TIFR Annual Report 2007-08
THEORETICAL PHYSICS

Condensed Matter and Statistical Physics

HIGHLIGHTS

Exact entropy of dimer coverings was determined for a class of lattices in three and higher dimensions.

An effective time-independent potential was derived for a confined Brownian particle subjected to a rapidly oscillating space-dependent force. Significant quantum effects were found in the underdamped regime.

The character of finite size effects was shown to change drastically across a non equilibrium phase transition in the zero range process.

Different schemes to derive effective time independent Hamiltonian were studied for rapidly and periodically driven quantum systems.

A new method was proposed for dynamically polarizing nuclei in quantum wires using spin-orbit interaction and a finite source- drain potential difference.

A one-third magnetization plateau with unusual low temperature nematic order was shown to arise in easy axis Kagome lattice antiferromagnets for a range of fields along the easy axis.

TEXT

Patterns made in growing sandpiles

Patterns produced by adding grains at a single site on a flat substrate in sandpile models provide examples of complex patterns produced from very simple deterministic local non-linear evolution rules (see Fig. 1). Some of the simpler patterns in Abelian sandpile models were studied when the stable heights are only 0 and 1. The diameter of the pattern formed increases with the number of grains added N as $\sqrt{N}$. The asymptotic pattern was characterized exactly, and the position of different triangles and quadrilaterals determined. [D. Dhar, T. Sadhu and S. Chandra]

The matrix structure of the particle addition operators in the one-dimensional Manna model

The matrix structure of the particle addition operators of the one dimensional Manna model was studied. In general, these matrices are not diagonalizable. Using the Abelian properties of the addition operators and algebraic relations between them, the eigen values and the Jordan block structure of the addition operators was determined. This procedure gives an efficient way to determine the probabilities of different final stable configurations when a grain is added at a specified site in a given recurrent configuration. Using numerically determined exact steady state of systems of size up to 12 sites, the mean density of sites as a function of their distance from the boundary was determined. Also the asymptotic value of density, and the exponent characterizing the deviation of the mean density from its asymptotic value as a function of the distance from the boundary was determined. [T. Sadhu and D. Dhar]

Coherent Control of Quantum States using Inverting Pulses

A qubit, arbitrarily coupled to its environment, evolving deterministically under a time-dependent magnetic field $\mathbf{B}(t)$ was studied. It was shown that one can choose the time-dependence so that decoherence effects are minimized, in the sense that over a finite time interval T, transition rates between different states of the qubit can be suppressed to any desired order in T. Moreover the optimal protocols are independent of the details of the interaction Hamiltonian and the state of the environment. Specializing to the case of a sequence of n ideal π-pulses, a simple proof was given that the sequence proposed by Uhrig suppresses the transition rates to $\mathcal{O}(T^{2(n+1)})$ for any number of pulses and any suitable coupling to, and state of, the environment. However this result also applies to more general pulses of finite duration, and specific protocols, which suppress the rates to $\mathcal{O}(T^4)$ and $\mathcal{O}(T^6)$ with 2 and 3 pulses respectively, were proposed. [J. Cardy (Oxford) and D. Dhar]

Emergence of quasi-units in the Zhang model

Many models of sandpiles dynamics have been studied in the last two decades as toy models for self-organized criticality. One of these is the so-called Zhang model, where the height of sand at each site is not a discrete, but a continuous variable. Zhang observed in numerical simulation, that for large lattices, the energy variables in the stationary state tend to concentrate around discrete values of energy; he called this the emergence of 'quasi-units'. He argued that in the Thermodynamic limit, the critical behaviour would be the same as in discrete model. The emergence of quasi-units has been proved recently by Redig et al., who showed that the distribution of energies tends to a delta-function as the number of sites in the chain tends to infinity.

In this work, the model in one dimension was studied. The probability distribution of

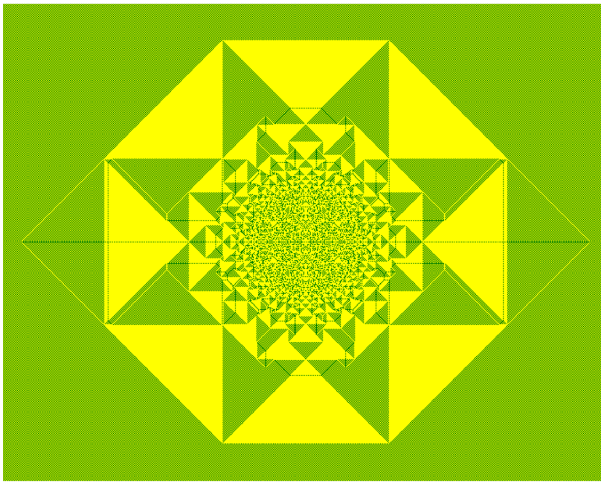

Figure 1: Pattern formed by addition of 10^6 number of grains at the central site. Dark (light) colour represents regions of average height 1(1/2).

energy variables was expressed in terms of the distribution of residence times of grains of grains in the corresponding discrete Abelian model. It was shown that the variance of the energy variables around mean value scales as $\frac{1}{L}$ in the stationary state of the model. Near the boundary of the pile, the variance of energy at site at a distance x from the boundary increases as $\log(L/x)$. [T. Sadhu and D. Dhar]

Exact Entropy of Dimer Coverings on three dimensional lattices

In how many ways can one cover an 8×8 chessboard with rectangular tiles of size 1×2 such that no square is left vacant, and no tiles overlap? This problem can be generalized to $m \times n$ chessboards, and also to other lattices. It is a well-studied problem, known as the dimer model in statistical physics, and the perfect matchings problem in computer science. It can be shown that if the number of sites N in the lattice is large, the number of ways increases as $\exp(CN)$. The problem is to calculate the constant C, called the entropy of dimer tilings per site, exactly for different lattices. In 1961, in a classic paper, Kasteleyn showed how to determine the number of dimer coverings for planar lattices. Since then, the two dimensional problem has been studied extensively. However, for three and more dimensions, no exact results were known so far. In the present work, a class of lattices was constructed in two and higher dimensions for which C can be determined exactly using elementary arguments. These lattices are a generalization of the two-dimensional Kagome lattice, and the method also works for graphs without translational symmetry. Some examples of the lattices for which the entropy can be determined are shown in Fig. 2 [D. Dhar and S. Chandra]

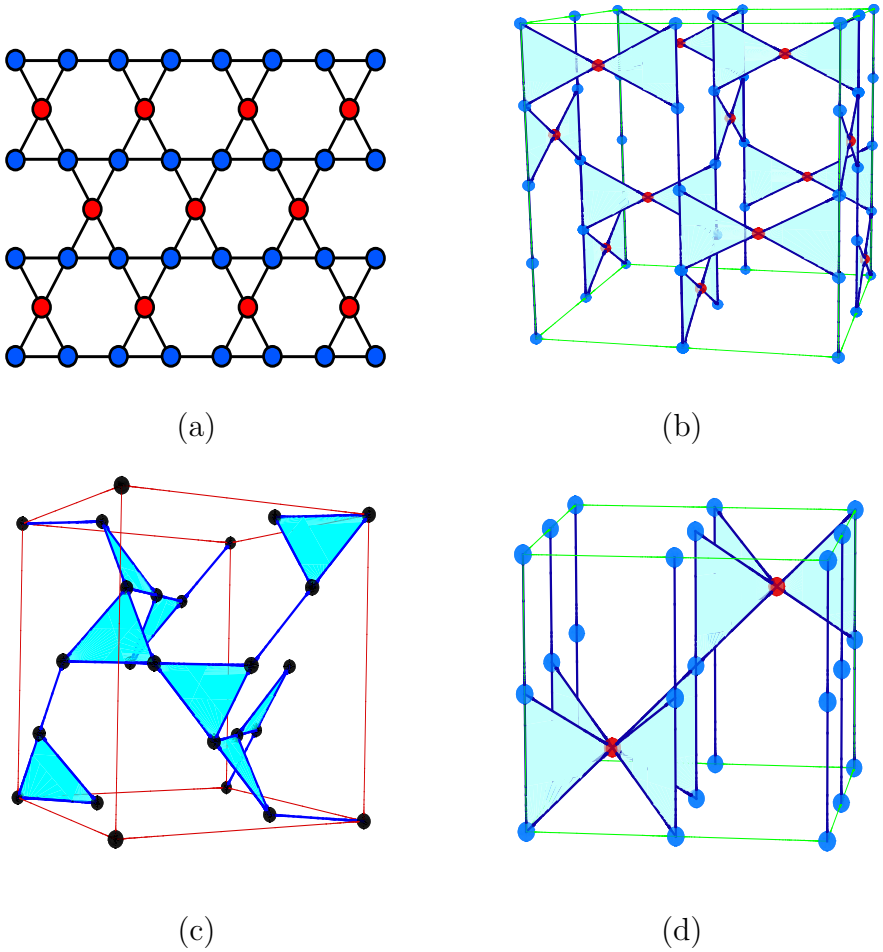

Figure 2: Some examples of lattices for which the exact entropy is calculated in this paper. (a) The Kagome lattice. (b) A three-dimensional lattice with corner-sharing triangles (c) The unit cell of the $Na_4Ir_3O_8$ lattice. There are 12 Iridium atoms per unit cell. Sodium and oxygen atoms are not shown. (d) A lattice with more than two triangles meeting at a vertex.

Unzipping a double stranded DNA (dsDNA) by pulling a single strand

Biological processes like DNA replication gets initiated by unzipping of a few base pairs at one end of the dsDNA and then continues till end. These processes are assisted by various enzymes, often by exerting force on DNA. It is known that by pulling the strands of the DNA in the opposite directions, the DNA undergoes an unzipping transition if the force exceeds a critical value. The transition sets in due to the competition between the base pairing energy which binds the complementary bases (or monomers) of the two strands and the orientation of the individual links connecting the monomers.

It was shown that the dsDNA can also be unzipped to two single strands if an external force is applied on a single strand while leaving the other strand free. The origin of the transition is, however, different from the both strands pulling case. In this case, the interplay between the binding energy and the entropy, which the free strand can gain if it is in the unzipped phase is responsible for the transition. The exact phase boundary between the zipped and the unzipped phases was obtained by using the generating function and the exact transfer matrix techniques. In the presence of an attractive surface near DNA, it was found that the phase diagram gets modified drastically and shows a critical end point and a triple point. [Rajeev Kapri]

Manipulating a single adsorbed DNA for a critical end point

A critical end point (CEP) occurs when a continuous transition line terminates on a first order transition line. A CEP is expected to occur in various mixtures and ferroelectrics. The existence of a CEP was shown in the phase diagram of unzipping of an adsorbed double-stranded polymer like DNA. It was found that the competition of base pairing, adsorption and stretching by an external force leads to the CEP. From exact results, the location of the CEP was determined and it was found that the two first order lines meet at the CEP with the same slope but with different curvatures. This discontinuity in second-order is consistent with the prediction of the Landau theory for the CEP. [Rajeev Kapri and Somendra M. Bhattacharjee (Institute of Physics, Bhubaneswar)]

Quantum Brownian motion with rapid periodic forcing

A confined quantum Brownian particle subjected to a rapidly oscillating force which varies in space was studied. In the classical case, when the period of the external force is small compared with the other time scales of the system, it is possible to separate the motion of the Brownian particle into slow and fast parts. To leading order in the period of driving, this enables a derivation of the equation of motion of the slow part from an effective time-independent Hamiltonian. In the quantum case, starting from the operator Langevin equation and averaging over initial product separable states, a c-number Langevin equation was derived and an effective time independent potential obtained again. The correction

brought in by quantum effects was found to be especially pronounced in the underdamped regime. [M. Bandyopadhyay and M. Barma]

Charged magneto-oscillator in a heat bath and third law of thermodynamics

Low temperature quantum thermodynamic behaviour of a charged magneto-oscillator coupled with a heat bath through different coupling schemes was obtained analytically. It was shown that finite dissipation substitutes the zero-coupling result of exponential decay of entropy by a power law behaviour at low temperature. For Ohmic bath, free energy and entropy vanishes linearly with temperature (T) in conformity with Nernst's theorem. It was seen that coordinate (velocity)-velocities (coordinates) coupling is much more beneficial than the coordinate-coordinates coupling to ensure third law of thermodynamics. It was observed that velocity-velocities coupling is the most advantageous coupling scheme in restoring the third law of thermodynamics. The low temperature thermodynamic functions are independent of magnetic field for all the above mentioned cases except the without dissipation case. [M. Bandyopadhyay, J. Kumar (IISER, Kolkata), S. Dattagupta (IISER, Kolkata) and P. A. Sreeram (IISER, Kolkata)]

Quantum mechanics of rapidly and periodically driven systems

This review dealt with the dynamics of quantum systems that are subjected to high frequency external perturbations. Though the problem may look very difficult, and poised on the extreme opposite side of adiabaticity, there exists a 'Kapitza Window' over which the dynamics can be treated in terms of effective time-independent Hamiltonians. The consequent results are important in the context of atomic traps as well as quantum optic properties of atoms in intense and high-frequency electromagnetic fields. [M. Bandyopadhyay and S. Dattagupta (IISER, Kolkata)]

Finite-size effects on the dynamics of the zero-range process

The zero-range process offers a paradigm to study nonequilibrium dynamics. The process involves particles on a lattice hopping preferentially to nearest neighbour sites; the hops are stochastic and occur with a rate that depends on the departure site occupancy. For certain classes of rates, the system undergoes a phase transition from a low-density disordered phase to a high-density condensed phase where a finite fraction of particles (the condensate) accumulates on a single site. In this work, the effect of finiteness of the system size on the stationary state dynamics of fluctuations on a one-dimensional periodic lattice was studied. The integrated number of particles crossing a bond in the lattice in a given time was monitored. In particular, the variance in the integrated current about its average over initial stationary ensemble and stochastic evolution was studied. It was found that the variance behaves very differently across a phase transition, reflecting different underlying

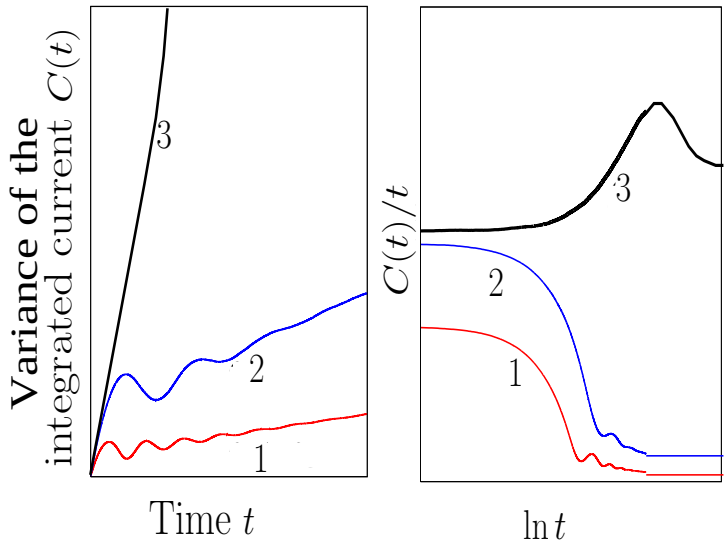

Figure 3: For the zero-range process on a one-dimensional periodic lattice, fluctuation dynamics are very different across a phase transition. The differences show up in the behaviour of the variance of the integrated current as a function of time in the disordered phase (1), at the critical point (2), and in the condensed phase (3).

dynamics. In the disordered phase, the variance has two size-dependent time scales, given by the circulation time and the decay time of a kinematic wave of density fluctuations. By contrast, in the condensed phase, the variance has four size-dependent time regimes, being governed by condensate relocation from site to site. See Fig. 3. [S. Gupta and M. Barma with S. N. Majumdar (LPTMS, Orsay, France)].

Theory of magnetization plateau in $S \geq 3/2$ Kagome lattice antiferromagnets with strong easy-axis anisotropy

Many experimentally studied frustrated magnets have higher spin ($S = 3/2$, $S = 5/2$ or higher) moments on triangular and Kagome lattices ('spin' here refers to the total angular momentum content of the ground state multiplet of the magnetically active ion). Since uniaxial *single ion* anisotropy is a generic feature of the magnetic Hamiltonian in layered materials especially in cases where the ground state angular momentum has significant orbital component, it is of considerable interest to understand the behaviour of such antiferromagnets in the presence of such strong uniaxial anisotropy on the two dimensional Kagome and triangular lattices.

With this general background in mind, results were obtained for one specific example,

namely the low temperature finite field state of the $S = 9/2$ easy axis Kagome antiferromagnet Nd-langasite. This work demonstrated that a one-third magnetization plateau (with magnetization equal to one-third of the saturation value) is stable for a range of magnetic fields along the easy axis.

A systematic (see below) perturbative expansion in $\lambda \equiv J/D$ (where J is the antiferromagnetic exchange, and D the single ion anisotropy) was then used to derive an effective Hamiltonian that encodes the low-energy dynamics within the macroscopically degenerate set of minimally frustrated one-third magnetization states. This effective Hamiltonian assigns a 'potential energy' to each state in this low energy subspace and can be cast in the form of an interacting dimer model.

In order to reliably understand this potential energy dominated low temperature regime, the recently developed loop algorithm of Sandvik, Moessner and Alet was adapted to simulate this interacting dimer model. Using this efficient numerical tool, the presence of an unusual 'sublattice ordered' low temperature state was established in which the fully equilibrated system breaks only the sublattice rotation symmetry of the Kagome lattice (but not translation symmetry of the underlying bravais lattice) and forms extended zig-zag stripe-like structures of alternating spins that fluctuate between the two possible alternating arrangements along the stripe.

Furthermore, by analyzing the behaviour of the system with realistic purely local dynamics (as opposed to the non-local dynamics used in the loop algorithm to rapidly equilibrate the system and sample the equilibrium Gibbs distribution), it was possible to demonstrate that the system exhibits slow glassy dynamics (well-described by the Vogel-Fulcher law) as the ordering transition is approached from the high temperature side—this constitutes an interesting example of glassy behaviour in the absence of disorder due to incipient extended structures (the striped pattern of alternating spins mentioned above) that give rise to a very slow relaxation rate. [K. Damle with A. Sen and A. Vishwanath (Berkeley)]

Variational wavefunction study of triangular lattice supersolid

Motivated by the surprising and unusual properties of the triangular lattice supersolid state studied earlier with D. Heidarian, A. Sen, an effort was made to develop a variational wavefunction based understanding of this supersolid state. The starting point was the observation that the equal amplitude superposition of all minimally frustrated states already has a finite condensate fraction, and is thus expected to be a reasonable description of the persistent superfluidity in the limit of strong interactions. Averages of various quantities in this wavefunction can be related to various expectation values in a classical dimer model on the honeycomb lattice. Using this mapping, it is possible to show that this wavefunction has quasi-long range power law ordering of the density at the three sublattice wavevector. However, it does not correctly reproduce the genuine long-range density-wave order of the supersolid state, nor does it account for the spontaneous deviation of the density from half-

filling.

To incorporate true long range density-wave order at the three sublattice wavevector, these observations were built upon by considering a class of wavefunctions which have a three-sublattice pattern of dimer fugacities (in dimer language). The search for the best wavefunction in this class was carried out using Pfaffian methods to calculate the variational energy. The conclusions are readily stated: The global variational mininum (within this one parameter family of states) is characterized by three-sublattice density wave order and accompanied by a small spontaneous deviation of the density from half-filling. The magnitude of the density wave order parameter as well as the spontaneous deviation of the density from half-filling at this variational minimum are in remarkable *quantitative* agreement with values obtained from quantum Monte-Carlo simulations in the large interaction energy limit. This variational treatment thus represents a remarkable instance in which a non-trivial constraint *on the lattice scale* imposed by a dominant term in the Hamiltonian can be taken into account in an exact manner in a variational calculation. [K. Damle, with A. Sen, P. Dutt, and R. Moessner (MPIPKS, Dresden)]

Resistance oscillations in disordered 2D electron gases

It was shown in earlier work reported in 2006-07 by the authors how at low densities, electrons confined to two dimensions in a delta-doped heterostructure can arrange themselves into droplets due to the interplay of disorder and screening effects. This droplet picture was used to study the dependence of magnetotransport on the electron density, in the regimes of weak and moderate magnetic fields. In the present work, it has been shown that Coulomb blockade effects in inter-droplet tunnelling give rise to resistance oscillations as a function of 2D electron density, and that a strong enough magnetic field enhances the amplitude of the resistance oscillations through the suppression of inter-droplet tunnelling. The main understanding from this (and the earlier reported) study is that many of the observations on magnetotransport in disordered 2D heterostructures reported recently by the Cavendish Laboratory group, and attributed to various novel electronic ordering scenarios such as a Wigner crystal and charge density wave, can be accounted qualitatively and quantitatively by this simpler picture of a droplet phase. [V. Tripathi and M. P. Kennett (Simon Fraser, Canada)]

Dynamic nuclear polarization in voltage-biased quantum wires with spin-orbit interaction

A new method was proposed for generating local dynamic nuclear polarization in a quasi one-dimensional quantum wire making use of the spin-orbit interaction and a finite source-drain potential difference, but avoiding any other external source such as a magnetic field or polarized light or electron spin resonance inducing radio frequency wave. Local control of nuclear polarization is useful for nuclear magnetic resonance and for control of nuclear

spins where external magnetic fields and/or optical sources will not be suitable. [V. Tripathi, Anson Cheung (Cambridge) and N. R. Cooper (Cambridge)]

Using NMR to detect localized electrons in semiconductor quantum point contact devices

The mutual interaction of electrons in quantum point contact (QPC) devices has long been appreciated as the cause of the "0.7 conductance anomaly", an unexpected shoulder like feature at a conductance of about 0.7 times the universal value for non-interacting electrons in a quasi one-dimensional conductor. Numerous, often mutually incompatible theoretical models have been proposed in the literature for explaining this conductance anomaly. A major point of view is that the anomaly is a Kondo effect arising from the unexpected formation of a bound electron state in the device. A probe based on nuclear relaxation and Knight shift measurements was proposed for the Kondo scenario for the 0.7 anomaly. It was shown that the presence of a bound electron in the QPC would lead to a much higher rate of nuclear relaxation compared to nuclear relaxation through exchange of spin with conduction electrons. Furthermore, it was found that the temperature dependence of this nuclear relaxation is very non-monotonic as opposed to the linear-T relaxation from coupling with conduction electrons. These observations, in combination with Knight shift measurements, can help in verifying whether the 0.7 anomaly is indeed due to the presence of a bound electron in the QPC. [V. Tripathi and N. R. Cooper (Cambridge)]