Light Dark Universe: Prospects for Light Particle Searches

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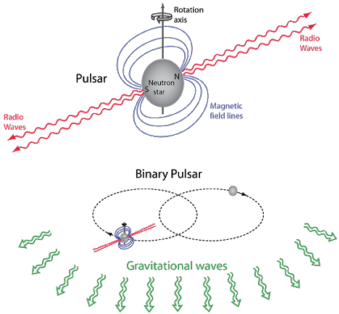
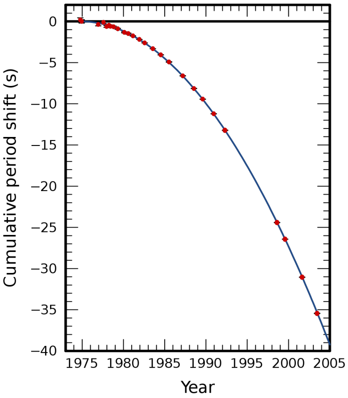
State of the Universe: TIFR

21st December, 2021

Indirect detection of Gravitational Wave





- Hulse and Taylor (1993): Orbital period loss of binary system (first indirect evidence of GW)
- GW150914: Merger of two steller mass black holes (first direct evidence of GW)

Quadrupole formula for GW radiation

$$P = \frac{G}{5c^5}\left(\frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3} - \frac{1}{3}\frac{d^3Q_{ii}}{dt^3}\frac{d^3Q_{jj}}{dt^3}\right)$$

Peters and Mathews (1963):

The energy loss for arbitrary eccentricity of Keplerian orbit

$$\frac{dE}{dt} = \frac{32G}{5}\Omega^6\left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 D^4(1-e^2)^{-7/2}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

The orbital period loss

$$\dot{P}_b = 6\pi G^{-\frac{3}{2}}(m_1 m_2)^{-1}(m_1 + m_2)^{-\frac{1}{2}} D^{\frac{5}{2}}\left(\frac{dE}{dt}\right)$$

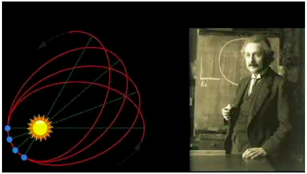
Hulse Taylor Binary System:

$$\dot{P}_{b\,GR} = (-2.40263 \pm 0.00005) \times 10^{-12} ss^{-1}$$

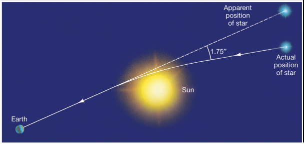
$$\dot{P}_{b\,observed} = -2.423(1) \times 10^{-12} ss^{-1}$$

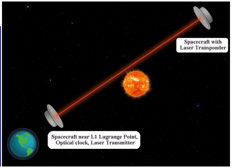
Matches in good agreement with the GR prediction.→ Indirect evidence of GW.

However there is less than 1% uncertainty!



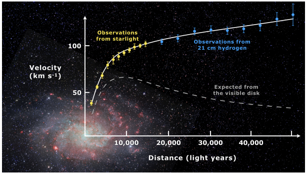
Perihelion precession of Mercury: Test of Einstein's GR Theory $\rightarrow$ Uncertainty in the measurement from the GR theory $\mathcal{O}(10^{-3})$!

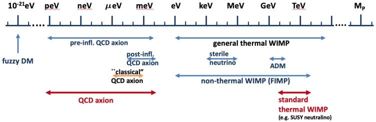


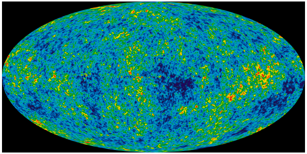


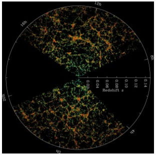
Gravitational light bending and Shapiro delay: Test of Einstein's GR Theory $\rightarrow$ Uncertainties in the measurement from the GR theory are $\mathcal{O}(10^{-4})$ and $\mathcal{O}(10^{-5})$ respectively !

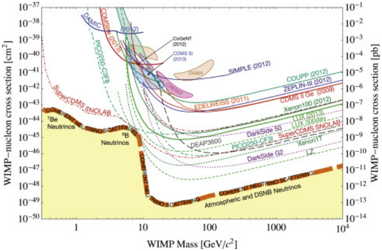
Dark Matter: Why do we need it?





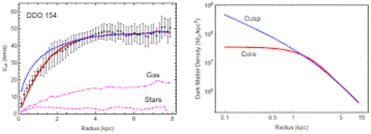


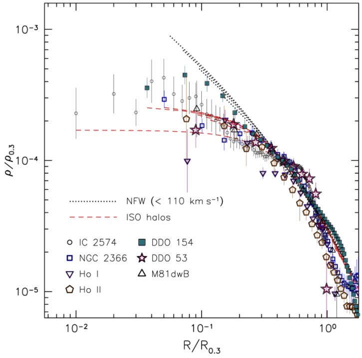




- Standard cold dark matter (WIMP)$\rightarrow$ Strong constraint from direct detection.
- small scale structure problem.
- other possible dark matter models.

Core-Cusp Problem

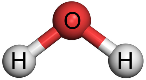




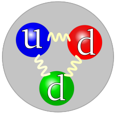
Radiation of light particles:

- Axions (Spin 0)
- light Z' (Spin 1)
- massive graviton (Spin 2)
- sterile neutrino (splin 1/2)
- others...

Strong CP problem Axion was first introduced to solve the strong CP problem. The most stringent probe of strong CP violation is the electric dipole moment of neutron.



Estimate: $d_{H_2O} \sim 10^{-8} e.cm$
Data: $d_{H_2O} \sim 0.5 \times 10^{-8} e.cm$



Estimate: $d_n \sim 10^{-16} e.cm$
Data: $|d_n| < 3 \times 10^{-26} e.cm$
(10.1103/PhysRevD.92.092003) The Strong CP problem!

QCD, the theory of strong interactions has a problem: The strong CP problem. The QCD Lagrangian is,

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{n} [\bar{q}_j i \gamma^\mu D_\mu q_j - (m_j q^\dagger_{Lj} q_{Rj} + h.c)]$$

$$+ \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

Where,

$$\tilde{G}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

The last term in the QCD Lagrangian violates the discrete symmetries CP,T and P.

1. θ term must be present if none of the quark masses vanishes.
2. QCD depends on θ through the combination of parameters.

$$\bar{\theta} = \theta + arg(det(M))$$

For a non-vanishing $\bar{\theta}$ the induced neutron electric dipole moment is,

$$d_n \simeq 5 \times 10^{-16} \bar{\theta} e.cm$$

(S.Profumo, An introduction to particle dark matter)
A general/natural $\bar{\theta} \sim \mathcal{O}(1)$ badly violates current experimental constraints,

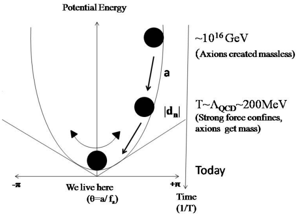
$$d_n < 3 \times 10^{-26} e.cm$$

(10.1103/PhysRevD.92.092003) by ten order of magnitude! So why nature chose such small $\bar{\theta}$.

Peccei-Quinn Solution, 1977

Peccei-Quinn Solution

- $\bar{\theta}$ is a dynamical field driven to zero by its own classical potential.
- Spontaneous breaking of $U(1)_{PQ}$ quasi symmetry at a scale f_a.
- explicit symmetry breaking at Λ_{QCD} due to non perturbative effects.
- pseudo nambu goldstone boson appears which is the QCD axion.



$\bar{\theta} \rightarrow \frac{a}{f_a}$ with scalar field a and scale f_a. Axions are created at this scale. Strong force generates axion potential,

$$V(a) \approx \frac{1}{2}\Lambda_{QCD}^4\left[1 - \cos\left(\frac{a}{f_a}\right)\right]$$

So the QCD axion mass,

$$m_a \approx \frac{\Lambda_{QCD}^2}{f_a} \approx 10^{-9} eV\left(\frac{10^{16} GeV}{f_a}\right)$$

(D.J.E Marsh, arXiv:1510.07633)

Axion Like Particles (ALPs) 1.They are not the exact QCD axions.

2.They have similar kind of interaction as QCD axions.

3. This ALPs are pNGB, spontaneous U(1) breaking scale f_a, explicit U(1) breaking scale at Λ by some non perturbative effects.

4. The potential is given as,

$$V(a) = \Lambda^4[1 - \cos\left(\frac{a}{f_a}\right)]$$

(D.J.E Marsh, arXiv:1510.07633)

Field Evolution of the Axionic FDM At the beginning of the universe how axion field evolve with the potential.

$$S = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\Big(\frac{f_a^2}{2}(\partial_\mu\theta)(\partial^\mu\theta) - V(\theta)\Big)$$

or,

$$= \int d^4x\sqrt{-g}\Big(\frac{1}{2}(\partial_\mu a)(\partial^\mu a) - V\Big(\frac{a}{f_a}\Big)\Big)$$

Where,

$$V\Big(\frac{a}{f_a}\Big) = m_a^2 f_a^2\Big[1 - \cos\Big(\frac{a}{f_a}\Big)\Big]$$

In the Fourier space for non relativistic or zero mode the equation of motion becomes,

$$\ddot{a_k} + 3H\dot{a_k} + m_a^2 a_k = 0$$

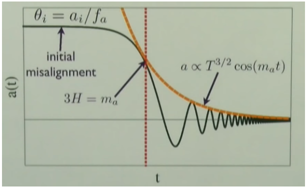
The energy-momentum tensor is given as,

$$T^{\mu}_{\nu} = (\partial^{\mu} a)(\partial_{\nu} a) - \mathcal{L}\delta^{\mu}_{\nu}$$

So the energy density and the pressure are,

$$\rho = \frac{1}{2}\dot{a}^2 + V(a)$$

$$p = \frac{1}{2}\dot{a}^2 - V(a)$$



So at the late time the axion field redshifts like a cold dark matter.
Assuming right Dark matter abundance $\Omega_a h^2 \sim 0.1$,Today,

$$\Omega_{DM} \sim 0.1\left(\frac{a_0}{10^{17}GeV}\right)^2\left(\frac{m_a}{10^{-22}eV}\right)^{\frac{1}{2}}$$

(L. Hui et al, Phys. Rev. D95,043541)

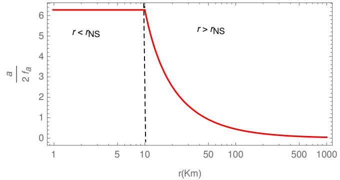
$$\frac{\lambda}{2\pi} = \frac{\hbar}{m_a v} = 1.92\mathrm{kpc}\left(\frac{10^{-22}\mathrm{eV}}{\mathrm{m_a}}\right)\left(\frac{10\mathrm{km/s}}{\mathrm{v}}\right)$$

Constraints on ultralight axions from compact binary systems(Subhendra Mohanty, Soumya Jana, T.K.P), Phys.Rev.D 101 (2020) 8, 083007.

$$\omega = \left[\frac{G(m_1+m_2)}{D^3}\right]^{\frac{1}{2}} \sim 10^{-19}\mathrm{eV},\ \mathrm{a} = -\frac{\mathrm{q_{eff}}}{2\mathrm{GM}} \ln\left(1-\frac{2\mathrm{GM}}{\mathrm{r}}\right),\ \mathrm{q_{eff}} = -\frac{8\pi \mathrm{GMf_a}}{\ln\left(1-\frac{2\mathrm{GM}}{\mathrm{r_{NS}}}\right)}$$

$$\frac{dE}{dt} = -\frac{32}{5}G\mu^2 D^4\omega^6(1-e^2)^{-\frac{7}{2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) - \frac{\omega^4 p^2}{24\pi}\frac{(1+e^2/2)}{(1-e^2)^{\frac{5}{2}}}$$

$$\Omega_{DM} \sim 0.1\left(\frac{a_0}{10^{17}GeV}\right)^2\left(\frac{m_a}{10^{-22}eV}\right)^{\frac{1}{2}}$$



If ALPs are FDM, they do not couple with quarks.

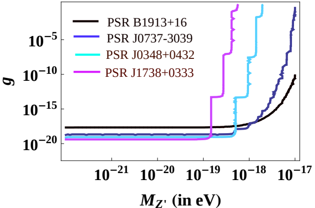
Compact binary system	f_a (GeV)	α
PSR J0348+0432	$\lesssim 1.66 \times 10^{11}$	$\lesssim 5.73 \times 10^{-10}$
PSR J0737-3039	$\lesssim 9.76 \times 10^{16}$	$\lesssim 9.21 \times 10^{-3}$
PSR J1738+0333	$\lesssim 2.03 \times 10^{11}$	$\lesssim 8.59 \times 10^{-10}$
PSR B1913+16	$\lesssim 2.12 \times 10^{17}$	$\lesssim 3.4 \times 10^{-2}$

Vector gauge boson radiation from compact binary systems in a gauged $L_\mu - L_\tau$ scenario(Subhendra Mohanty, Soumya Jana, T.K.P), Phys.Rev.D 100 (2019) 12, 123023.

$$N_\mu \approx 10^{55}(\mathrm{R.Garani, J.Heeck; 2019}).$$

$$\frac{dE}{dt} = \frac{g^2}{6\pi} a^2 M^2 \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)^2 \Omega^4 \sum_{n>n_0} 2n^2 \left[J_n'^2(ne) + \frac{(1-e^2)}{e^2} J_n^2(ne)\right] \left(1 - \frac{n_0^2}{n^2}\right)^{\frac{1}{2}} \left(1 + \frac{1}{2}\frac{n_0^2}{n^2}\right).$$

Compact binary system	g(fifth force)	g(orbital period decay)
PSR B1913+16	$\leq 4.99 \times 10^{-17}$	$\leq 2.21 \times 10^{-18}$
PSR J0737-3039	$\leq 4.58 \times 10^{-17}$	$\leq 2.17 \times 10^{-19}$
PSR J0348+0432	–	$\leq 9.02 \times 10^{-20}$
PSR J1738+0333	–	$\leq 4.24 \times 10^{-20}$

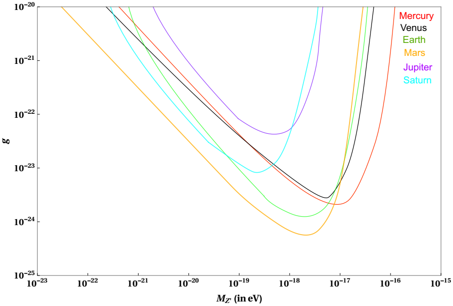


Constraints on long range force from perihelion precession of planets in a gauged $L_e - L_{\mu,\tau}$ scenario(Subhendra Mohnaty, Soumya Jana, T.K.P), Eur.Phys.J.C 81 (2021) 4, 286.

$$M_{Z'} \ll \frac{1}{a} \sim \mathcal{O}(10^{-19}\mathrm{eV}), \quad \frac{\mathrm{d}^2\mathrm{u}}{\mathrm{d}\phi^2} + \mathrm{u} = \frac{\mathrm{M}}{\mathrm{L}^2} + 3\mathrm{M}\mathrm{u}^2 + \frac{\mathrm{g}^2\mathrm{N}_1\mathrm{N}_2}{4\pi\mathrm{L}^2\mathrm{M}_\mathrm{p}}\mathrm{e}^{-\frac{\mathrm{M}_{\mathrm{Z}'}}{\mathrm{u}}} + \frac{\mathrm{g}^2\mathrm{N}_1\mathrm{N}_2\mathrm{E}\mathrm{M}_{\mathrm{Z}'}}{4\pi\mathrm{L}^2\mathrm{M}_\mathrm{p}\mathrm{u}}\mathrm{e}^{-\frac{\mathrm{M}_{\mathrm{Z}'}}{\mathrm{u}}}$$

$$\Delta\phi = \frac{6\pi GM}{a(1-e^2)} + \frac{g^2 N_1 N_2 |E| M_{Z'}^2 a^2 (1-e^2)}{4M_p(GM + \frac{g^2 N_1 N_2}{4\pi M_p})(1+e)}$$

$$\frac{g^2 N_1 N_2 |E| M_{Z'}^2 a^2 (1-e^2)}{4M_p(GM + \frac{g^2 N_1 N_2 |E|}{4\pi M_p})(1+e)}\left(\frac{\mathrm{century}}{T}\right) < 3.0 \times 10^{-3}\mathrm{arcsecond/century}$$



Probing the angle of birefringence due to long range axion hair from pulsars(Subhendra Mohanty,T.K.P), Phys.Rev.D 102 (2020) 8, 083029

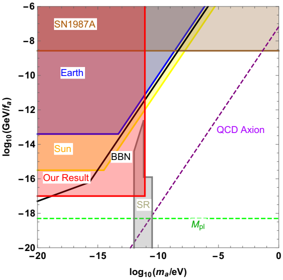
$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \nabla_\mu \nabla^\mu \mathbf{B} = -g_{a\gamma\gamma}(\nabla a) \times \frac{\partial \mathbf{B}}{\partial t}$$

$$\omega^2\left(1 - \frac{2GM}{r}\right)^{-1} - k_r^2\left(1 - \frac{2GM}{r}\right) = \pm g_{a\gamma\gamma}(\partial_r a)\omega$$

$$\Delta\phi = -\frac{c\alpha_{em}}{2\pi f_a}\frac{q_a e^{-m_a R}}{R}\left[1 + \frac{GM}{R}\{1 - m_a R\ln(m_a R) + m_a R e^{2m_a R}E_i(-2m_a R)\}\right]$$

$$q_a = 4\pi f_a R e^{m_a R}\left[1 + \frac{GM}{R}\{1 - m_a R\ln(m_a R) + m_a R e^{2m_a R}E_i(-2m_a R)\}\right]^{-1}$$

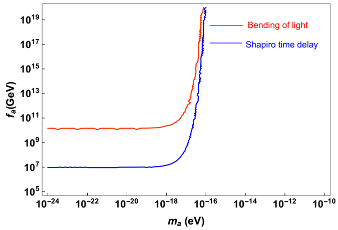
$$\Delta\theta = -c\alpha_{em} = 0.42^\circ$$



Constraints on axionic fuzzy dark matter from light bending and Shapiro time delay(T.K.P), JCAP09(2021)041

$$\Delta\phi_{axions} = \frac{\frac{4M}{b^2} + \frac{q_1 q_2}{2\pi M_p L^2}(1 - 0.347 m_a^2 b^2)}{\frac{1}{b} + \frac{q_1 q_2 m_a^2 b^2}{8\pi M_p L^2}} - \frac{4M}{b}$$

$$\Delta T_{axions} = \Big[4M\Big[\ln\Big(\frac{4r_e r_v}{r_0^2}\Big) + 1\Big] + 2b_0 c_0(-1 + c_0 M)(r_e + r_v) + \frac{b_0 c_0^2}{2}(r_e^2 + r_v^2) + 2b_0 - 4c_0 M b_0 + 2a_0(r_e + r_v) + \frac{b_0}{24}(48 + 36 c_0^2 r_0^2)[Ei(-c_0 r_e) + Ei(-c_0 r_v)]\Big] - 4M\Big[\ln\Big(\frac{4r_e r_v}{r_0^2}\Big) + 1\Big]$$

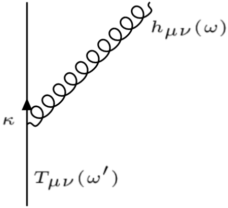


If ALPs are FDM, they do not couple with quarks.

Experiments	axion decay constant (f_a)	α
Light bending	$\lesssim 1.58 \times 10^{10}\mathrm{GeV}$	$\lesssim 10^{-2}$
Shapiro time delay	$\lesssim 9.85 \times 10^{6}\mathrm{GeV}$	$\lesssim 4.12 \times 10^{-9}$

Gravitational radiation from binary systems in massive graviton theories(Subhendra Mohanty, Soumya Jana, T.K.P arXiv:2105.13335)

Energy loss by massless graviton radiation from binaries



$$S_{EH} = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R + \mathcal{L}_m\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \qquad \kappa = \sqrt{32\pi G}$$

$$S_{EH} = \int d^4x\Big[-\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + \frac{1}{2}(\partial_\mu h)^2 - (\partial_\mu h)(\partial^\nu h^\mu_\nu) + (\partial_\mu h_{\nu\rho})(\partial^\nu h^{\mu\rho}) + \frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}\Big]$$

Polarization sum of massless spin-2 gravitons

$$\sum_{\lambda=1}^{2} \epsilon^{\lambda}_{\mu\nu}(k)\epsilon^{*\lambda}_{\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

Emission rate of massless graviton

$$d\Gamma = \frac{\kappa^2}{4}\sum_{\lambda=1}^{2} |T_{\mu\nu}(k')\epsilon^{\mu\nu}_{\lambda}(k)|^2 2\pi\delta(\omega-\omega')\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}$$

Rate of energy loss

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2}\int \left[|T_{\mu\nu}(k')|^2 - \frac{1}{2}|T^{\mu}{}_{\mu}(k')|^2\right]\delta(\omega-\omega')\omega^2 d\omega d\Omega_k$$

The stress tensor or the current density for this compact binary system is

$$T_{\mu\nu}(x') = \mu\delta^3(\mathbf{x}' - \mathbf{x}(t))U_\mu U_\nu$$

We can write the Keplerian orbit in the parametric form as

$$x = a(\cos\xi - e), \quad y = a\sqrt{(1-e^2)}\sin\xi, \quad \Omega t = \xi - e\sin\xi$$

We can write

$$\left[|T_{\mu\nu}(k')|^2 - \frac{1}{2}|T^{\mu}{}_{\mu}(k')|^2\right] = \Lambda^0_{ij,lm} T^{ij*} T^{lm}$$

where,

$$\Lambda^0_{ij,lm} = \left[\delta_{il}\delta_{jm} - 2\hat{k}_j\hat{k}_m\delta_{il} + \frac{1}{2}\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m - \frac{1}{2}\delta_{ij}\delta_{lm} + \frac{1}{2}\Big(\delta_{ij}\hat{k}_l\hat{k}_m + \delta_{lm}\hat{k}_i\hat{k}_j\Big)\right]$$

The angular integral becomes

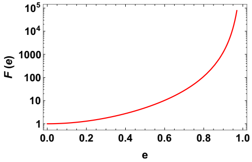
$$\int d\Omega_k \Lambda^0_{ij,lm} T^{ij*}(\omega') T^{lm}(\omega') = \frac{8\pi}{5}\Big(T_{ij}(\omega') T^*_{ji}(\omega') - \frac{1}{3}|T^i{}_i(\omega')|^2\Big)$$

$$T_{xx}(\omega') = -\frac{\mu\omega'^2 a^2}{4n}\Big[J_{n-2}(ne) - 2eJ_{n-1}(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\Big]$$

$$T_{yy}(\omega') = \frac{\mu\omega'^2 a^2}{4n}\Big[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{4}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\Big]$$

$$T_{xy}(\omega') = \frac{-i\mu\omega'^2 a^2}{4n}(1-e^2)^{\frac{1}{2}}\Big[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)\Big]$$

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2}\int \frac{8\pi}{5}\left[T_{ij}(\omega')T^*_{ji}(\omega') - \frac{1}{3}|T^i{}_i(\omega')|^2\right]\delta(\omega-\omega')\omega^2 d\omega,$$
$$= \frac{32G}{5}\sum_{n=1}^{\infty}(n\Omega)^2\mu^2 a^4 (n\Omega)^4 f(n,e)$$
$$= \frac{32G}{5}\Omega^6\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 a^4(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$



- Einstein's quadrupole radiation matches with the Peter Mathews result.
- Massless graviton emission $\rightarrow$ about 1% uncertainty from the GR prediction of orbital period loss can be accounted by other light particle radiation: Massive graviton.

Fierz Pauli massive gravity theory

$$S = \int d^4x \Big[-\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + \frac{1}{2}(\partial_\mu h)^2 - (\partial_\mu h)(\partial^\nu h^\mu_\nu) + (\partial_\mu h_{\nu\rho})(\partial^\nu h^{\mu\rho}) + \frac{1}{2}m_g^2\Big(h_{\mu\nu}h^{\mu\nu} - h^2\Big) + \frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}\Big]$$

The polarisation sum for the FP massive gravity theory can be written as

$$\sum_\lambda \epsilon^\lambda_{\mu\nu}(k)\epsilon^{*\lambda}_{\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\nu\alpha}\eta_{\mu\beta}) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\mu\nu} + (k-\text{dependent terms})$$

There is an extra contribution of $(1/6)\,T^*T'$ to the amplitude in the FP theory in $m_g \to 0$ limit. $\to$ vDVZ discontinuity.

Newtonian potential between two massive bodies: The amplitude:

$$\mathcal{A}_{GR} = \frac{\kappa^2}{4} T^{\mu\nu} D^{(0)}_{\mu\nu\alpha\beta}(k) T'^{\alpha\beta}$$

$$\begin{aligned} V_{GR} &= \frac{\kappa^2}{4} \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot r} \frac{1}{-k^2} \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\alpha_\alpha \right) T'^{\mu\nu} \\ &= \frac{GM_1 M_2}{r} \end{aligned}$$

$$\mathcal{A}_{FP} = \frac{\kappa^2}{4} \frac{1}{-k^2 + m_g^2} \left(T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^\alpha_\alpha \right) T'^{\mu\nu}$$

$$\begin{aligned} V_{FP} = &= \frac{\kappa^2}{4} \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot r} \frac{1}{-k^2 + m_g^2} \left(T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^\alpha_\alpha \right) T'^{\mu\nu} \\ &= \left(\frac{4}{3} \right) \frac{GM_1 M_2}{r} e^{-m_g r} \end{aligned}$$

The bending of light and perihelion precession of planets: The extra factor of (4/3) in the Newtonian potential of FP theory cannot be absorbed by redefining G.

Graviton radiation from binaries in Fierz Pauli theory

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2}\int \left[|T_{\mu\nu}(k')|^2 - \frac{1}{3}|T^{\mu}{}_{\mu}(k')|^2\right]\delta(\omega-\omega')\omega^2\left(1-\frac{m_g^2}{\omega^2}\right)^{\frac{1}{2}} d\omega d\Omega_k$$

For the massive graviton, the dispersion relation is

$$|\mathbf{k}|^2 = \omega^2\left(1-\frac{m_g^2}{\omega^2}\right)$$

$$\frac{dE}{dt} = \frac{32G}{5}\mu^2 a^4\Omega^6\sum_{n=1}^{\infty} n^6\sqrt{1-\frac{n_0^2}{n^2}}\Bigg[f(n,e)\left(1+\frac{4}{3}\frac{n_0^2}{n^2}+\frac{1}{6}\frac{n_0^4}{n^4}\right) - \frac{5J_n^2(ne)}{36n^4}\frac{n_0^2}{n^2}\left(1-\frac{n_0^2}{4n^2}\right)\Bigg] +$$
$$\frac{32G}{5}\mu^2 a^4\Omega^6\sum_{n=1}^{\infty} n^6\sqrt{1-\frac{n_0^2}{n^2}}\left[\frac{1}{18}f(n,e)\left(1-\frac{n_0^2}{n^2}\right)^2+\frac{5J_n^2(ne)}{108n^4}\left(1+\frac{n_0^2}{2n^2}\right)^2\right]$$

First term denotes the energy loss in the massive gravity theory without vDVZ discontinuity. Second term denotes the contribution due to the scalar mode.

Massive gravity without vDVZ discontinuity

$$S = \int d^4x \Big[\frac{1}{2} h_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} h_{\alpha\beta} + \frac{1}{2} m_g^2 h_{\mu\nu} (\eta^{\mu(\alpha} \eta^{\beta)\nu} - (1-a)\eta^{\mu\nu}\eta^{\alpha\beta}) h_{\alpha\beta} + \frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}\Big]$$

$$D^{1/2}_{\alpha\beta\mu\nu}(k) = \frac{1}{-k^2 + m_g^2} \Big(\frac{1}{2} (\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \frac{1}{2}\eta_{\alpha\beta}\eta_{\mu\nu}\Big) + (k - \text{dependent terms})$$

The gravitational potential in this theory takes the Yukawa form

$$V^{(1/2)}(r) = \frac{GM_1 M_2}{r} \, e^{-m_g r}$$

We get the expression for the rate of energy loss as

$$\frac{dE}{dt} = \frac{32G}{5} \mu^2 a^4 \Omega^6 \sum_{n=1}^{\infty} n^6 \sqrt{1 - \frac{n_0^2}{n^2}} \Big[f(n,e) \Big(1 + \frac{4}{3}\frac{n_0^2}{n^2} + \frac{1}{6}\frac{n_0^4}{n^4}\Big) - \frac{5J_n^2(ne)}{36n^4} \frac{n_0^2}{n^2} \Big(1 - \frac{n_0^2}{4n^2}\Big)\Big]$$

Dvali-Gabadadze-Porrati (DGP) theory

$$\mathcal{S} \supset \int d^4x dy \Big(\frac{M_5^3}{4} \sqrt{-{}^{(5)}g}\,{}^{(5)}R + \delta(y) \Big[\sqrt{-g} \frac{M_{pl}^2}{2} R[g] + \mathcal{L}_m(g, \psi_i) \Big] \Big)$$

$$\omega^2 = |\mathbf{k}|^2 - m_0^2$$

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2} \int \Big[|T_{\mu\nu}(k')|^2 - \frac{1}{3} |T^{\mu}{}_{\mu}(k')|^2 \Big] \delta(\omega - \omega') \omega^2 \Big(1 + \frac{m_0^2}{\omega^2} \Big)^{\frac{1}{2}} d\omega d\Omega_k$$

$$\frac{dE}{dt} = \frac{32G}{5} \mu^2 a^4 \Omega^6 \sum_{n=1}^{\infty} n^6 \sqrt{1 + \frac{\tilde{n}_0^2}{n^2}} \Big[f(n,e) \left(\frac{19}{18} - \frac{11}{9} \frac{\tilde{n}_0^2}{n^2} + \frac{2}{9} \frac{\tilde{n}_0^4}{n^4} \right) + \frac{5 J_n^2(ne)}{108 n^4} \left(1 + \frac{\tilde{n}_0^2}{n^2} \right)^2 \Big]$$

Constraints from observations

Vainshtein radius and limits of linear theory Vainshtein radius for FP theory

$$r_V = \left(\frac{R_s}{m_g^4}\right)^{1/5}$$

$$\lambda \sim \pi\Omega^{-1} > r_V = \left(\frac{R_s}{m_g^4}\right)^{1/5}$$

$$m_g > \frac{\Omega^{5/4}}{\pi^{5/4}}(2GM)^{1/4}$$

- PSR B1913+16: $m_g > 3.06 \times 10^{-22}\,\mathrm{eV}$
- PSR J1738+0333 $m_g > 2.456 \times 10^{-22}\mathrm{eV}$

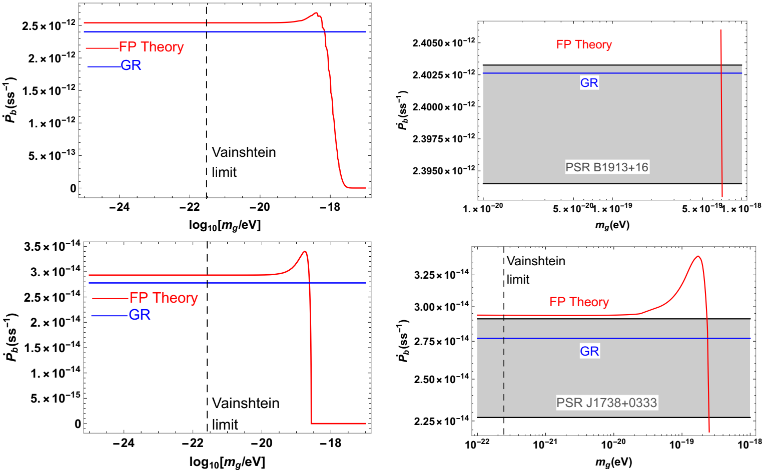
Vainshtein radius for DGP theory

$$r_V = \left(\frac{R_g}{m_g^2} \right)^{1/3}$$

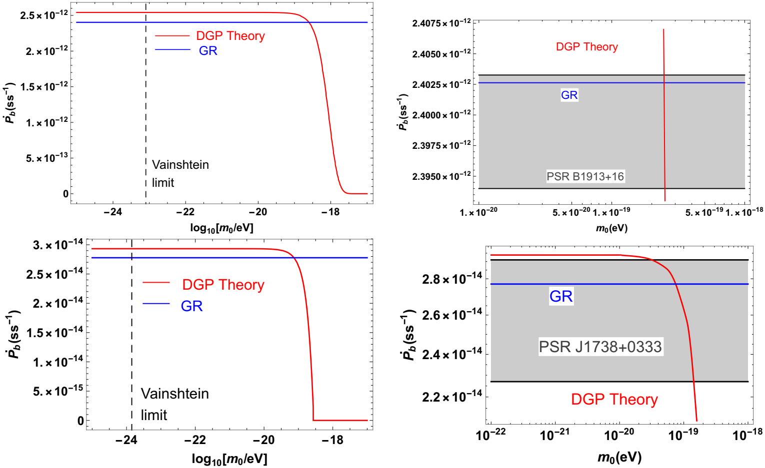
- PSR B1913+16: $m_g > 7.84 \times 10^{-24}\,\mathrm{eV}$
- PSR J1738+0333 $m_g > 1.406 \times 10^{-24}\mathrm{eV}$

For the case of $a = 1/2$: no Vainshtein limiting radius.

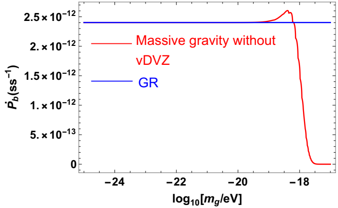
Constraints from observation for FP theory

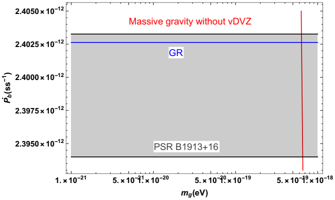


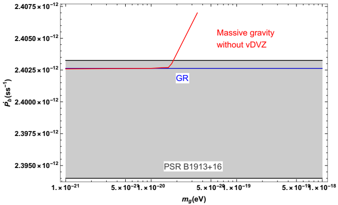
Constraints from observation for DGP theory

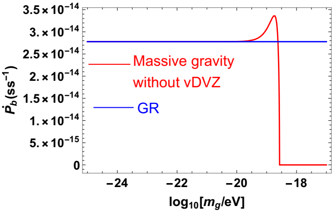


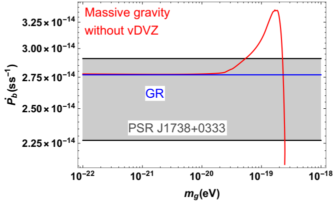
No vDVZ discontinuity theory











Results

- We calculate the energy loss due to massive graviton radiation for single graviton vertex process and study vDVZ discontinuity for FP, DGP and modified FP theories from Feynman Diagram techniques.
- Observations from PSR B1913+16 and PSR J1738+0333 rule out all value of graviton mass and from the Vainshtein limit we can put the upper bounds $m_g < 3.06 \times 10^{-22}$ eV for the FP theory and $m_0 < 1.406 \times 10^{-24}$ eV for the DGP theory.
- For the No-vDVZ discontinuity theory the upper bound from combined PSR B1913+16 and PSR J1738+0333 data is $m_g < 1.81 \times 10^{-20}$eV.
- The diagramatic method can also be used for computing the wave-form of gravitational waves observed in direct detection experiments like LIGO and VIRGO.

Freeze-in sterile neutrino dark matter in a class of $U(1)'$ models with inverse seesaw(T.K.P, Srubabati Goswami, Arindam Das, Vishnudath K.N, arXiv:2104.13986)

The Model

	Q_{L_i}	u_{R_i}	d_{R_i}	ℓ_{L_i}	e_{R_i}	ν_{R_α}	H	Φ	S
$\mathrm{SU(3)_C}$	3	3	3	1	1	1	1	1	1
$\mathrm{SU(2)_L}$	2	1	1	2	1	1	2	1	1
$\mathrm{U(1)_Y}$	1/6	2/3	−1/3	−1/2	−1	0	1/2	0	0
$U(1)'$	x_q	x_u	x_d	x_l	x_e	x_ν	$\frac{x_h}{2}$	$-x_\phi$	0

The most general Yukawa Lagrangian invariant under this new gauge group is,

$$-\mathcal{L}_{\rm Yukawa} = Y_e \bar{\ell}_L H e_R + Y_\nu \bar{\ell}_L \tilde{H} \nu_R + Y_u \bar{Q}_L \tilde{H} u_R + Y_d \bar{Q}_L H d_R + y_{NS} \bar{\nu_R} \Phi S + \frac{1}{2} \bar{S^c} M_\mu S + \text{h.c.}$$

Freeze-in production of dark matter:

The Boltzmann equation to solve the relic density

$$\dot{n} + 3Hn = <\sigma v(ab \to NN)> n_a n_b.$$

The final abundance of the RHN DM is,

$$\frac{dY}{dT} = \frac{<\sigma v(ab \to NN)> n_a n_b}{sHT}.$$

The thermal average dark matter annihilation cross section

$$<\sigma v> = \frac{1}{(sY_{EQ})^2} g_N^2 \frac{m_N}{64\pi^2 x} \int_{4m_N^2}^{\infty} ds \times 2(s - 4m_N^2)\sigma(s)\sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right),$$

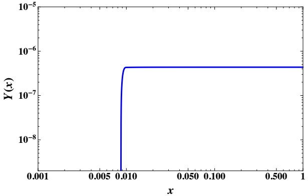
Dark matter in our $U(1)'$ model:

For the process $f\bar{f} \to NN$ in the s-channel mediated by the Z' as,

$$|\bar{\mathcal{M}}|^2 = \frac{g'^4 y_N'^2}{(k^2 - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}\Big[c_V^2\{s^2 + (s-4m_f^2)(s-4m_N^2)\cos^2\theta - 4s(m_N^2 - m_f^2) - 16m_N^2 m_f^2\} + c_A^2\{s^2 + (s-4m_f^2)(s-4m_N^2)\cos^2\theta - 4s(m_N^2 + m_f^2) + 16m_N^2 m_f^2 + \frac{16m_f^2 m_N^2}{M_{Z'}^4}(s - M_{Z'}^2)^2\}\Big]$$

Heavy Z' ($M_Z' \gg T_R \gg m_N$):

$$\Omega_N h^2 \simeq 0.12 \times \left(\frac{m_N}{1\mathrm{MeV}}\right)\left(\frac{10}{g_*}\right)^{\frac{3}{2}}\left(\frac{T_R}{5\mathrm{MeV}}\right)^3\left(\frac{9.7\mathrm{TeV}}{M_Z'/g'}\right)^4 \times \frac{x_\phi^2}{3}\left[2\left\{\left(\frac{3}{4}x_h + x_\phi\right)^2 + \left(\frac{x_h}{4}\right)^2\right\} + \left\{\left(x_\phi + \frac{x_h}{4}\right)^2 + \left(\frac{x_h}{4}\right)^2\right\}\right].$$



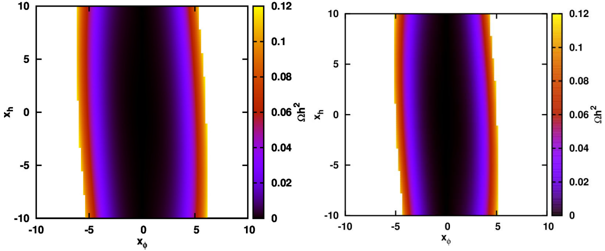


Figure: Relic density plot in the $x_h - x_\phi$ plane. In the left panel, we have fixed the values of the parameters as $m_N = 1$ MeV, $T_R = 10$ MeV, $M_{Z'} = 10$ TeV and $g' = 0.1$ whereas the right panel is for $m_N = 10$ MeV, $T_R = 50$ MeV, $M_{Z'} = 5$ TeV and $g' = 0.01$.

Light Z' $(M'_Z \ll m_N)$:

$$\Omega_N h^2 = 0.12 \times \left(\frac{106.75}{g_*}\right)^{\frac{3}{2}} \left(\frac{g'}{3.04 \times 10^{-6}}\right)^4 x_\phi^2 (10x_h^2 + 13x_\phi^2 + 16x_H x_\phi).$$

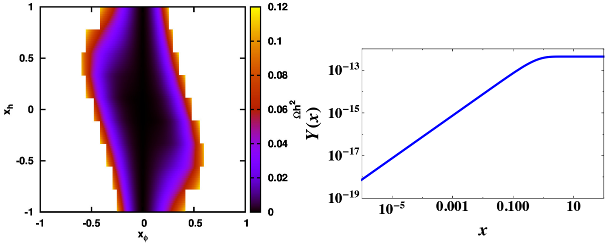


Figure: Relic density plot in $x_h - x_\phi$ plane. In the left panel, We have varied both x_h and x_ϕ from -1 to $+1$ for $g' = 3.04 \times 10^{-6}$ and in right panel, we have varied both x_h and x_ϕ from -200 to $+200$ for $g' = 10^{-8}$.

Intermediate Z' ($m_N \ll M_{Z'} \ll T_R$):

$$\Omega_N h^2 \simeq 0.12\left(\frac{g'}{6.54\times 10^{-9}}\right)^2\left(\frac{m_N}{10\mathrm{keV}}\right)\left(\frac{10\mathrm{GeV}}{M_{Z'}}\right)\left[\frac{x_\phi^2(10x_h^2+13x_\phi^2+16x_hx_\phi)}{\frac{1}{36}(241x_h^2+418x_\phi^2+436x_hx_\phi)+x_\phi^2}\right].$$

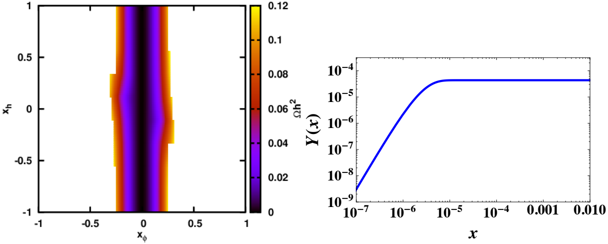
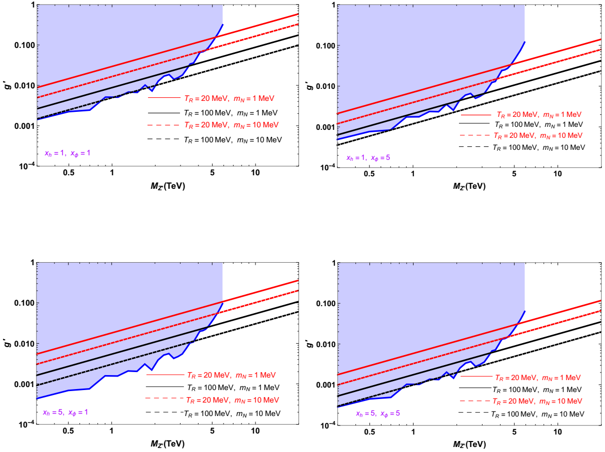
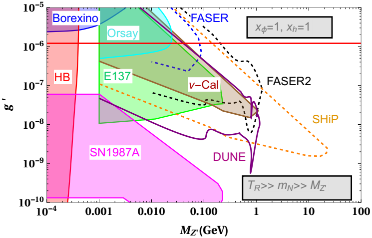


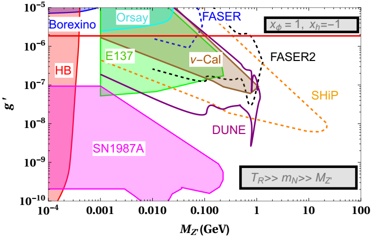
Figure: Relic density plot in $x_h - x_\phi$ plane. In the left panel, We have varied both x_h and x_ϕ from -1 to $+1$ for $g' = 6.54\times 10^{-9}$, $m_N = 10\mathrm{keV}$, $M_{Z'} = 1\mathrm{GeV}$ and in right panel, we have varied both x_h and x_ϕ from -3000 to $+3000$ for $g' = 10^{-12}$, $m_N = 10\mathrm{keV}$, $M_{Z'} = 1\mathrm{GeV}$.

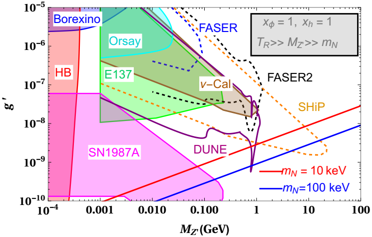
Bounds on coupling for heavy Z' from Experiments

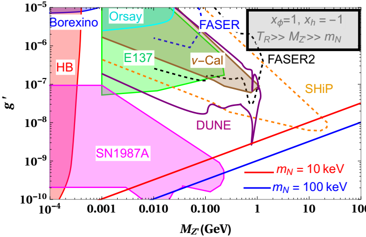


Bounds on coupling for light Z' from Experiments









Discussions

- The precision measurements of GW and other astrophysical experiments demands a possibility of radiation of light particles like axions, gauge bosons, gravitons etc.

- One can probe $U(1)_{L_i - L_j}$ from those above experiments.

- Sterile neutrino is one of the promising candiates of DM. Can also explain INTEGRAL anomaly. The associated gauge boson can be probed in LHC and FASERs, SHiP and DUNE experiments.

- Physics of those light particles is interesting as it can be a possible dark matter candidate.

Publications

- Constraints on ultralight axions from compact binary systems, Phys.Rev.D 101 (2020) 8, 083007, Subhendra Mohanty, Soumya Jana, T.K.P.
- Vector gage boson radiation from compact binary systems in a gauged $L_\mu - L_\tau$ scenario, Phys. Rev. D 100 (2019)12, 123023, Subhendra Mohanty, Soumya Jana, T.K.P.
- Implications of the dark large mixing angle solution and a fourth sterile neutrino for neutrinoless double beta decay, Phys.Rev.D 102 (2020) 1, 015020, Srubabati Goswami, K.N Deepthi, Vishnudath K.N., T.K.P.
- Constraints on long range force from perihelion precession of planets in a gauged $L_e - L_{\mu,\tau}$ scenario, Eur.Phys.J.C 81(2021)4,286Subhendra Mohanty, Soumya Jana, T.K.P.
- Probing the angle of birefringence due to long range axion hair from pulsars, Phys.Rev.D 102 (2020) 8, 083029, Subhendra Mohanty,T.K.P.

Contd...

- Constraints on axionic fuzzy dark matter from light bending and Shapiro time delay, JCAP 09 (2021) 041, T.K.P.
- Freeze-in sterile neutrino dark matter in a class of $U(1)'$ models with inverse seesaw, arXiv:2104.13986, Arindam Das, Srubabati Goswami, Vishnudath K.N., T.K.P.
- Gravitational radiation from binary systems in massive graviton theories, arXiv:2105.13335, Subhendra Mohanty, Soumya Jana, T.K.P.
- Constraints on ultralight axions, vector gauge bosons, and unparticles from geodetic and frame-dragging effects arXiv:2111.05632, T.K.P.

Future Directions

- Propagation of GW through fuzzy dark matter medium and constraining FDM models...The study of refractive index
- Studying GW waveform for both massless and massive gravity theories field theoretic perspective
- DM can be accreted by NS and it can affect GW...Effect of eccentricity.

Thank You!