Self-interacting neutrinos in the light of cosmological and IceCube observations

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Outline

1. H_0 **Tension?**

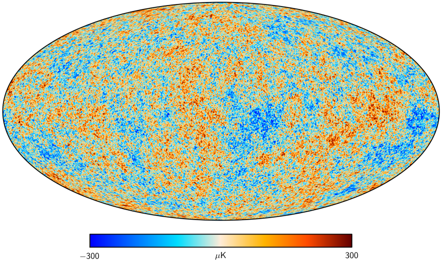
2. **Self interacting neutrinos**

3. **IceCube and self-interacting neutrinos**

4. **Self-interacting sterile neutrinos**

ΛCDM and Planck CMB

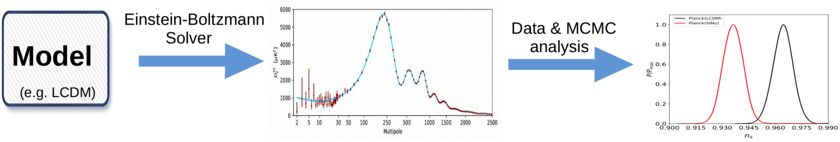
- CMB ⇒ "Cosmic Microwave Background"
  Snapshot of photons in our universe leftover after it's decoupling.



ΛCDM model

- CDM, baryons, dark energy, radiation
- Six standard parameters (ω_b, ω_c, A_s, n_s, τ_{reio}, θ)
- Two derived parameters (H_0 and σ_8)

- Angular power spectrum



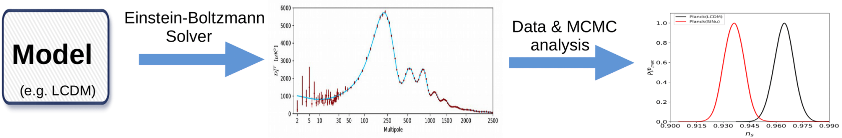
Model

(e.g. LCDM)

Einstein-Boltzmann Solver

Data & MCMC analysis

ΛCDM and Planck CMB



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- Two derived parameters (H_0 and σ_8)

Most of the parameters determined at (sub-) percent level.

Parameter	ΛCDM
$\boldsymbol{\omega_b}$	$2.212 \pm 0.022 \times 10^{-2}$
$\boldsymbol{\omega_c}$	0.1202 ± 0.0021
$\boldsymbol{A_s}$	$2.091 \pm 0.034 \times 10^{-9}$
$\boldsymbol{n_s}$	0.9643 ± 0.0057
$\boldsymbol{\tau_{\rm reio}}$	0.0522 ± 0.0079
$\mathbf{100}\ \boldsymbol{\theta}$	1.04185 ± 0.00045
$\boldsymbol{H_0}$	67.08 ± 0.92
$\boldsymbol{\sigma_8}$	0.8101 ± 0.0092

ΛCDM is a good fit to the Planck data.

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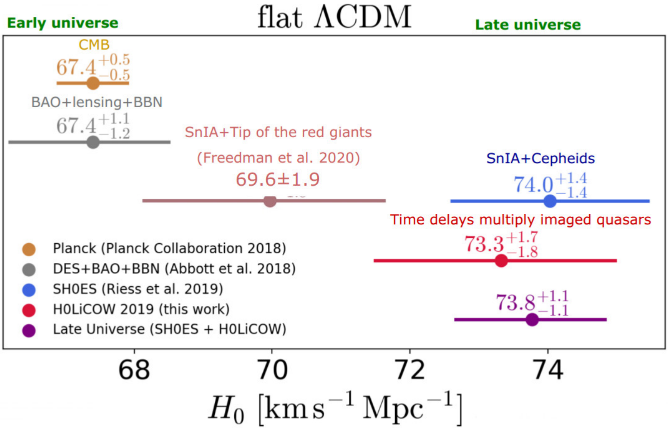
[1] Planck Collaboration arXiv:1807.06209

ΛCDM and Planck CMB

- Overall ΛCDM is remarkably consistent with CMB, large scale structure and supernova data. However, there are few discrepancies within ΛCDM framework.
- Within ΛCDM cosmology, the derived value of σ_8 from Planck CMB observation is in tension with the value of σ_8 obtained from low redshift observations (cluster count, week lensing, etc.) at 2-σ level.
- Another discrepancy within ΛCDM framework is the mismatch between the H_0 value inferred from Planck CMB observation and those obtained from direct local measurements.

H_0 Tension?

- The value of the Hubble constant (H_0) inferred from Planck CMB data is lower than the value obtained by the direct local measurements by around 4σ.



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[1] H0LiCOW arxiv:1907.04869

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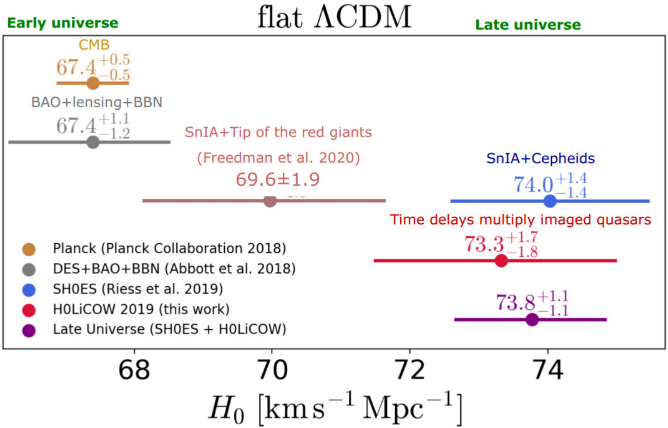

Systematic or Hint of new physics??

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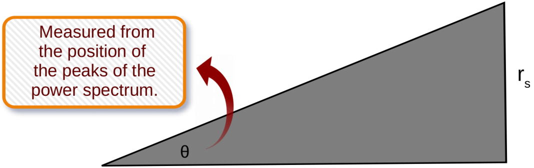
[1] H0LiCOW arxiv:1907.04869

Measurement of H_0 from CMB

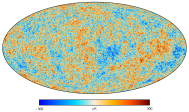
- ❐ Calculate the sound horizon at recombination : assumes model for sound speed and expansion of the universe before recombination

$$r_s = \int_{z_s}^{0} \frac{c_s(z)}{H(z)} dz$$



Measured from the position of the peaks of the power spectrum.

$D_A(z) = \int_0^z dz' / H(Z')$



- ❐ Infer angular diameter distance to last scattering surface ⇔ depends on H_0
- ❐ Expansion rate after recombination

$$H(z)^2 = H_0^2[\Omega_m * (1+z)^3 + \Omega_r * (1+z)^4 + \Omega_\Lambda]$$

This is model dependent ⇒ New physics can change H_0 significantly.

How to resolve H_0 Tension?

❒ Peak position in the angular power spectrum is fixed from the observations ⇒ θ_s is fixed.

- **Late Universe solution:** (e.g modified DE equation of state, dynamical dark energy (Planck 2015, 2018), Interacting dark matter dark energy (Di Valentino+2019), decaying DM (Poulin+2018, Pettis+2019), Modified gravity (Raveri+ 2019)etc.)
  - Change in late-time dynamics of dark matter and/or dark energy

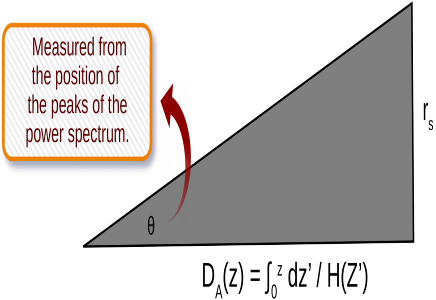
$$\Downarrow$$

r_s is fixed and $H(z)$ at small z changes.

$$\Downarrow$$

Increase H_0 to keep $D_A(z)$ fixed.

- Highly constrained by BAO, Supernovae and other observations.



Measured from the position of the peaks of the power spectrum.

$$D_A(z) = \int_0^z dz' / H(Z')$$

How to resolve H_0 Tension?

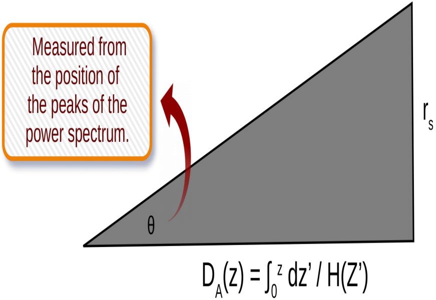
Early Universe solution:

- Change the sound horizon

$$r_s = \int_{z_s}^{0} \frac{c_s(z)}{H(z)} dz$$

- Change the expansion history $H(z)$ of the Universe at early times. (e.g. Early dark energy (Polin+ 2018, Agarwal+2019), dark radiation, etc.)
- Change the Sound speed $c_s(z)$ (e.g. DM-baryon interaction (Boddy+ 2018))
- Change the time of decoupling. (e.g. changing recombination (Hart&chluba 2019, Ivanov+ 2020))

- Problem with many other observations



Measured from the position of the peaks of the power spectrum.

- Change the inference of θ_s from CMB. (e.g. Neutrino interaction (Kreisch et.al. 2019), Dark neutrino interaction (Ghosh et.al. 2019))

Self interacting neutrinos

- Neutrinos free stream through the photon-baryon plasma at early times.
- Free-streaming neutrinos ⇒ Phase shift in the CMB power spectra towards larger scales and slight suppression in the amplitude.

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- Neutrinos free stream through the photon-baryon plasma at early times.
- Free-streaming neutrinos ⇒ Phase shift in the CMB power spectra towards larger scales and slight suppression in the amplitude.
- Neutrino self-interactions can be mediated by scalars and gauge bosons which are motivated from different particle physics models.

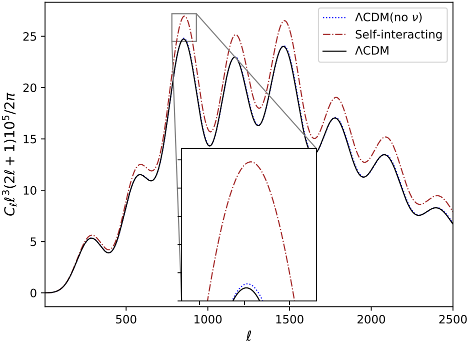
$$\mathcal{L} = g_\phi \sum_{\alpha,\beta} g_{\alpha\beta} \phi \bar{\nu}_\alpha \nu_\beta \,, \quad \mathcal{L} = g_X \sum_{\alpha,\beta} \bar{\nu}_\alpha g_{\alpha\beta} \gamma^\mu P_L \nu_\beta X_\mu \,,$$

- Neutrino self interaction delays the time at which neutrinos begin to free-stream.

$$\Downarrow$$

Modes entering the horizon while neutrinos are still tightly-coupled will not receive the associated phase shift and amplitude reduction.

Self interacting neutrinos



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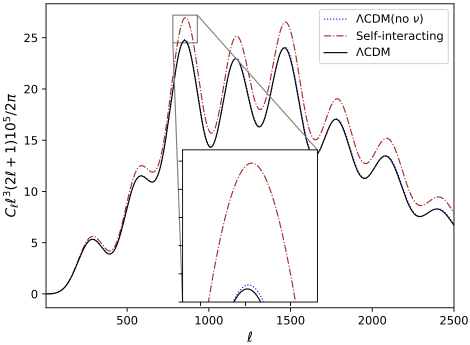
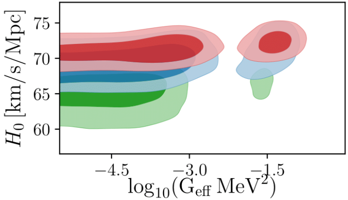
[1] Kreisch et. al. arxiv:1902.00534

Self interacting neutrinos





- Bimodal posterior (Moderate self-interaction (MI) and strong self-interaction (SI) region)
- Flavour universal self interaction is strongly constrained by BBN and Lab bounds

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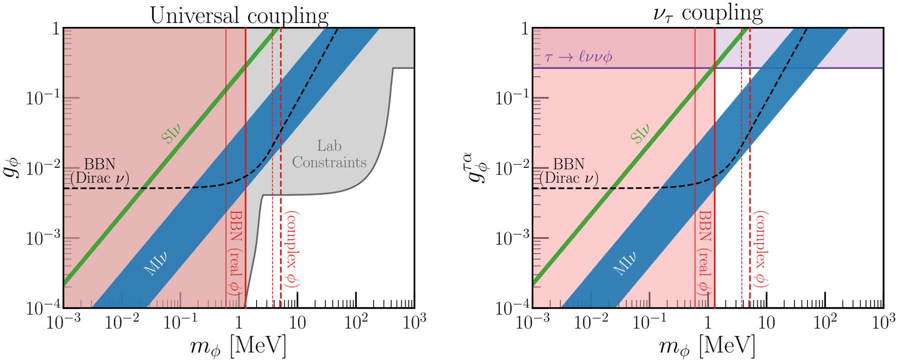
Self interacting neutrinos



- ❐ Flavor-universal G_{eff} is excluded by lab and BBN constraints .
- ❐ Only some part of the cosmological allowed MI region of ν_τ interaction is allowed by lab and BBN constraints.
- ❐ ν_e and ν_μ self-interactions is also disfavored by the lab and BBN constraints.
- ❐ Cosmological bounds assume flavour universal interaction.

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[1] Blinov et. al. Phys. Rev. Lett. 123, 191102 (2019)

Flavour specific self interaction

- Flavour specific self interaction

$$\mathcal{L} = g_\phi \sum_{\alpha,\beta} g_{\alpha\beta}\phi\bar{\nu}_\alpha\nu_\beta\,, \quad \mathcal{L} = g_X \sum_{\alpha,\beta} \bar{\nu}_\alpha g_{\alpha\beta}\gamma^\mu P_L \nu_\beta X_\mu\,,$$

- Four cases : ν_e, ν_μ, ν_τ and the flavour universal self-interactions.

The matrix $g_{\alpha\beta}$ defines the flavour dependence of the interactions. $g_{\alpha\beta}$ will be $\delta_{\alpha\beta}$ for universal interaction and for interaction in a particular flavour it will be a diagonal matrix with only one among $g_{ee}, g_{\mu\mu}$ or $g_{\tau\tau}$ set to be one.

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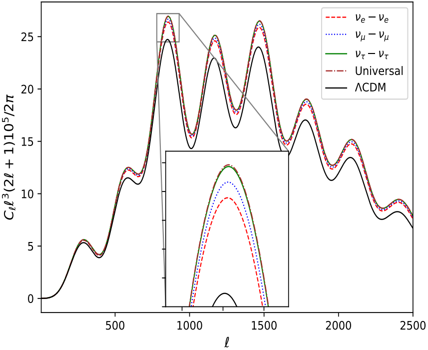
- The perturbed Boltzmann equation

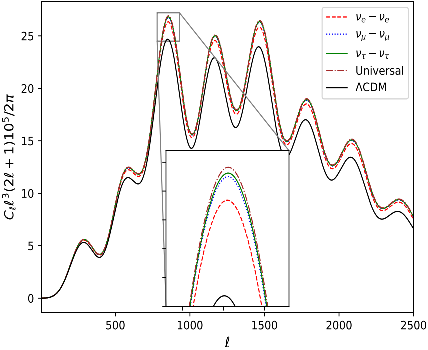
$$\frac{\partial\Psi_i}{\partial\tau} + i\frac{q(\vec{k}\cdot\hat{n})}{\epsilon}\Psi_i + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h}+6\dot{\eta}}{2}\left(\hat{k}\cdot\hat{n}\right)^2\right] = -\Gamma_{ij}\Psi_j\,,$$

where $\Gamma_{ij} = U^\dagger g_{\alpha\beta} U \tau_\nu^{-1}$. and

$$\tau_\nu^{-1} = a n_\nu\langle\sigma v\rangle = \frac{3}{2}\frac{\zeta(3)}{\pi^2} a G_{\text{eff}}^2 T_\nu^5\,,$$

Effect of Flavour specific SI on CMB

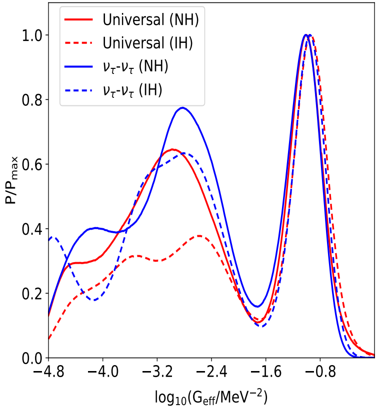


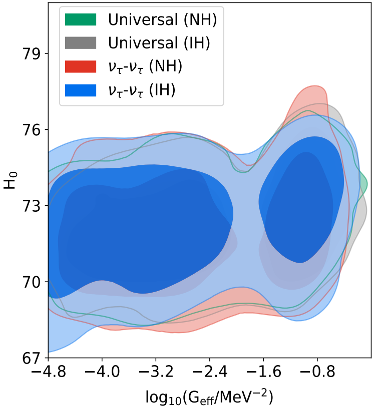


Effect of self-interaction $\propto \sum_{i,j} \Gamma_{ij}$.

Hierarchy	Universal Int.	ν_τ Int.	ν_μ Int.	ν_e Int.
NH	$3\tau_\nu^{-1}$	$2.965\,\tau_\nu^{-1}$	$2.643\,\tau_\nu^{-1}$	$2.308\,\tau_\nu^{-1}$
IH	$3\tau_\nu^{-1}$	$2.88116\,\tau_\nu^{-1}$	$2.809\,\tau_\nu^{-1}$	$2.309\,\tau_\nu^{-1}$

Parameters in flavour specific self interaction





- ❒ Similar to the Universal case, $G_{\rm eff}$ has a bimodal posterior. We call the two modes strong interaction (MI) and moderate interaction (MI) region.
- ❒ In case of ν_τ-ν_τ interaction, $H_0 = 73.3 \pm 1.8$ and $H_0 = 73.3^{+2.0}_{-1.4}$ for NH and IH respectively.

- Parameters for the Moderate self Interaction.

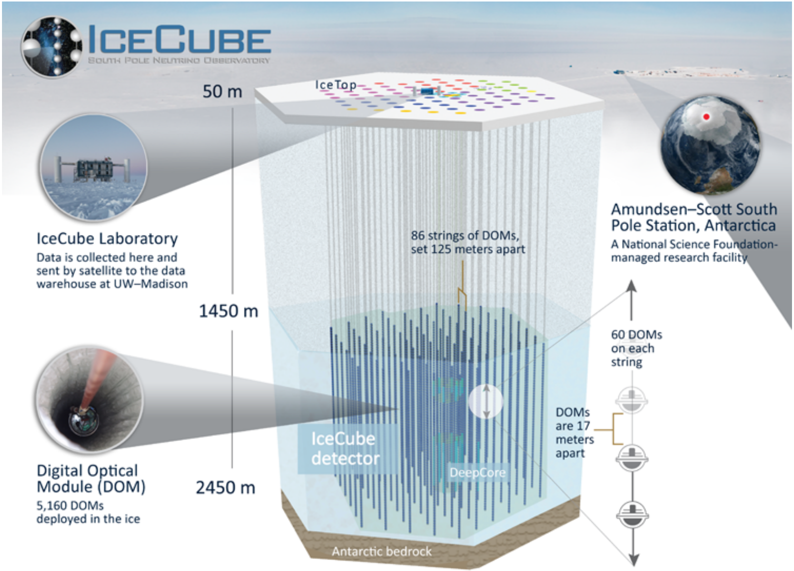
Parameters	Universal Int. (NH)	Universal Int. (IH)	ν_τ-ν_τ Int. (NH)	ν_τ-ν_τ Int. (IH)
$\boldsymbol{100\theta_s}$	$1.0412^{+0.0009}_{-0.0011}$	$1.0411^{+0.0010}_{-0.0012}$	$1.0413^{+0.0010}_{-0.0011}$	1.0411 ± 0.0010
$\boldsymbol{n_s}$	0.986 ± 0.014	$0.988^{+0.015}_{-0.013}$	$0.984^{+0.016}_{-0.013}$	$0.986^{+0.015}_{-0.013}$
$\boldsymbol{m_0}$	$0.057^{+0.022}_{-0.050}$	$0.058^{+0.025}_{-0.055}$	$0.052^{+0.017}_{-0.049}$	$0.051^{+0.018}_{-0.042}$
$\boldsymbol{\log_{10} G_{\rm eff}}$	$-3.25^{+0.94}_{-0.65}$	-3.23 ± 0.75	$-3.20^{+1.3}_{-1.5}$	$-3.33^{+1.1}_{-0.75}$
$\boldsymbol{N_{\rm eff}}$	$3.75^{+0.31}_{-0.38}$	$3.77^{+0.29}_{-0.32}$	$3.69^{+0.28}_{-0.33}$	$3.74^{+0.33}_{-0.37}$
$\boldsymbol{H_0}$	$72.0^{+1.7}_{-1.9}$	71.8 ± 1.7	71.6 ± 1.8	71.9 ± 1.9

- Parameters for the strong self Interaction.

$\boldsymbol{100\theta_s}$	1.0456 ± 0.0012	$1.0454^{+0.0012}_{-0.0014}$	1.0453 ± 0.0011	1.0451 ± 0.0010
$\boldsymbol{n_s}$	0.959 ± 0.014	0.959 ± 0.013	$0.962^{+0.015}_{-0.011}$	0.964 ± 0.012
$\boldsymbol{m_0}$	$0.081^{+0.038}_{-0.069}$	$0.082^{+0.045}_{-0.054}$	$0.084^{+0.044}_{-0.060}$	0.092 ± 0.062
$\boldsymbol{\log_{10} G_{\rm eff}}$	$-1.01^{+0.21}_{-0.18}$	$-0.95^{+0.22}_{-0.18}$	$-1.05^{+0.26}_{-0.17}$	$-0.97^{+0.19}_{-0.12}$
$\boldsymbol{N_{\rm eff}}$	3.93 ± 0.35	3.97 ± 0.36	$4.07^{+0.39}_{-0.35}$	$4.09^{+0.42}_{-0.21}$
$\boldsymbol{H_0}$	72.8 ± 1.9	72.9 ± 1.7	73.3 ± 1.8	$73.3^{+2.0}_{-1.4}$

IceCube and self-interacting neutrinos

IceCube Neutrino Observatory



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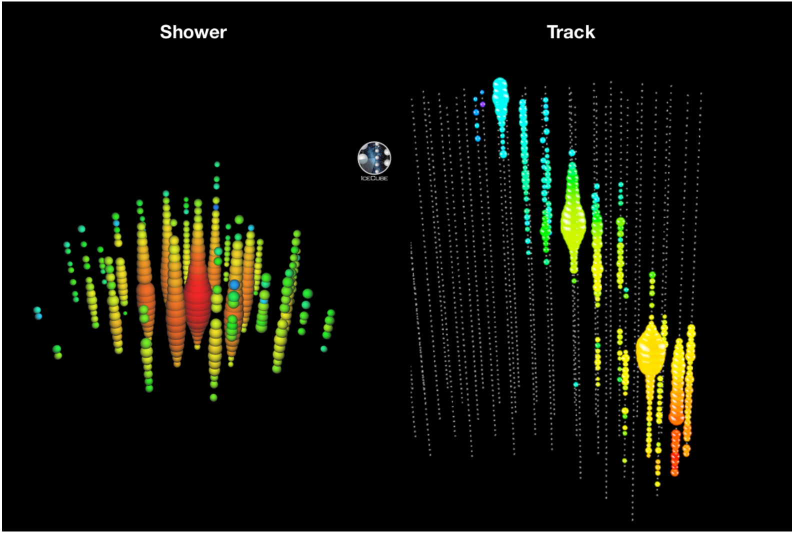
[1] From:https://icecube.wisc.edu/science/icecube/detector

IceCube Neutrino Observatory



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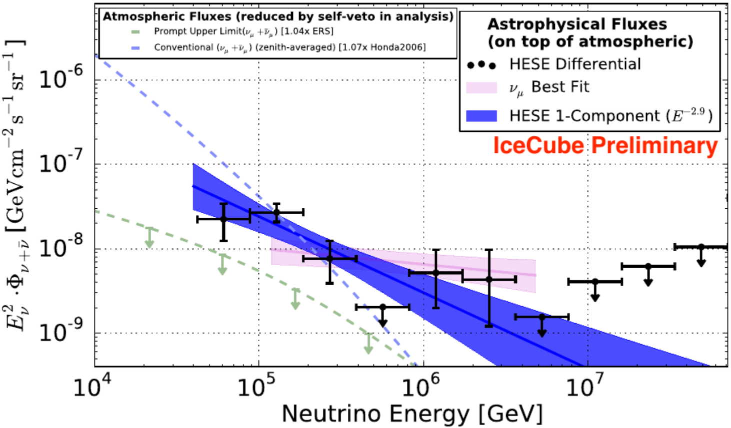
[0] From: http://gallery.icecube.wisc.edu/

IceCube Neutrino Observatory



The best fit for single power law flux is

$$E_0^2\phi = (2.46 \pm 0.8) \times 10^{-8} \left(\frac{E_0}{100\ \mathrm{TeV}}\right)^{-0.92}\ \mathrm{GeV\,cm^{-2}s^{-1}sr^{-1}}$$

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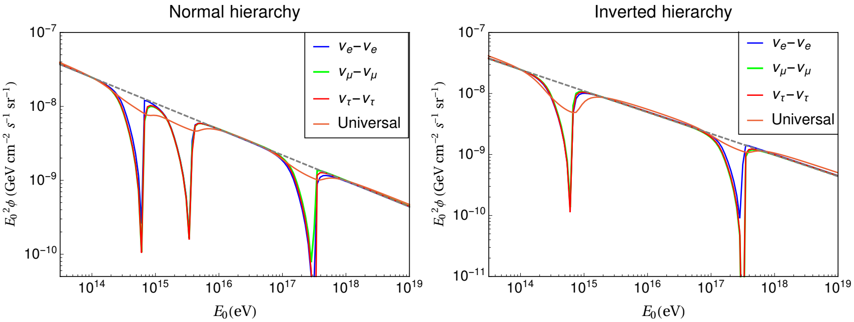
[1] Icecube 2017

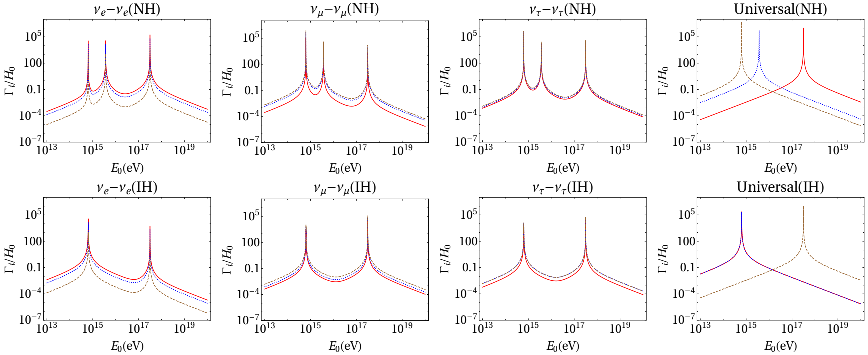
Absorption of Neutrinos

- The astrophysical neutrinos can interact with cosmic neutrinos and produce an on shell mediator $\nu\nu \to \phi$, when the neutrino energy $E_\nu = m_\phi^2/(2m_\nu)$.
- The resonant absorption of astrophysical neutrinos will show up as dips in the IceCube flux.



$$m_\phi = 10^{6.9}\text{eV},\ g_\phi = 10^{-1.5},\ \gamma = 2.35 \text{ and } m_0 = 10^{-4}\text{eV}$$

Absorption of Neutrinos

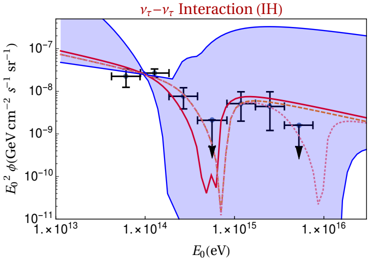


$$\sigma_{ijkl} = \sigma\left(\bar{\nu}_i\nu_j \to \bar{\nu}_k\nu_l\right) = \frac{1}{4\pi}|g_{kl}|^2|g_{ij}|^2\frac{g_\phi^4 s_j}{(s_j - m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2},$$

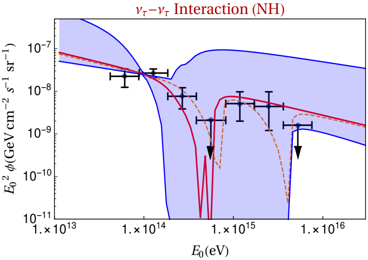
where $s \approx 2E_i m_i$ and $g_{ij} = g_{\alpha\beta}U^*_{\alpha i}U_{\beta j}$ $\quad \Gamma_i = \sum_j \tilde{n}_j\sigma_{ij}$

Parameter estimation from flux at IceCube

- Four parameters (m_ϕ, g_ϕ, m_0 and γ) have been varied to do the MCMC analysis with IceCube data.



$\nu_\tau - \nu_\tau$ Interaction (IH)



$\nu_\tau - \nu_\tau$ Interaction (NH)

Parameters	NH	IH
$\boldsymbol{m_0}$**(eV)**	$0.062^{+0.042}_{-0.046}$	$0.062^{+0.038}_{-0.058}$
$\boldsymbol{\log_{10} m_\phi/\mathrm{eV}}$	6.92 ± 0.11	$6.936^{+0.089}_{-0.14}$
$\boldsymbol{\log_{10} g_\phi}$	-1.40 ± 0.13	-1.25 ± 0.17
$\boldsymbol{\gamma}$	2.50 ± 0.17	$2.46^{+0.18}_{-0.16}$

- Both one dip and two dip solution are possible.

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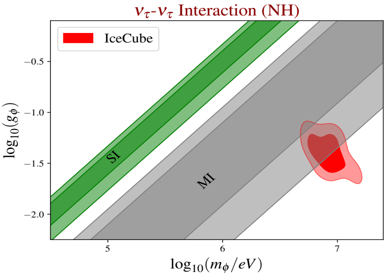
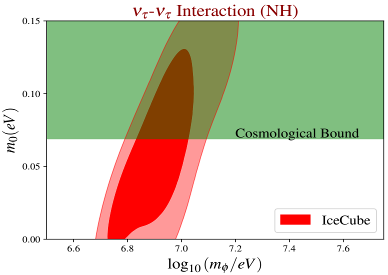

ν_τ-ν_τ Interaction (NH)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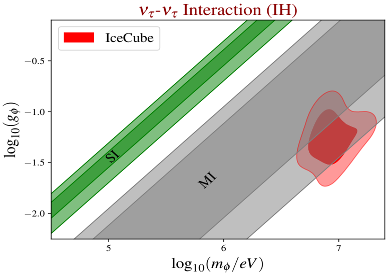
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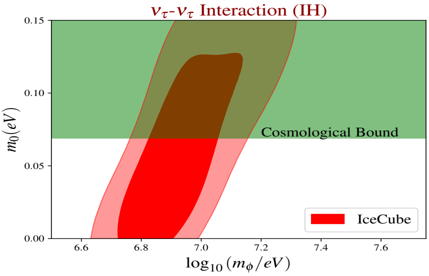
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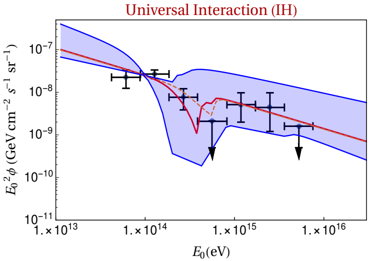
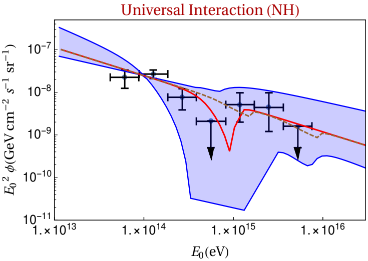
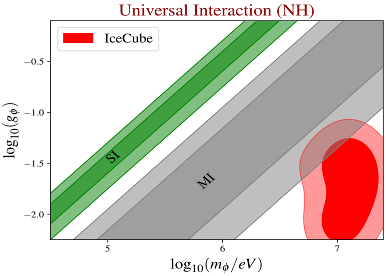

Universal Interaction (IH)

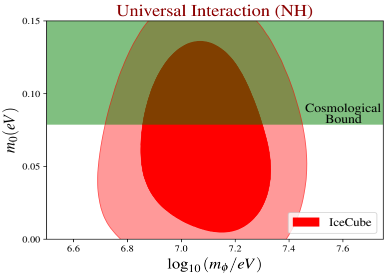

Universal Interaction (NH)

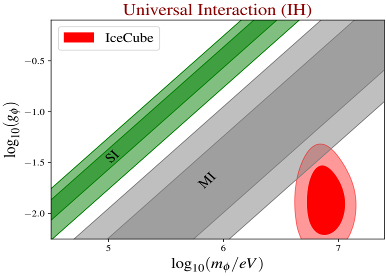
Parameters	NH	IH
$\boldsymbol{m_0}$**(eV)**	$0.067^{+0.038}_{-0.046}$	$0.052^{+0.034}_{-0.040}$
$\boldsymbol{\log_{10} m_\phi/\mathrm{eV}}$	7.09 ± 0.15	6.883 ± 0.091
$\boldsymbol{\log_{10} g_\phi}$	-1.75 ± 0.31	-1.87 ± 0.20
$\boldsymbol{\gamma}$	$2.66^{+0.21}_{-0.18}$	2.63 ± 0.17

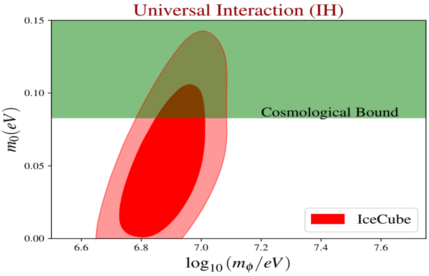
Only single dip solution possible.

Parameter estimation from flux at IceCube









Self-interacting sterile neutrinos

eV sterile neutrinos in cosmology

- Short baseline neutrino experiments, like LSND and MiniBooNE experiments, pointed towards the existence of eV mass scale sterile neutrinos. (PhysRevD.64.112007, PhysRevLett.12.221801)
- There is a long-standing problem of accommodating eV mass sterile neutrinos in the cosmological context.
- Non zero mixing between active and sterile neutrinos ⇒ Thermal equilibrium ($T_s = T_\nu$)

$$\Downarrow$$

Contribute to $N_{\rm eff}$

Current bound

$$N_{\rm eff} = 2.96^{+0.34}_{-0.33} \ \ \text{(CMB + lensing + BAO)}$$

$$N_{\rm eff} = 2.93 \pm 0.23 \ \ \text{(CMB + lensing + BAO + BBN)}$$

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- ❒ In simple 3+1 neutrinos scenario with eV sterile neutrinos and non-zero mixing angle $N_{\rm eff} \approx 4$

Conflict with the existing bounds ⇒ Hints for new physics??

Self interacting sterile neutrinos

- Self-interaction between sterile neutrinos has been proposed to alleviate this tension. [1]

$$\mathcal{L}_{\text{int}} = g_X \bar{\nu}_s \gamma_\mu \frac{1}{2}(1 - \gamma_5)\nu_s X^\mu$$

- The self-interaction generates a effective potential that suppresses the effective mixing of the sterile and active states in the early universe.
- Mixing angle is large at low temperature, sterile and active neutrino recouple at low temperature.

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[3] Hannestad et al. Phys. Rev. Lett. 112(3):031802
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- Mixing angle is large at low temperature, sterile and active neutrino recouple at low temperature.
- Quantitatively the effect of self interaction can be studied using QKEs (Phys. Rev. D86 (2012) 053009)

$$\mathrm{i}\,\frac{d\varrho}{dt} = [\Omega, \varrho] + C[\varrho]$$

where $C[\varrho]$ is the collision term and

$$\Omega = \frac{1}{2p} U^\dagger M^2 U - \sqrt{2}\frac{8pG_F}{3}\left[\left(\frac{\mathcal{E}_l}{M_W^2} + \frac{\mathcal{E}_\nu}{M_Z^2}\right)\right] - \sqrt{2}G_X\left[\frac{8p}{3}\frac{\mathcal{E}_s}{M_X^2}\right]$$

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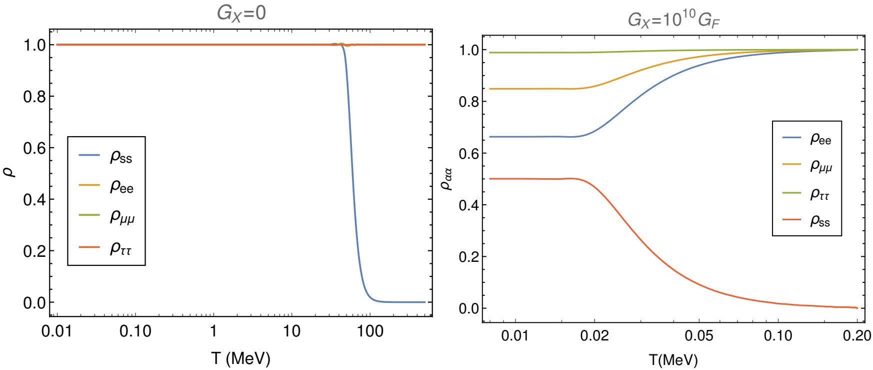

① Self interaction delays the mixing between active and sterile neutrinos till sub MeV temperatures.

② N_{eff} is always within the allowed range.

Self interacting sterile neutrinos

- ❒ The self-interaction between the sterile neutrinos induces scattering among them and the scattering rate is given as

$$\Gamma_X = n_{\nu_s}\langle \sigma v\rangle \simeq G_X^2 T_{\nu_s}^5$$

- ❒ Free-streaming neutrinos suppress the growth of perturbations.
- ❒ Self interaction delays the free-stream regime .
- ❒ Sufficiently large $G_x \Rightarrow$ No free stream regime till neutrino turn non-relativistic.

$$G_X^2 T_{\nu_s}^5 \approx H(T_\gamma). \quad \Rightarrow G_X \sim 10^{10} G_F$$

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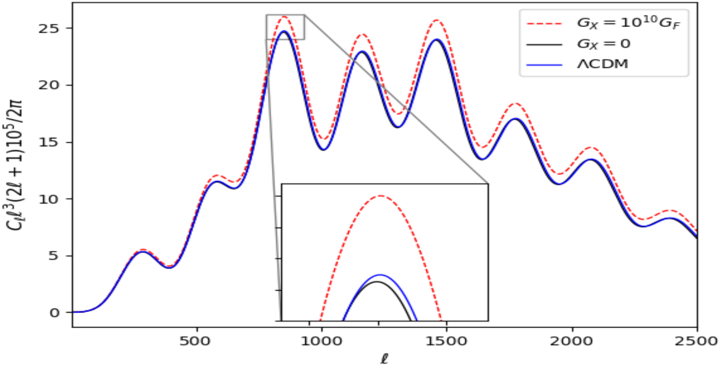
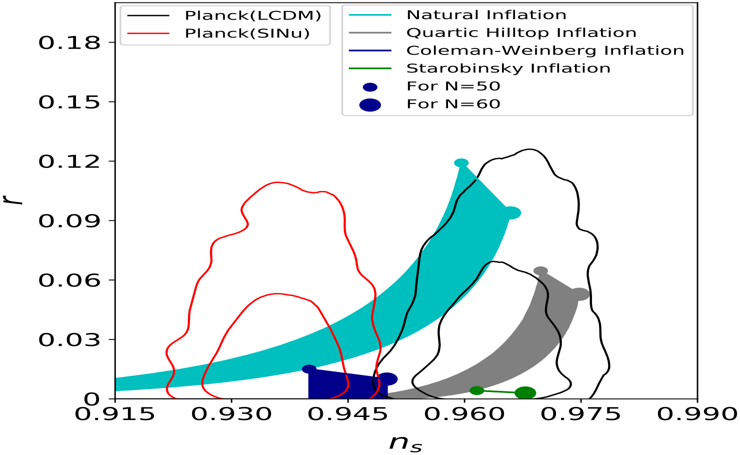
[5] Mirizzi et al. PhysRevD.91.025091
Xiaoyong Chu et al. JCAP10(2105)011

Effect of SIν on temperature power spectrum

Effect of self interacting sterile neutrinos on the CMB temperature anisotropy power spectrum.



Cosmological parameters



Cosmological parameters

Parameter	ΛCDM	SIν model
$\boldsymbol{\omega_b}$	$2.212 \pm 0.022 \times 10^{-2}$	$2.177 \pm 0.022 \times 10^{-2}$
$\boldsymbol{\omega_{cdm}}$	0.1202 ± 0.0021	0.1220 ± 0.0021
$\boldsymbol{100\theta_s}$	1.04185 ± 0.00045	1.04481 ± 0.00045
$\boldsymbol{n_s}$	0.9643 ± 0.0057	0.9361 ± 0.0055
$\boldsymbol{\tau_{\mathrm{reio}}}$	0.0522 ± 0.0079	0.0476 ± 0.0074
$\boldsymbol{r}$	< 0.0460	< 0.0383
$\boldsymbol{H_0}$	67.08 ± 0.92	66.00 ± 0.91
$\boldsymbol{A_s}$	$2.091 \pm 0.034 \times 10^{-9}$	$1.988 \pm 0.030 \times 10^{-9}$
$\boldsymbol{\sigma_8}$	0.8101 ± 0.0092	0.7769 ± 0.0086

Table: The best fit values with 1-σ error for all the parameters obtained from the MCMC analyses for both the models considered are listed here.

Conclusions

1. Flavour specific self interactiopn (ν_τ-ν_τ) interaction can resolve the H_0 tension. Posterior of $G_{\rm eff}$ is bimodal in nature.
2. Flavour specific self interaction can also explain the dip in neutrino flux at IceCube.
3. The values of self-interaction coupling constant and mediator mass required for explaining the IceCube dips are inconsistent with the strong neutrino self-interactions preferred by the cosmological data.
4. The required amount of self-interaction between tau neutrinos (ν_τ) in inverted hierarchy for explaining IceCube dips is consistent with the moderate self-interaction region of cosmological bounds at 1-σ level.
5. For the case of other interactions and hierarchies, the IceCube preferred amount of self-interaction is consistent with moderate self-interaction region of cosmological bounds at 2-σ level only.
6. Self interaction between sterile neutrinos had been proposed to make eV mass sterile neutrinos viable with cosmological observations .
7. The value of spectral index (n_s) shifts to a lower value in SIν model, which makes a significant impact on the validity of inflation models.

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Thanks You!!