Quark mass effects on the QCD equation of state

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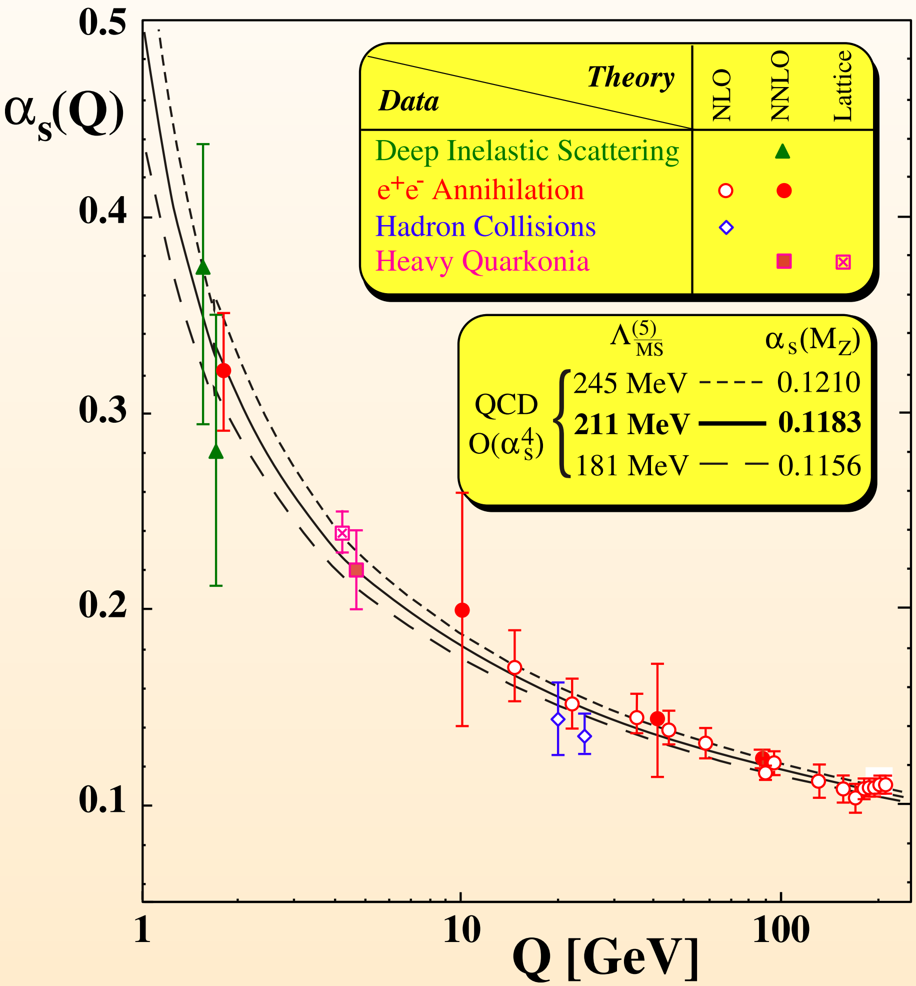
TIFR Mumbai, 12 Feb 2008

Quantum Chromodynamics (QCD)

''zoom'' into part of SM gauge system: QCD



[PDG; LEP EWWG]

- central feature: asymptotic freedom
- smash atoms $\longrightarrow$ e^- get emitted
  basics of our electronics
- smash protons (p) $\longrightarrow$ get more p
  + exotic particles; never a quark
- strong force rises with distance
- quarks closer together (high E)
  $\Rightarrow$ force weaker
- unexpected! (em force opposite)
- beautiful theory result
  Nobel price 2004 G/P/W
- experiment?! ($\leftarrow$ see left)

Quantum Chromodynamics (QCD)

Reality check?!

- outraged claim: none of qu, gl ever seen!
  - ▷ *have to explain confinement*

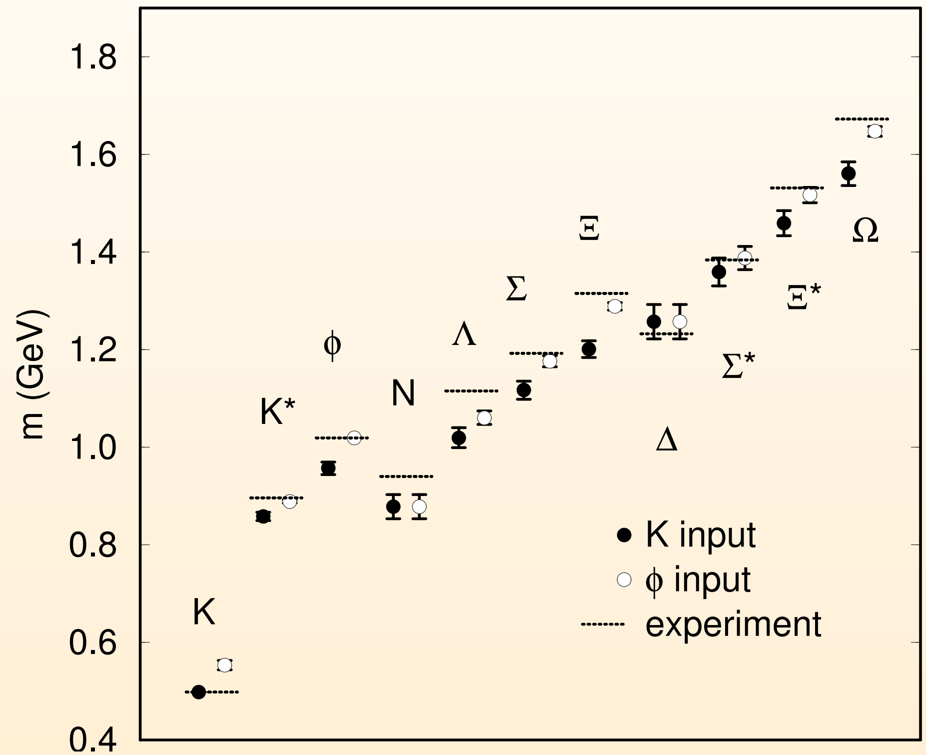
how to check QCD vs Reality?

- (a) just solve its eqs ($\longrightarrow$ see next slide)
  - ▷ *by computer (lattice); tough; ''oracle''; understand?!*
- (b) consider models ''close to QCD''
  - ▷ *fewer dims; different sy groups; diff particle content*
- (c) consider circumstances in which eqs simplify
  - ▷ *remainder of this talk*

QCD reality check (a:computer)

look at hadron spectrum (hadrons: bound states of quarks; e.g. K=$s\bar{d}$, p=uud, Λ=uds)

- solve QCD eqs by computer
  [e.g. S. Aoki et.al., CP-PACS 1999]
- what does not come out:
  - ▷ *gluons*
  - ▷ *fractional charges*
  - ▷ *enlarged multiplets*
- what one gets:
  - ▷ *just the observed particles + masses*
  - ▷ *no more, no less!*



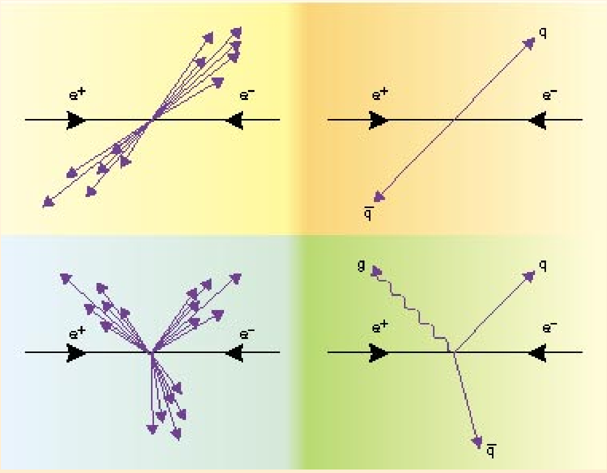
- punchline: obtain amazingly realistic spectrum, with 10% error
  - ▷ *QCD lite; need to add remaining quark effects + quark masses*
  - ▷ *much development here; teraflop speeds, worldwide effort*

QCD reality check (c:collider)

- e.g. LEP, $e^+e^- \to X$ (stuff hitting detector): find 2 broad classes of events (QM!)

- (1) $X = e^+e^-$ or $\tau^+\tau^-$ or … l^+l^-
  - ▷ *leptons: no color charge → mainly QED interactions*
  - ▷ *simple final state: coupling small ($\alpha = e^2/(4\pi) \approx 1/137$) most of the time (99%) nothing happens*
  - ▷ $e^+e^-\gamma \sim$ *1% → check details of QED*
  - ▷ $e^+e^-\gamma\gamma \sim$ *0.01% → …*

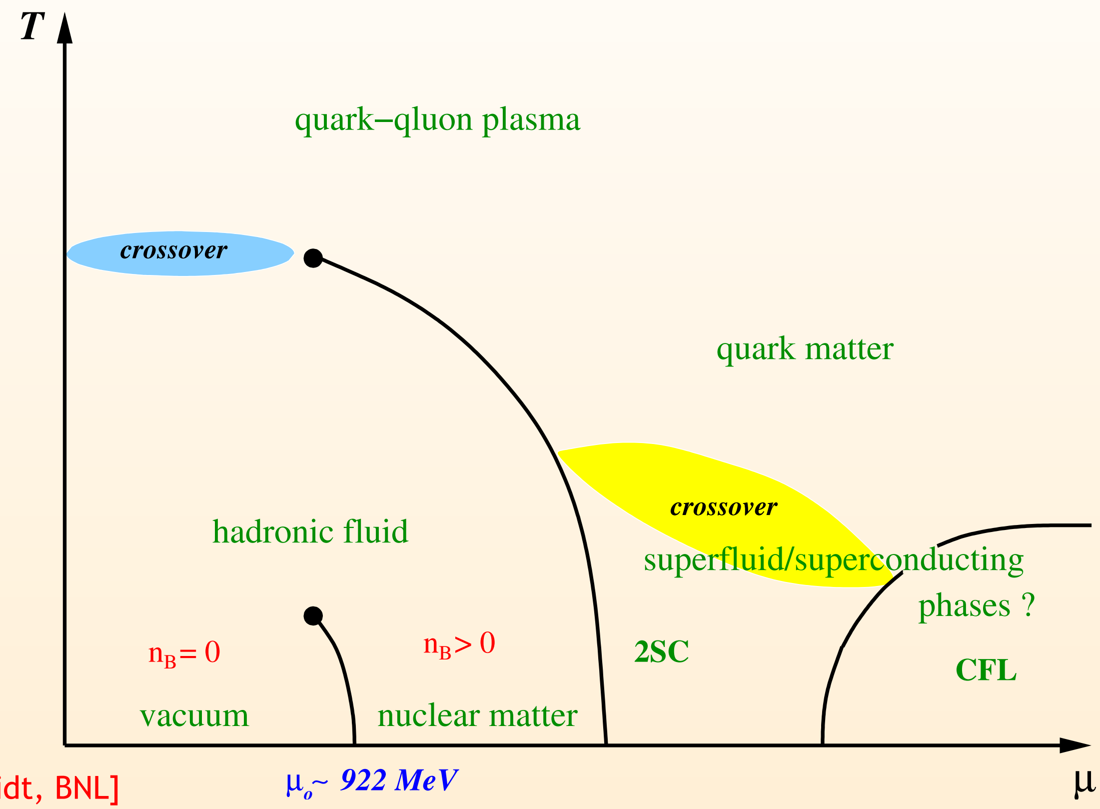
- (2) $X > 10$ particles: π, ρ, p, $\bar{p}$, …
  - ▷ *''greek+latin soup'' constructed from qu+gl*
  - ▷ *pattern: flow of E+momentum in ''jets''*
  - ▷ *2 jets ∼ 90%; 3 jets ∼ 9%; 4 jets ∼ 0.9%*
  - ▷ *direct confirmation of asy. freedom!*
  - ▷ *hard radiation is rare → # of jets*
  - ▷ *soft radiation is common → broadens jet*



- nowadays: ''testing QCD'' → ''calculating backgrounds'' in search for new phenomena

QCD reality check (c:extremes)

childlike questions: what happens when I heat or squeeze matter?



[Ch. Schmidt, BNL]

nature: early univ, μ tiny ($\sim \frac{\#baryons}{entropy}$), $T_c \sim 170MeV \sim 10\mu s$
neutron/quark stars

lab expt.: SPS / RHIC $\mu_B \sim \frac{\#baryons}{pions} \sim 45MeV$ / LHC / GSI

basic thermodynamic observable: pressure $p(T)$

$p(T)$ important for cosmology:

- cooling rate of the universe

$$\partial_t T = -\frac{\sqrt{24\pi}}{m_{\mathsf{Pl}}} \frac{\sqrt{e(T)}}{\partial_T \ln s(T)}$$

- with entropy $s = \partial_T\, p$ and energy density $e = Ts - p$
- $\Rightarrow$ cosmol. relics (dark matter, background radiation etc.) originate when an interaction rate $\tau(T)$ gets larger than the age of the universe $t(T)$.
  - ▷ *Ex.: ''sterile'' ν_R with $m_\nu \sim$ keV can be warm dark matter, and decouple around $T \sim 150$ MeV* *[Abazajian, Fuller 02; Asaka, Shaposhnikov 05]*

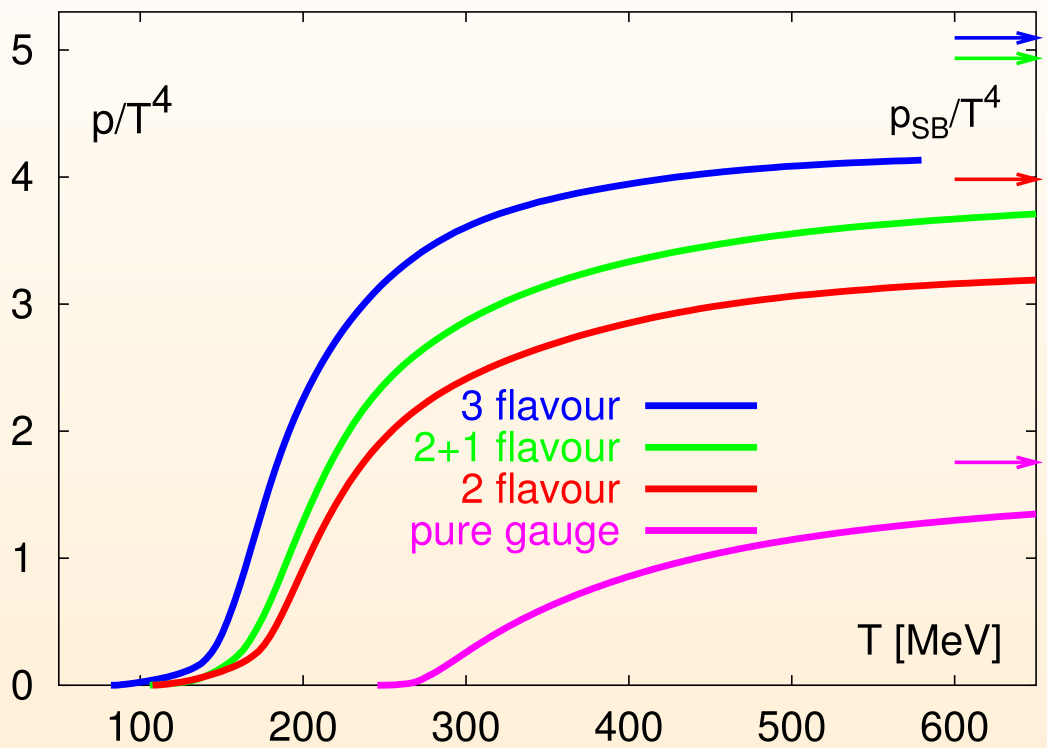
$p(T)$ in heavy ion collisions:

- expansion rate (after thermalization) given by

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad T^{\mu\nu} = [p(T) + e(T)]\, u^\mu u^\nu - p(T) g^{\mu\nu}$$

- with flow velocity $u^\mu(t, x)$
  - ▷ *hydrodynamic expansion: hadronization at $T \sim 100 - 150$ MeV*
    $\Rightarrow$ observed hadron spectrum depends (indirectly) on $p(T)$

$p(T)$ via (large) computer ($\mu_B = 0$)



[lattice data from Karsch et.al.]

at $T \to \infty$, expect ideal gas: $p_{\mathrm{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$

confirms simplicity: 3 dofs (π) $\to$ 52 ($3 \times 3 \times 2 \times 2$ qu + 8×2 gl)

$p(T)$ via weak-coupling expansion

need to explain 20% deviation from ideal gas at $T \sim 4T_c$

structure of pert series is non-trivial !

- Ex.: $$p(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A_\mu^a, \psi, \bar{\psi}] \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x\, \mathcal{L}_{\text{QCD}}\right)$$ $$= g^2 + g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g + \ldots$$

reason: interactions make QCD a multiscale system

dynamically generated scales ($|k| \sim \pi T$ is called ''hard''):
color-electric screening at $|k| \sim m_{\text{E}} \sim gT$ (''soft'')
color-magnetic screening at $|k| \sim g^2 T$ (''ultrasoft'')

expansion parameter

$$g^2\, n_b(|k|) = \frac{g^2}{e^{|k|/T} - 1} \overset{|k| \lesssim T}{\approx} \frac{g^2 T}{|k|}$$

treatment of a multiscale system: effective field theory !

Effective theory prediction for $p(T)$

- collect contributions to $p(T)$ from all physical scales
  - *weak coupling, effective field theory setup*
  - *faithfully adding up all Feynman diagrams*
  - *get long-distance input from clean 3d lattice observable:*

$$p_{\mathsf{G}}(T) \quad \equiv \quad \frac{T}{V} \ln \int \mathcal{D}[A_k^a] \exp\Big(-S_{\mathsf{M}}\Big) = T \# \, g_{\mathsf{M}}^6$$

only one non-perturbative (but computable!) coeff needed

$$\frac{p_{\mathsf{QCD}}(T)}{p_{\mathsf{SB}}} = \frac{p_{\mathsf{E}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{M}}(T)}{p_{\mathsf{SB}}} + \frac{p_{\mathsf{G}}(T)}{p_{\mathsf{SB}}} \quad , \quad p_{\mathsf{SB}} = \left(16 + \frac{21}{2} N_f\right) \frac{\pi^2 T^4}{90}$$

$$= 1 + g^2 \quad + g^4 \quad + g^6 \quad + \ldots \qquad \Leftarrow \text{4d QCD}$$

$$+ g^3 + g^4 + g^5 + g^6 + \ldots \qquad \Leftarrow \text{3d adj H}$$

$$+ \frac{1}{p_{\mathsf{SB}}} \frac{T}{V} \int \mathcal{D}[A_k^a] \exp\Big(-S_{\mathsf{M}}\Big) \qquad \Leftarrow \text{3d YM}$$

$$= c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

[c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03]

Outlook: 08 → 10 → 12

$$\frac{p_{\mathsf{G}}}{p_{\mathsf{SB}}} = \#_{(6)} \left(\frac{g_{\mathsf{M}}^2}{T}\right)^3 + [\delta\mathcal{L}_{\mathsf{M}}]_{(9)}$$

$$g_{\mathsf{M}}^2 = g_{\mathsf{E}}^2 \left[1 + \#_{(7)}\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}} + \left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^2 \left(\#_{(8)} + \#_{(10)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right) + \dots_{(9)}\right]$$

$$\frac{p_{\mathsf{M}}}{p_{\mathsf{SB}}} = \frac{m_{\mathsf{E}}^3}{T^3}\left[\#_{(3)} + \frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\left(\#_{(4)} + \#_{(6)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right) + \left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^2\left(\#_{(5)} + \#_{(7)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2} + \#_{(9)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^2\right)\right.$$
$$+ \left(\frac{g_{\mathsf{E}}^2}{m_{\mathsf{E}}}\right)^3\left(\#_{(6)} + \#_{(8)}\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2} + \#_{(10)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^2 + \#_{(12)}\left(\frac{\lambda_{\mathsf{E}}}{g_{\mathsf{E}}^2}\right)^3\right)$$
$$\left. + [\text{3d 5loop 0pt}]_{(7)} + [\delta\mathcal{L}_{\mathsf{E}}]_{(7)} + [\text{3d 6loop 0pt}]_{(8)} + \dots_{(9)}\right]$$

$$m_{\mathsf{E}}^2 = T^2\left[\#_{(3)}g^2 + \#_{(5)}g^4 + [\text{4d 3loop 2pt}]_{(7)} + \dots_{(9)}\right]$$

$$\lambda_{\mathsf{E}} = T\left[\#_{(6)}g^4 + \#_{(8)}g^6 + \dots_{(10)}\right]$$

$$g_{\mathsf{E}}^2 = T\left[g^2 + \#_{(6)}g^4 + \#_{(8)}g^6 + \dots_{(10)}\right]$$

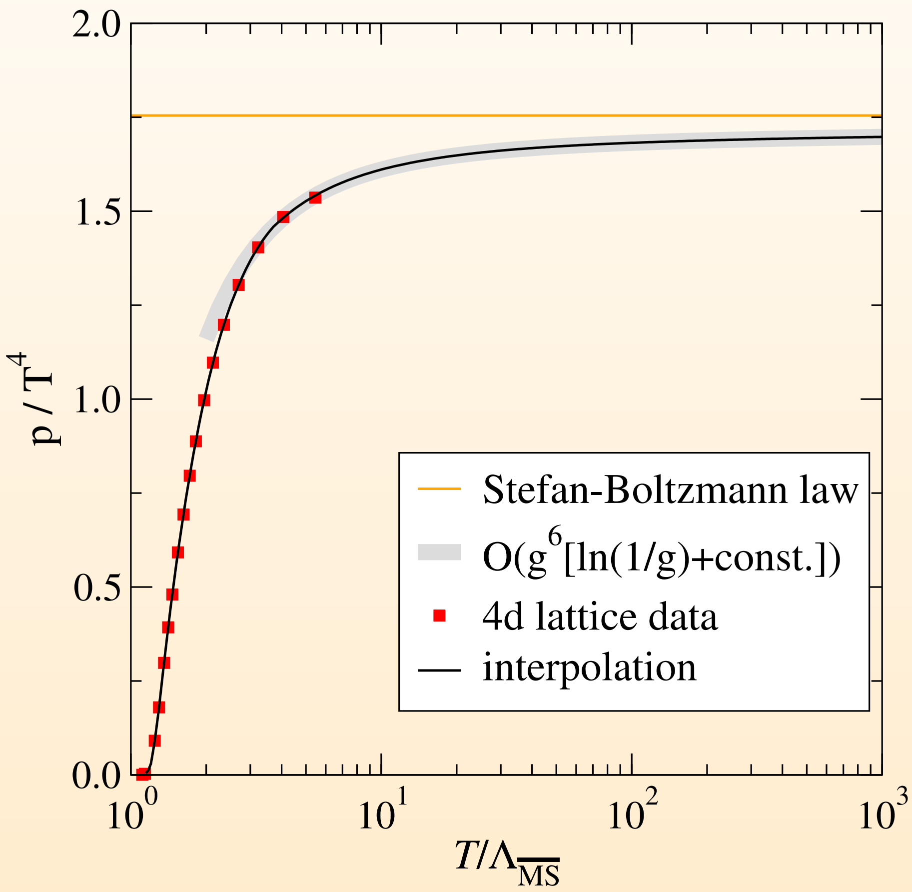
$$\frac{p_{\mathsf{E}}}{p_{\mathsf{SB}}} = \#_{(0)} + \#_{(2)}g^2 + \#_{(4)}g^4 + \#_{(6)}g^6 + [\text{4d 5loop 0pt}]_{(8)} + \dots_{(10)}$$

notation: $\#_{(n)}$ enters p_{QCD} at g^n

[cave: no $\frac{1}{\epsilon} + 1 + \epsilon$, no IR/UV, and no logs shown above]

matching $p(T)$ at $N_f = 0$

want best possible description of pure-glue sector



- fix unknown perturbative $\mathcal{O}(g^6)$ coeff
- use available lattice data
- translate via $T_c/\Lambda_{\overline{\rm MS}} \approx 1.20$
- match at intermediate $T \sim 3 - 5T_c$

Quark mass dependence

analyze quark mass dependence to NLO

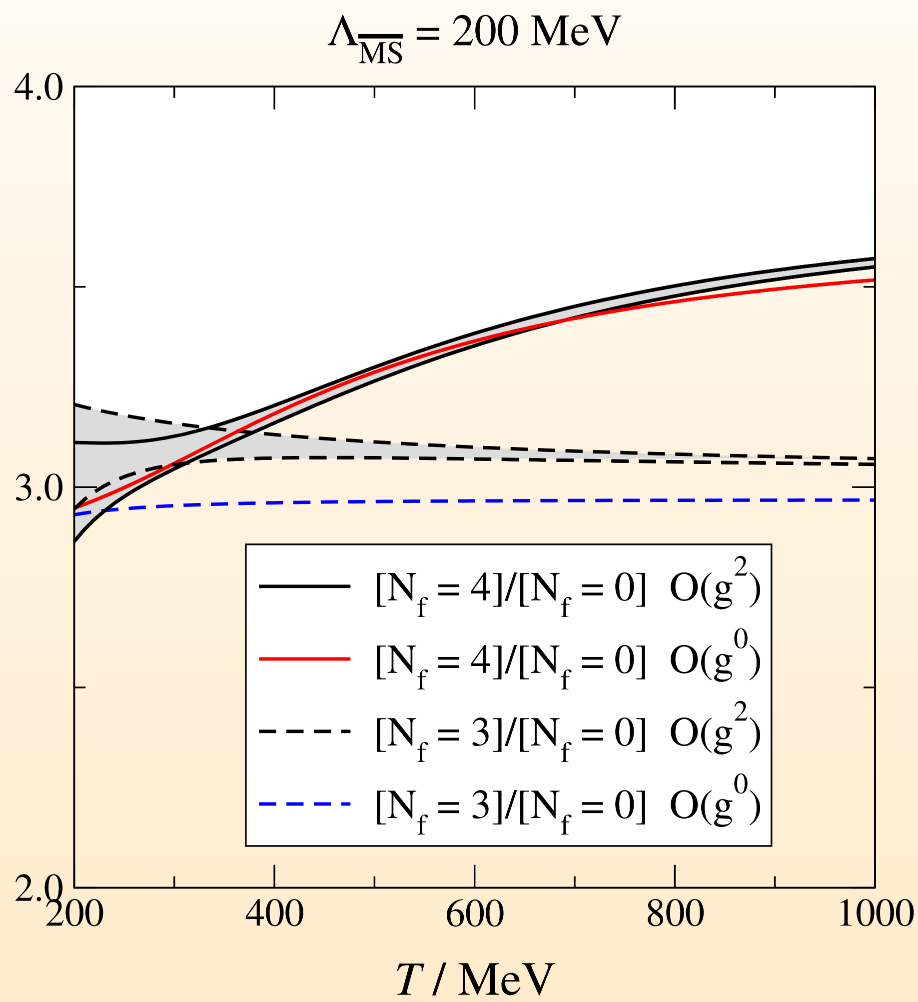
- strategy: ''unquenching''
  start from $N_f = 0$, i.e. $m_q = \infty$
  lower N_f quark masses to $m_{q,phys}$
  p at any T increases

- estimate this ''correction factor''

- aproach is systematic
  LO: $c_0(N_f)/c_0(0)$
  NLO: $[c_0 + g^2 c_2](N_f)$ / $[c_0 + g^2 c_2](0)$

- computed $c_{0,2}(T, N_c, N_f, m_i, \mu_i)$

- good convergence LO$\longrightarrow$NLO
  - ▷ $N_f = 3$: *5% effect*
  - ▷ $N_f = 4$: *even better*



charm quark contributes already at low $T \sim 350 MeV$

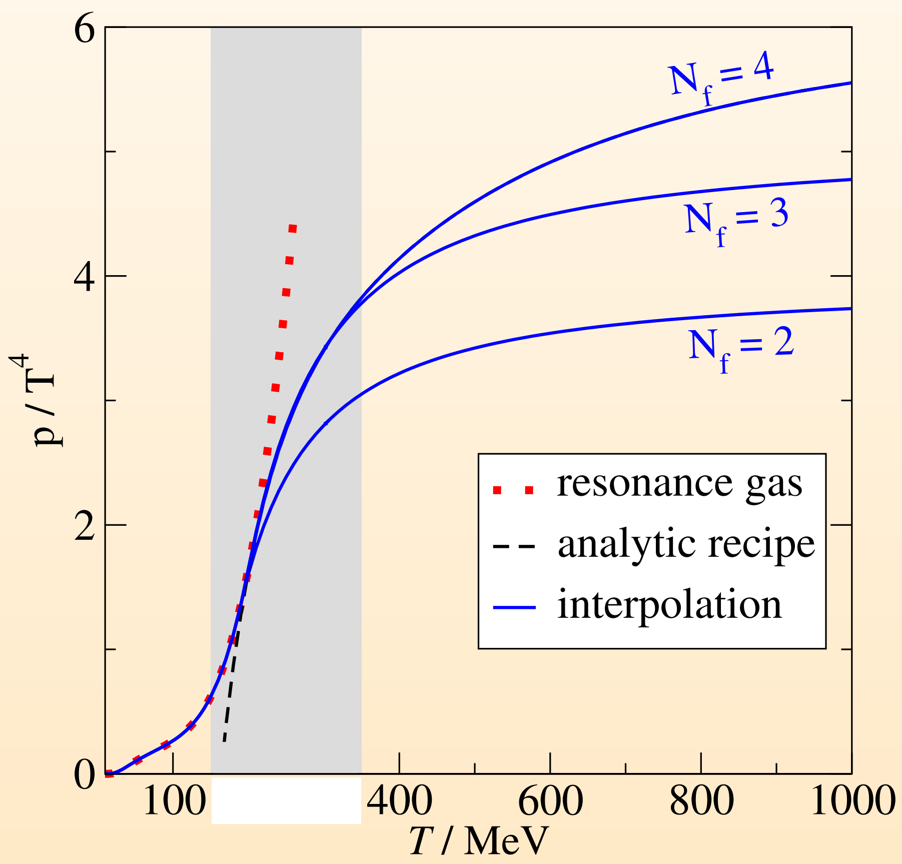
setting the scale

now ready to estimate thermodynamic quantities

multiply best $N_f = 0$ result with correction factor

$$g^2(\bar{\mu}) = \frac{24\pi^2}{(11C_A - 2N_f)\ln(\bar{\mu}/\Lambda_{\overline{MS}})}\,,\quad m_i(\bar{\mu}) = m_i(\bar{\mu}_{\rm ref})\left[\frac{\ln(\bar{\mu}_{\rm ref}/\Lambda_{\overline{MS}})}{\ln(\bar{\mu}/\Lambda_{\overline{MS}})}\right]^{\frac{9C_F}{11C_A-2N_f}}$$

- need to fix $\Lambda_{\overline{MS}}$ in physical units!
- strategy: matching
  take p of hadronic resonances
  match p and p' to our recipe
- obtain $\Lambda^{(eff)}_{\overline{MS}} \approx 175...180MeV$
- shaded: lattice simulations needed!



hadron resonance gas

from PDG, get `http://pdg.lbl.gov/2007/mcdata/mass_width_2006.csv`:

```
*MASS(MeV)     ,Err+    ,Err-    ,WIDTH(MeV)  ,I   ,G,J    ,P,C,A,Chrg,R,S,Name                ,Quarks
1.3957018E+02 ,3.5E-04,3.5E-04,2.5284E-14  ,1   ,-,0    ,-, ,B,   +, ,R,pi                  ,uD
1.349766E+02  ,6.0E-04,6.0E-04,7.8E-06     ,1   ,-,0    ,-,+, ,   0, ,R,pi                  ,(uU-dD)/sqrt(2)
5.4751E+02    ,1.8E-01,1.8E-01,1.30E-03    ,0   ,+,0    ,-,+, ,   0, ,R,eta                 ,x(uU+dD)+y(sS)
8.0E+02       ,4.0E+02,4.0E+02,8.0E+02     ,0   ,+,0    ,+,+, ,   0, ,R,f(0)(600)           ,Maybe non-qQ
7.755E+02     ,4.0E-01,4.0E-01,1.4940E+02  ,1   ,+,1    ,-, ,B,   +, ,R,rho(770)            ,uD
...
```

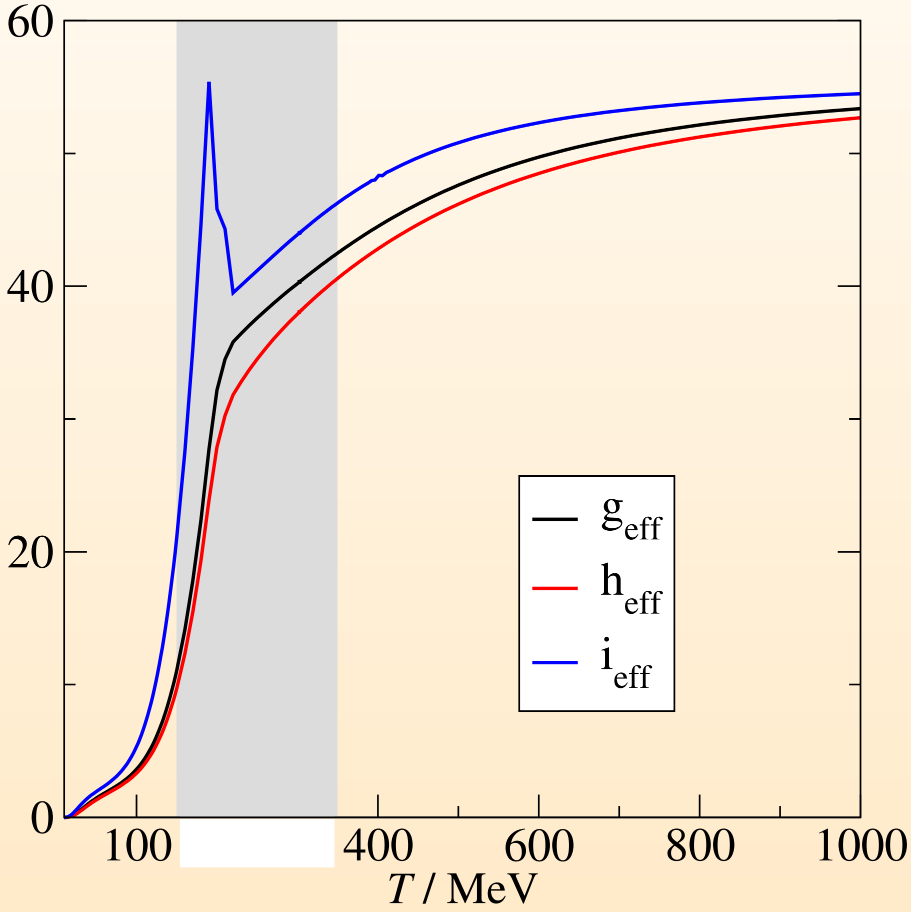
extract list of Mesons and Baryons, incl masses + deg.factors

$$\frac{p_{had}(T)}{T^4} = \sum_{i\in Baryons} \frac{d_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left(1+e^{-\sqrt{p^2+m_i^2/T^2}}\right) - \sum_{i\in Mesons} \frac{d_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left(1-e^{-\sqrt{p^2+m_i^2/T^2}}\right)$$

thermodynamic quantities

now use the recipe $\boxed{p(N_f{=}0) \times corr.fct}$ to obtain

$$s(T) = p'(T)\ ,\ \ e(T) = Ts(T) - p(T)\ ,\ \ c(T) = e'(T) = Tp''(T)$$



- use eff numbers of bosonic dof's

  $g_{\text{eff}}(T) \equiv e(T) / \left[\frac{\pi^2 T^4}{30}\right]$

  $h_{\text{eff}}(T) \equiv s(T) / \left[\frac{2\pi^2 T^3}{45}\right]$

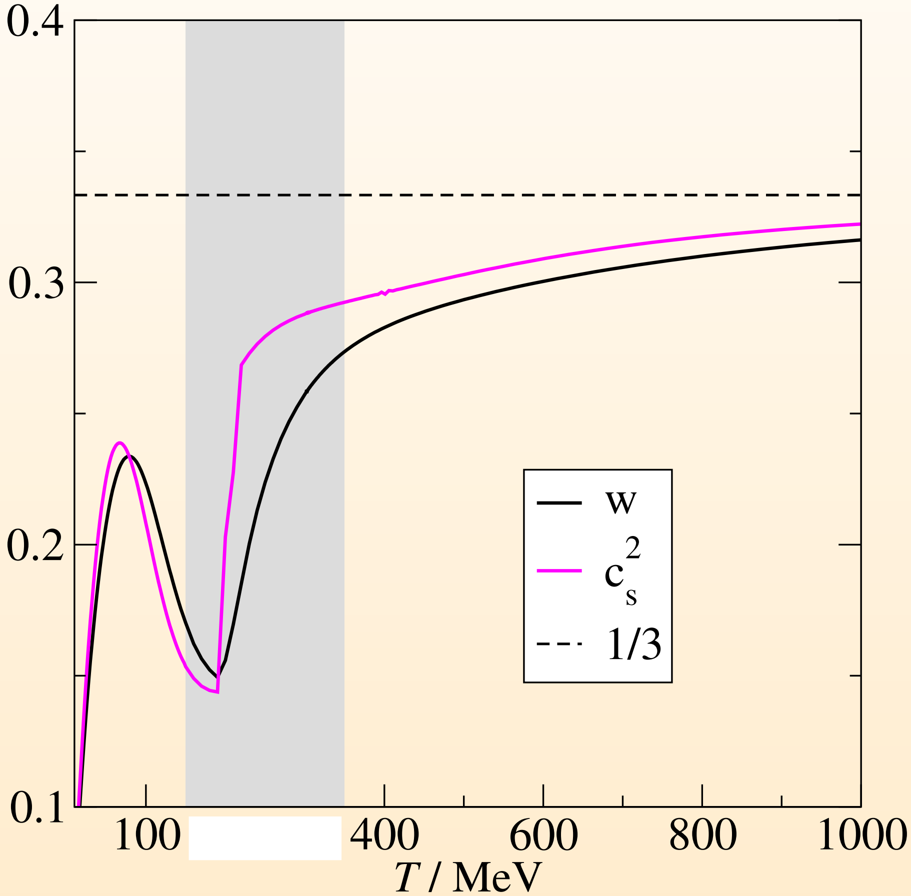
  $i_{\text{eff}}(T) \equiv c(T) / \left[\frac{2\pi^2 T^3}{15}\right]$

- observe significant structure

- at 2nd order phase transition $i(T) \sim (T - T_c)^{-\gamma}$

Equation of state

consider dimensionless ratios of thermodynamic quantities



- equation of state

$$w(T) \equiv \frac{p(T)}{e(T)} = \frac{p(T)}{Tp'(T) - p(T)}$$

- sound speed (squared)

$$c_s^2(T) \equiv \frac{p'(T)}{e'(T)} = \frac{p'(T)}{Tp''(T)} = \frac{s(T)}{c(T)}$$

- $\left(\frac{1}{3} - w(T)\right) \propto$ ''trace anomaly''
(or ''interaction measure'')

peak around 70MeV not (yet) visible in lattice simulations

Summary

- QCD contains an extremely rich structure
- thermodynamic quantities of QCD are relevant for cosmology and heavy ion collisons
- these quantities can be determined numerically at $T \sim 200$ MeV, and analytically at $T \gg 200$ MeV; multi-loop sports, eff. theories convenient
- effective field theory opens up tremendous opportunities: analytic treatment of fermions, universality, superrenormalizabilty
- quark mass dependence shows good convergence
- charm quark contributes already at fairly low T
- need reliable lattice simulations in transition region