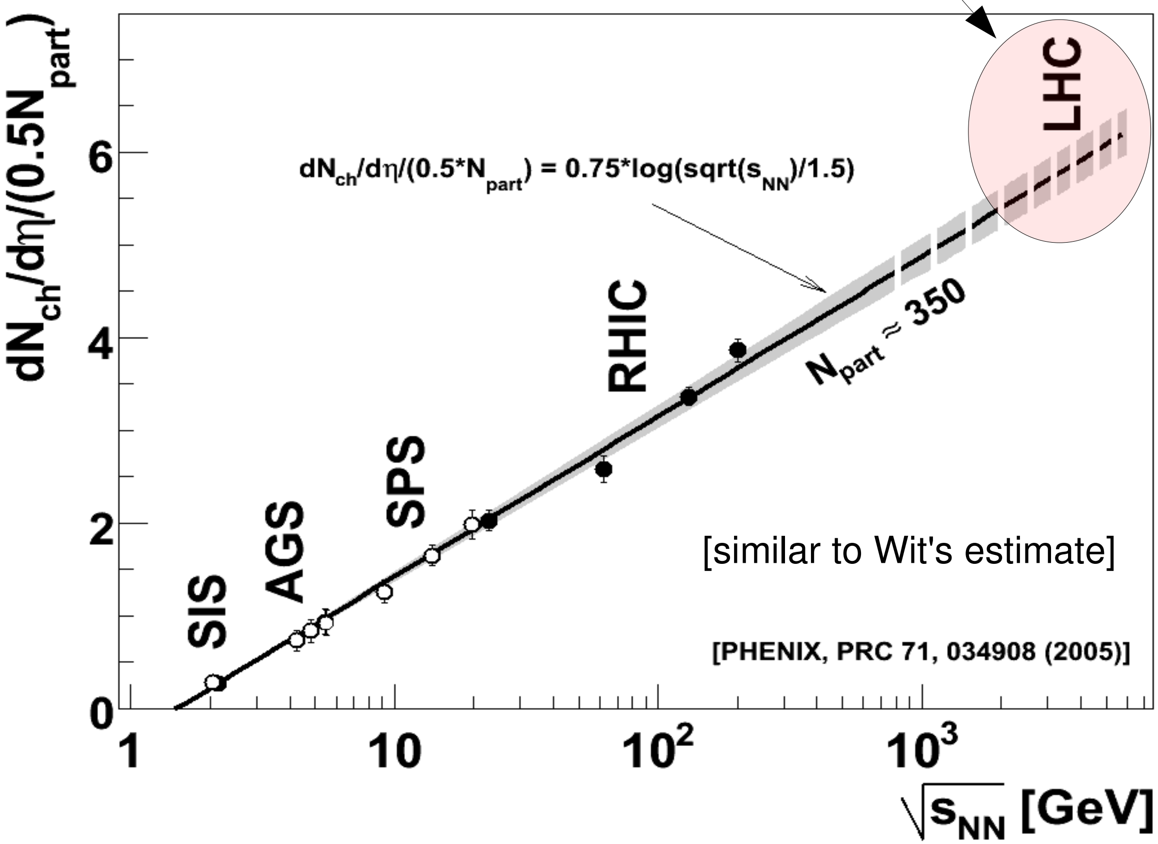
Arleo, DdE, Peressounko
arXiv:0707.2356; arXiv:0707.2357

Hydrodynamics: :LHC initial conditions

- Hydro input parameters for head-on (b=0 fm) Pb-Pb @ 5.5 TeV:

$\tau_0 = 1/Q_s \sim 0.1$ fm/c	(time-scale for secondary parton-parton collisions)
$s_0 = 1120$ fm^{-3}	($dN_{tot}/dy \sim 2200$ extrapolation) $\Rightarrow \varepsilon_0 \propto s_0^{4/3} \sim 650$ GeV/fm^3
$\mu_B = 5$ MeV	(~ baryon-free)

$T_0 = 770$ MeV ($\langle T_0 \rangle = 470$ MeV)



$dN/dy = N_{tot}/N_{ch} * d\eta/dy * dN_{ch}/d\eta =$
$= 3/2*1.2*1300 \sim 2200$

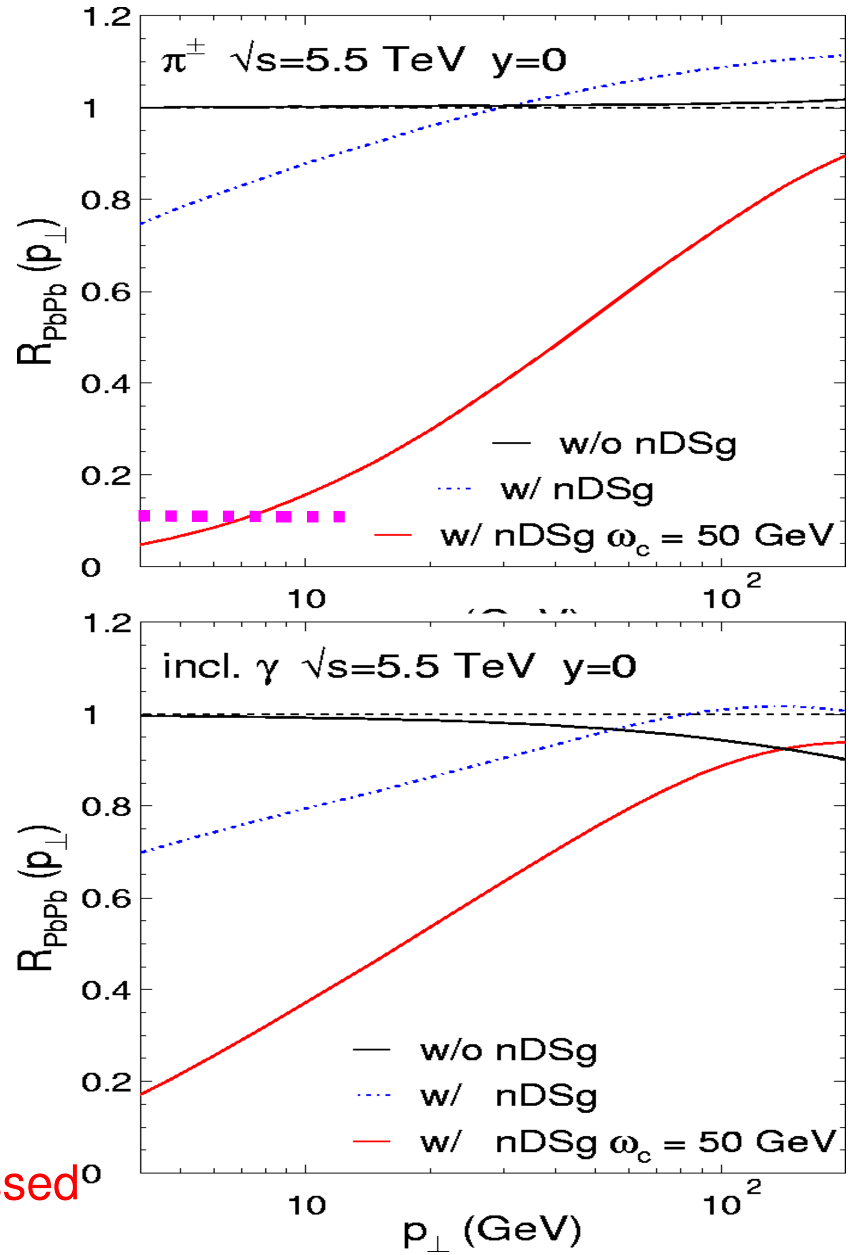
- Rest as of RHIC: b-dependence, freeze-out (T_{chem}=155 MeV, T_{fo} = 120 MeV)

pQCD + parton energy loss

F. Arleo, JHEP 0609:015 (2006)

- NLO pQCD (PHOX code). Scales: $\mu = p_T$
- Parton Distrib. Functions: CTEQ6M + nDSg
  (DeFlorian-Sassot "strong" shadowing)
- Fragmentation functions:
  AKK (newest set for kaons, protons)
- Final-state parton energy loss:
  Medium-modified FFs.
  BDMPS quenching weights: $P(\varepsilon, E)$
  $\omega_c = \hat{q} \cdot L^2$, $\quad dN/dy \sim \alpha_s(Q_s^2)\, Q_s^2$
  $\omega_c(\text{LHC}) = \omega_c(\text{RHIC}) * (5500/200)^{\lambda=0.3}$
  $\omega_c(\text{LHC}) \sim 50$ GeV (for $dN_{ch}/dy \sim 1700$)

  Max. quenching ("corona emission"):
  R_{AA} (b=0 fm) = $N_{part}/N_{coll} \sim 400/2200 \sim 0.15$

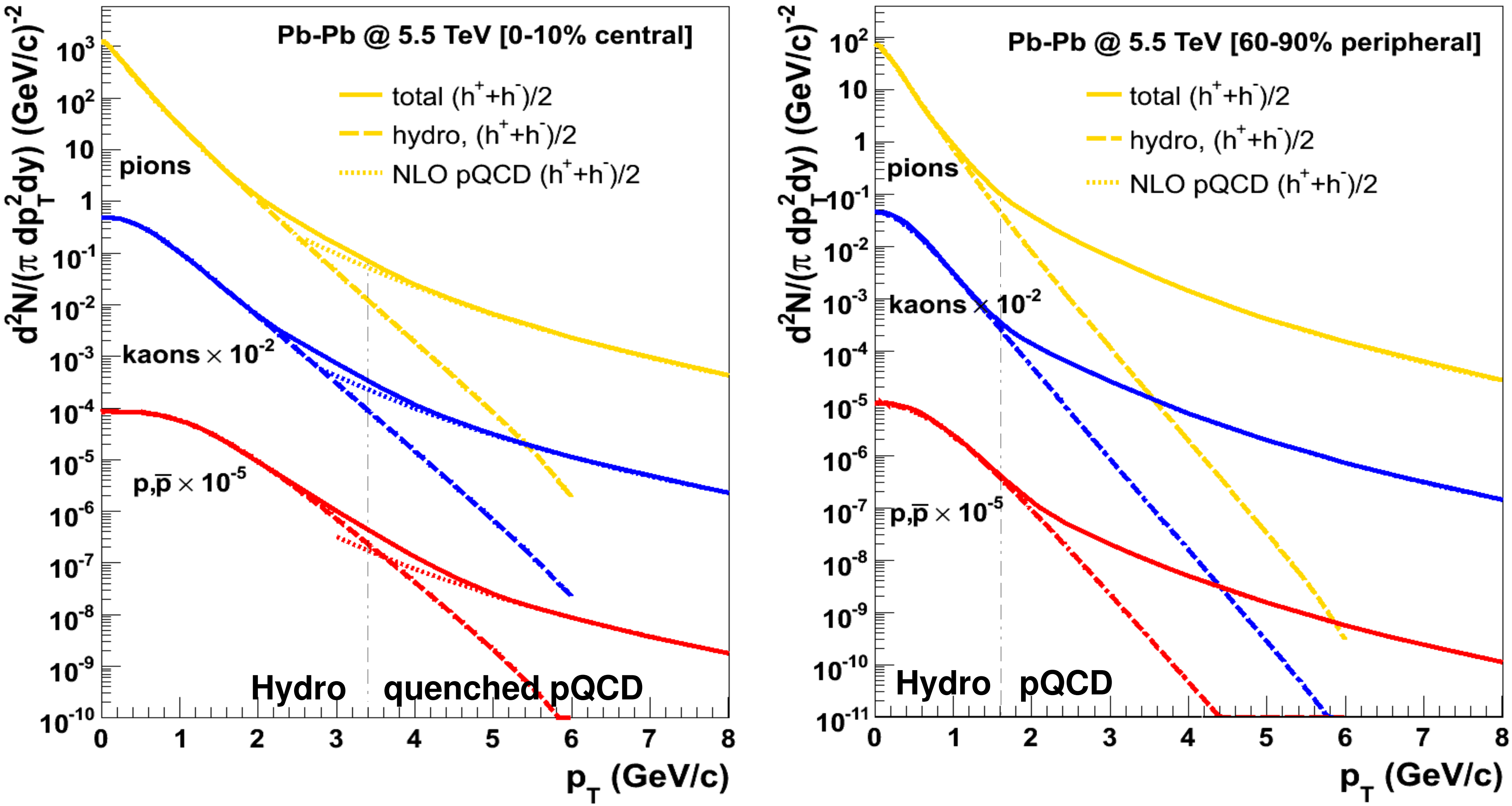


- Fragmentation γ (dominant at low p_T) also suppressed

Hadron spectra: hydro+pQCD predictions

- $\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$ spectra: hydro + quenched NLO pQCD:

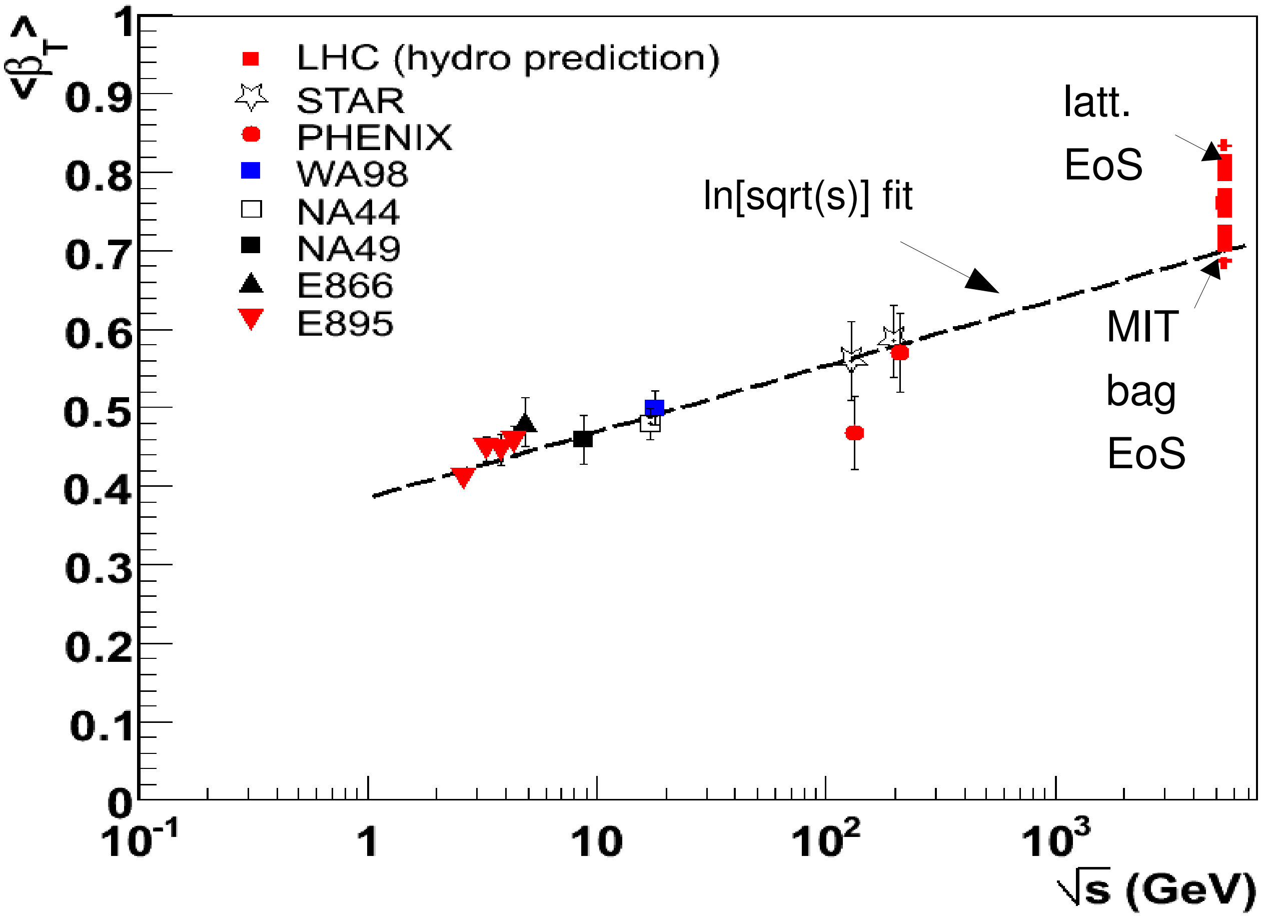
Pb-Pb 0-10% central (<b> ~ 3 fm) Pb-Pb 60-90% periph. (<b> ~ 13 fm)



- Quenched pQCD: NLO (PHOX), CTEQ6 PDFs, nDSg, AKK FFs, R_{AA} ~ 0.1

Hadron spectra: hydro (LHC)

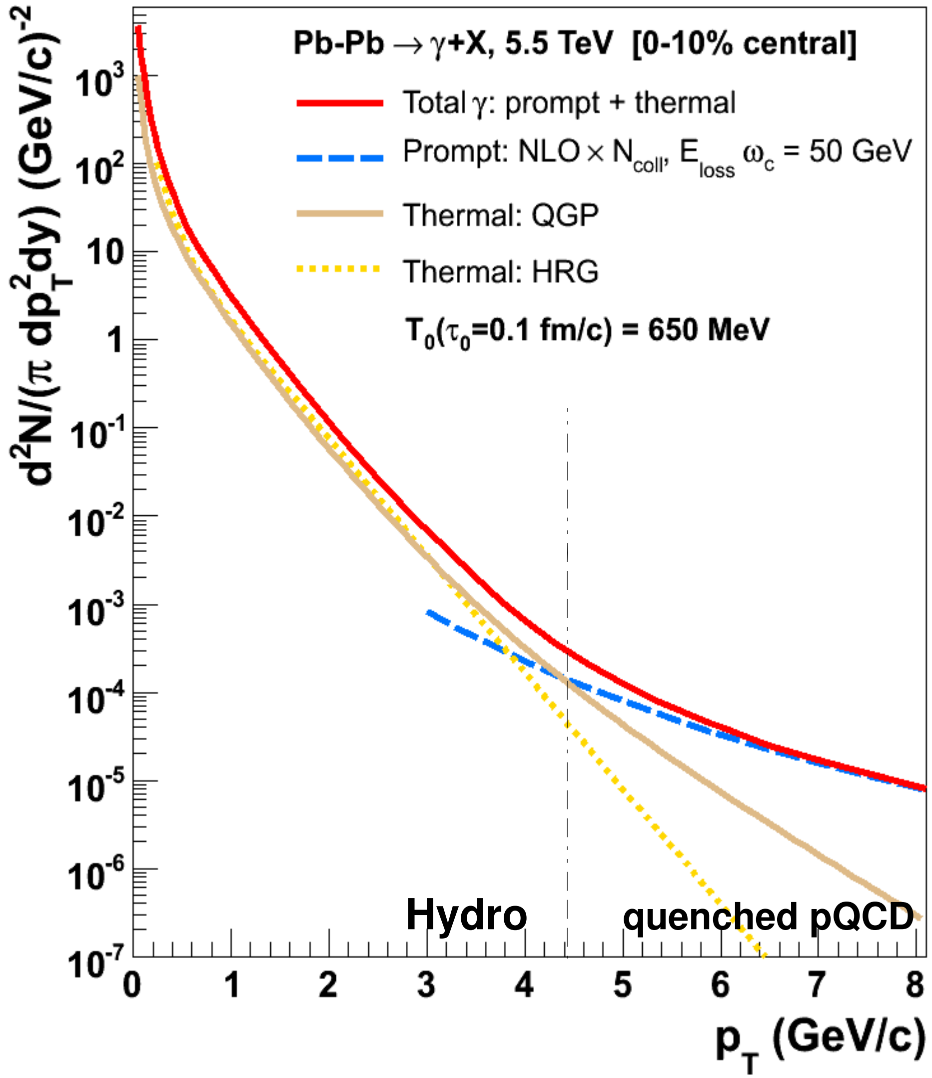
- Collective transverse flow of hadrons:



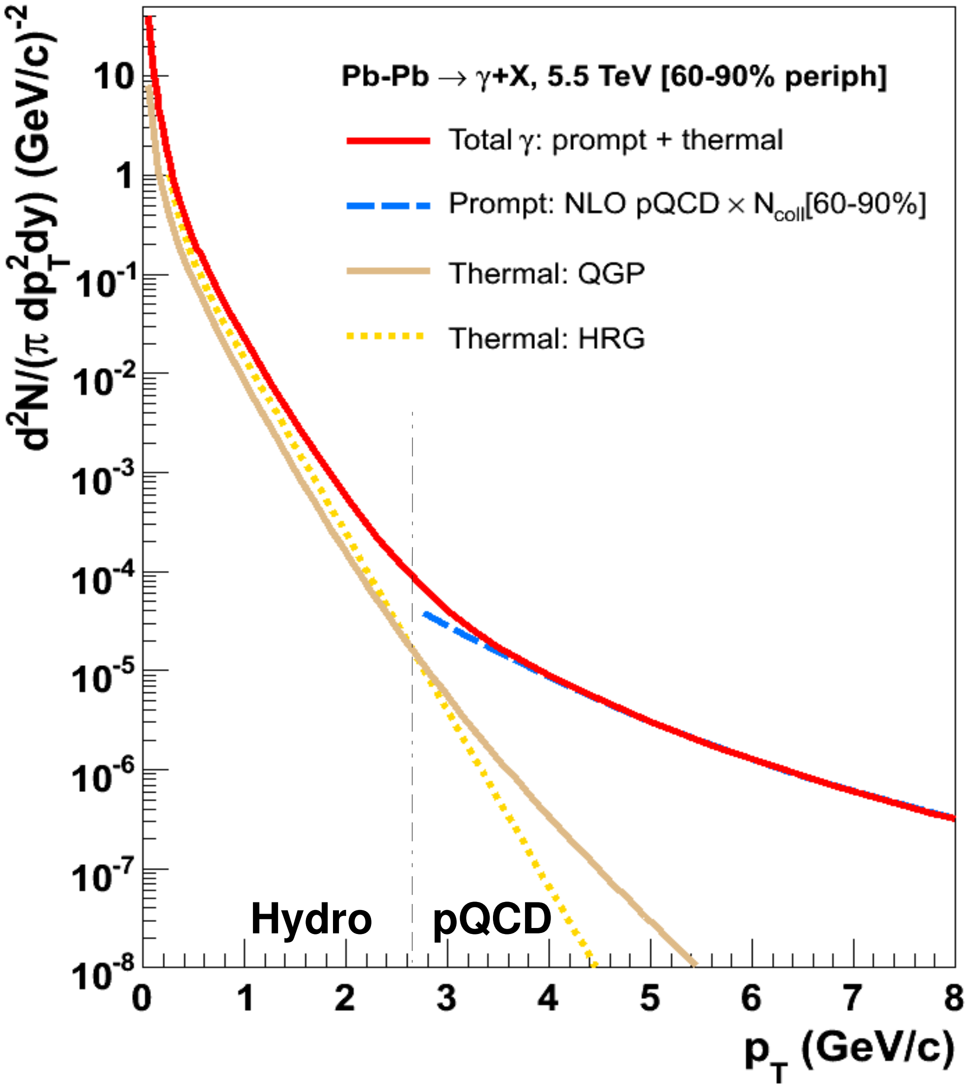
Direct γ spectra: hydro+pQCD prediction (LHC)

- Photon spectra: hydro + (quenched) NLO pQCD:

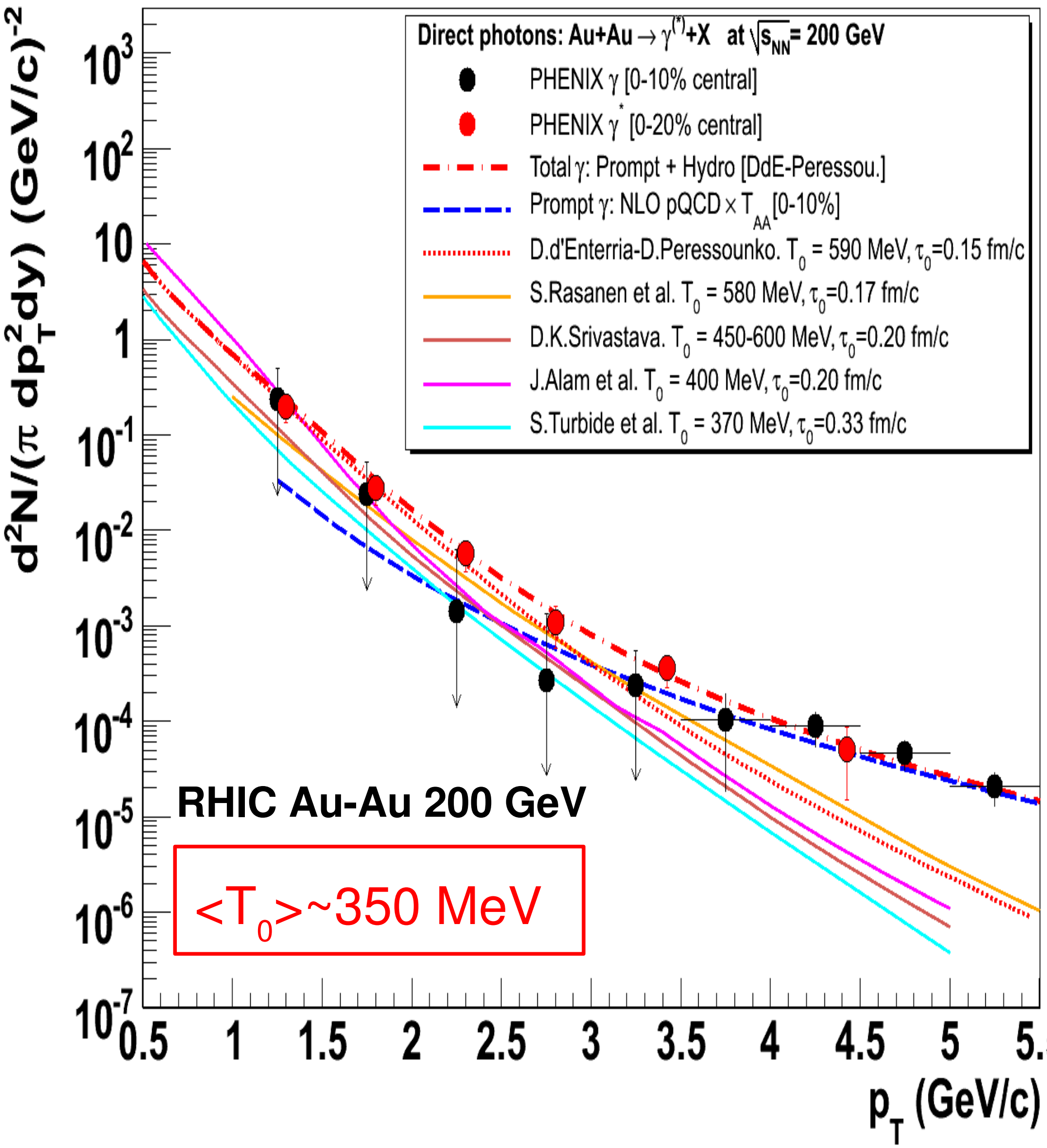
Pb-Pb 0-10% <u>central</u> (<b> ~ 3 fm)

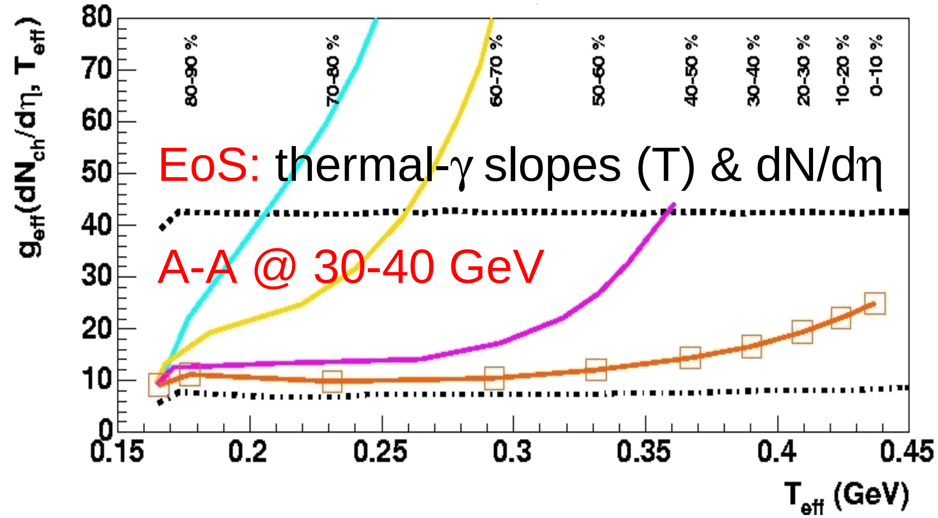


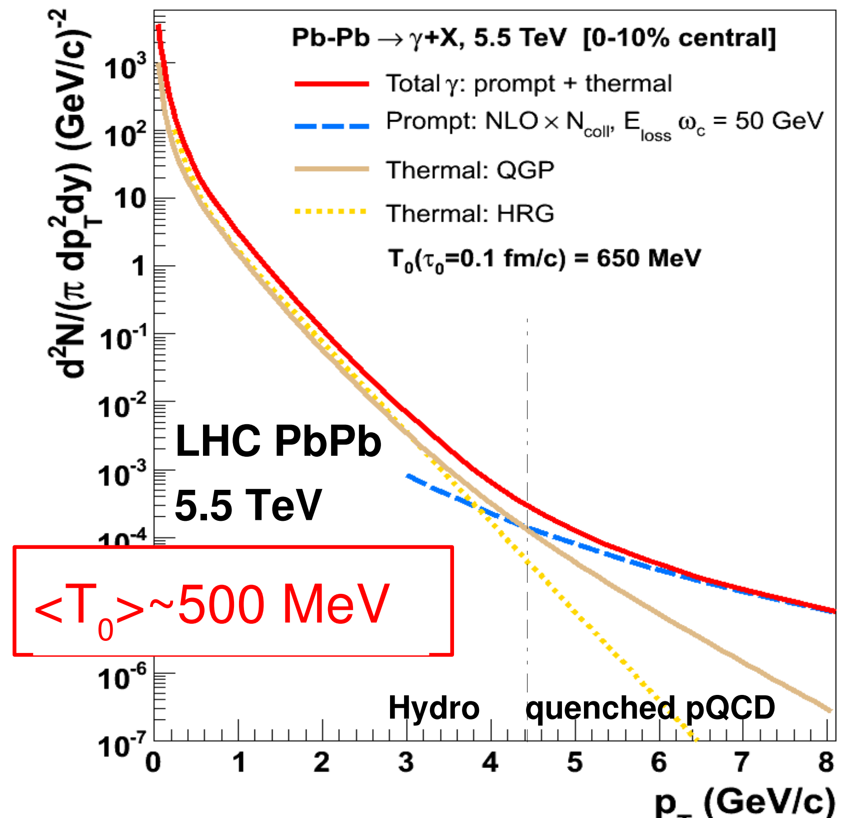
Pb-Pb 60-90% <u>periph</u>. (<b> ~ 13 fm)



Summary


Direct photons: Au+Au → $\gamma^{(*)}$+X at $\sqrt{s_{NN}}$= 200 GeV
PHENIX γ [0-10% central]
PHENIX γ^* [0-20% central]
Total γ: Prompt + Hydro [DdE-Peressou.]
Prompt γ: NLO pQCD × T_{AA} [0-10%]
D.d'Enterria-D.Peressounko. T_0 = 590 MeV, τ_0=0.15 fm/c
S.Rasanen et al. T_0 = 580 MeV, τ_0=0.17 fm/c
D.K.Srivastava. T_0 = 450-600 MeV, τ_0=0.20 fm/c
J.Alam et al. T_0 = 400 MeV, τ_0=0.20 fm/c
S.Turbide et al. T_0 = 370 MeV, τ_0=0.33 fm/c
$d^2N/(\pi\, dp_T^2 dy)$ (GeV/c)$^{-2}$
p_T (GeV/c)
RHIC Au-Au 200 GeV
$\langle T_0 \rangle$~350 MeV


$g_{eff}(dN_{ch}/d\eta, T_{eff})$
T_{eff} (GeV)
80-90 %
70-80 %
60-70 %
50-60 %
40-50 %
30-40 %
20-30 %
10-20 %
0-10 %
EoS: thermal-γ slopes (T) & dN/dη
A-A @ 30-40 GeV


Pb-Pb → γ+X, 5.5 TeV [0-10% central]
Total γ: prompt + thermal
Prompt: NLO × N_{coll}, E_{loss} ω_c = 50 GeV
Thermal: QGP
Thermal: HRG
$T_0(\tau_0$=0.1 fm/c) = 650 MeV
$d^2N/(\pi\, dp_T^2 dy)$ (GeV/c)$^{-2}$
p_T (GeV/c)
LHC PbPb
5.5 TeV
Hydro
quenched pQCD
$\langle T_0 \rangle$~500 MeV

Backup slides ...

Summary

0. Hadron & direct photon production in high-energy A-A collisions:

- Prompt (pQCD): T_{AA}-scaled p-p (NLO) + parton energy loss
- Hydrodynamics: thermal emission from hot expanding medium (i.c., EoS)

1. Hydrodynamics:

- 2D+1 ideal fluid, longitud. boost-invariant, cylindrical symm.
- EoS: QGP (ideal gas w/ MIT bag or latt. parametrization) + HRG
  + 1st order phase transition (T_c=165 MeV)
- Initial conditions: ϵ_0~650 GeV/fm^3 (dN/dy~2200 RHIC extrapolation), T_0~770 MeV,
  τ_0~$1/Q_s$=0.1 fm/c, μ_B ~ 5 MeV
- Freeze-out: T_{chem}=155 MeV, Cooper-Frye at T_{fo} = 120 MeV
- Photon rates parametrizations: AMY (QGP), Turbide-et-al (HRG)

2. pQCD:

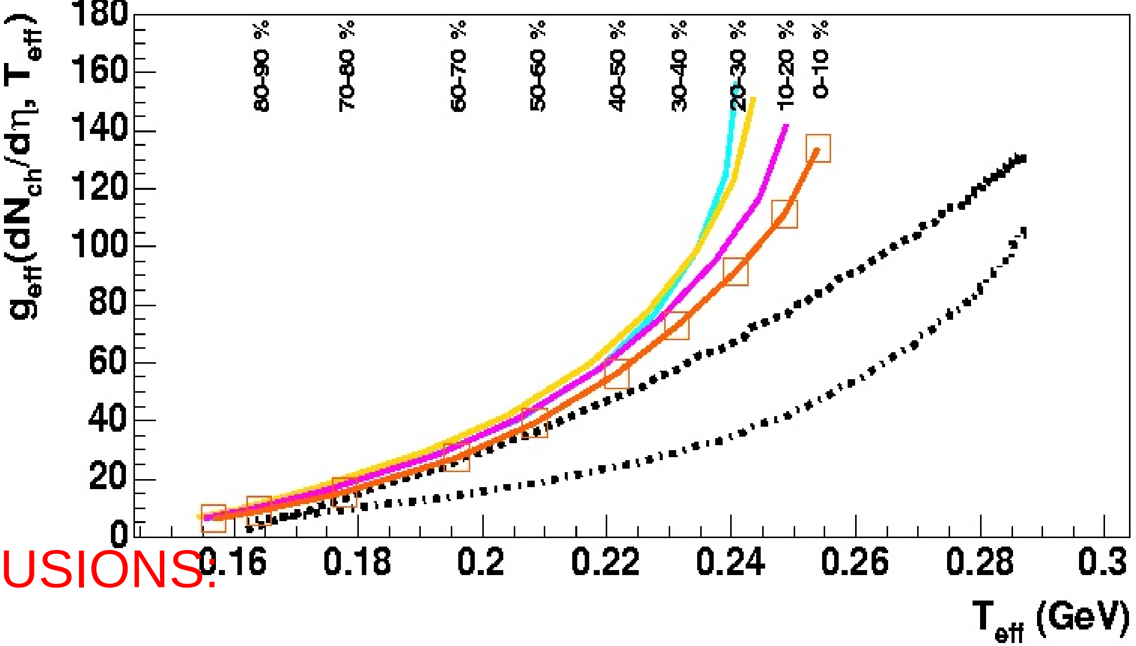
- NLO (scale μ=p_T). PDF: CTEQ6M + nDSg shadowing. FF: AKK
- Final-state suppr.: BDMPS parton energy-loss (ω_c~50 GeV) for hadrons & γ-fragm.

3. LHC predictions (validated in Au-Au @ RHIC):

- Hydro-pQCD crossing line at p_T~3 - 4 GeV/c for hadrons & photons.

QGP EoS from thermal γ & hadron multiplicities (II)

- Different medium EoS (e.g. HRG-like) should result in significantly different $g_{eff}(dN_{ch}/d\eta,T_{eff})$ evolution with AuAu centrality:



CONCLUSIONS:

- Concurrent reproduction of hydro models of experimental data on: (i) thermal γ, and (ii) hadron spectra imposes severe constraints in the initial thermodynamical conditions of the system.

- Combined measurement of (i) thermal γ inverse slopes, and (ii) $dN_{ch}/d\eta$ in diff. AuAu centralities provides direct information on the evolution (with T) of the # of degrees of freedom (EoS) of the produced medium

Overview

1. Photon production in high-energy A-A collisions:
   - Prompt photon (pQCD): T_{AB}-scaled p+p (or NLO) reference
   - Thermal photon (Hydro) $\rightarrow$ Connection to QCD thermodynamics
2. Hydrodynamical evolution:
   - Relativistic fluid-dyn. (2D+1): longitudinally boost-invariant, cylindrical symm.
   - EoS: QGP (ideal gas) + HRG + 1^{st} order phase transition.
   - Initial & final conditions fixed by soft hadron data: s_0, τ_0, μ_B; Cooper-Frye at T_{fo}
3. Thermal photon production:
   - Photon rates parametrizations: QGP, HRG.
   - Hydro vs. data: Hydro (+ pQCD) vs dN_γ/dpT in AuAu @ RHIC-200 GeV
4. Thermal γ (exponential inverse) slopes $\rightarrow$ initial QGP Temperature
5. Thermal γ slopes, hadron $dN/d\eta$ $\rightarrow$ T, entropy $\rightarrow$ QCD EoS
6. Outlook: RHIC(-II) light A+A at lower sqrt(s) ?

Prompt (pQCD) photon in p+p @ √s = 5.5 TeV

Photon production in p+p @ 200 GeV:

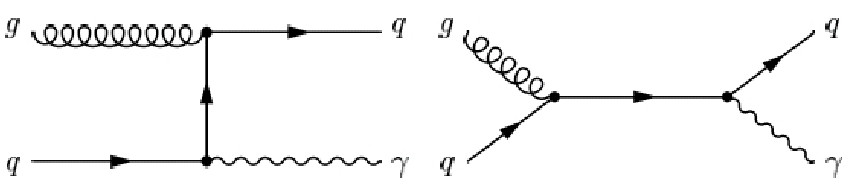


Figure 2.1: Compton diagrams.

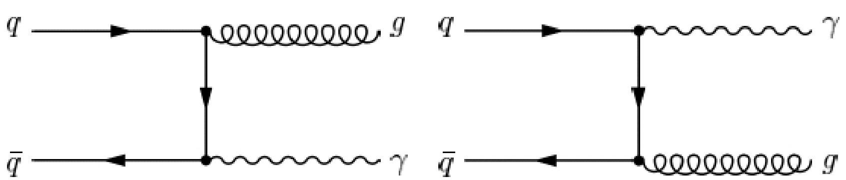


Figure 2.2: Annihilation diagrams.

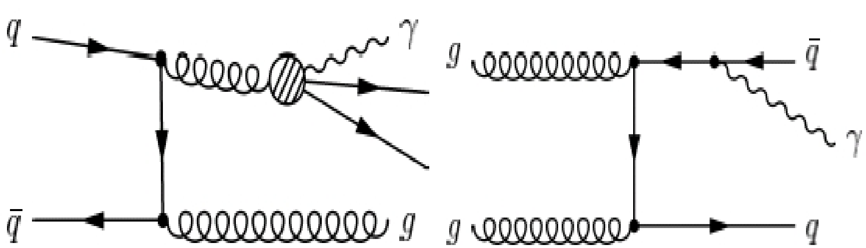
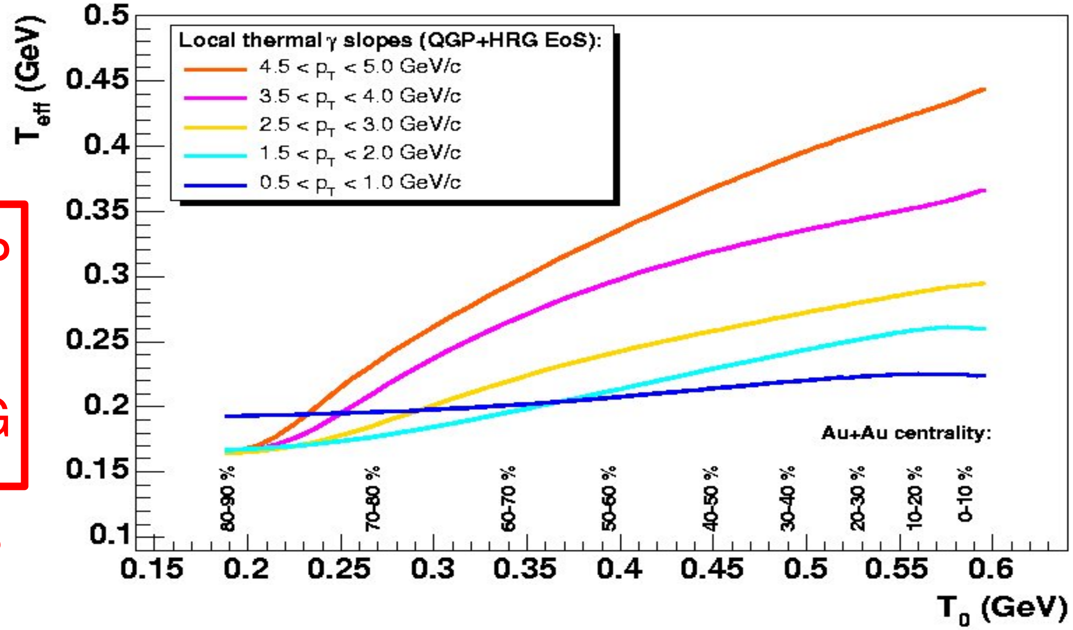


Figure 2.3: Bremsstrahlung diagrams.

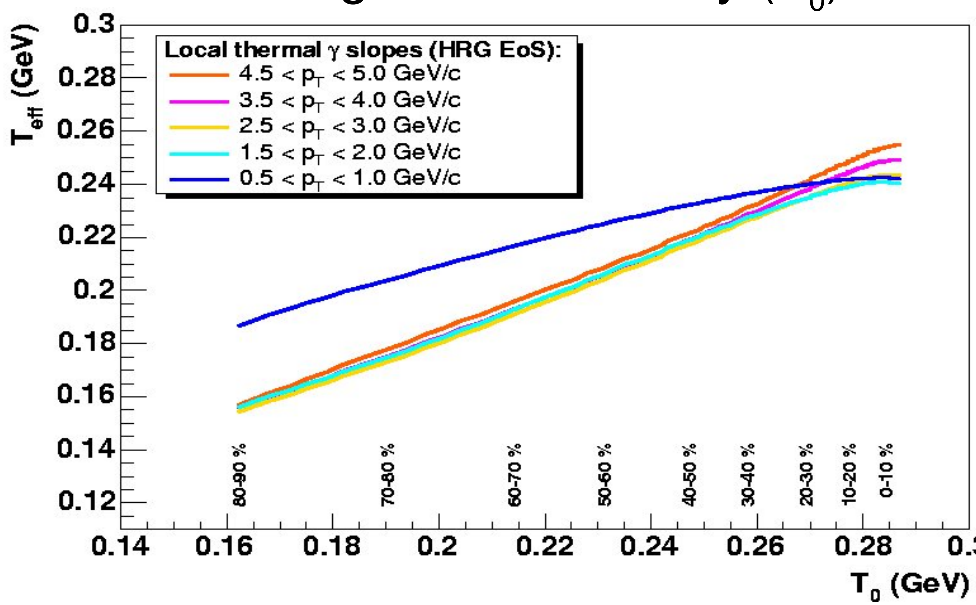
Thermal photon slopes vs. initial temperature

QGP + HRG EoS



Increasing AuAu centrality (T_0) →

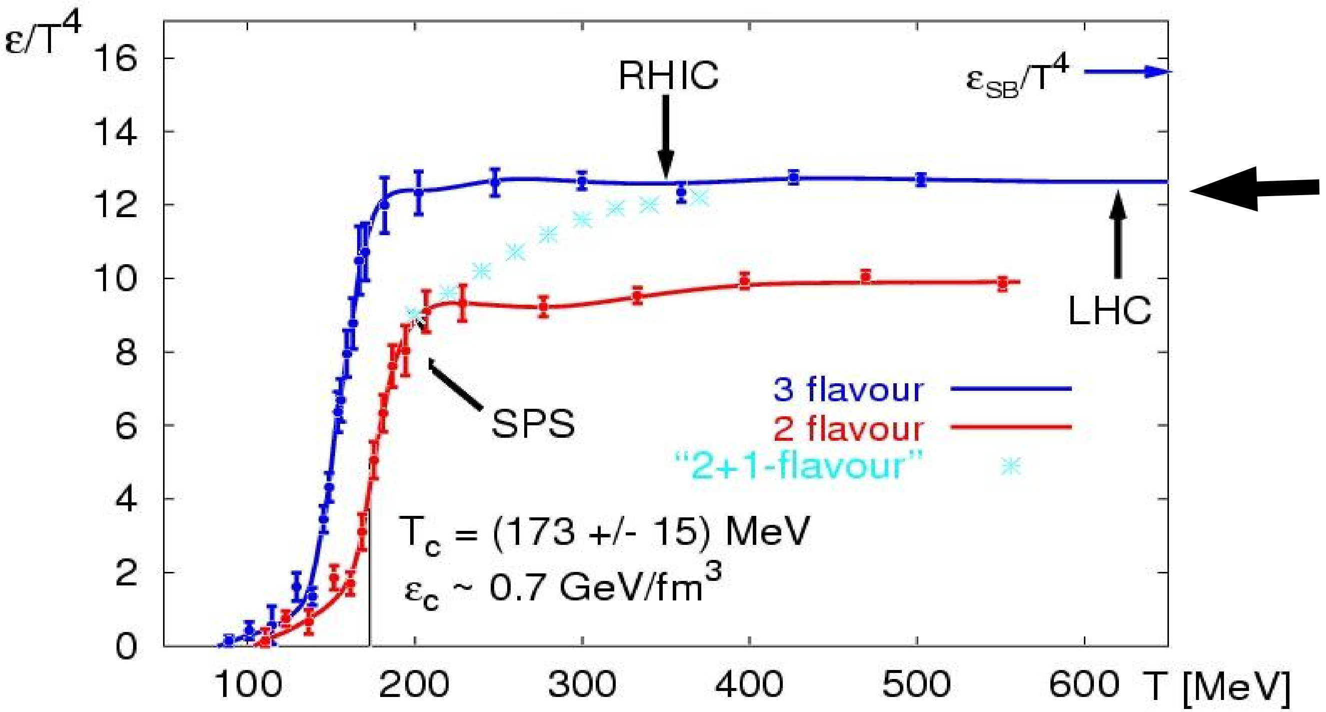
HRG EoS



- Photon expo inverse slopes proportional to initial (max.) temperature of the system at diff. centralities.
- For QGP+HRG EoS, the higher the p_T range where the slope is measured (= fitted), the closer is the apparent T to the initial QGP T_0 ~ 500 MeV
- HRG EoS alone (w/ lower initial T_0):
  (i) results in systematically smaller photon slopes compared to QGP + HRG (T_{eff} < 250 MeV)
  (ii) All p_T ranges yield the same apparent temperature (specified by expo pre-factors in HRG rates).

Text-book plot in high-energy heavy-ion physics

- Probe the phase diagram of hot & dense QCD matter.
- Study the properties of deconfined quark-gluon matter (QGP).



[Karsch & Laermann, hep-lat/0305025]

Only plot that would unambigously prove QGP at RHIC/LHC:

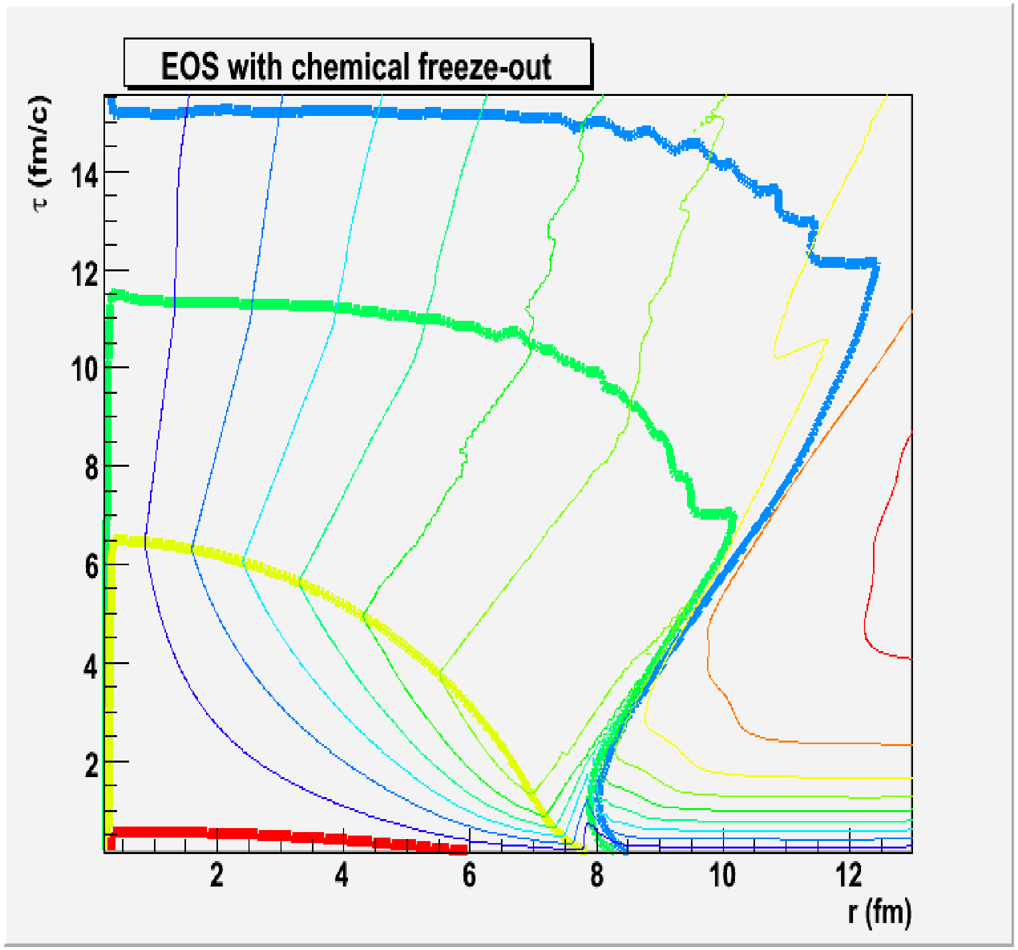
(i) rise of degs. of freedom at T_C

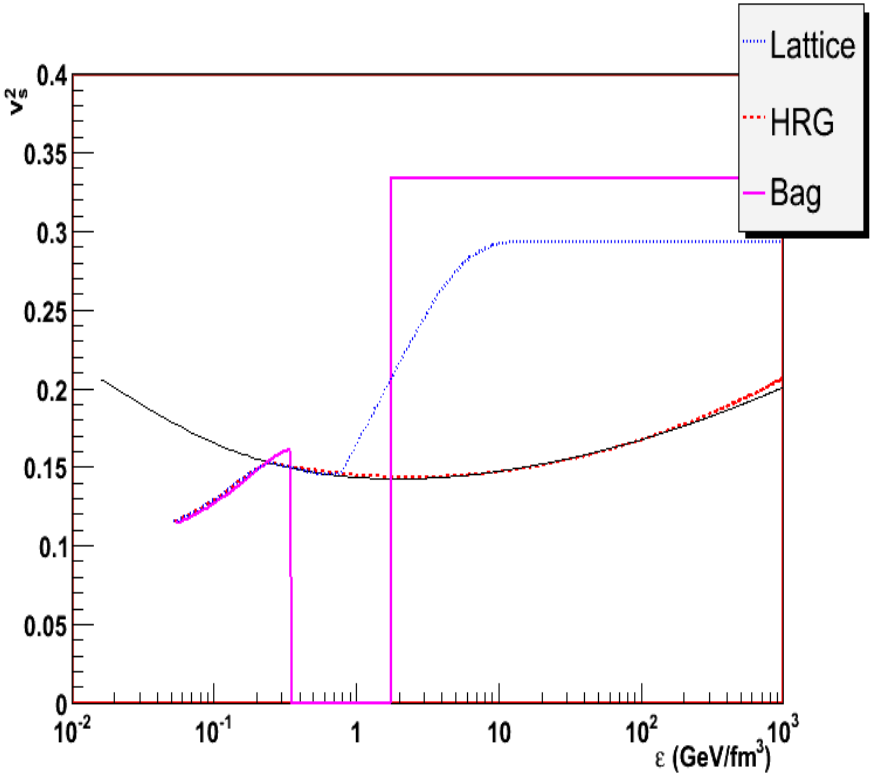
(ii) "plateau" at high ε

Q: Are there any exp. observables that provide unambiguous info on the thermodynamical properties & the EoS of the matter produced in HI colls. ?

A: Combine thermal photons (slope = direct measure of T) w/ hadron multiplicities (~ entropy density) for diff. A-A centralities. Plot: $s/T^3 = f(T)$

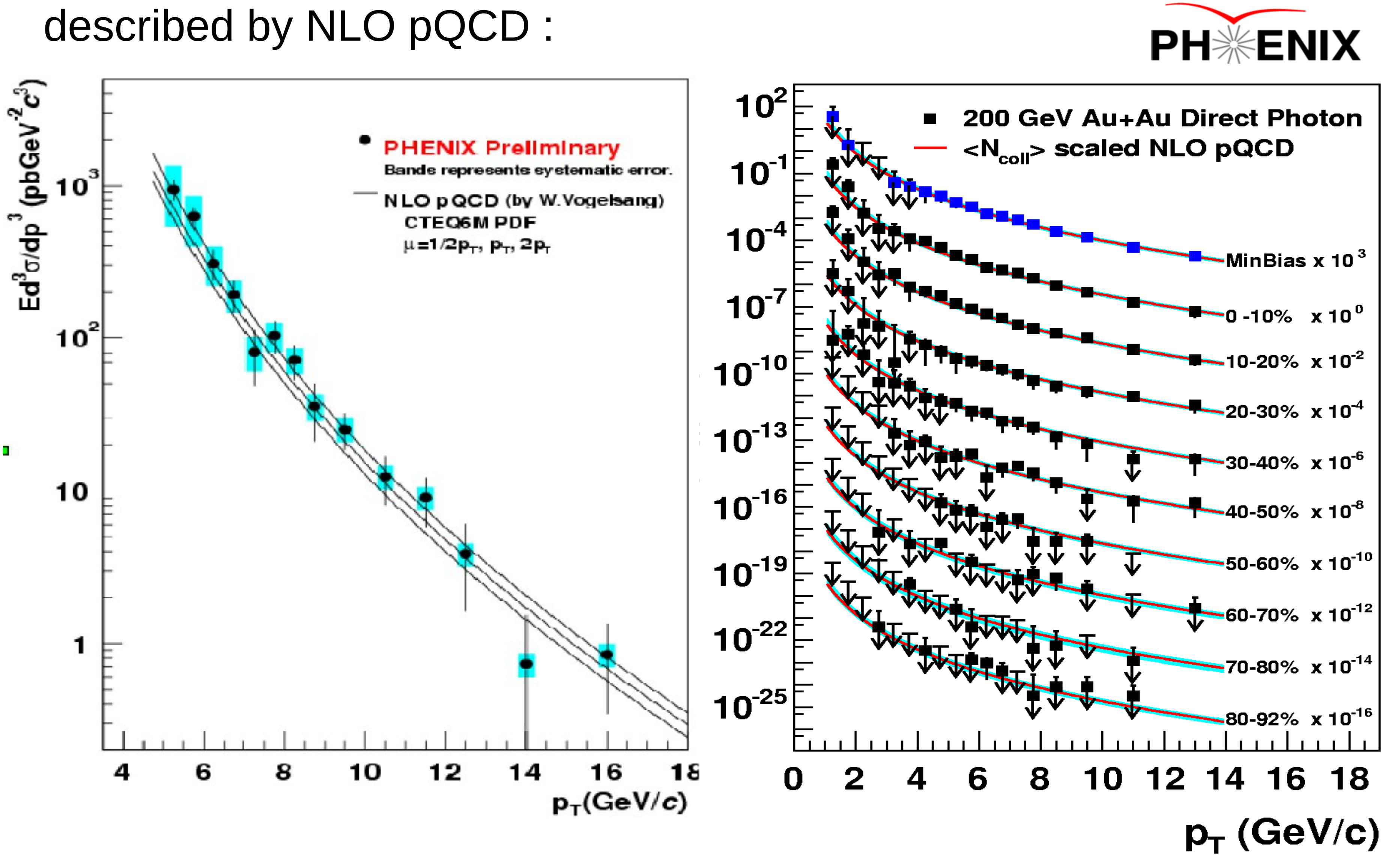
Hydro details





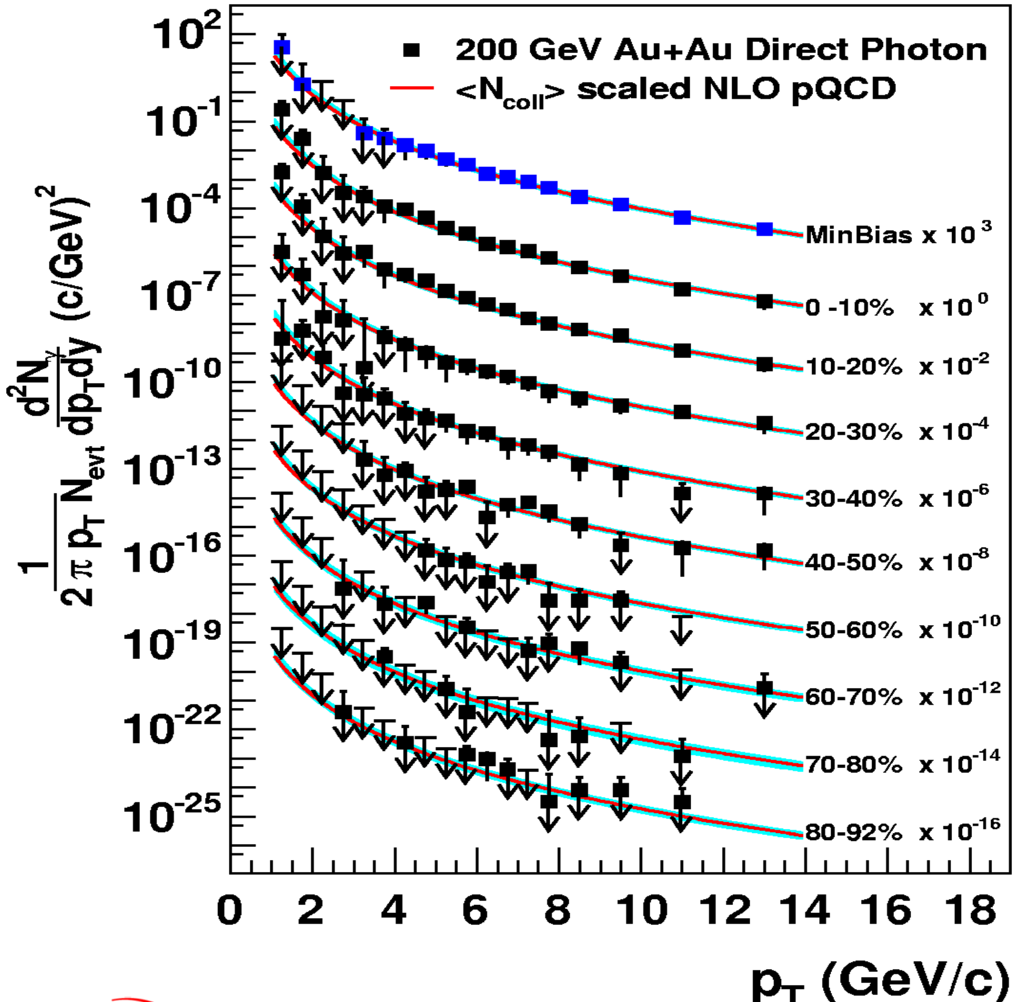
Prompt photons: p+p, Au+Au @ 200 GeV

- AuAu and (reference) pp photon production above p_T~4 GeV/c well described by NLO pQCD :

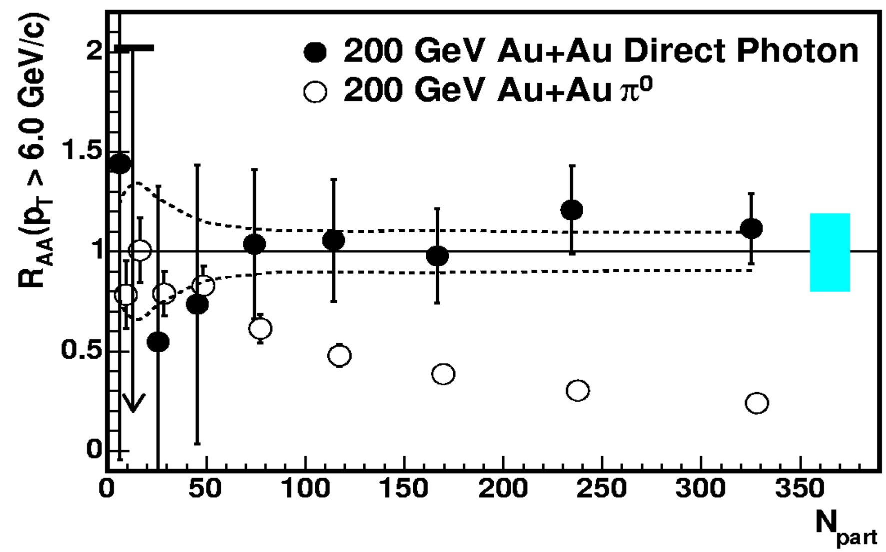


NN scaling in Au+Au @ 200 GeV: Direct Photons

- Direct photon production in Au+Au (all centralities) consistent w/ p+p incoherent scattering (NN-scaled pQCD) predictions:



$$R_{AA}(p_T, y; b) = \frac{\text{“hot/dense QCD medium”}}{\text{“QCD vacuum”}} = \frac{d^2N_{AA}/dydp_T}{\langle T_{AA}(b)\rangle \cdot d^2\sigma_{pp}/dydp_T},$$

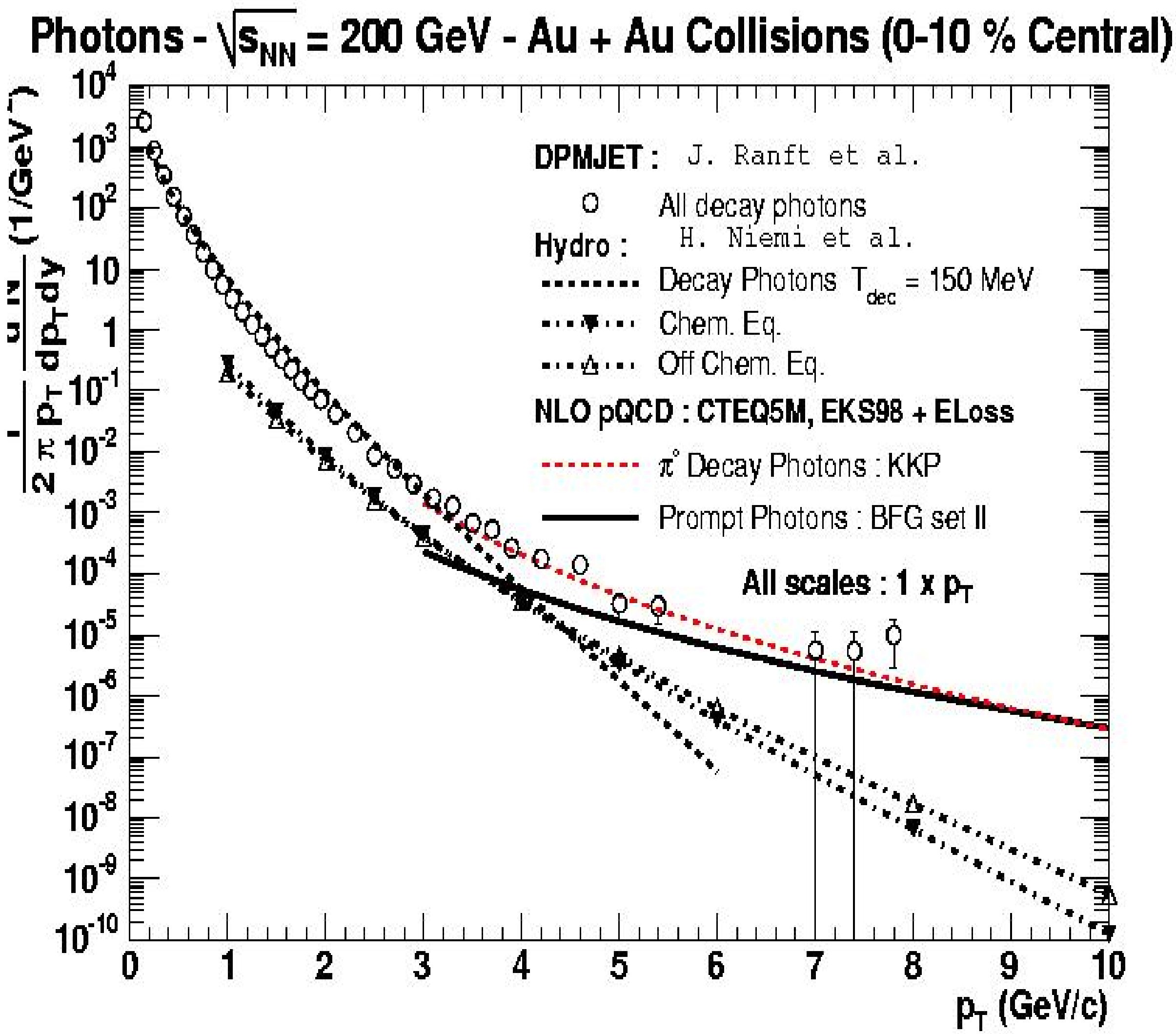


- Direct photon production in Au+Au unmodified by QCD medium.

PHENIX Submitted to PRL nucl-ex/0503003

Thermal photons from other hydros

- Our predictions are very similar to those of Jyvaskyla group :



In Arleo et al.
CERN Yellow Report
HI @ LHC Photon Physics

hep-ph/0311131

Disentangling "thermal" γ from quenched prompt γ

Step 1: Measure p+p → γ(isolated) + X

down to p_T = 1 GeV/c

with uncertainties ~10%

Handle on γ from qg-Compton, qqbar annihilation

Step 2: Measure p+p → γ(total) + X

down to p_T = 1 GeV/c

with uncertainties ~10%

Handle on fragmentation γ production

Step 3: Measure Au+Au → γ(total) + X

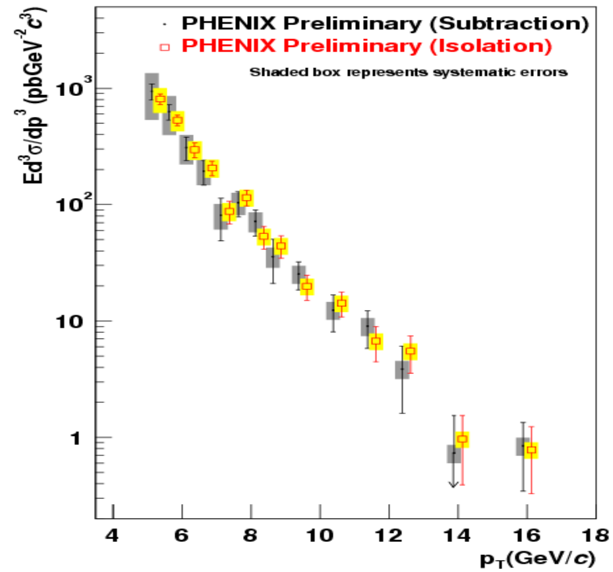
down to p_T = 1 GeV/c

with uncertainties ~10%

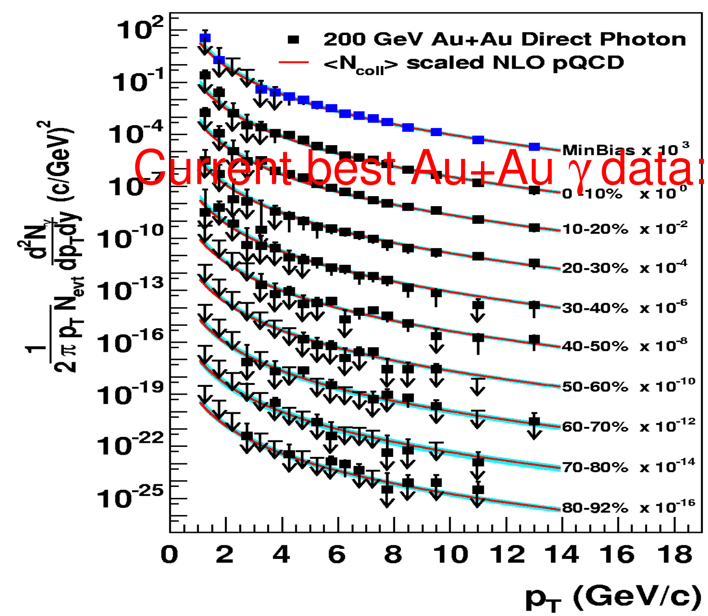
Step 4: (AuAu γ_{total}) – T_{AB}•(pp $\gamma_{isolated}$)

Upper limit on thermal spectrum.

Current best p+p γ data:

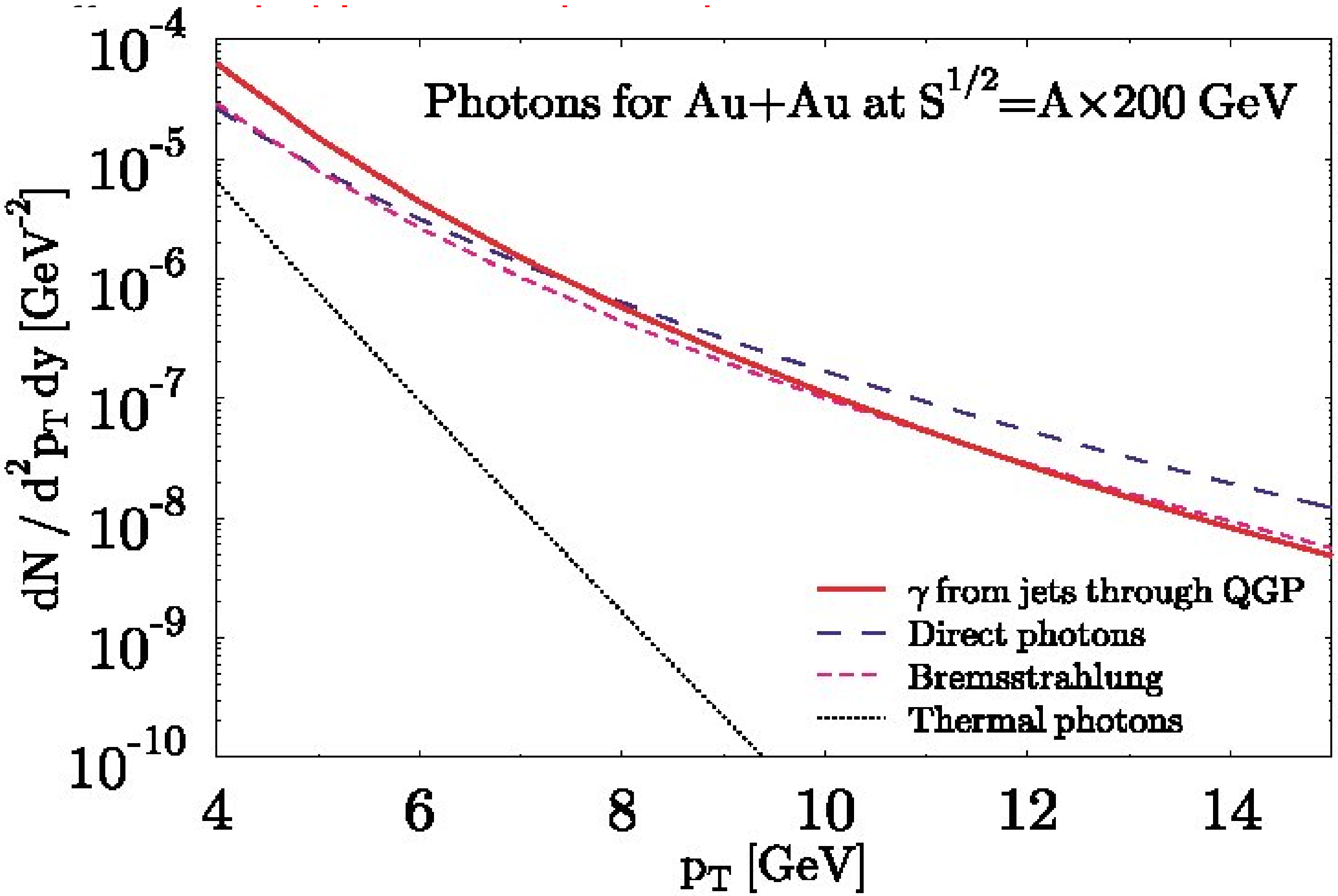


Current best Au+Au γ data:



Photons from quark jets in the medium ?

- Duke group predictions for Compton & annih. of fast quark in medium
- LO for photons (& not most recent thermal photon rates)
- But NLO (K = 2.5) for jets, no energy loss taken into account ...



Fries, Muller, Srivastava
PRL 90 132301 (2003)

$q_{hard} + \bar{q}_{QGP} \rightarrow \gamma + g$

$q_{hard} + g_{QGP} \rightarrow \gamma + q$

Energy loss in Au+Au→ γ + X @ $\sqrt{s}$ = 200 GeV ?

(Part of the) prompt photons can be distorted by the dense QCD medium (esp. in the region p_T< 4 GeV/c).

Photon production in p+p @ 200 GeV:

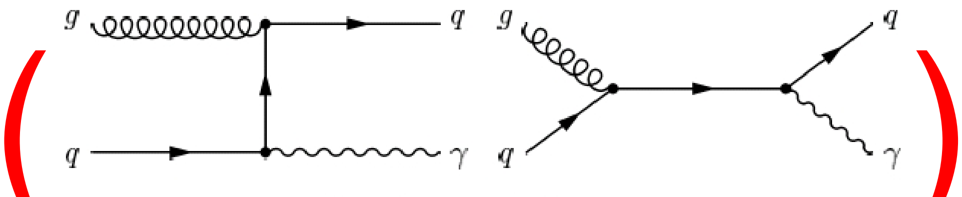


Figure 2.1: Compton diagrams.

+

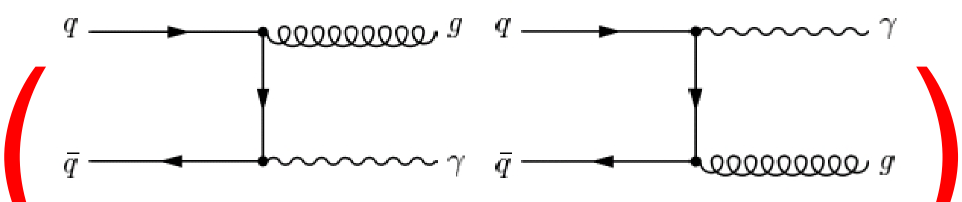


Figure 2.2: Annihilation diagrams.

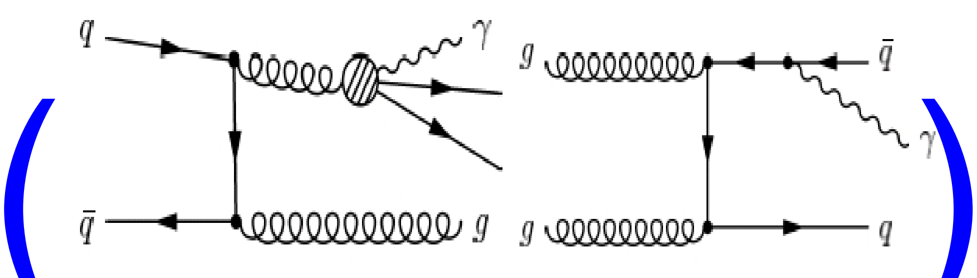
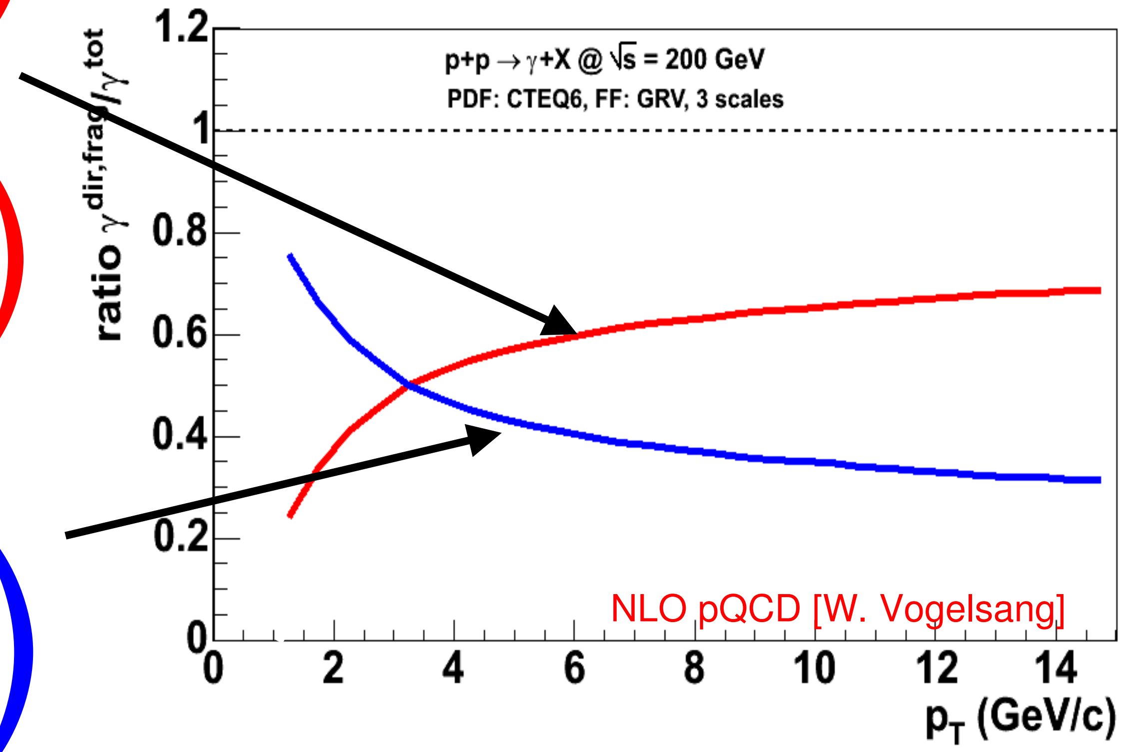


Figure 2.3: Bremsstrahlung diagrams.

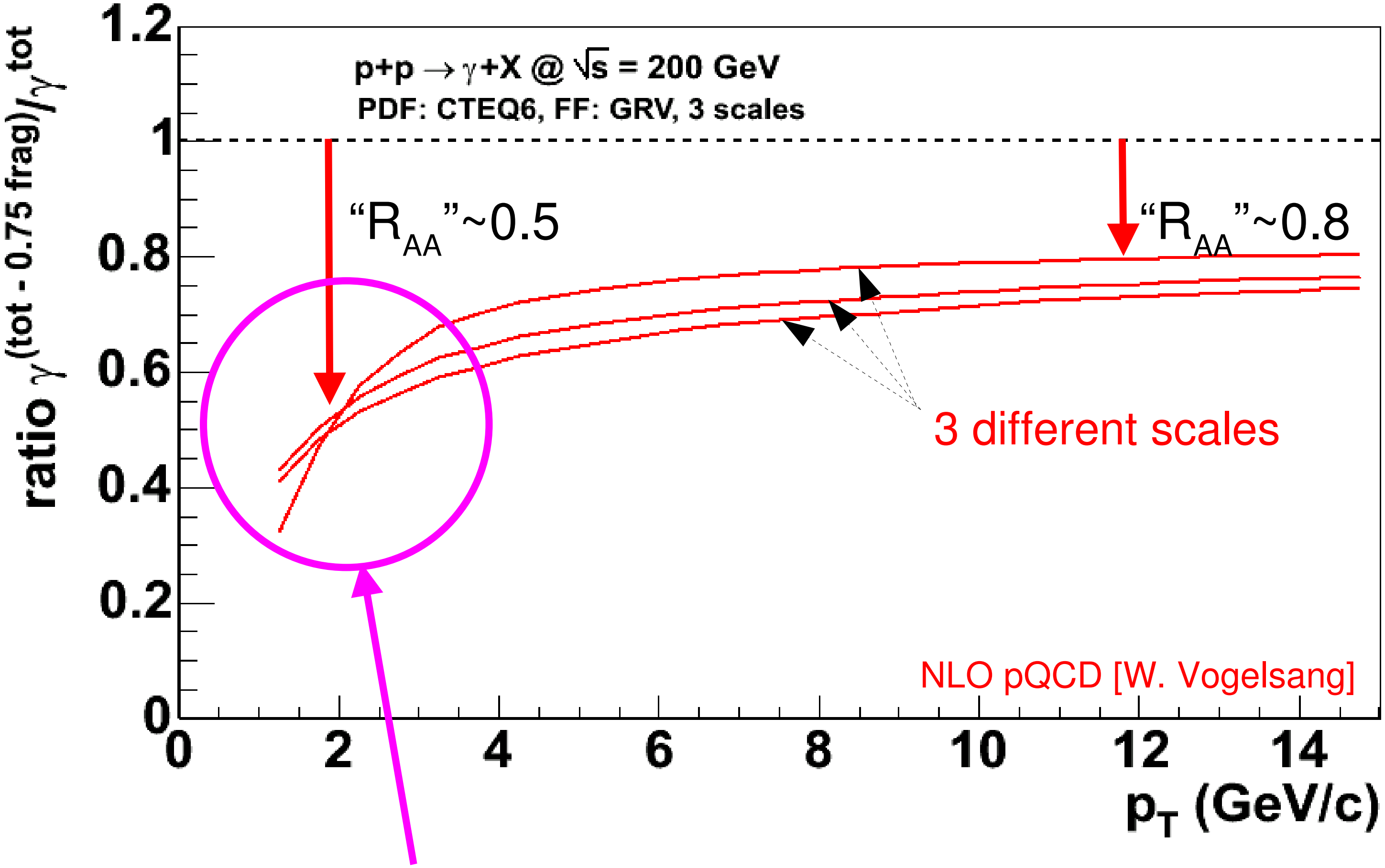
Below $p_T \approx$ 4. GeV/c dominated by γ from collinear q,g fragmentation



R_{AA} for photons @ $\sqrt{s}$ = 200 GeV (I)

Back-of-the-envelope ansatz for γ suppression: R_{AA}(γ frag.) = R_{AA}(q,g) ≈ 0.25

R_{AA} ≈ Ratio of γ(tot – 0.75*frag)/γ(tot):



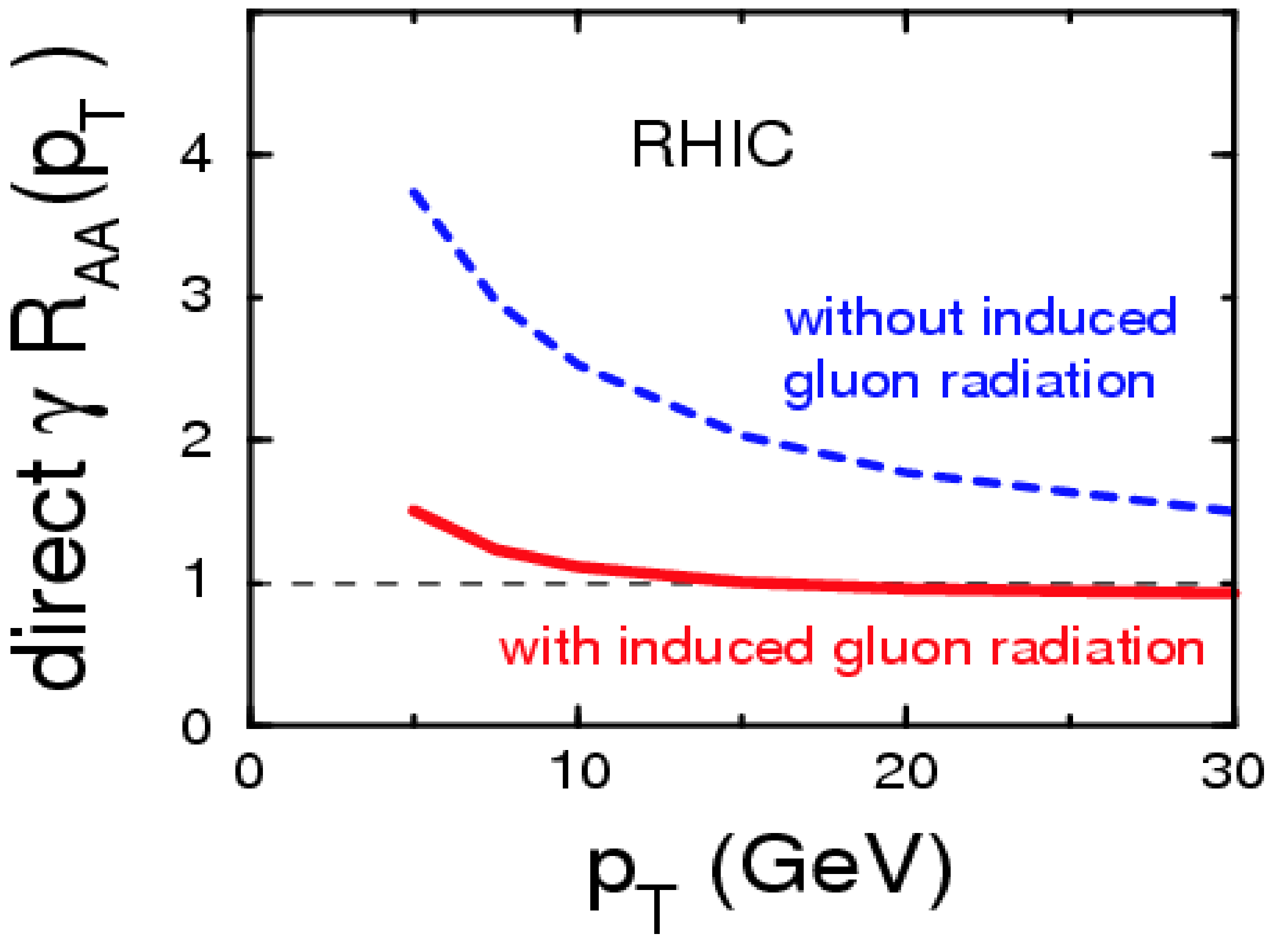
(No shadowing corrs. included)

Results confirmed by more involved calculations: F.Arleo: hep-ph/0406291

R_{AA} for photons @ $\sqrt{s}$ = 200 GeV (II)

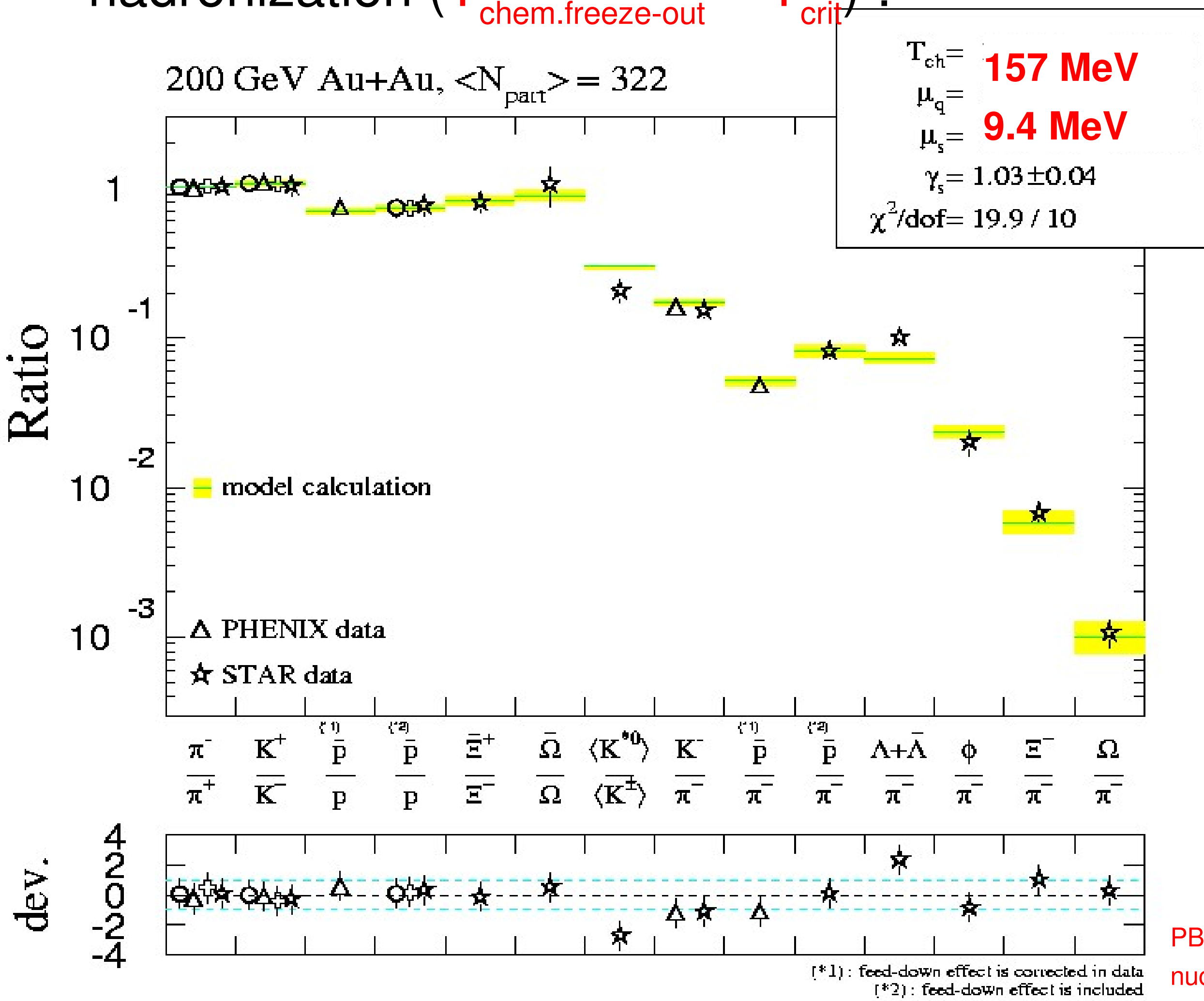
Zakharov does NOT predict any suppression (but enhancement):

Zakharov, hep-ph/0405101



Ratios of particle yields

- Ratios of hadron yields consistent w/ system at chemical equilibrium at hadronization ($T_{chem.freeze-out} \sim T_{crit}$) :



- Assume all distrib. described by one T and one μ:

$$dN \sim e^{-(E-\mu)/T} d^3p$$

- 1 ratio (e.g. $\bar{p}/p$) determines μ/T

$$\bar{p}/p \sim e^{-(E+\mu)/T}/e^{-(E-\mu)/T} = e^{-2\mu/T}$$

- 2^{nd} ratio (e.g. K/pi) provides T,μ.

- Then predict all other hadronic yields and ratios

PBM, Redlich, Stachel nucl-th/0304013
Kaneta, Xu nucl-th/0405068