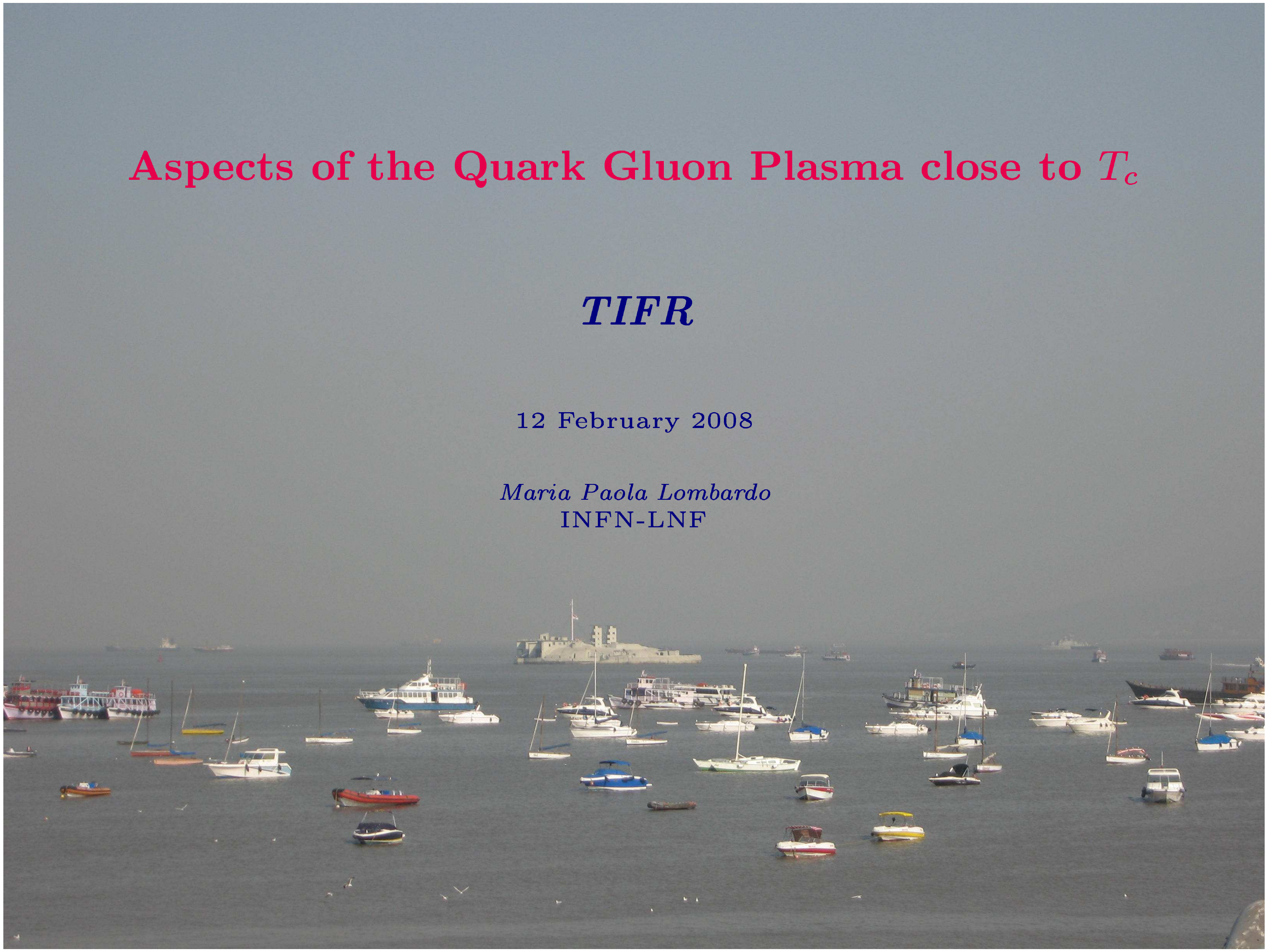


Aspects of the Quark Gluon Plasma close to T_c

TIFR

12 February 2008

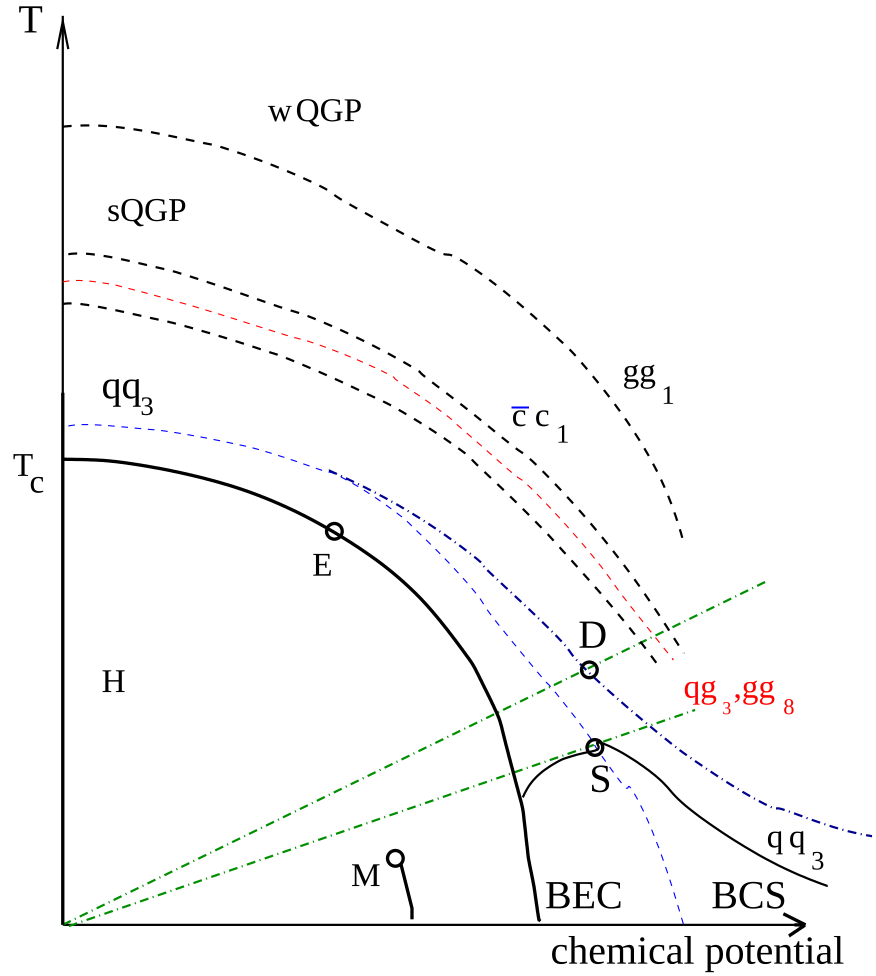
Maria Paola Lombardo
INFN-LNF

Based on
M. D'Elia , F. Di Renzo, M. P. Lombardo, PRD2007
M.D'E, F.D.R, MpL, A. Vuorinen, work in progress

PLAN

- Introduction
- Effects of the critical line on thermodynamics close to T_c
- Comparison with other approaches
  - Taylor expansion
  - Susceptibility analysis
  - Hadron Resonance Gas Model
  - Quasi Particle Models
- Summary, Outlook

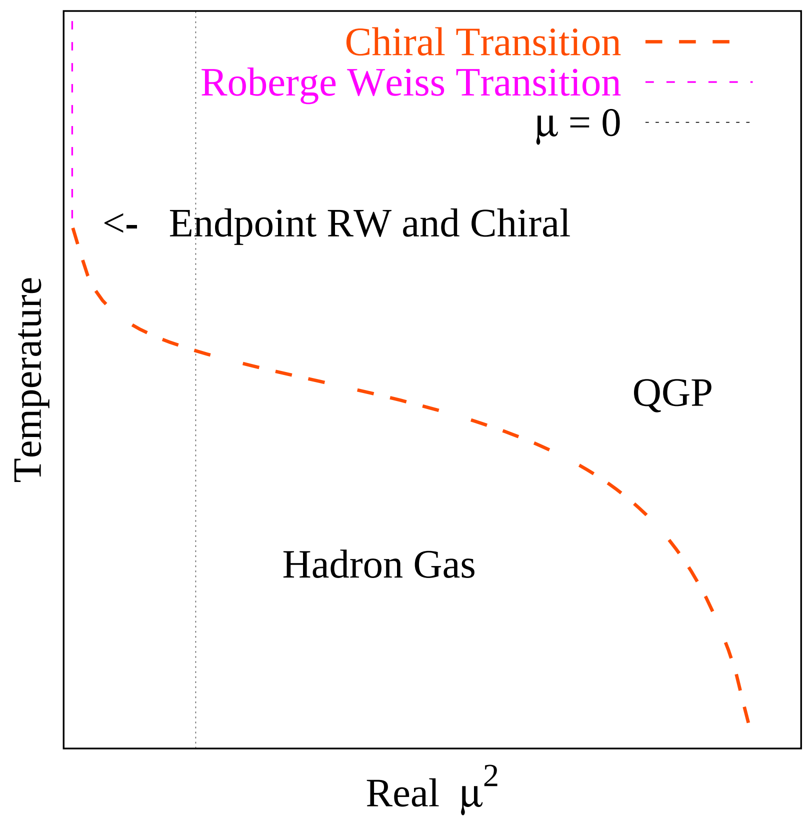
THE QCD PHASE DIAGRAM



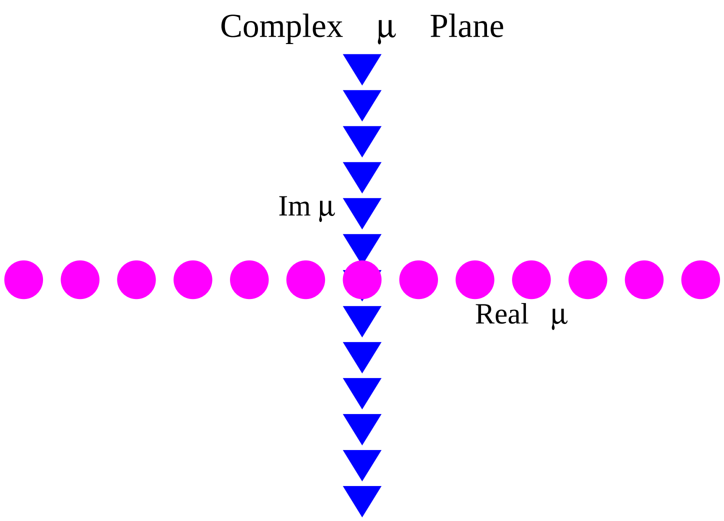
E. Shuryak, 2006, and closing talk at QM2008

...the critical lines continue in the complex μ plane

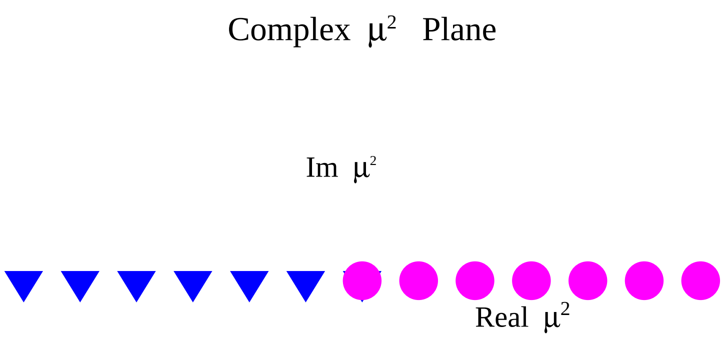
The Phase Diagram in the T, μ_B^2 Plane



Because of the QCD symmetries, the complex μ_B plane

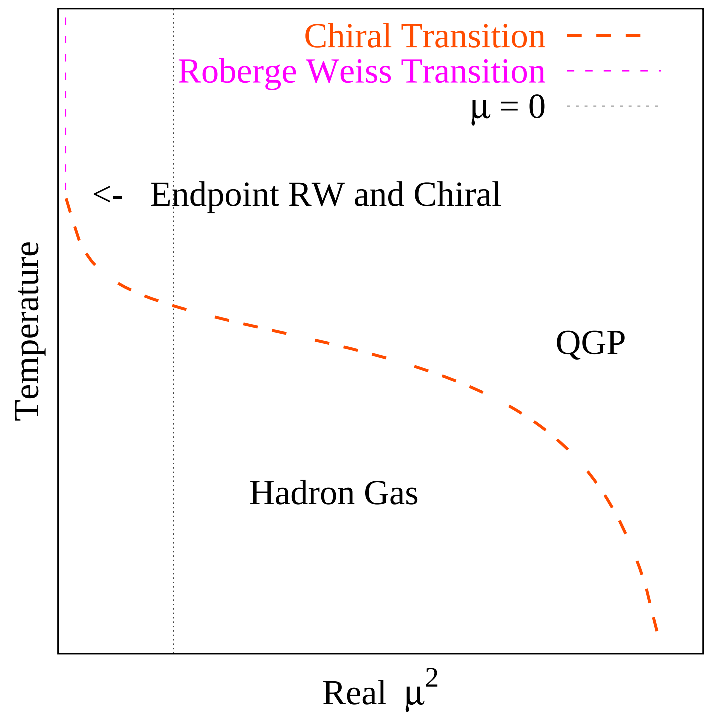


can be mapped onto the complex μ_B^2 plane



QCD at imaginary μ_B at the endpoint of the chiral transition $T = 1.1T_c$:

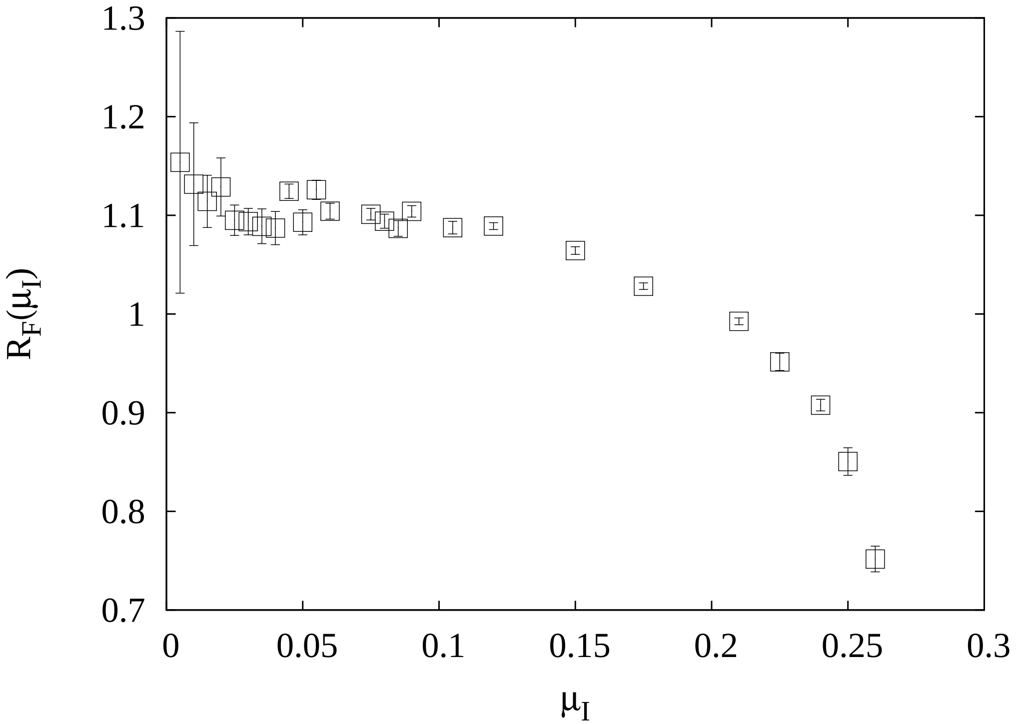
- Direct Access to Critical Behaviour at Non-zero chemical potential
- Insight into Thermodynamics
- Interplay EoS/Critical behaviour



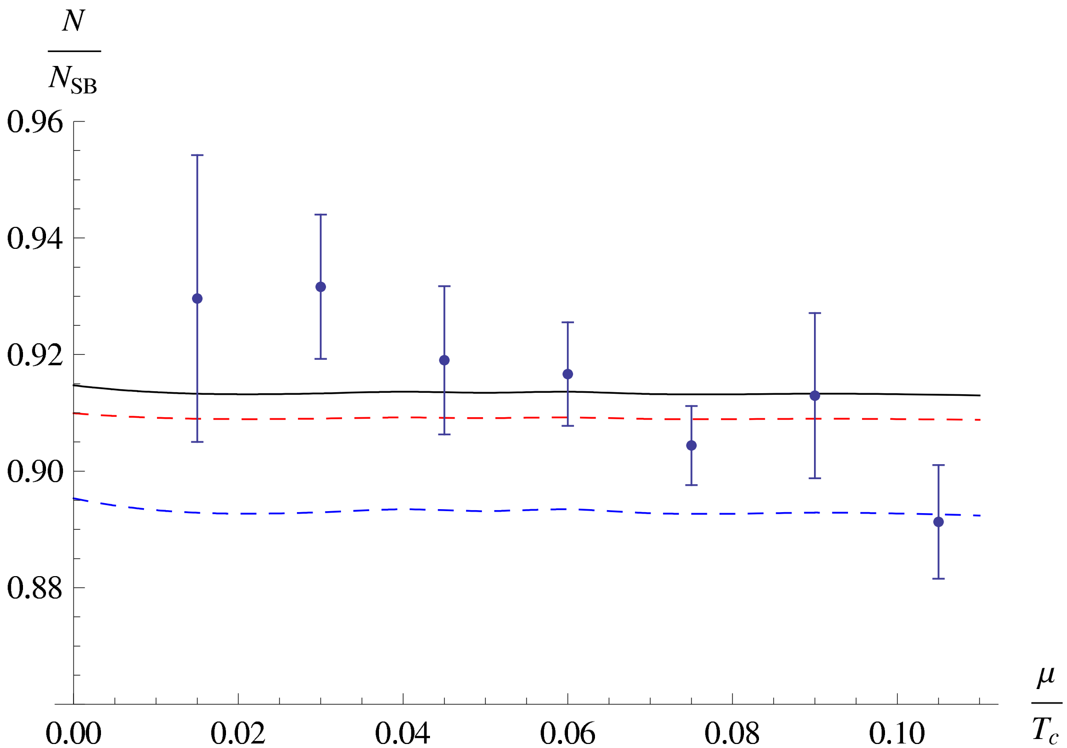
RATIO OF n(μ) TO THE FREE RESULT $n(\mu_I)_{SB}$

$$R_F(\mu_I) = n(\mu_I)/n(\mu_I)_{SB}$$

shows a very clear evidence of a deviation from free field $R_F(\mu_I) = 1$

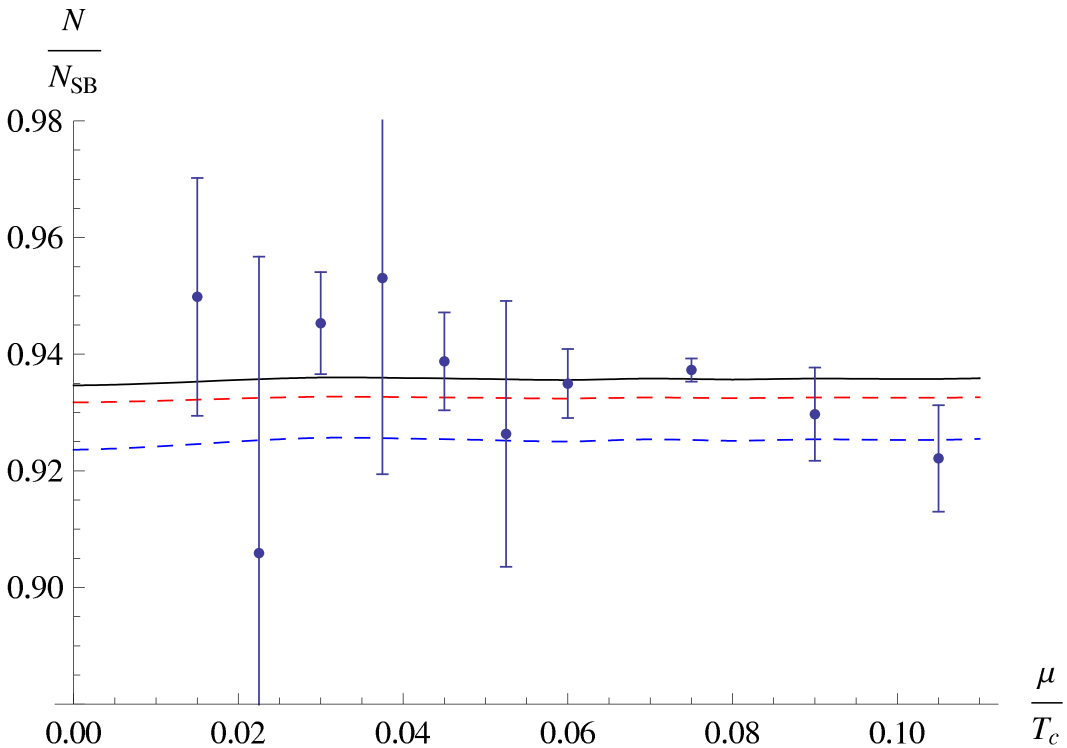


APPROACH TO THE STEFAN-BOLTZMANN LIMIT
ANALYTIC RESULTS VS. LATTICE DATA
Based on A. Vuorinen, 2004



$T = 1.5T_c$

D'Elia, Di Renzo, Lombardo, Vuorinen, in progress

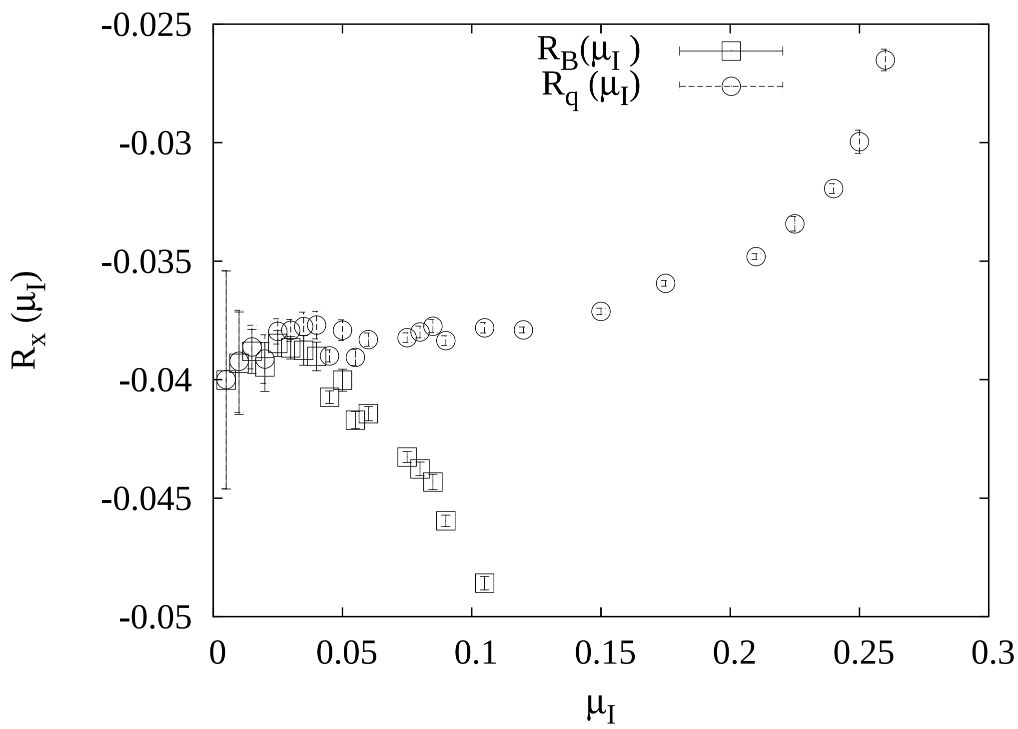


$T = 3.5 T_c$

D'Elia, Di Renzo, Lombardo, Vuorinen, in progress

NAIVE COMPARISON WITH HRG

- $R_B(\mu_I) = \frac{n(\mu_I)}{\sin((3)\mu_I/T)}$
  should be a constant for a simple hadron gas
- $R_q(\mu_I) = \frac{n(\mu_I)}{(3)\sin(\mu_I/T)}$
  should be a constant for a "hadron gas" made of quarks.



CRITICAL FITS AT THE CEP

$$n(\mu_I) = A\mu_I({\mu_I^c}^2 - \mu_I^2)^\alpha \,.$$

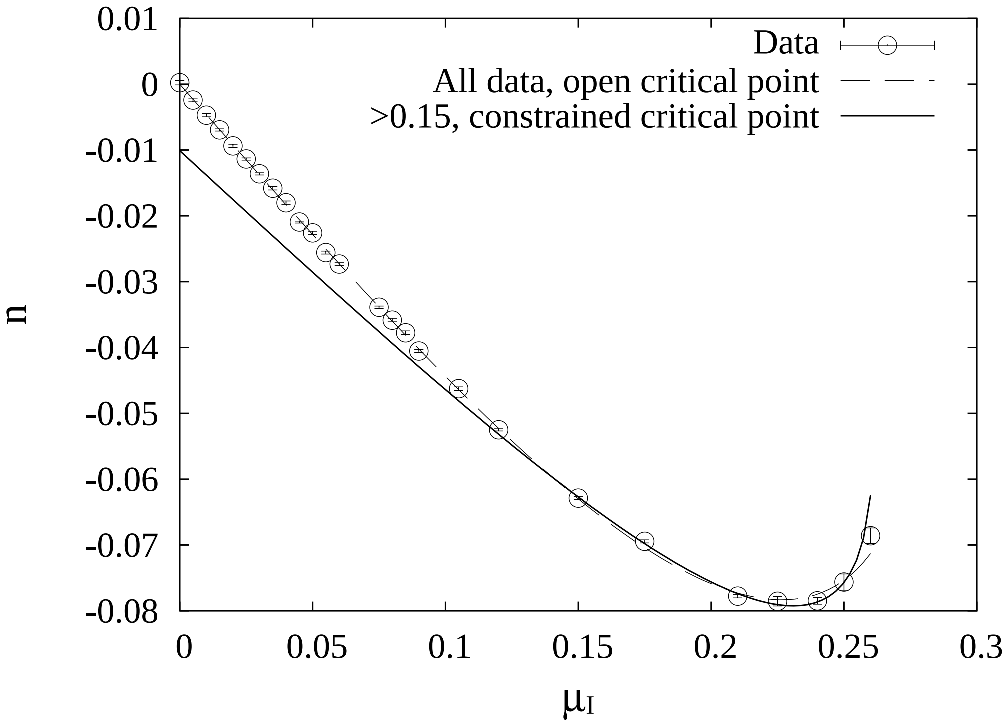
Implying:

$$\chi_q(\mu_I) \propto \frac{1}{({\mu_I^c}^2 - \mu_I^2)^\gamma}$$

where $\gamma = 1 - \alpha$

- A fit to our entire interval with unconstrained μ_I^c gives $A = -0.94(4)$, ${\mu_I^c}^2 = 0.0804(2)$, $\alpha = 0.28(2)$ with a reduced $\tilde{\chi}^2 = 2.4$.
- If we constrain ${\mu_I^c}^2 = (\pi/12)^2$ the quality of the fits decreases giving a reduced $\tilde{\chi}^2 \simeq 12$.
- We checked the stability of these results by choosing different ranges in chemical potential, and we obtained the exponent α ranging between 0.34(8) and 0.26(3), ${\mu_I^c}^2$ between 0.078(4) and 0.091(12), with reduced $\tilde{\chi}^2$ ranging between 1.8 and 5.

RESULTS OF CRITICAL FITS FOR $n(\mu)$ AT THE CEP



FIRST OBSERVATION OF (A) CEP BEHAVIOR IN QCD

$$n(\mu_I) \propto \mu_I({\mu_I^c}^2 - \mu_I^2)^{(\alpha)}$$

FIT I : $\gamma = 0.23(3)\alpha = 0.77(3)$
FIT II : $\gamma = 0.18(2)\alpha = 0.82(2)$

Consistent with a crossover from mean field to CEP

CRITICAL FITS IN THE CRITICAL REGION : Further checks

- If we limit the fitting interval to $\mu_I > 0.15$, we need to add a constant to the function to approximate the regular component.

$$n(\mu_I) = A\mu_I({\mu_I^c}^2 - \mu_I^2)^\alpha \ + B,$$

We obtain $A = -0.54(4), \alpha = 0.14(1), B = -0.010(3)$ a reduced $\tilde{\chi}^2 = 1.79$.

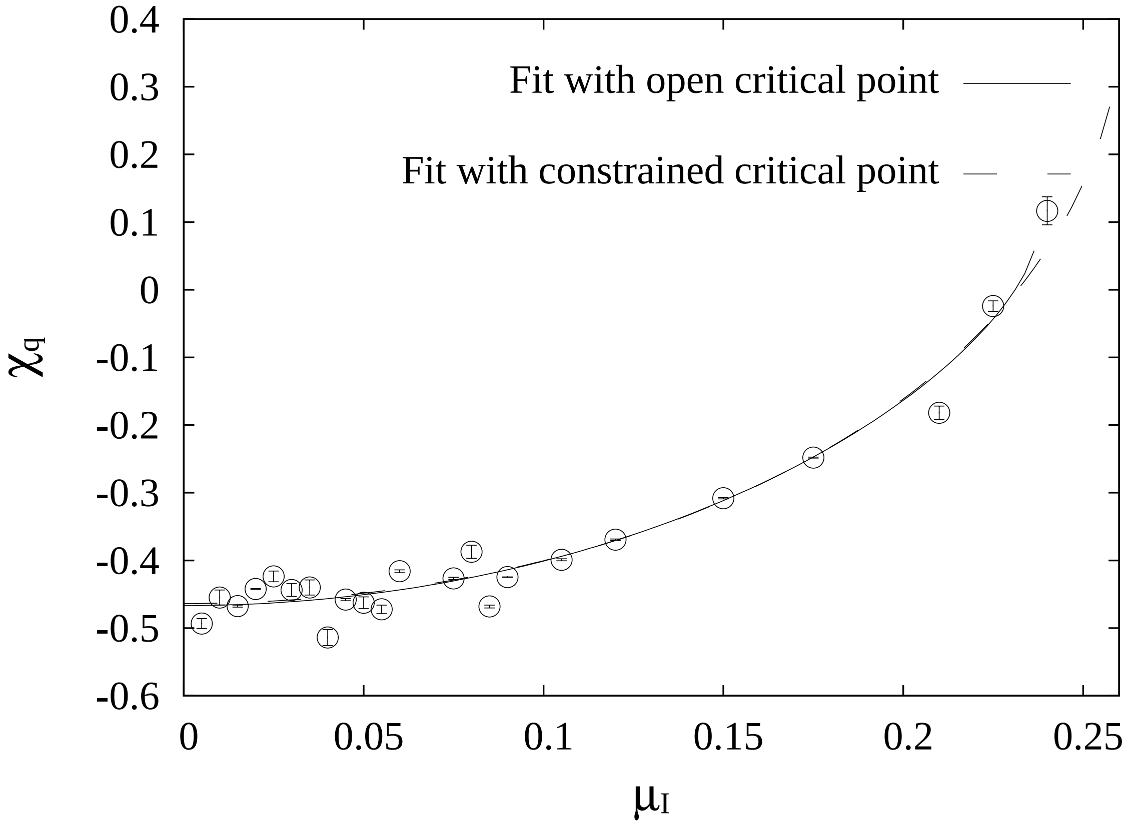
- We also consider a critical behaviour supplemented by a regular linear term

$$n(\mu_I) = A\mu_I({\mu_I^c}^2 - \mu_I^2)^\alpha \ + B\mu_I, .$$

This form of the regular term respects the symmetries of $n(\mu)$ hence can be used for analytic continuation, at variance with the simple modification considered above, when we supplemented $n(\mu)$ by a constant. The results for this fit are $A = -1.04(7)$, $\mu_I^2 = 0.62(7)$, $\alpha = 0.62(7)$ and $B = -0.26(4)$, with a reduced $\chi^2 = 1.74$.

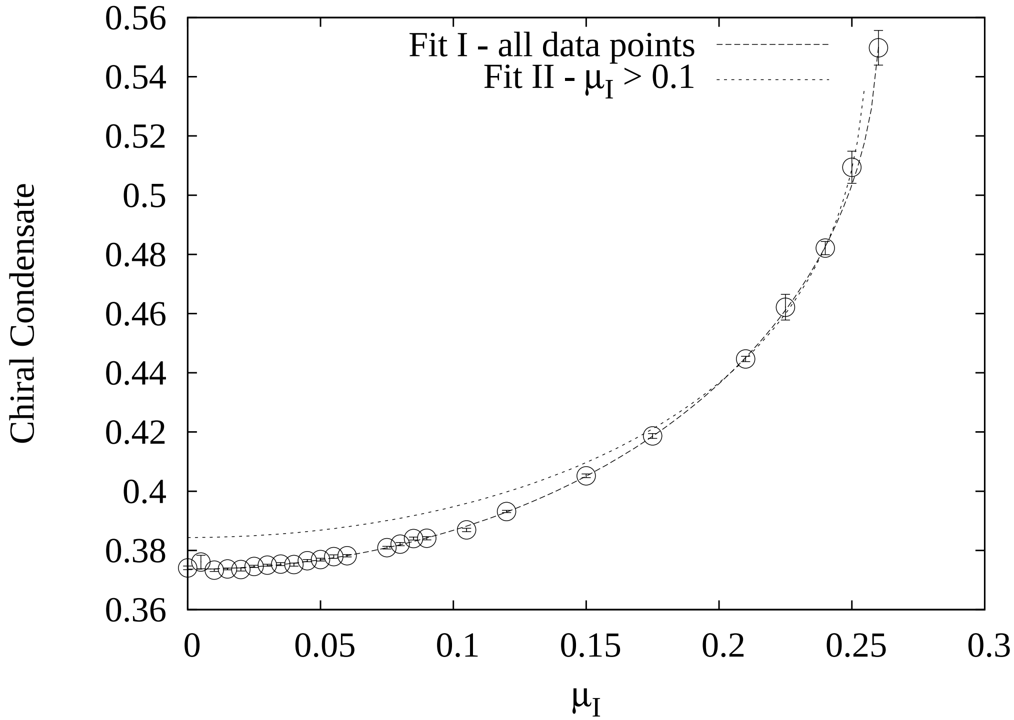
QUARK NUMBER SUSCEPTIBILITY AT A CEP

see Hatta Ikeda, Gavai Gupta for a discussion in QCD



open $\mu_I : \gamma = 0.66(16)$ constrained $\mu_I : \gamma = 0.44(22)$

CHIRAL CONDENSATE



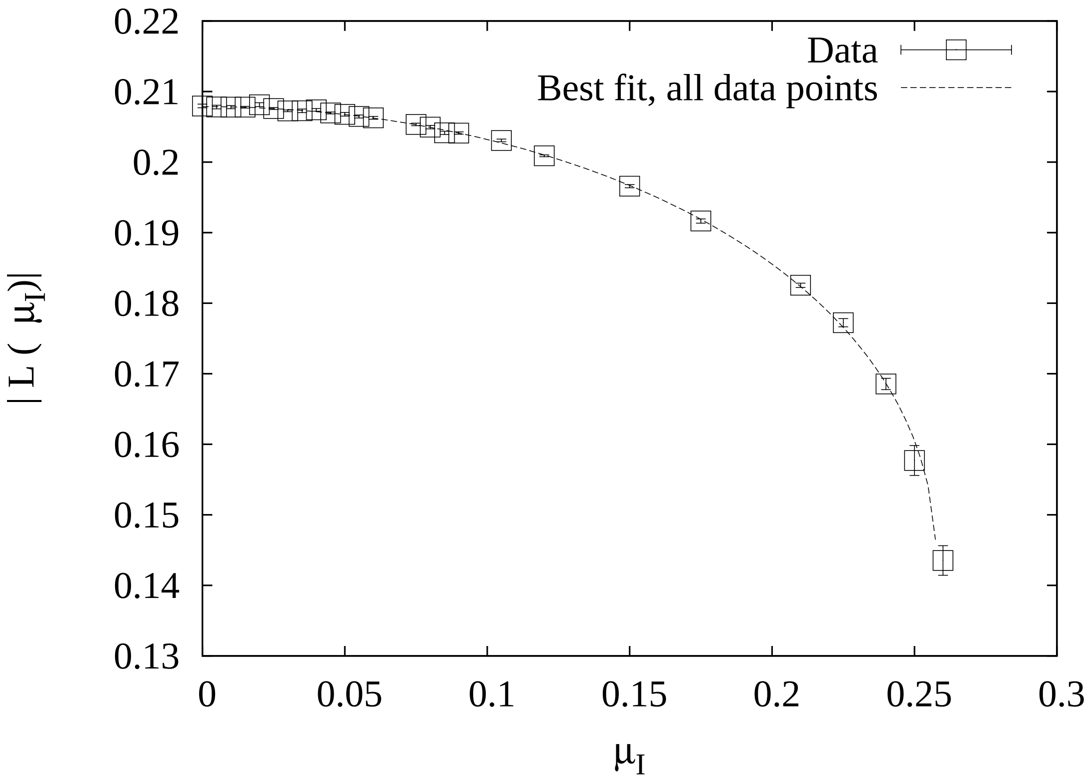
Maxwell relations:

$$\frac{\partial n(\mu, m)}{\partial m} = \frac{\partial < \bar{\psi}\psi > (\mu, m)}{\partial \mu}.$$

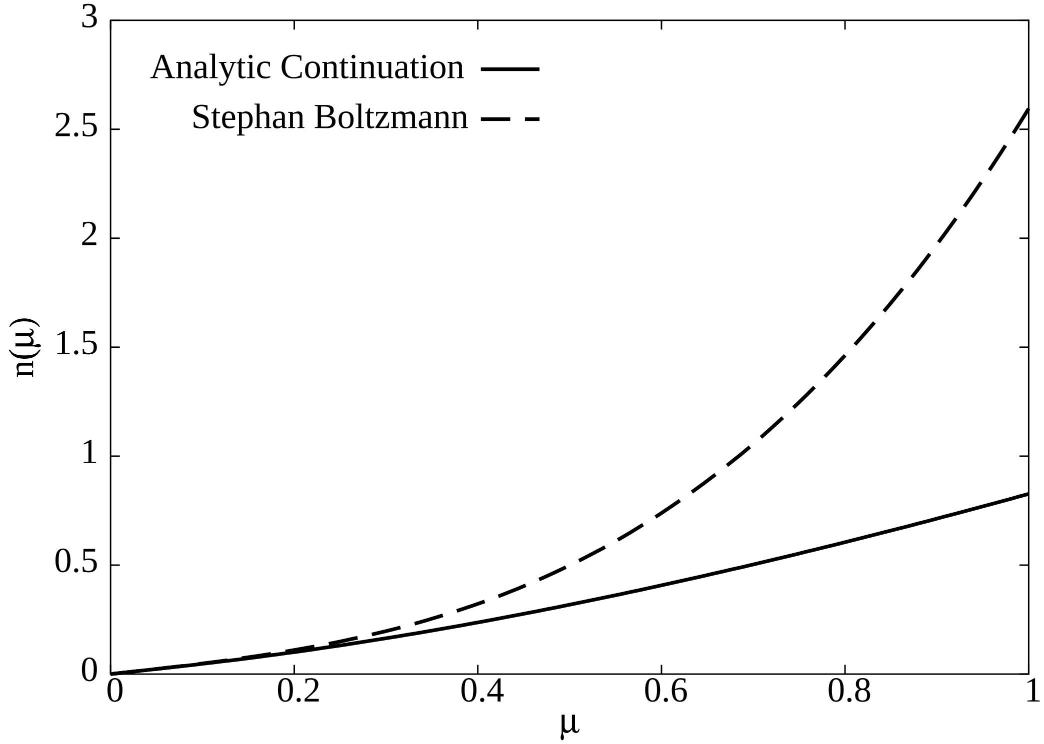
POLYAKOV LOOP

$L = < \mathrm{Tr} P >$

$L(\mu_I) \propto (\mu_I^{c\,2} - \mu_I^2)^{\beta}$



ANALYTIC CONTINUATION: EOS AT $T = 1.1Tc$

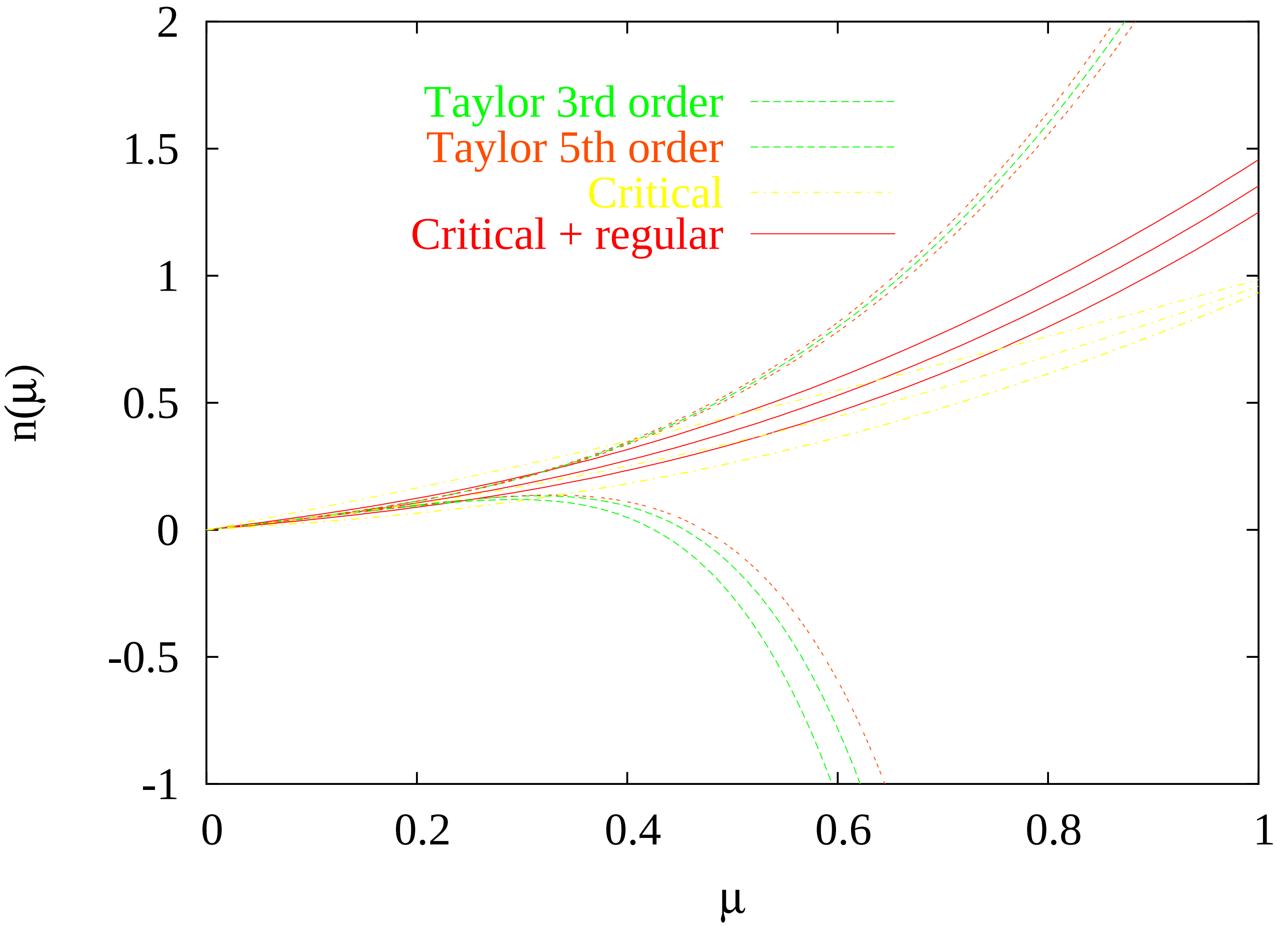


- $n(\mu)$ from analytic continuation, together with a free field behaviour.
- The fits suggest that the slower increase observed in the interacting case with respect to the free case can be described by on overall exponent smaller than one.

COMPARISON WITH OTHER APPROACHES

- Taylor expansion
- Susceptibility analysis
- Hadron Resonance Gas Model
- Quasi Particle Models

......VS. TAYLOR EXPANSION



FINITE RADIUS OF CONVERGENCE OF THE TAYLOR EXPANSION!

.....vs HADRON RESONANCE GAS

Ejiri , Karsch, Redlich proposed the following parametrization for the contribution of the coloured states to the subtracted pressure

$$\Delta P_C = P_C(T,\mu) - P_C(T,0)$$

$$\begin{aligned} \frac{\Delta P}{T^4} &= F_q(T)(\cos(\mu/T)) - 1) + F_{qq}(T)(\cos(2\mu/T) - 1.) \qquad (1) \\ &+ F_{qqq}(T)(\cos(3\mu/T) - 1) + +F_{qqqq}(T)(\cos(4\mu/T) - 1.) \end{aligned}$$

giving in turn:

$$\begin{aligned} n(\mu_I, T) &= F_q(T)\sin(\mu_I/T) + 2F_{qq}(T)\sin(2\mu_I/T) \qquad (2) \\ &+ 3F_{qqq}(T)\sin(3\mu_I/T) + 4F_{qqq}(T)\sin(3\mu_I/T) \end{aligned}$$

Table 1: Parameters of the Trigonometric Fits

F_1	F_2	F_3	F_4	$\chi^2/d.o.f.$
-0.110(1)	–	–	–	84
-0.071(3)	-0.023(2)	–	–	11.11
0.028(15)	- 0.114(14)	0.029(4)	–	4.18
0.43(11)	-0.55(13)	0.257(66)	-0.049(14)	2.85

SIMPLE STRATEGY : FIT TO

$$n_K(\mu_I) = \sum_{j=1}^{K} F_j \sin(j\mu_I/T)$$

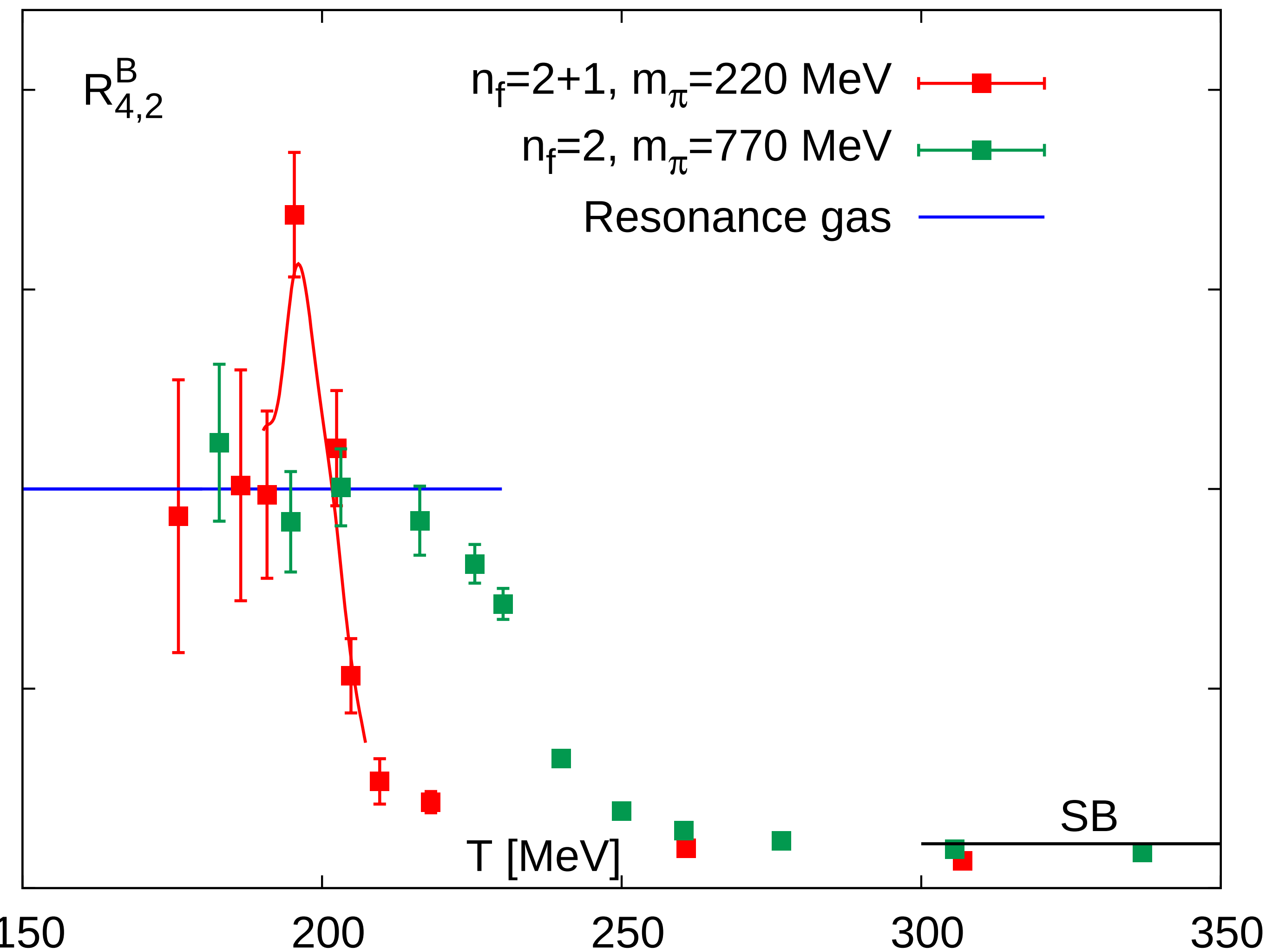
.....vs. SUSCEPTIBILITIES ANALYSIS

- The susceptibilities at zero chemical potential can be easily computed and we recognise that their ratios allow the identifications of the relevant degrees of freedom. **Gavai, Gupta,2001 2008**
- These prediction for the susceptibilities ratio for quarrk,diquark, etc. around T_c was contrasted with the numerical results, finding a poor agreement . **Ejiri, Karsch, Redlich, 2007**
- The derivatives of the masses with respect to the chemical potential should depend on μ:

$$M''(T) = \frac{\partial^2 M(T,\mu)}{\partial \mu^2}(T, \mu = 0)$$

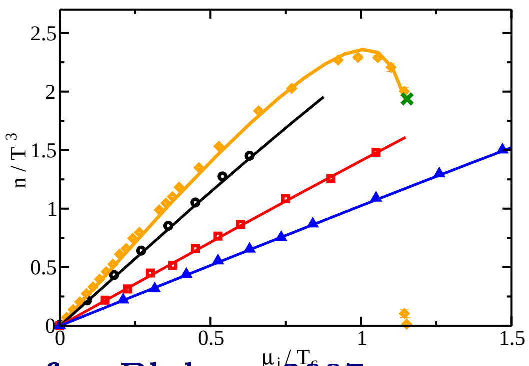
Miao, Shuryak, 2006-2008

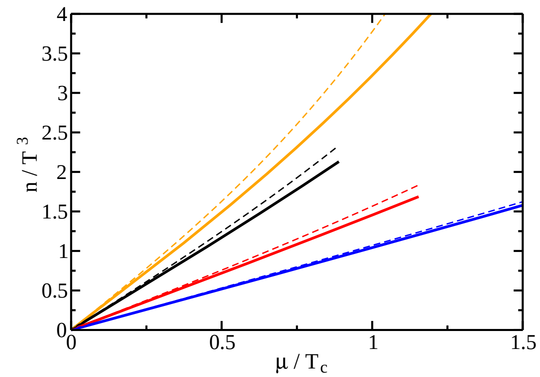
- **When M"(T) is large enough, the simple interpretation of the zero chemical potential susceptibilities as probes of particle contents has to be revised**
- New results for 2+1 Flavor **RBC-Bielefeld collaboration QM2008; presented by C. Schmidt; plot (next pg) courtesy C. Schmidt.**


$R^B_{4,2}$
n_f=2+1, m_π=220 MeV
n_f=2, m_π=770 MeV
Resonance gas
SB
T [MeV]
150
200
250
300
350

..VS QUASIPARTICLE MODELS
Kämpfer, Bluhm Proposal

Quasiparticlemodel vs Imaginary Chemical Potential Lattice Data, and analytic continuation to Real Chemical Potential





Kämpfer, Bluhm , 2007

Crucial ingredients:

- Explicit dependence of the self-energy parts on $\mu_i = \mu_{u,d}$ and T
- Implicit dependence via the effective coupling $G^2(T, \mu_u, \mu_d)$.

$$\omega_i^2 = k^2 + m_i^2 + \Pi_i, \quad \Pi_i = \frac{1}{3}\left(T^2 + \frac{\mu_i^2}{\pi^2}\right) G^2(T, \mu_u, \mu_d) .$$

SUMMARY

- Observed the critical behaviour of the system in proximity of the critical endpoint of the chiral line in the negative μ^2.
- Simple description of the non-perturbative features of the sQGP phase, based on the analysis of the critical behaviour in the imaginary μ plane, with EOS of the form

$$n(\mu) = A\,\mu\,({\mu_I}_c^2 + \mu^2)^\alpha$$

  with $\alpha \simeq 0.3$
- The exponent would read $\alpha = 1$ for a Stefann–Boltzmann-like law.
- The approach to the SB law can be followed within resummed P.T.
- A fit including colored states cannot afford any definite conclusion, suggesting that the masses themselves, hence the coefficients, should depend on μ.
- This conclusion is supported by a recent analysis by Kämpfer and Bluhm within a quasiparticle approach.

OUTLOOK

- improve on quantitative agreement with resummed chiral perturbation theory
- control of the chiral behaviour
- approach to the critical line from the hadronic phase at imaginary μ
- flavor content dependence.