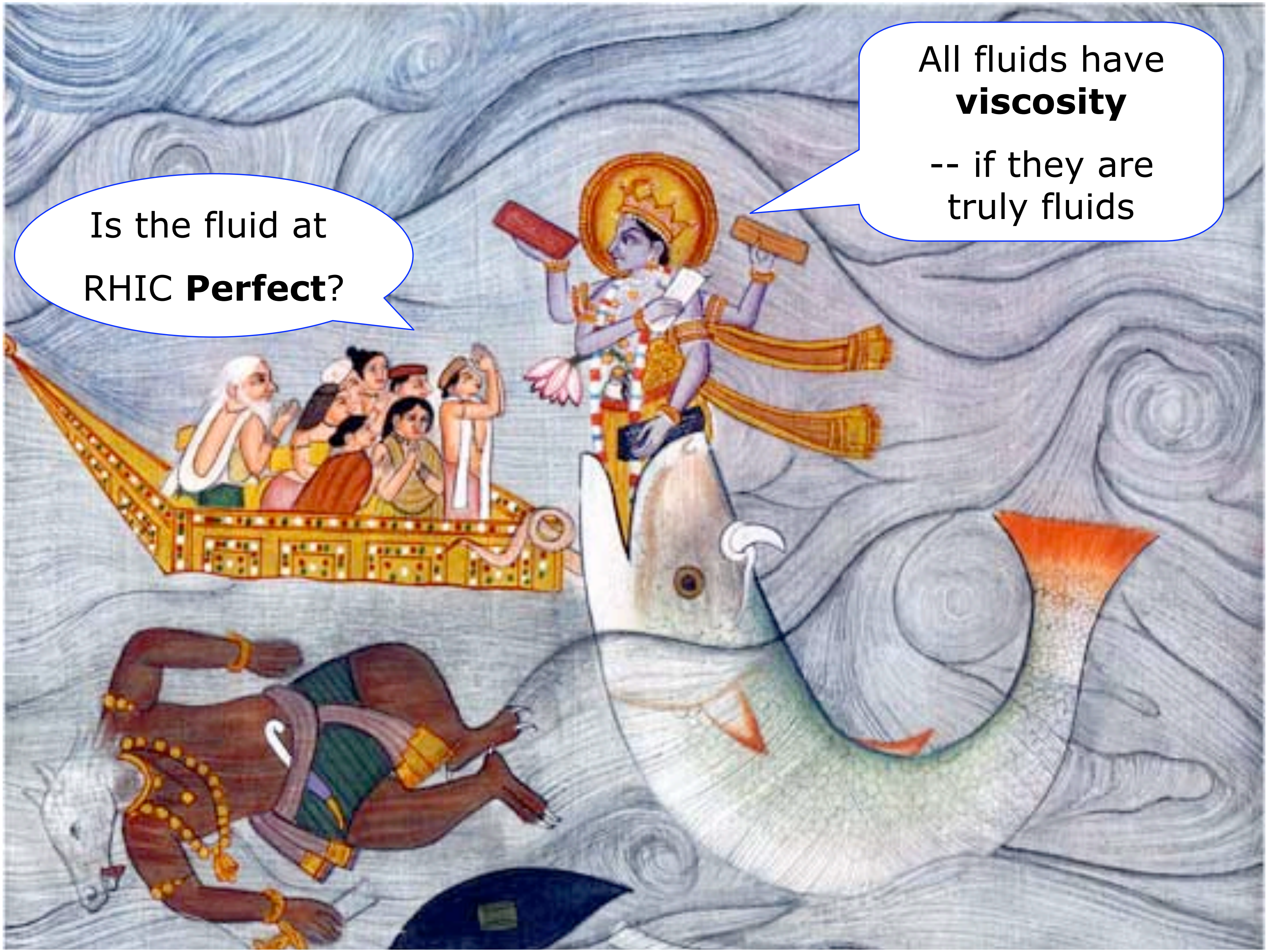

Is the fluid at RHIC **Perfect**?


Is the fluid at RHIC **Perfect**?
All fluids have **viscosity**
-- if they are truly fluids

# Measuring Viscosity at RHIC

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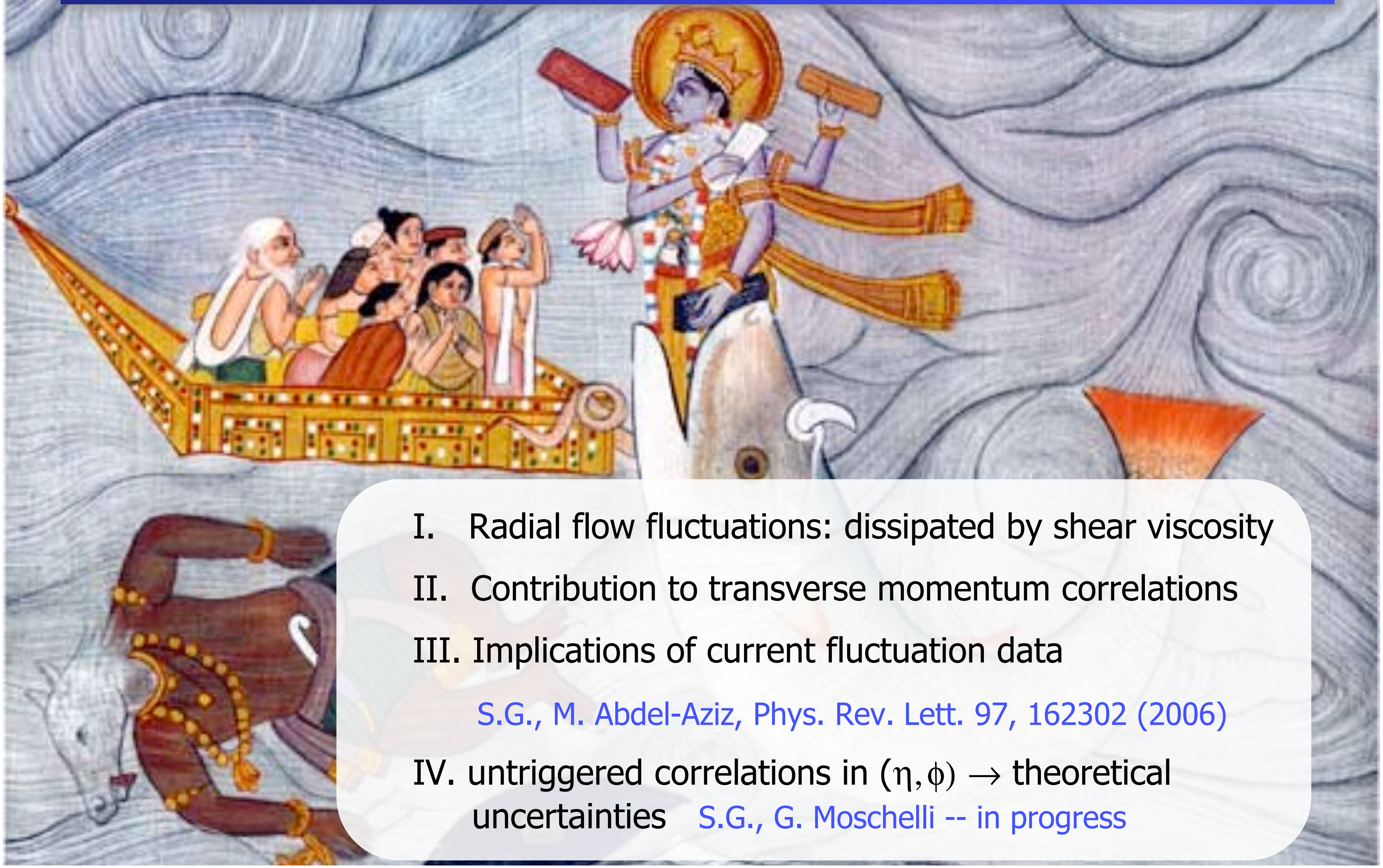


I. Radial flow fluctuations: dissipated by shear viscosity

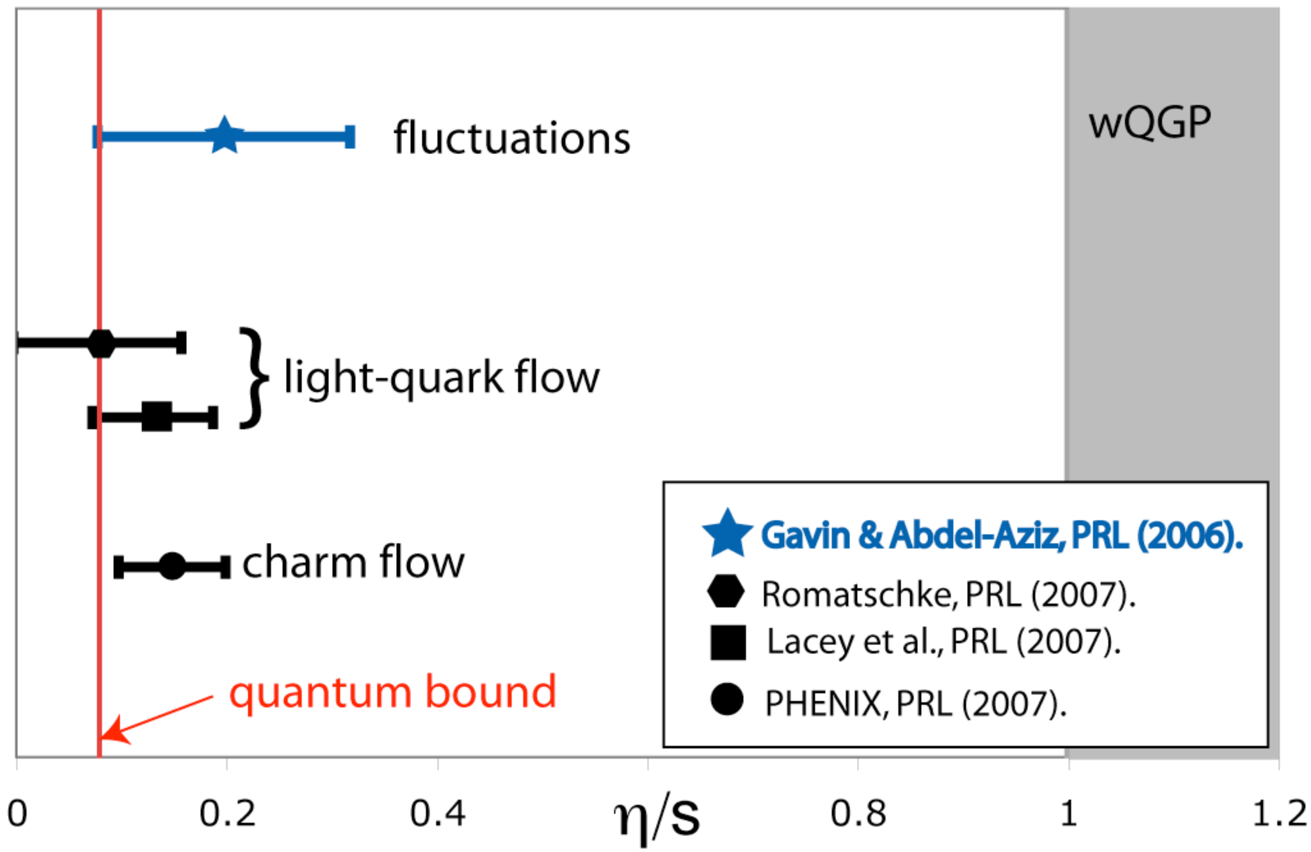
II. Contribution to transverse momentum correlations

III. Implications of current fluctuation data

S.G., M. Abdel-Aziz, Phys. Rev. Lett. 97, 162302 (2006)

IV. untriggered correlations in $(\eta, \phi) \rightarrow$ theoretical uncertainties S.G., G. Moschelli -- in progress

# Shear Viscosity Measurements




- **consensus:** viscosity is extremely small
- light quark $v_2$ only a **bound** -- ideal hydro works
- **theoretical uncertainties** -- first steps

# Measuring Shear Viscosity

Elliptic and radial flow suggest small shear viscosity

- Teaney; Kolb & Heinz; Huovinen & Ruuskanen

**Problem:** initial conditions unknown

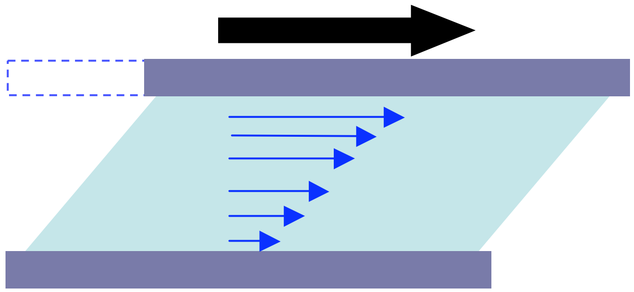
- CGC $\Rightarrow$ more flow? larger viscosity? Hirano et al.; Kraznitz et al.; Lappi & Venugopalan; Dumitru et al.

## Additional viscosity probes?

What does shear viscosity do? **It resists shear flow.**

flow $v_x(z)$ $\quad\Rightarrow\quad$ $T_{zx} = -\eta \dfrac{\partial v_x}{\partial z}$

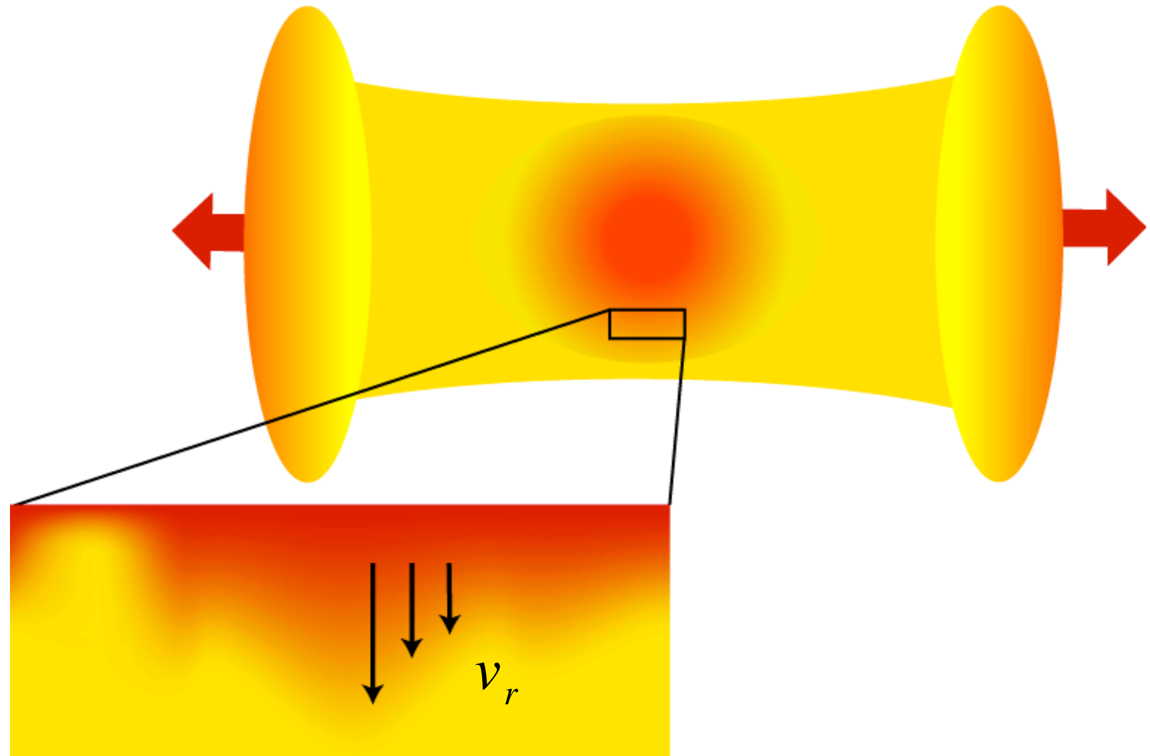
shear viscosity $\eta$



# Transverse Flow Fluctuations

small variations in radial flow in each event

neighboring fluid elements flow past one another
⇒ viscous friction

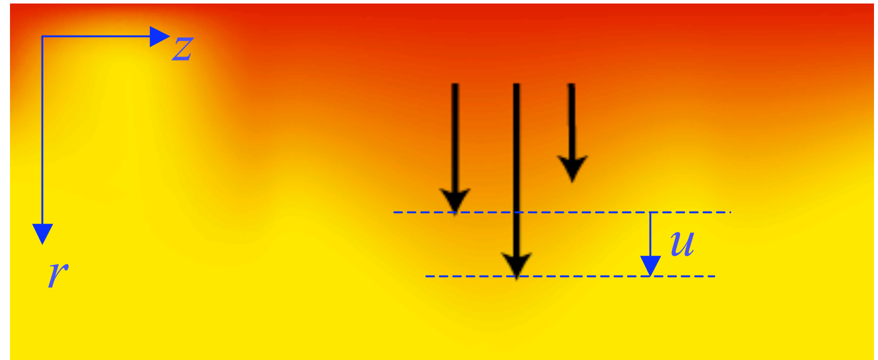



**shear viscosity drives velocity toward the average**

$$T_{zr} = -\eta\, \partial v_r / \partial z$$

**damping of radial flow fluctuations ⇒ viscosity**

# Evolution of Fluctuations




$$u(z,t) \approx v_r - \langle v_r \rangle$$

**momentum current** for small fluctuations

$$g_t \equiv T_{0r} - \langle T_{0r} \rangle \approx \langle Ts \rangle u$$

shear stress

$$T_{zr} \approx -\eta \frac{\partial u}{\partial z} \approx -\frac{\eta}{Ts} \frac{\partial g_t}{\partial z}$$

momentum conservation

$$\frac{\partial}{\partial t} T_{0r} + \frac{\partial}{\partial z} T_{zr} = 0$$

**diffusion equation** for momentum current

$$\left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) g_t = 0$$

kinematic viscosity $\nu = \eta / Ts$

shear viscosity $\eta$

entropy density $s$, temperature $T$

# Hydrodynamic Momentum Correlations

**fluctuating momentum current** $\frac{\partial}{\partial t} g_t = \nu \nabla^2 \left( g_t + \text{noise} \right)$

particles jump between fluid cells $\rightarrow$ Langevin noise

**momentum flux density correlation function**

$$r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

**deterministic diffusion equation for $\Delta r_g = r_g - r_{g,eq}$**

**fluctuations diffuse through volume, driving $r_g \rightarrow r_{g,eq}$**

width in relative rapidity
grows from initial value $\sigma_0$

$$\sigma^2 = \sigma_0^{\ 2} + 4\nu \left( \frac{1}{\tau_0} - \frac{1}{\tau_F} \right)$$

# Transverse Momentum Covariance

**observable:**

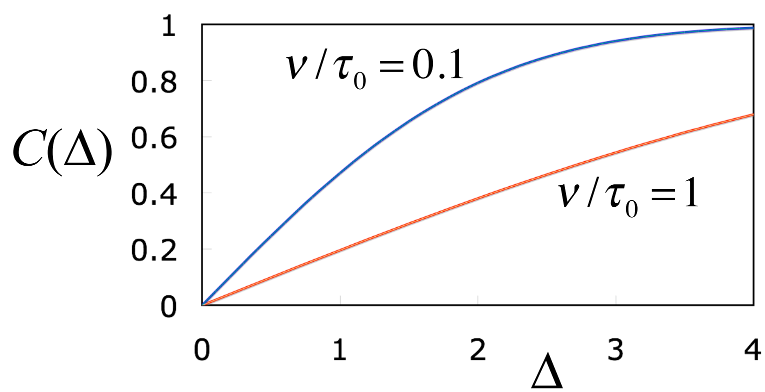
$$C = \langle p_{t1} p_{t2} \rangle - \langle p_t \rangle^2$$

$$\langle p_{t1} p_{t2} \rangle \equiv \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs } i \neq j} p_{ti} p_{tj} \right\rangle \qquad \langle p_t \rangle \equiv \frac{1}{\langle N \rangle} \left\langle \sum p_{ti} \right\rangle$$

**measures momentum-density correlation function**

$$C = \frac{1}{\langle N \rangle^2} \int \Delta r_g \, dp_1 dp_2$$

$C$ depends on rapidity interval $\Delta$




**propose:** measure $C(\Delta)$ to extract width $\sigma^2$ of $\Delta r_g$

# Current Data?

**STAR measures rapidity width of $p_t$ fluctuations** J.Phys. G32 (2006) L37

$$\Delta\sigma_{p_t:n} = \frac{1}{\langle N\rangle}\left\langle \sum_{i\neq j}\left(p_{ti} - \langle p_t\rangle\right)\left(p_{tj} - \langle p_t\rangle\right)\right\rangle$$

find width $\sigma_*$ increases in central collisions

- most peripheral $\sigma_* \sim 0.45$
- central $\sigma_* \sim 0.75$

**naively identify** $\sigma_*$ with $\sigma$

$$\sigma^2_{central} - \sigma^2_{peripheral} = 4\nu\left(\frac{1}{\tau_{f,p}} - \frac{1}{\tau_{f,c}}\right)$$

freezeout $\tau_{f,p} \sim 1$ fm, $\tau_{f,c} \sim 20$ fm $\Rightarrow \nu \sim 0.09$ fm

at $T_c \sim 170$ MeV $\Rightarrow$ $\eta/s \sim 0.08 \sim 1/4\pi$

# Current Data?

**STAR measures rapidity width of $p_t$ fluctuations** J.Phys. G32 (2006) L37

$$\Delta\sigma_{p_t:n} = \frac{1}{\langle N \rangle}\left\langle \sum_{i\neq j}\left(p_{ti} - \langle p_t \rangle\right)\left(p_{tj} - \langle p_t \rangle\right)\right\rangle$$

find width $\sigma_*$ increases in central collisions

- most peripheral $\sigma_* \sim 0.45$
- central $\sigma_* \sim 0.75$

**naively identify** $\sigma_*$ with $\sigma$ (strictly, $\Delta\sigma_{p_t:n} = \langle N \rangle C + \text{corrections}$)

$$\sigma^2_{central} - \sigma^2_{peripheral} = 4\nu\left(\frac{1}{\tau_{f,p}} - \frac{1}{\tau_{f,c}}\right)$$

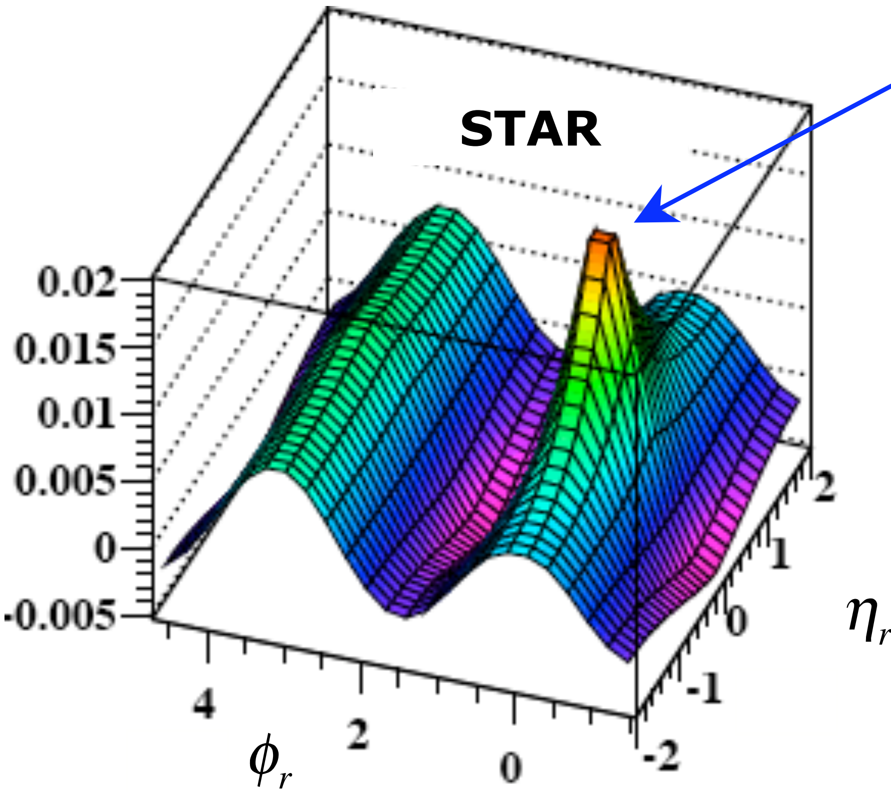
**but** maybe $\sigma_n \approx 2\sigma_*$ STAR, PRC 66, 044904 (2006)

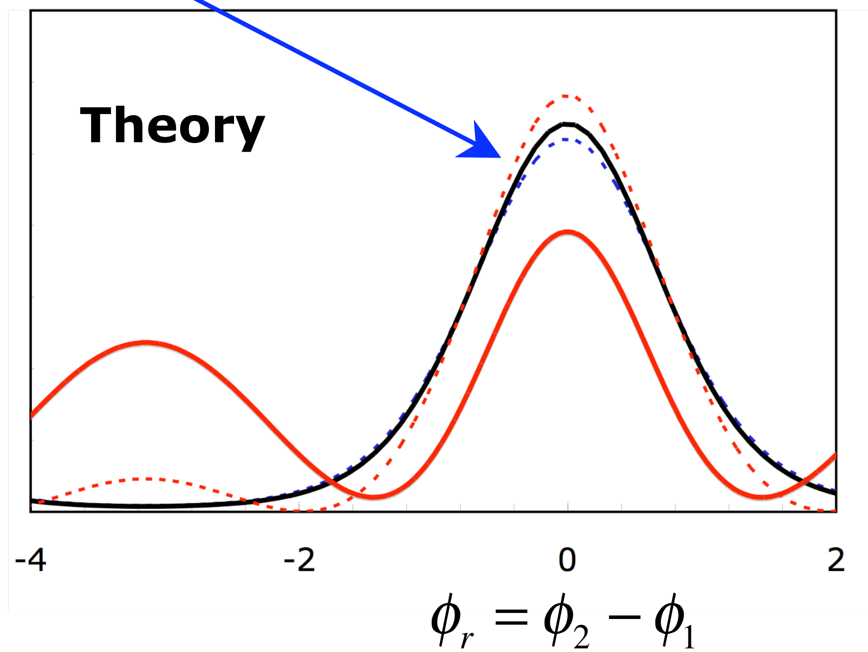
uncertainty range $\sigma_* \leq \sigma \leq 2\sigma_*$ $\Rightarrow$ $0.08 < \eta/s < 0.3$

# Behavior part of a Correlation Landscape

**near side peak**

S.G. + G. Moschelli
in progress



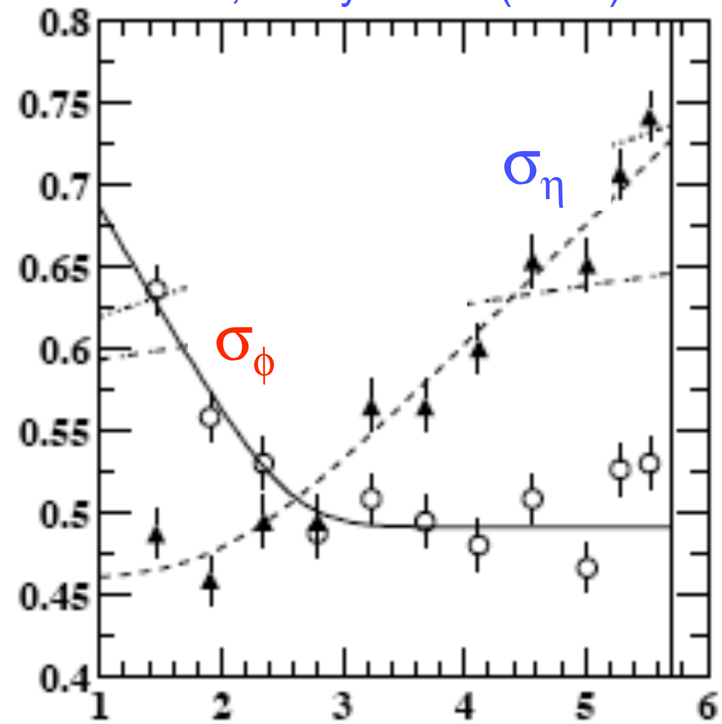




**untriggered correlations: no jet tag**

**near side peak:** similar to **ridge** with jet tag **but** at ordinary $p_t$ scales

# Near Side Peak: Centrality Dependence

STAR, J.Phys. G32 (2006) L37




centrality, $2n_{bin}/n_{part}$

**near side peak**

- pseudorapidity width $\sigma \rightarrow \sigma_\eta$
- azimuthal width $\sigma_\phi$

centrality dependence:

path length $\sim 2n_{bin}/n_{part}$

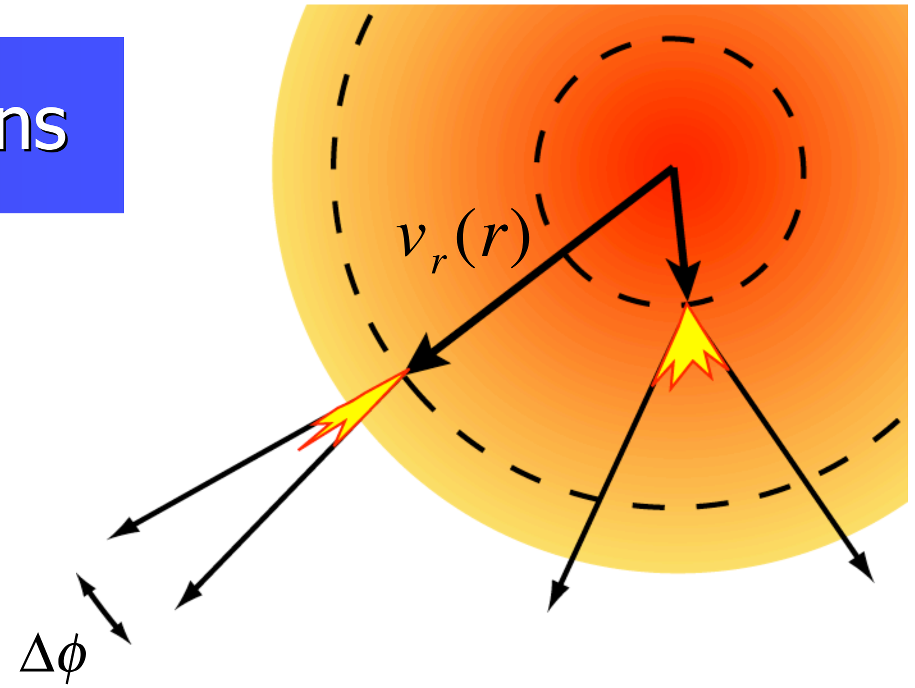
**trends:**

- rapidity broadening (viscosity)
- azimuthal narrowing

**common explanation of trends?**

**find:** $\sigma_\phi$ **requires radial and elliptic flow plus viscosity**

# Flow ⇒ Azimuthal Correlations




**mean flow depends on position**

blast wave $\vec{v}_r \sim \lambda\vec{r}$

**opening angle** for each fluid element depends on $r$

$$\Delta\phi \sim v_{th}/v_r \sim (\lambda r)^{-1}$$

**correlations:** $r(p_1, p_2) = \text{pairs} - (\text{singles})^2 = \iint_{x_1, x_2} f(p_1, x_1) f(p_2, x_2) r(x_1, x_2)$

gaussian spatial $r(x_1, x_2)$:

- $\sigma_t$ -- width in $\vec{r} = \vec{r}_{t2} - \vec{r}_{t1}$
- $\Sigma_t$ -- width in $\vec{R} = (\vec{r}_{t1} + \vec{r}_{t2})/2$

momentum distribution:

$$f = e^{-\gamma(E - \vec{p}\cdot\vec{v})/T}$$

$$\sigma_\phi{}^2 = \langle \Delta\phi^2 \rangle \propto \lambda^{-2}(\Sigma_t^2 - \sigma_t^2/4)^{-1}$$

# Measured Flow Constrains Correlations

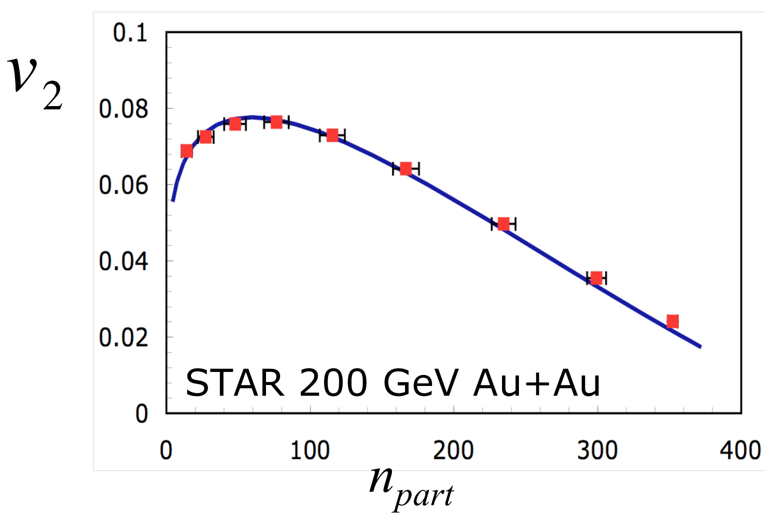
**diffusion + flow for correlations**

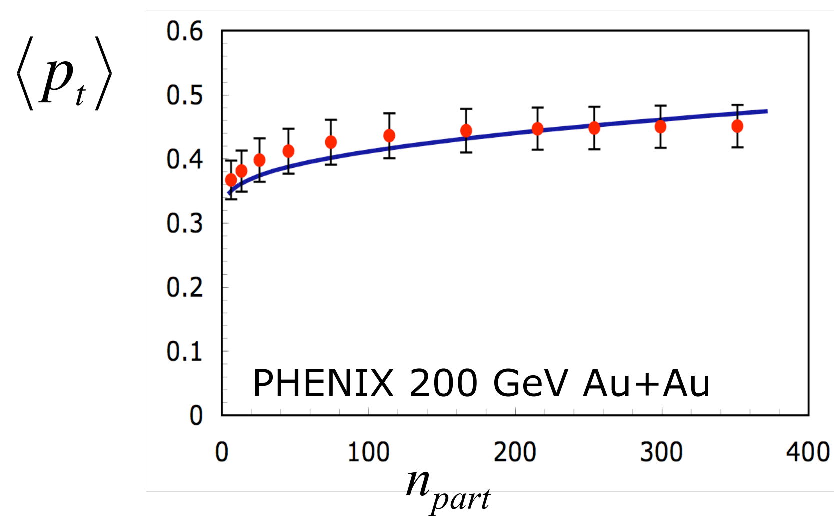
$$\frac{\partial g_t}{\partial \tau} + \vec{v}_r \cdot \vec{\nabla}_t g_t + g_t \frac{\partial v_t}{\partial r} = \nu \nabla^2 g_t \quad \rightarrow \quad \Sigma_t(\tau),\ \sigma_t(\tau)$$

**radial plus elliptic flow:**
"eccentric" blast wave
Heinz et al.

$$\vec{v}_r = \varepsilon_x(\tau)\, x\hat{x} + \varepsilon_y(\tau)\, y\hat{y}$$

**constraints:** flow velocity, radius from measured $v_2$, $\langle p_t \rangle$ vs. centrality



# Rapidity and Azimuthal Trends

G. Moschelli + S.G., *in progress*

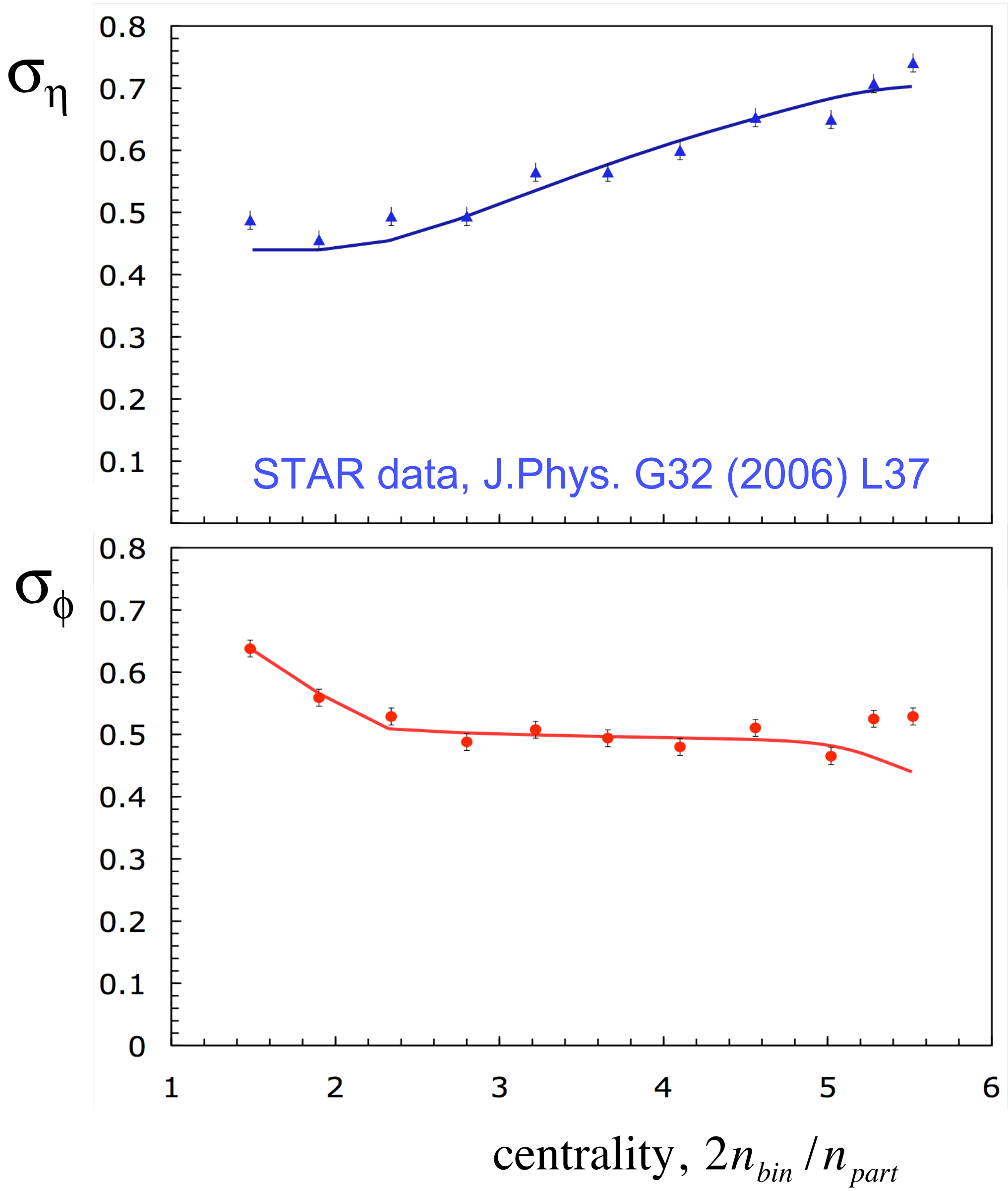
**rapidity width:**

- viscous broadening, $\eta/s \sim 1/4\pi$
- assume local equilibrium for all impact parameters

**azimuthal width:**

- flow dominates
- viscosity effect tiny

agreement with data easier if
we ignore peripheral region
-- nonequilibrium?



# Summary: small viscosity or strong flow?

perfect fluid? need viscosity info!

- viscosity broadens $p_t$ correlations in rapidity
- $p_t$ **covariance** measures these correlations

current correlation data compatible with

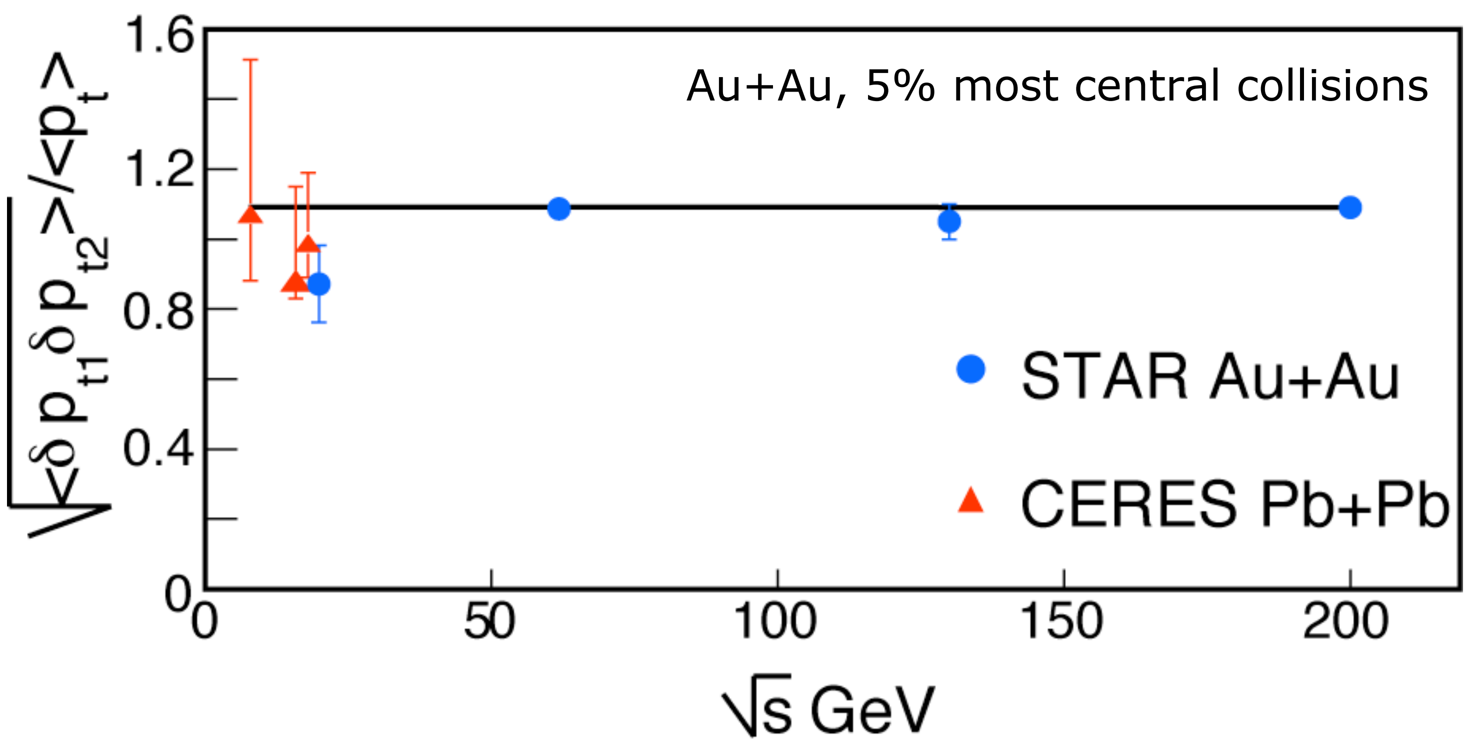
$$0.08 < \eta / s < 0.3$$

comparable to other observables with different uncertainties

untriggered $(\eta, \phi)$ correlations

- flow + viscosity works! hydro OK
- relation of $(\eta, \phi)$ structure to jet-tagged ridge?

# $p_t$ Fluctuations Energy Independent



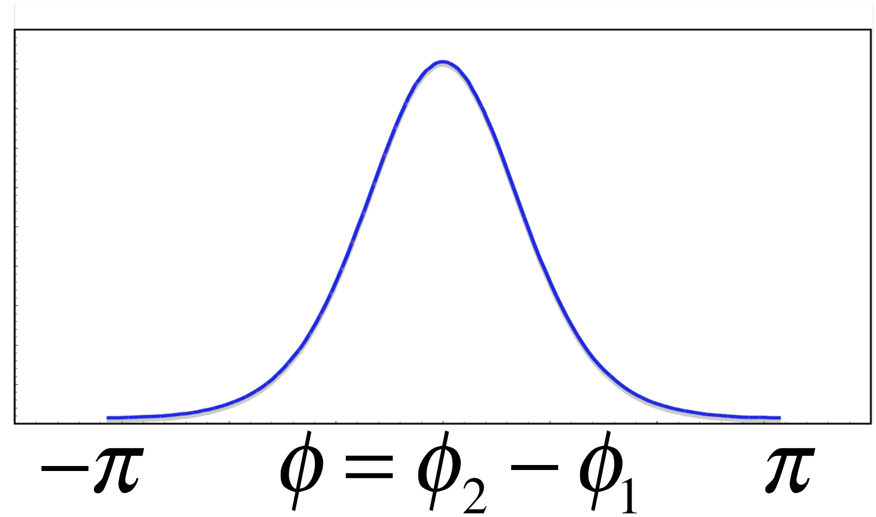

sources of $p_t$ fluctuations: thermalization, flow, jets?

- central collisions ⇒ thermalized
- energy independent bulk quantity ⇒ jet contribution small

# Azimuthal Correlations from Flow

**transverse flow:** narrows angular correlations

- no flow $\Rightarrow \sigma_\phi = \pi/\sqrt{3}$
- $\sigma_\phi \propto 1/v_{rel}$



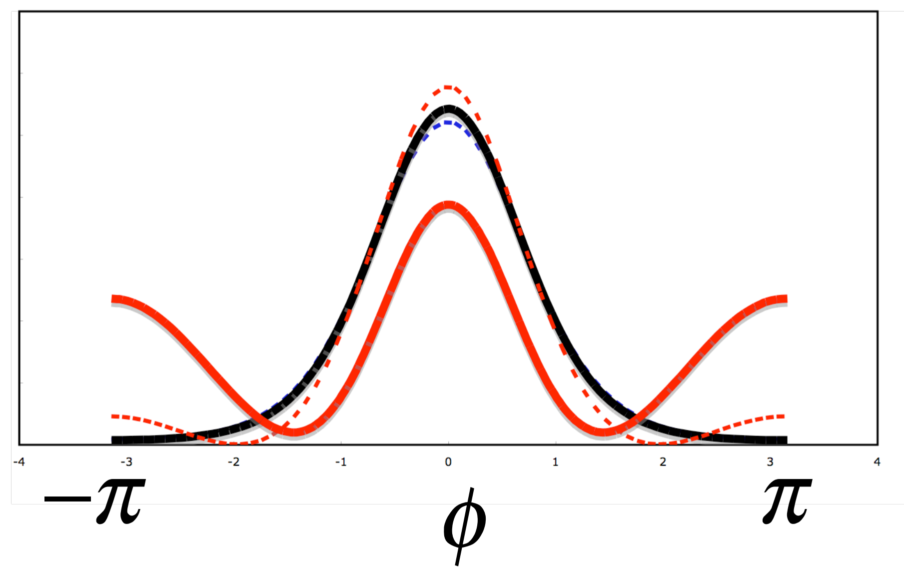

**elliptic flow:**

- $v_2$ contribution
- STAR subtracted

**momentum conservation:**

- $\propto \sin\phi$ ; subtracted

  Borghini, et al

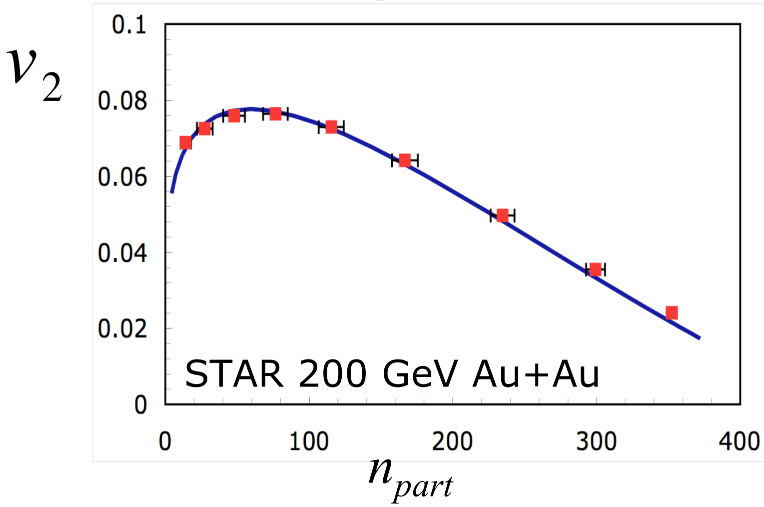



**viscous diffusion:**

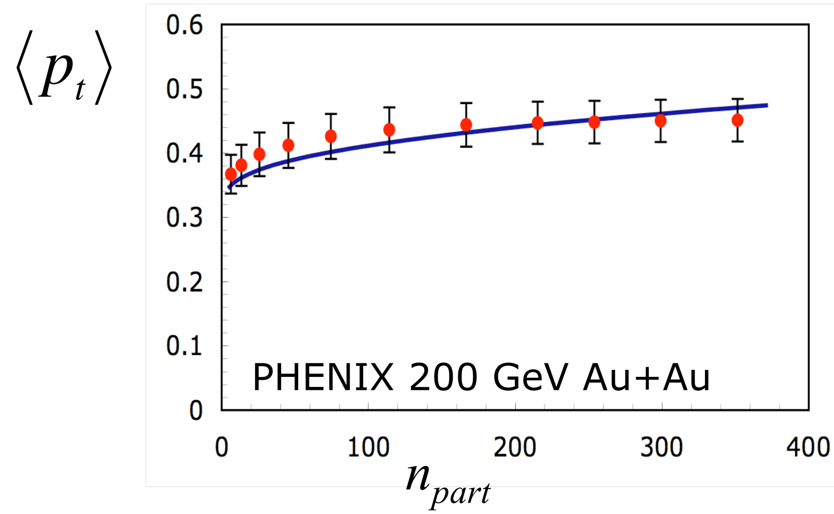
- increases spatial widths $\Sigma_t$ and $\sigma_t$
- $\sigma_\phi \propto (\Sigma_t^2 - \sigma_t^2/4)^{-1/2}$

# Elliptic and Transverse Flow

**elliptic flow**




**transverse flow**




**blast wave:** $n(p) \propto \int f(p,x)\rho(x) \propto \int e^{-\gamma(E-\vec{p}\cdot\vec{v})/T} e^{-r^2/2R^2}$

**transverse plus elliptic flow:** "eccentric" blast wave $\quad \vec{v}_r = \varepsilon_x\, x\hat{x} + \varepsilon_y\, y\hat{y}$

**flow observables:** fix $\varepsilon_x$, $\varepsilon_y$, and $R$ vs. centrality

# Blast Wave Azimuthal Correlations

$$r(p_1,p_2)=\iint_{x_1,x_2} f(p_1,x_1)f(p_2,x_2)r(x_1,x_2)$$

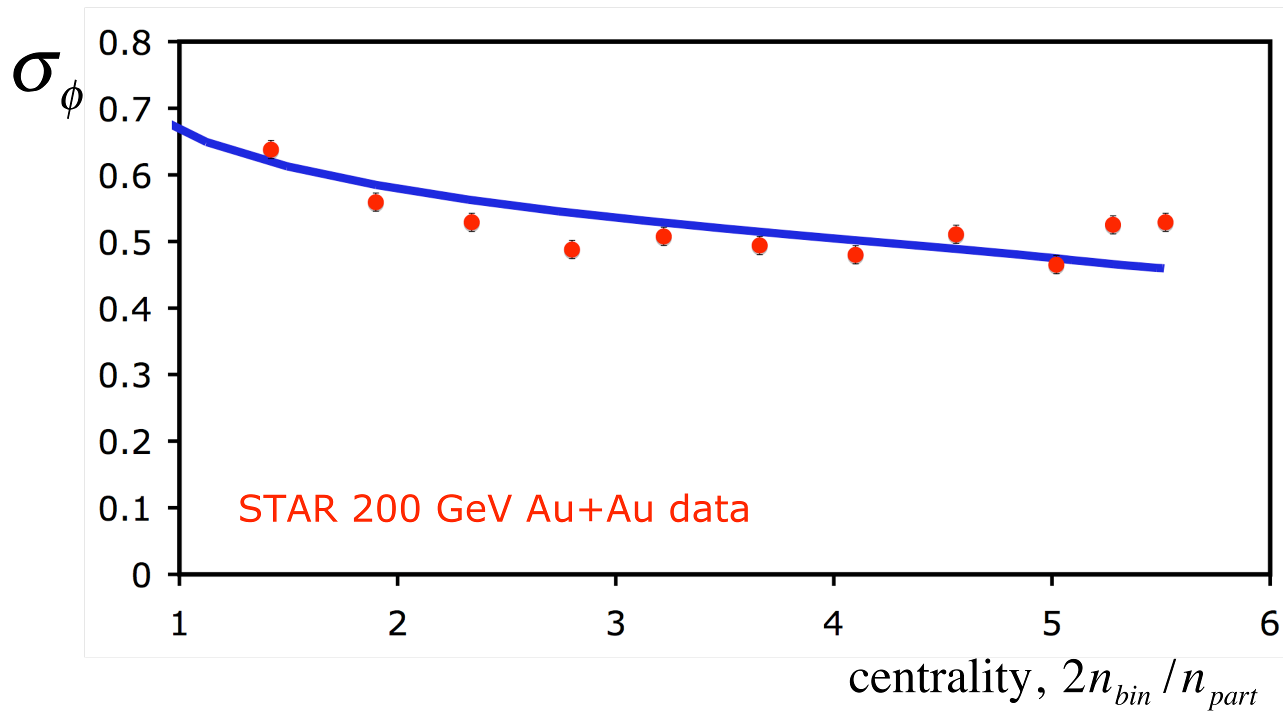
gaussian spatial $r(x_1, x_2)$:

- $\sigma_t$ -- width in $\vec{r}=\vec{r}_{t2}-\vec{r}_{t1}$
- $\Sigma_t$ -- width in $\vec{R}=(\vec{r}_{t1}+\vec{r}_{t2})/2$

azimuthal trend :

- $\Sigma_t \propto$ system size
- $\sigma_t$ constant
- roughly:

$$\langle\Delta\phi^2\rangle \propto \lambda^{-2}(\Sigma_t^2-\sigma_t^2/4)^{-1}$$



# Uncertainty Range

we want: $$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{i \neq j} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2$$

STAR measures: $$\langle N \rangle \Delta\sigma_{p_t:n} = \left\langle \sum_{i \neq j} (p_{ti} - \langle p_t \rangle)(p_{tj} - \langle p_t \rangle) \right\rangle$$

$$= \int dx_1 dx_2 \left[ \Delta r_g(x_1,x_2) - \langle p_t \rangle^2 \Delta r_n(x_1,x_2) \right]$$

momentum density correlations

density correlations

**density correlation function**

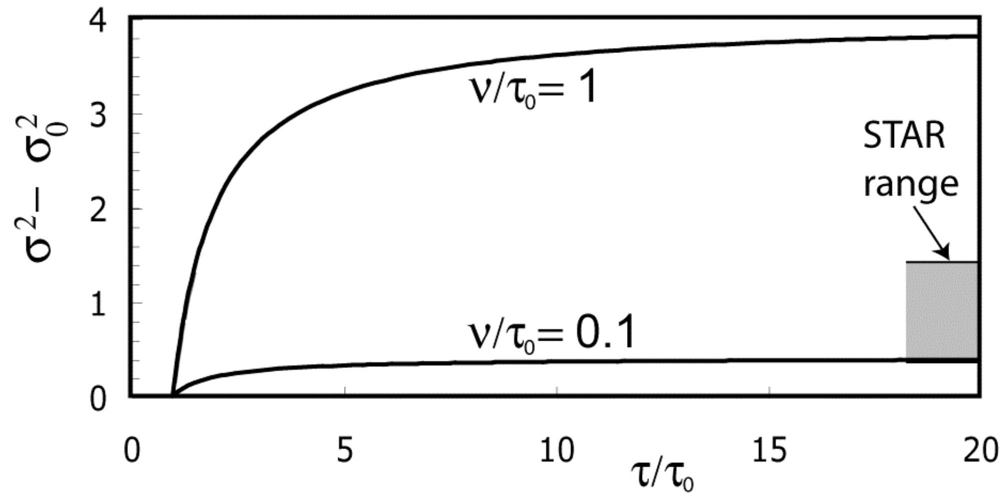
$\Delta r_n = r_n - r_{n,eq}$ may differ from $\Delta r_g$

maybe $\sigma_n \approx 2\sigma_*$

STAR, PRC 66, 044904 (2006)

uncertainty range $\sigma_* \leq \sigma \leq 2\sigma_*$

$\Rightarrow 0.08 \leq \eta/s \leq 0.3$



# Covariance ⇒ Momentum Flux

covariance $$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs } i \neq j} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2$$

unrestricted sum: $$\sum_{\text{all} i,j} p_{ti} p_{tj} = \int p_{t1} p_{t2} dn_1 dn_2$$

$$= \int dx_1 dx_2 \left( \int dp_1\, p_{t1} f_1 \right) \left( \int dp_2 p_{t2} f_2 \right)$$

$$dn = f(x,p) dp dx$$
$$g_t(x) = \int dp\; p_t \Delta f(x,p)$$

$$\rightarrow \int g(x_1) g(x_2) dx_1 dx_2$$

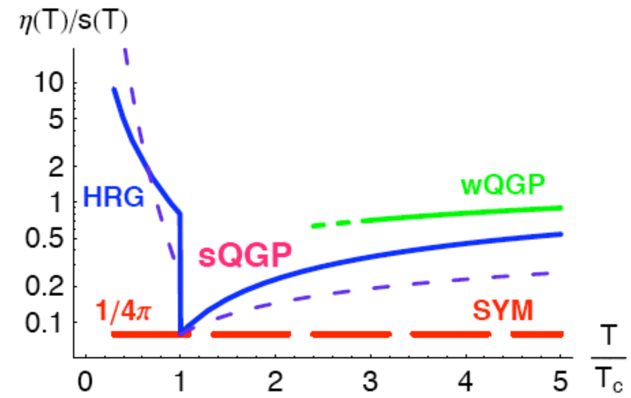
correlation function: $$r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

$$\int r_g dx_1 dx_2 = \left\langle \sum p_{ti} p_{tj} \right\rangle - \langle N \rangle^2 \langle p_t \rangle^2 = \left\langle \sum p_{ti}{}^2 \right\rangle + \langle N \rangle^2 C$$

$C = 0$ in equilibrium ⇒ $$C = \frac{1}{\langle N \rangle^2} \int (r_g - r_{g,eq}) dx_1 dx_2$$

# sQGP + Hadronic Corona

**viscosity in collisions** -- Hirano & Gyulassy



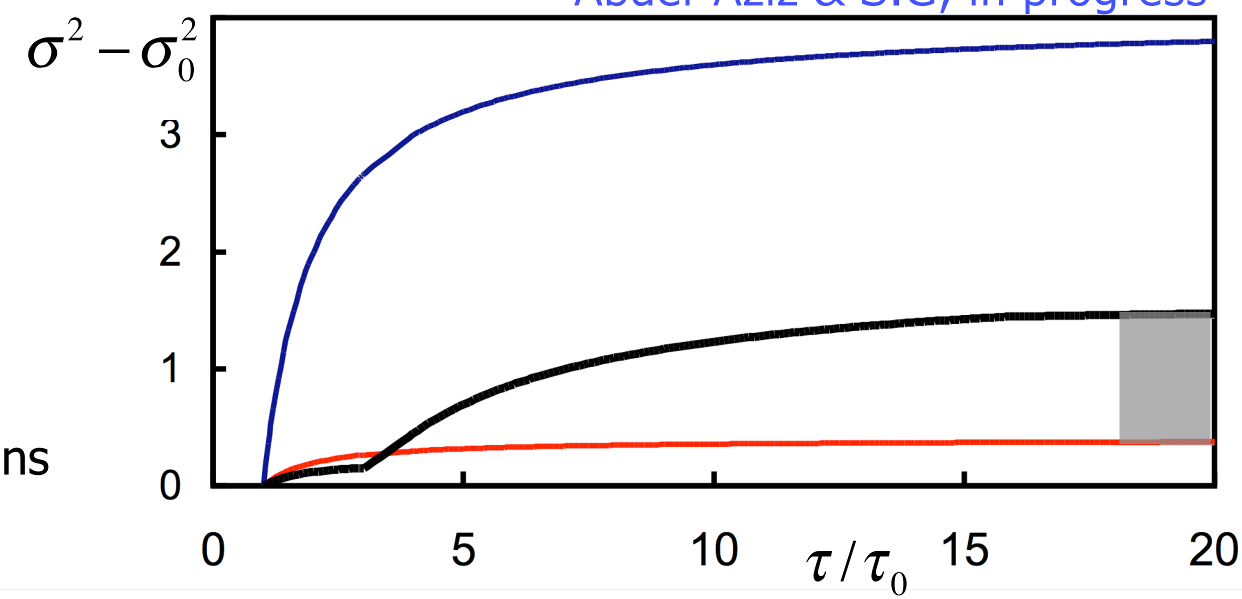

supersymmetric Yang-Mills: $\eta/s = 1/4\pi$

pQCD and hadron gas: $\eta/s \sim 1$

**Broadening from viscosity**

$$\frac{d}{d\tau}\sigma^2 \approx \frac{4\nu(\tau)}{\tau^2},$$

Abdel-Aziz & S.G, in progress




H&G $\rightarrow$ $\nu = T^{-1}(\eta/s)$

QGP + mixed phase + hadrons $\rightarrow T(\tau)$