TIFR Annual Report 2004-05

THEORETICAL PHYSICS

Condensed Matter and Statistical Physics

Research in this field was conducted in the general areas of non-equilibrium statistical physics, disordered systems, high temperature-superconductivity and strongly correlated systems.

Clustering in particles advected by a fluid

The problem of particles advected by a Burgers flow can be mapped onto particles sliding on a fluctuating surface. An unusual kind of phase ordering with strong fluctuations occurs in the presence of hard-core repulsion between particles. Monte-Carlo simulations and analytic calculations on a surface model have shown that in steady state, the density auto-correlation function has a cusp singularity at small values of time (scaled appropriately with system size); this is in contrast to the linear decay seen in ordinary equilibrium phase-ordering systems. On the other hand, the approach to steady state was seen to be characterized by an auto-correlation function which is a function of time scaled by a time-dependent domain size. The scaling function, for large argument, shows a power law decay, while for small argument, it shows a cuspy decay as in the steady state .

If the hard core constraint is removed and the passive scalars are non-interacting, a new kind of steady state with extreme clustering was observed. Monte-Carlo simulations on a lattice model show that the two-point density correlation function is a scaling function of distance divided by system size and diverges at small argument. This is an indicator of very strong clustering. Further, the effect of changing the ratio of particle to surface updates was studied and it was seen that the strong clustering steady state is robust, with universal scaling exponents. The details of the scaling function, however, do depend on the ratio of updates, and one sees stronger clustering as one increases the particle update speed. [Apoorva Nagar, Sakuntala Chatterjee, M. Barma (TIFR) and S. N. Majumdar (Universit'e de Paris - Sud, Orsay, France)]

Entropy driven ordering in a 2-D system of hard rods

An assembly of infinitely thin hard rods (needles) is a simple model of a system with many long molecules e.g. liquid crystals. This 2-D model was studied using Monte Carlo simulations, allowing for deposition and evaporation of rods from a substrate. The ratio of deposition to evaporation rates determines the equilibrium density and is the control parameter in this problem. If the ratio is spatially uniform, there is a phase transition from an istropic phase to a phase with power law decay of nematic correlations.

Interesting new effects were observed if the deposition-evaporation ratio is spatially inhomogeneous, as for instance induced by variations of substrate temperature. The simulations showed that needles tend to align along the local spatial gradient of the ratio. The root of this effect is entropic: an average alignment of needles along the gradient is preferred as it leads to an enhancement of entropy of rotational excursions around the mean. In effect, the gradient in density (along one direction only) acts like a field that induces the orientational ordering of the needles. This was illustrated by a simple 1-D model where needles are found to orient preferentially perpendicular to the line. In the framework of continuum theory, the gradient suggests new terms in the Frank-like free energy which imply orientational ordering. Finally, random variations of deposition-evaporation rates in both directions induce frustration, resulting in a state with glassy characteristics. [M. Khandkar and M. Barma]

Linear and Branched Polymers on Fractals

Determination of the behavior of linear and branched polymers on deterministic fractals using exact renormalization group techniques provides a very useful way to understand these theoretically. It also provides a constructive way to understand well-known formal analytical techniques like the ϵ-expansion. Work of the last twenty-five years in this area was reviewed (to appear as a chapter in a book titled "Polymers in Disordered Media").

It was shown that the average number of branched polymers with n sites on the Given-Mandelbrot family of fractals varies as $\lambda^n \exp(cn^\psi)$. The critical exponent psi was determined exactly for all members of this family characterized by an interger parameter $b, 2 \leq b \leq \infty$.

Deterministic fractals are inhomogenous objects. Thus, rooted polymers on these fractals behave like rooted polymers in a disordered random medium. Different kinds of averages (quenched and annealed averages) were calculated exactly for rooted polymers on fractals, and these were shown to provide a very useful theoretical model to understand the corresponding objects in a disordered medium. [D. Dhar and Sumedha with Y. Singh of B.H.U.]

Residence times of grains in ricepiles

It has recently been shown that in the Oslo ricepile model, the probability that fluctuations of the mean slope about the average value have remarkably unsymmetrical behavior for large deviations from mean. For a system of size L in one dimension, the probability of a slope σ decreases as $\exp(-L)$ for σ greater than the mean slope σ_0, but as $\exp(-L^3)$ for $\sigma < \sigma_0$. This

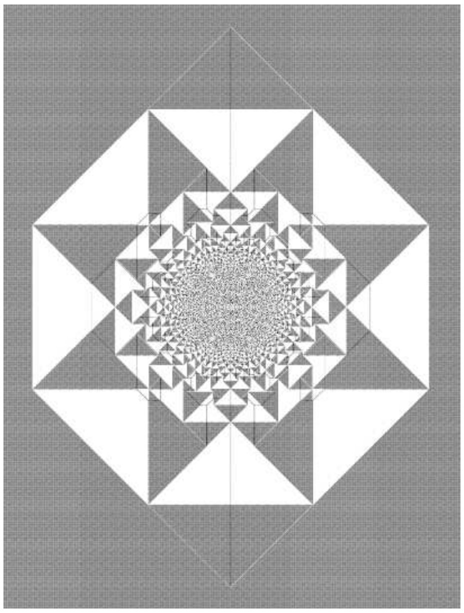

Figure 1: Pattern formed by adding 150000 grains at the center of two dimensional sandpile on F-lattice, and relaxing. The starting state was of uniform grain density 1/2. Here white and black denote occupied and empty sites respectively.

implies that the residence time distribution varies as $1/T$ for a very wide range of residence times T. [P. Pradhan and D. Dhar]

Patterns made in growing sandpiles

Patterns produced by adding grains at a single site on a flat substrate in simple sandpile models provide very simple examples of complex patterns produced from very simple deterministic local non-linear evolution rules. We have studied some of the simpler patterns in abelian sandpile models when the stable heights are only 0 and 1. These patterns are much simpler than those produced in the standard Bak-Tang-Wiesenfeld model on a square lattice, and their theoretical characterization is in progress. [S. B. Singha and D. Dhar]

Spiral Trees in d dimensions

Spiral structures are common in nature. Some examples are spiral structures in galaxies, shoot arrangement in plants, polymers with spiral structure etc. In statistical mechanics, lattice models of spiral self avoiding walks have been studied and can be solved exactly in two dimension, unlike the usual case.

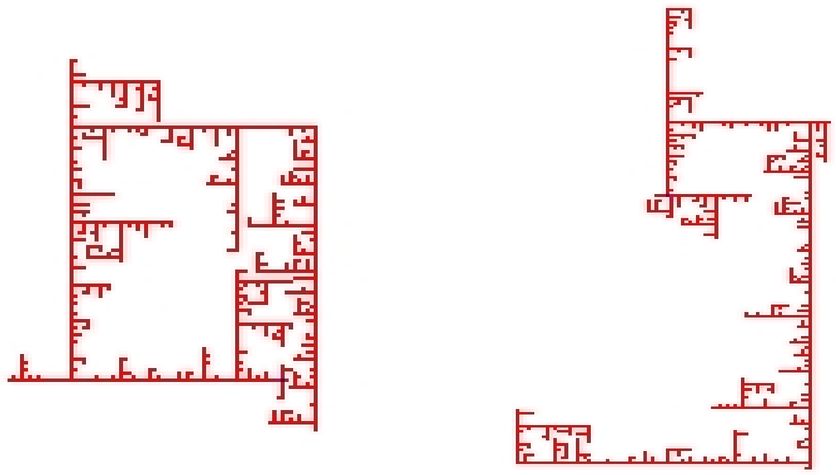

Figure 2: The figure shows randomly generated spiral trees of one thousand sites in 2-dimensions. These trees are generated using recently proposed improved incomplete-enumeration Monte-Carlo algorithm. Their structure is very different from lattice trees without the spiral constraint. Because of the spiral constraint they tend to branch much less.

Rooted spiral trees (a tree is a cluster of connected sites which contains no loops) in $2, 3$ and 4 dimensions on a hyper cubic lattice were studied using exact enumeration and Monte-Carlo techniques. This model was conjectured to show a dimensional reduction by four, which would have implied $\theta = -2\nu$ in 2-dimensions in this problem. Our series expansion studies give $\theta = -1.3667 \pm 0.0010$ and $\nu = 0.6574 \pm 0.0010$ on a square lattice. With Monte-Carlo simulations the estimates as $\theta = -1.364 \pm 0.010$, and $\nu = 0.656 \pm 0.010$. These results are numerical evidence against the proposed dimensional reduction by four in this problem. In dimensions higher than two, the spiral constraint can be implemented in two ways. In either case, our series expansion results do not support the proposed dimensional reduction. On the square lattice, we also obtain exact lower bound of 1.93565 on the growth constant λ (our numerical estimate for λ is 2.1143 ± 0.0001). [Sumedha]

Persistence of fluctuations in growing interfaces

A particular model of an interface growth process (linear Langevin dynamics in the two dimensional plane with arbitrary dynamical exponent z) was studied in radial and cylindrical geometries to understand the persistence of surface fluctuations in these geometries. It was shown analytically that the probability $p(t)$ that a surface fluctuation in a given direction does not change sign decays to zero at large times following a power law, $p(t) \sim t^{-\theta}$, with smaller value of θ in the radial geometry as compared to in cylindrical geometry. The numerical estimates of θ for radial growth are near 0.5 for z lying between 2 and 10. The corresponding values of θ are substantially higher (near 1.5) for cylindrical growth. This implies that relaxation of surface fluctuation is slower in radial growth. Both analytical and numerical results were used to argue that this behaviour persists in radial growth of surfaces described by KPZ equation. [Subhendu B. Singha]

Thermodynamics of glassy systems

A simple 1-d lattice gas model that exhibits nonergodicity was analyzed. In systems with broken ergodicity, the system only explores disconnected sectors of the available phase space; this may be described using the previously introduced notion of *picocanonical ensembles*. It was shown that the introduction of the piston at a boundary still results in decomposition of phase space into disconnected sectors. This was used to calculate the equation of state of this system in the non-equilibrium glassy phase. [Anandamohan Ghosh and D. Dhar]

Ferromagnetism in Gallium-Manganese-Arsenide

The effects of disorder in the position of Manganese dopants, Coulomb repulsion between holes, and spin-orbit coupling were studied in detail using a variational and mean-field approach. It was demonstrated that the combined effects of disorder and spin-orbit coupling were responsible for the anomalously low ground state magnetization of GaMnAs at low Mn concentrations. [Kedar Damle, with G. Fiete (KITP UC Santa Barbara, U.S.A), G. Zarand, and P. Moca (Technical University, Budapest, Hungary)]

Griffiths effects in the Random Bond Ising model

The Random Bond Ising model was mapped on to a non-hermitean, but unitary random hopping 'Hamiltonian'. The spectrum of psedo-energies of this Hamiltonian was shown to exhibit a Griffiths effect whereby rare long string-like configurations of the disorder produce a divergent power-law density of low energy states in a certain parameter regime. [Kedar Damle, with A. Mildenberger, F. Evers, R. Narayanan, and A. Mirlin (Institute for Nanotechnology, Karlsruhe, Germany)]

Disorder and interaction in the 2D electron gas

The interplay of disorder and interaction in a 2D electronic gas was studied using an inhomogeneous Hartree-Fock approximation. The main results obtained were: (i) At low disorder the system is a Mott insulator with a hard gap in the single particle density of states. The states at the Fermi surface are localized. There exists antiferromagtic (AF) long range order. (ii) In an intermediate disorder regime, the system has metallic behavior with no gap in the density of states and extended states at the Fermi surface. The AF order persists upto the classical percolation threshold of weak disorder sites. (iii) At large disorder the system is an Anderson insulator with localized states at the Fermi energy. [Dariush Heidarian, N. Trivedi, and M. Barma].

Ionic Hubbard Model

The ionic Hubbard model was studied within dynamical mean-field theory approximation (DMFT). This is a tight binding model with an onsite repulsion term U and an ionic potential term Δ which has alternate sign on the two sublattices. The model was studied both in the paramagnetic(PM) and antiferromagnetic (AFM) phases using DMFT to capture the effect of strong interactions and quantum fluctuations more accurately. In the PM phase for both the Bethe lattice of infinite connectivity and 2D square lattice a continuous phase transition from band insulator to metal was seen. This was followed by another continuous transition from metal to Mott insulator on increasing the onsite repulsion U for a fixed ionic potential Δ. A clear indication of two critical points and the metallic phase intervening between the two insulating phases was also seen for the first time. [Arti Garg and Mohit Randeria, with H.R.Krishnamurthy (Dept. of Physics, I.I.Sc. Bangalore)].

Effect of Disorder on d-wave Superconductors

The effect of non-magnetic impurities on d-wave superconductors was studied. For weak disorder, a spread in the nodal points with increase in disorder strength was seen. The single particle density of states $N(\omega)$ at $\omega = 0$ was seen to increase as the disorder strength is increased which indicates breaking of pairs. The local density of states at the points where disorder is large does not have any coherence peak and it resembles that in the zero temperature pseudo gap phase of the cuprates. [Arti Garg, Mohit Randeria, and Nandini Trivedi]