Harnessing the Power of Unresolved Lensed Systems

Detecting Strong Lenses and Measuring Time Delays from Unresolved Light Curves

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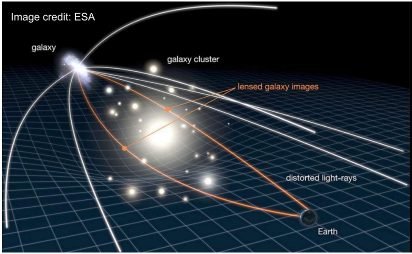
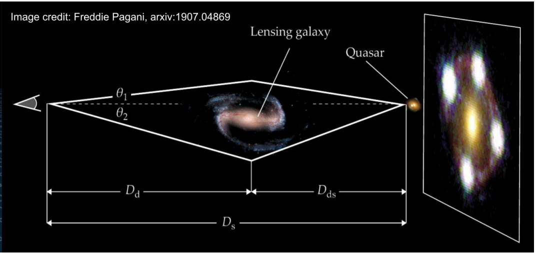
27th May, 2022

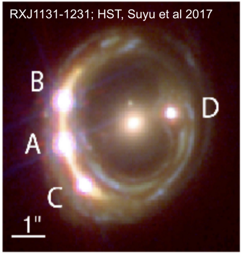
- ΛCDM model fits individual data sets very well.
  - also – SNe1A + BAO + CMB
- But Tensions in some combination: cannot be explained by minimal modifications.
- H_0 **tension:** early universe vs local measurements 67.4 ± 0.5 vs 73.04 ± 1.04 km/s/Mpc
- S_8 **tension** (clustering).
- **Systematics? New Physics? or Both?**
- Need independent probes: Strong Lensing, GW cosmology etc.

Strong lensing

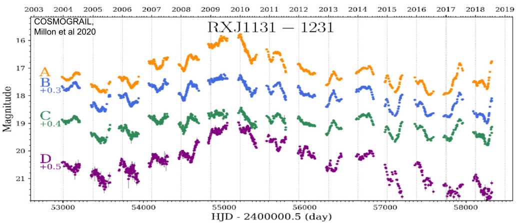


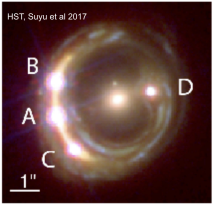


- Multiple images.
- Different Magnifications.
- Time delays.
  - Different geometrical paths.
  - Different gravitational potential.
- Flux ratio anomalies $\Longrightarrow$ Dark Matter.



Variable sources : Time delays

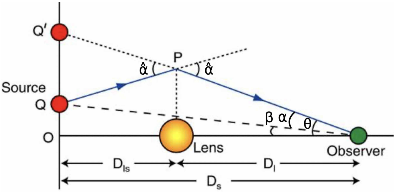




- Time delay:

$$\Delta t_{A,B} = \frac{1}{c}\frac{D_l D_s}{D_{ls}}(1+z_l)\left[\phi(\vec{\theta}_A,\vec{\beta}) - \phi(\vec{\theta}_B,\vec{\beta})\right]$$

$$\phi(\vec{\theta},\vec{\beta}) = \left[\frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2 - \psi(\vec{\theta})\right]$$

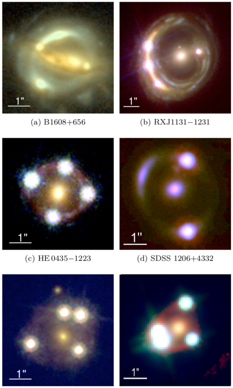


Credit: Zalesky & Ebeling

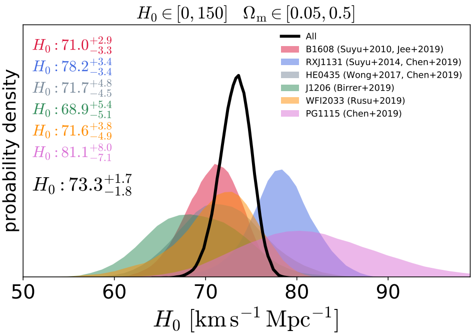
- Also needed:
  - Angular positions: $(\vec{\theta},\vec{\beta})$.
  - Lens mass distribution: $\psi(\vec{\theta})$.
  - Redshifts.
- Time delays can be measured: **variable sources – QSOs, SNe**

Time Delay Cosmography

- Observed: few tens of Lensed QSO's and 3 confirmed lensed SNe.
- H0LiCOW with 6 LQSO's: $H_0 = 73.3^{+1.7}_{-1.8}$ km/s/Mpc (ΛCDM) [Wong+ 2020]



(a) B1608+656 (b) RXJ1131−1231 (c) HE 0435−1223 (d) SDSS 1206+4332 (e) WFI2033−4723 (f) PG 1115+080

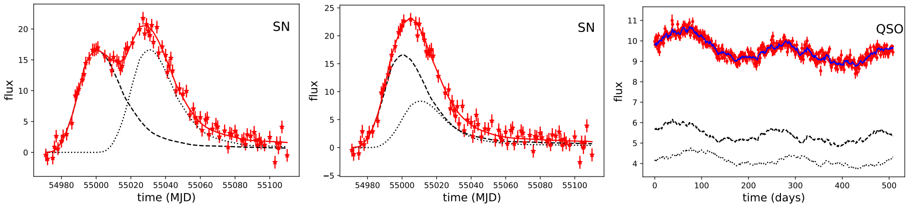


Credit: H0LiCOW team, Martin Millon, Vivian Bonvin

- Model independent: $H_0 = 72.8^{+1.6}_{-1.7}$ km/s/Mpc [Liao, Shafieloo+ 2020]
- **More Samples → Better Accuracy**

Unresolved light curves

- **When spatially unresolved, we observe a joint light curve.**



Simulated examples

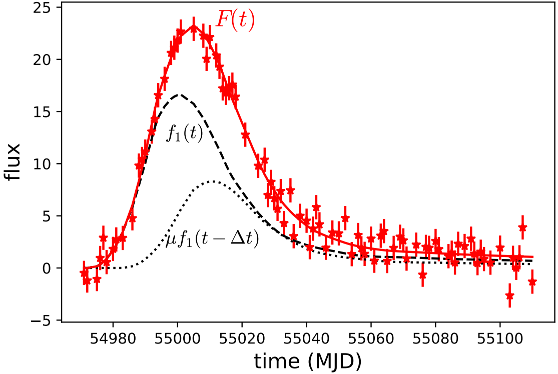
- Working with unresolved light curves has many advantages.
  - No need for resolving images a priori.
  - No need for high-resolution telescope monitoring individual images. Smaller telescopes can be used.
  - Fast scan through light curve databases.
  - No confusion between lensed and binary QSO's.
- **ZTF, Rubin, Roman etc will observe many lensed systems but a lot can remain unresolved.**

- Lensed SNe and QSO's have their own advantages.
- In favour of Lensed SNe: emerging field.
  - well understood light curves,
  - Type1A : standard candle
  - one needs to monitor for months
- Rare: only three lensed SNe have been confirmed yet.
- In favour of Lensed QSOs: more abundant, not transients.

Outline

1. Unresolved lensed SNe
2. Unresolved lensed quasars

Unresolved SNe: example



- **For simplicity we consider only two images**

$$F(t) = f_1(t) + \mu\, f_1(t - \Delta t)\ ,$$

$$f_1(t) \equiv a_1\, \mathcal{F}(t - t_1)$$

(Observed lightcurve of the 1st image)

$$\mu = \mathbf{a_2/a_1}\ , \quad \mathbf{\Delta t = t_2 - t_1}$$

magnification and time delay relative to the 1st image

Modelling the lensed SNe

$$F(t) = f_1(t) + \mu\, f_1(t-\Delta t)\ ,\quad f_1(t) = \text{(Observed flux of the 1st image)}$$

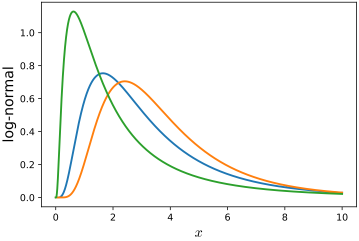
$$\mu = \mathbf{a_2}/\mathbf{a_1}\ ,\quad \mathbf{\Delta t} = \mathbf{t_2} - \mathbf{t_1}\quad \text{magnification and time delay relative to the 1st image}$$

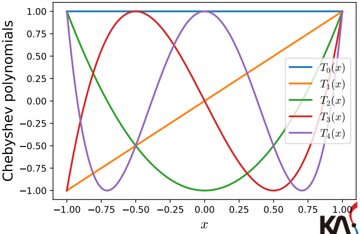
- Modelling the SN with a generic **log-normal template** with **Chebyshev polynomials** (crossing statistics [Shafieloo et al 2012, Shafieloo 2012]):

$$f_{1,j}(t) = N_j\,\frac{1}{t}\exp\left[-\frac{(\ln t - b_j)^2}{2\sigma_j^2}\right] \times \text{(Crossing terms)}$$

$$\begin{aligned}\text{(Crossing terms)} = \Big[&1 + C_{1,j}t_{s,j} + C_{2,j}(2t_{s,j}^2 - 1)\\ &+ C_{3,j}(4t_{s,j}^3 - 3t_{s,j})\\ &+ C_{4,j}(8t_{s,j}^4 - 8t_{s,j}^2 + 1)\Big]\ .\end{aligned}$$

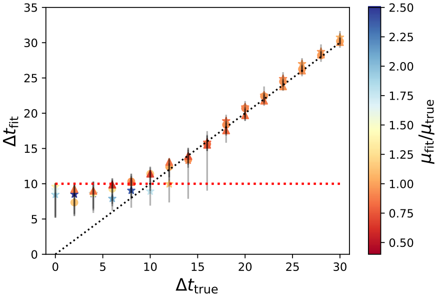
We focus on **ZTF-1a:** 3 filters : g, r, i.
Total fit parameters: $3\times(2+1+4)+2+1=24$
$\Longrightarrow$ **HMC** used in fitting.





Systematic test

- **Simulations using 'SNCosmo'.**
  - $\Delta t_{\rm true} = 0, 2, 4, 6, ..., 28, 30$ days. Total 16 time delays.
  - $\mu_{\rm true} = 0.5, 1.0, 2.0$. Different noise levels considered.
- 5% noise level

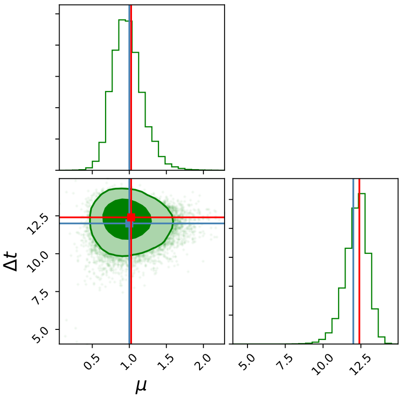


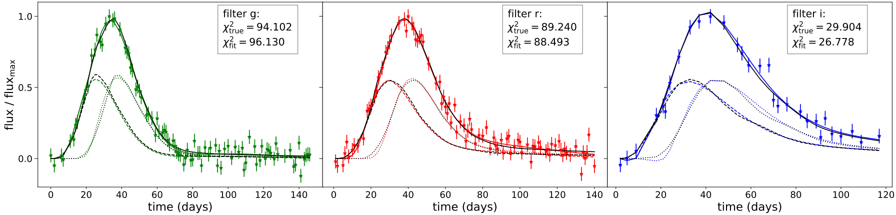
- For $\Delta t_{\rm true} \geq 10$ days: $\Delta t_{\rm fit} \approx \Delta t_{\rm true}$ **Good match!!!**
- For $\Delta t_{\rm true} < 10$ days: $\Delta t_{\rm fit} \lesssim 10$ days. **Not trustworthy!!!**

SB, Kim, Linder and Shafieloo, ApJ 910 (2021) 65

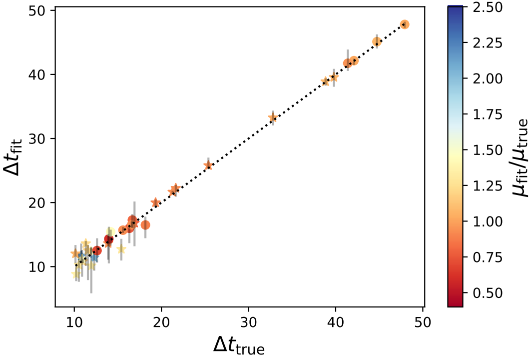
5% noise level

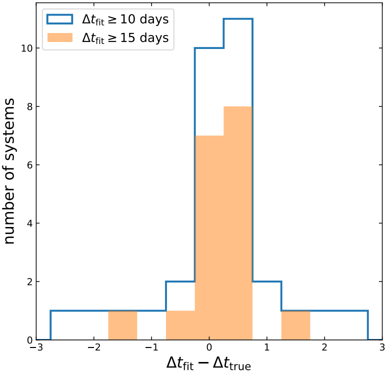
$\Delta t_{\text{true}} = 12$ days
$\mu_{\text{true}} = 1.0$





Testing on systems from Goldstein et al 2018 1809.10147

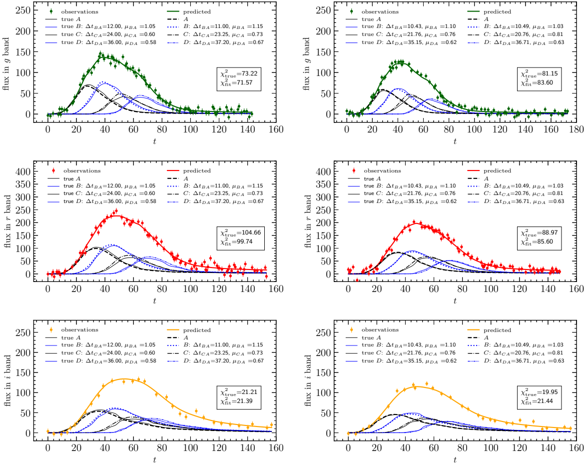




Time delay selection	$\epsilon_{\Delta t}$
$\Delta t_{\rm true} \geq 10$ days	**4.95%**
$\Delta t_{\rm true} \geq 15$ days	**2.65%**

SB, Kim, Linder and Shafieloo, ApJ 910 (2021) 65

Suspected type-1a: Hsiao+free-form

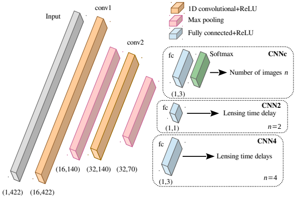


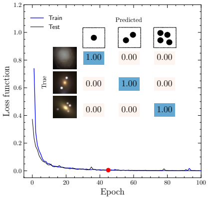
- For suspected type-1a cases
  - Hsiao SN1a template + free-form
  - Higher precision
  - Works for Quad systems

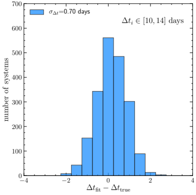
Denissenya, **SB**, Kim, Linder, Shafieloo, MNRAS 511 (2022) 1210

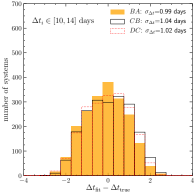
Another approach: Convolutional Neural Network

- Denissenya, & Linder 2022

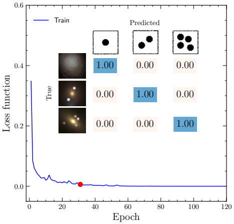


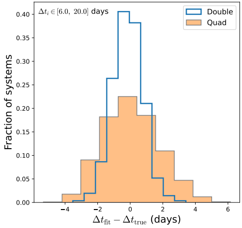






- Fairly insensitive to: noise $\sim 20\%$, small time delays, missing early data.
- But trained and tested on same Hsiao template.
- Work in progress: smooth the noisy data
- Mix SN templates (Hsiao + SALT2) and/or types.
- Vary redshift and noise levels.
- Test how sensitive to microlensing, cadence etc.





Preliminary results

- CNN can be used for unresolved lensed SNe.

Summary: Unresolved SNe

- Lensed SNe can be the next frontier in the cosmic probes.
- Growing in number but many can remain unresolved in wide field surveys.
- Discussed three flexible methods to detect unresolved SNe and to extract the time delays from the joint light curves.
- **Crossing statistics:**
  - Model agnostic, suitable for unclassified SNe.
- **Free-form:**
  - For suspected type-1a cases: more precise, tested for quad systems.
- **Convolutional Neural Network:**
  - Work in progress.
- **Can be used for lensed SNe detection in ZTF, LSST, Roman etc.**

- Incorporate **microlensing**: both perform well for toy microlensing effects.

  SB, Denissenya, Kim, Linder, Shafieloo+, in progress

- Employing Deep Learning (CNN).

- Build pipelines for ZTF and LSST.

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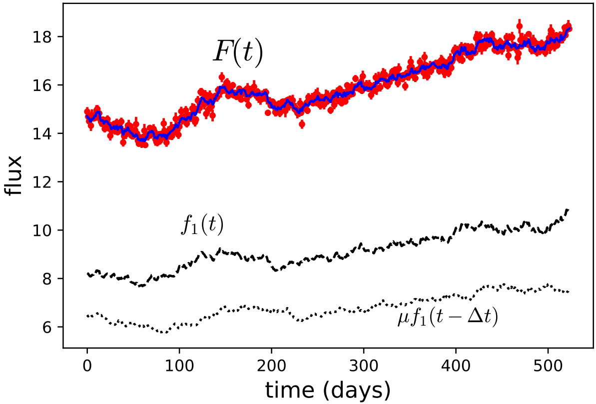


- $F(t) = f_1(t) + \mu f_1(t - \Delta t)$

- We know very little about the time variability of quasar light curves.
  No constraint on $f_1(t)$.

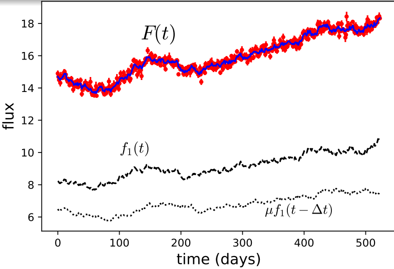
- **Is it possible to detect lensing from $F(t)$, at all?**

Methods proposed recently

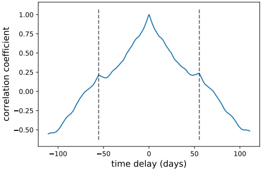
- [Springer and Ofek (2021)] assume the power spectrum to be a power law (red noise).
- [Shu et al 2021] employed with auto correlation.
- [Biggio et al 2021] use CNN: trained and tested on light curves with same time variability.
- **Not completely model independent.**

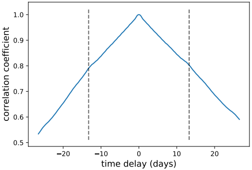
Unresolved QSO: Auto correlation (negligible noise)

$$\mathrm{ACF}(t_{\mathrm{lag}}; F) = \frac{\langle [F(t) - \langle F(t) \rangle][F(t + t_{\mathrm{lag}}) - \langle F(t) \rangle] \rangle}{\langle F^2(t) \rangle - \langle F(t) \rangle^2}$$

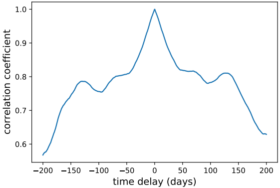


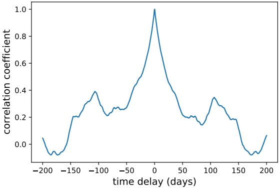
- When $t_{\mathrm{lag}} \approx \pm \Delta t_{\mathrm{true}}$ one expects excess power in ACF.
  - Lensed systems (No noise):





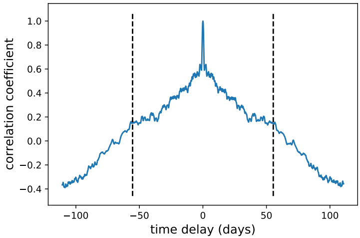
  - Unlensed systems (No noise):

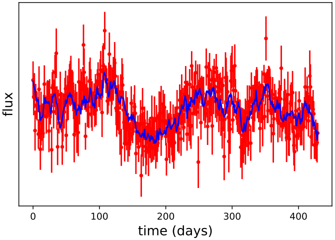




Unresolved QSO: Auto correlation (with ZTF-like noise)

- Lensed systems (with noise):





- **Auto-correlation is not reliable!**
- See also [Geiger & Schneider 1996], [Shu et al 2021]
- [Shu et al 2021] use DRW template for both simulation and validation.
- **Not model independent**.

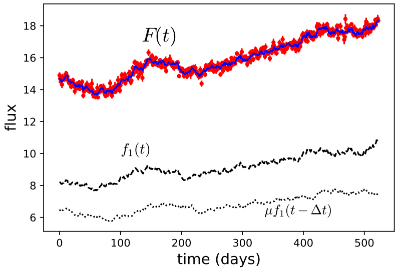
Image reconstruction

- Consider 2-image lensed system for simplicity.

$$F(t) \quad = \quad f_1(t) + \mu f_1(t - \Delta t)$$

$$f_{1,\mathrm{rec}}\,(t|\Delta t, \mu) = \sum_{n=0}^{\infty} (-\mu)^n \; F(t - n \cdot \Delta t)$$

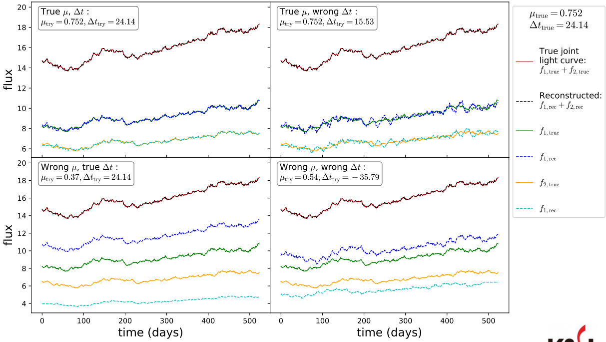
- **converges for $\mu < 1$: Unique solution**



- Let us call **brighter image as first image** $\Longrightarrow \mu \leq 1$.

- **One can reconstruct the images ($f_1(t)$) for any choice of $(\mu_{\mathrm{try}}, \Delta t_{\mathrm{try}})$.**

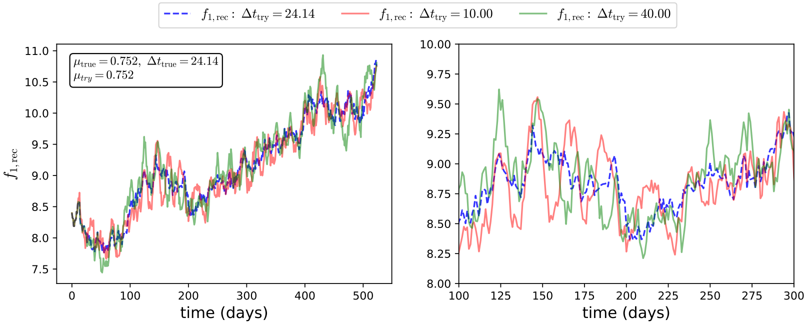
  see also [Geiger & Schneider 1996]



$$F_{\mathrm{rec}}(t) = f_{1,\mathrm{rec}} + \mu_{\mathrm{try}} f_{1,\mathrm{rec}}(t - \Delta t_{\mathrm{try}}) = F_{\mathrm{obs}}(t) \quad \text{(Mathematical degeneracy!!)}$$

- One can reconstruct the images ($f_1(t)$) for any choice of ($\mu_{\rm try}, \Delta t_{\rm try}$).

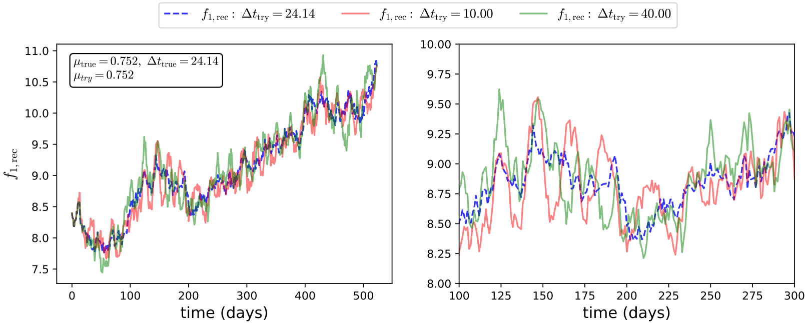
BUT!!



- **More fluctuation in $f_{1,\rm rec}$ for wrong Δt.**

- One can reconstruct the images ($f_1(t)$) for any choice of ($\mu_{\rm try}, \Delta t_{\rm try}$).

BUT!!



- **More fluctuation in $f_{1,\rm rec}$ for wrong Δt.**

- Measure fluctuation: $\epsilon(\Delta t_{\rm try}) = \sum_i^{N_D} \left[f_{1,\rm rec}(t_i) - f_{1,\rm rec}(t_{i+1})\right]^2$

Quantifying fluctuation

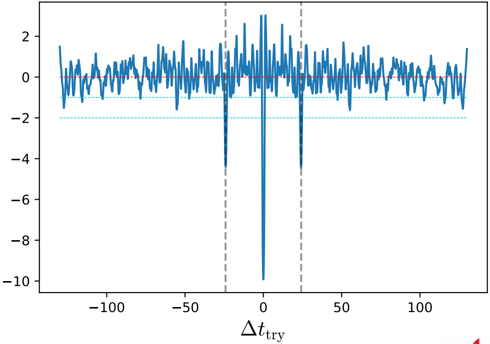
- Fix $\mu_{\rm try}$ to any arbitrary value.

$$f_{1,\rm rec}\left(t|\Delta t_{\rm try}\right) = \sum_{n=0}^{\infty}\left(-\mu_{\rm try}\right)^n F(t - n\cdot\Delta t_{\rm try}) \Rightarrow$$

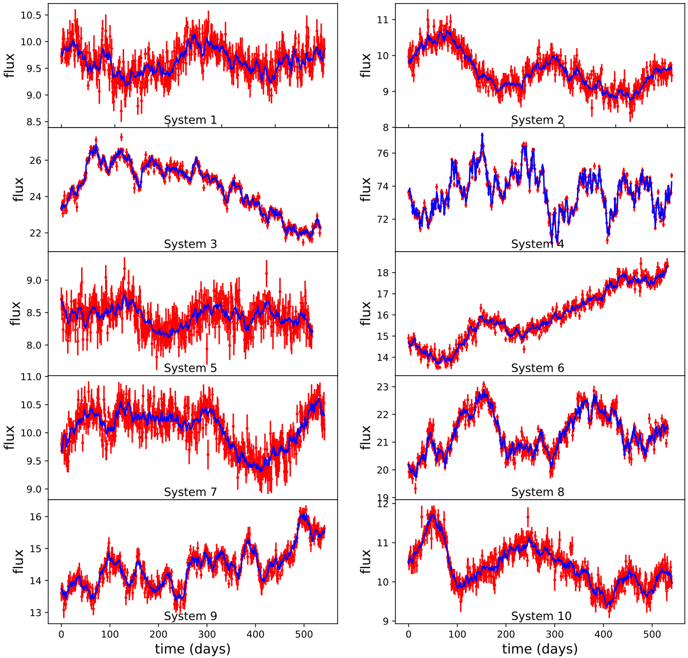
$$\epsilon(\Delta t_{\rm try}) = \sum_{i}^{N_D}\left[f_{1,\rm rec}(t_i) - f_{1,\rm rec}(t_{i+1})\right]^2$$

- Symmetric around $\Delta t_{\rm try} = 0$.
- Global minima at $\Delta t_{\rm try} = 0$: unlensed solution.
- At $\Delta t_{\rm try} = \pm\Delta t_{\rm true}$: **two prominent secondary minima.** **Lensed systems can be identified.**
- Valid for different $\mu_{\rm try}$'s.
- Mathematical proof under progress in a separate work.
  SB, Sohn, Shafieloo, Liao and Treu (2022), in preparation

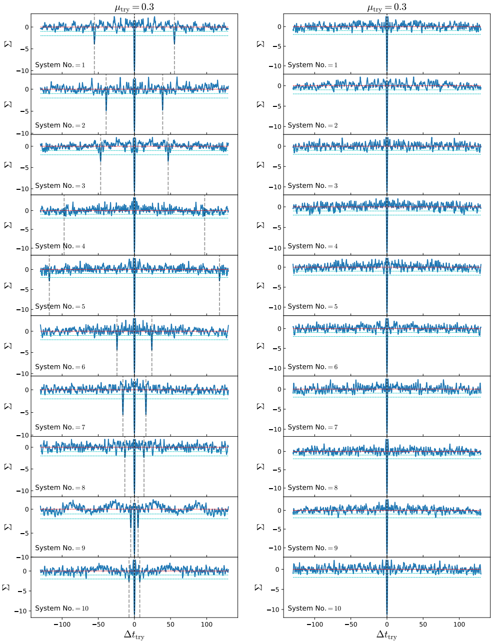
Normalise: $$\Sigma(\Delta t_{\rm try}) = \frac{\epsilon(\Delta t_{\rm try}) - \langle\epsilon(\Delta t_{\rm try})\rangle}{\sigma_\epsilon}$$



Validation set (10 lensed + 10 unlensed systems)



Validation set: 10 lensed + 10 unlensed systems (marginal noise)



Validation + blind set: 20 lensed + 20 unlensed systems (marginal noise)

- **Validation set (10 lensed + 10 unlensed systems):**

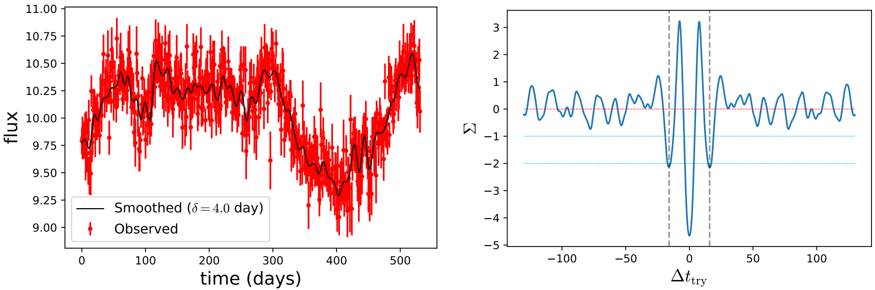
System No.	True time delay $\Delta t_{\rm true}$ in days	Estimated time delay $\Delta t_{\rm est}$ in days
1	55.37	55.4 ± 0.1
2	39.1	39.0 ± 0.1
3	46.7	46.7 ± 0.1
4	97.19	$--$
5	117.7	117.7 ± 0.1
6	24.14	24.1 ± 0.1
7	15.9	15.9 ± 0.1
8	13.27	13.2 ± 0.1
9	5.13	5.0 ± 0.1
10	7.42	7.5 ± 0.1

- 9 out of 10 lensed systems and all 10 unlensed systems are identified correctly with accurate time delay measurement.

- **Blind set (20 systems):** we could recovred all 10 lensed and 10 unlensed systems blindly.

- Validation + blind set: **precision** $= 100\%$, **recall** $= 19/20 = 95\%$, **accuracy** $= 39/40 = 97.5\%$.

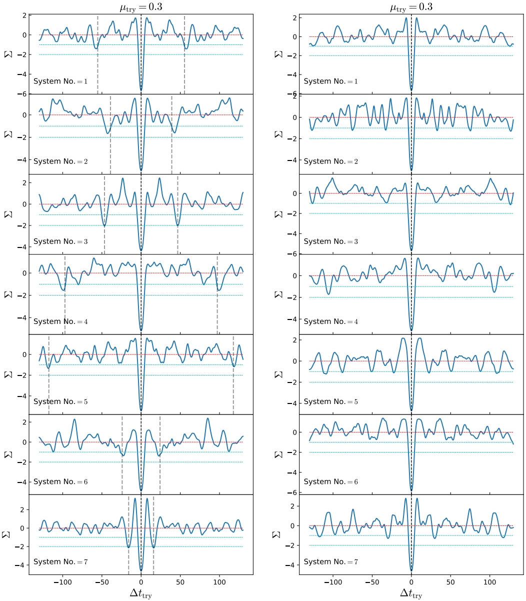
- Iterative smoothing [Shafieloo at al 2006, 2007]



- Use multiple smoothing scale; $\delta = 3.0, 4.0, 5.0$ day.
- Combine $\epsilon(\Delta t_{\rm try})$ curves $\Longrightarrow$ get final $\Sigma(\Delta t_{\rm try})$

Validation + blind set (data with ZTF like noise)

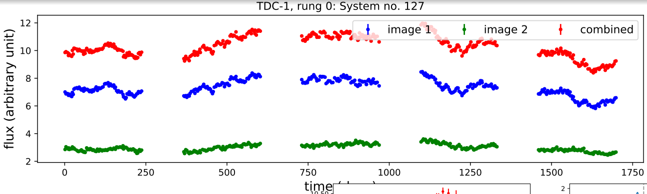
- Validation set



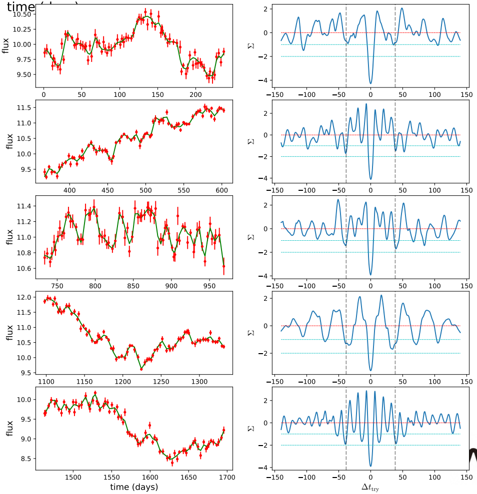
- Known set:
  7 out of 10 lensed detected.
- Blind set:
  5 out of 10 lensed detected.
- Combining known and blind set:
  12 out of 20 lensed identified,
  one probable false positive case.
- **Precision** $= 92.3\%$.
  Recall $= 60\%$.
- Time delays are accurately recovered (most within 3%).

SB, Shafieloo, Liao and Treu, ApJ 927 (2022) 191 [2110.15315]

Realistic simulation: Time Delay Challenge 1 (TDC 1), rung 0

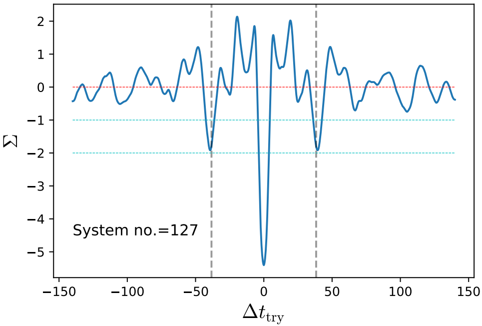


- So far, continuous observation with cadence 1 day.
- Real data: poorer cadence, seasonal observation, gaps in the data.
- TDC 1 (rung 0) simulations : use all 5 patches separately. [Liao et al. 2015]

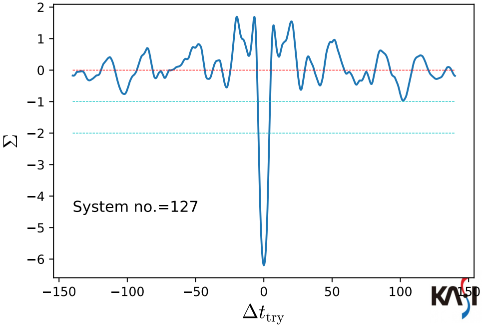


System 127 from TDC 1, rung 0

- Using the joint light curve
- Our estimation: $\Delta t_{\text{est}} = 39.35$ day
- Truth: $\Delta t_{\text{est}} = 38.33$ day .



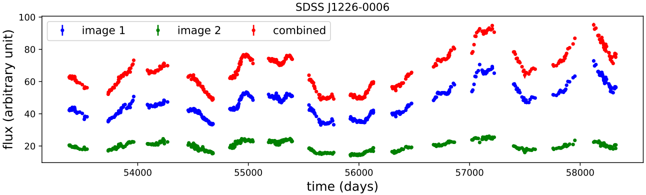
- Using the light curve of the brightest image only
- No spurious lensing found.



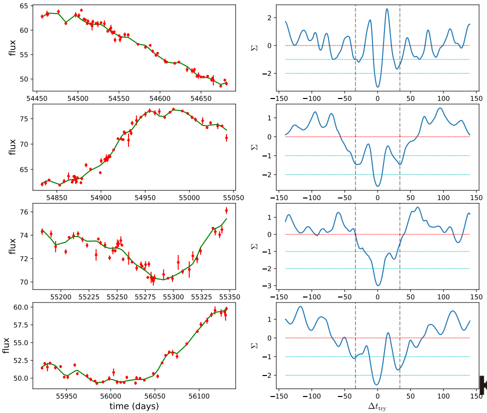
TDC 1 : rung 0 and rung 1

System No.	True time delay $\Delta t_{\rm true}$ in days	Estimated time delay $\Delta t_{\rm est}$ in days
TDC1 (rung 0): observed in ~ 400 epochs over a period of 5 years		
27	-40.79	43.15
105	23.73	23.80
125	32.47	30.85
127	38.33	39.35
131	-45.03	45.55
204	29.1	28.75
TDC1 (rung 1): observed in ~ 400 epochs over a period of 10 years		
5	32.47	33.00
102	-13.04	13.50
202	50.81	49.35
208	39.93	39.80
246	31.38	32.25
254	-44.97	44.10
358	47.26	46.10

Observed lensed QSO: SDSS J1226-0006 from COSMOGRAIL

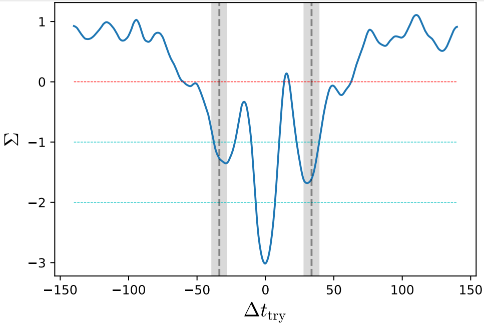


- Observed by Euler telescope over 14 years. [Millon et al. 2020]
- Use only 4 ‘good’ patches separately.

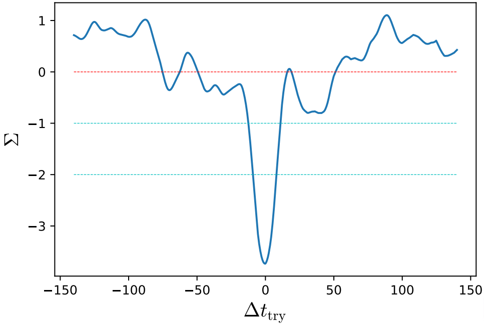


SDSS J1226-0006 from COSMOGRAIL

- Using the joint light curve
- Our estimation: $\Delta t_{\rm est} = 29.6$ day
- COSMOGRAIL: $\Delta t_{\rm est} = 33.7 \pm 2.7$ day using the resolved image light curves
- Error estimation in progress.



- Using the light curve of the brightest image only
- No spurious lensing found.



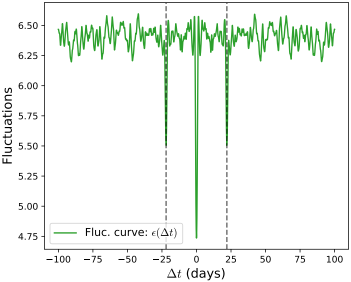
SB, Shafieloo, Liao and Treu, ApJ 927 (2022) 191 [2110.15315]

Why minimal fluctuation for correct time delay?

$$f_{1,\mathrm{rec}}\left(t|\Delta t,\mu_{\mathrm{try}}\right)=\sum_{n=0}^{\infty}\left(-\mu_{\mathrm{try}}\right)^n F(t-n\cdot\Delta t_{\mathrm{try}}) \Longrightarrow \epsilon(\Delta t_{\mathrm{try}}|\mu_{\mathrm{try}})=\sum_i^{N_D}\left[f_{1,\mathrm{rec}}(t_i)-f_{1,\mathrm{rec}}(t_{i+1})\right]^2$$

$$\epsilon(\Delta t_{\mathrm{try}}|\mu_{\mathrm{try}})=\sum_i H(t_i)^2-2\mu_{\mathrm{try}}\left(\sum_i H(t_i)H(t_i-\Delta t_{\mathrm{try}})\right)+\mu_{\mathrm{try}}^2(\ldots)+\ldots$$

where $H(t_i)\equiv F(t_{i+1})-F(t_i)$ **(derivative of the joint light curve)**

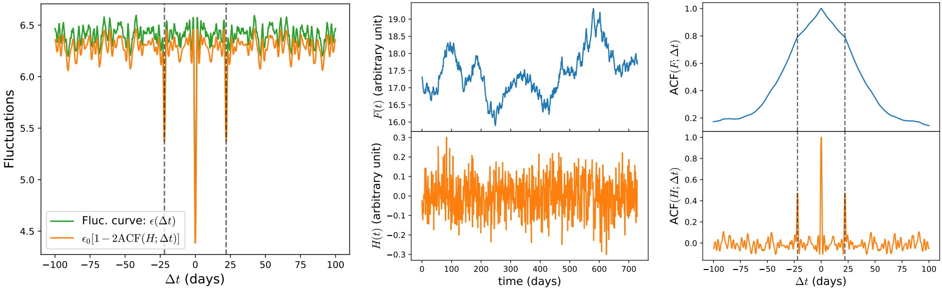


Why minimal fluctuation for correct time delay?

$$f_{1,\mathrm{rec}}\left(t|\Delta t, \mu_{\mathrm{try}}\right) = \sum_{n=0}^{\infty} \left(-\mu_{\mathrm{try}}\right)^n F(t - n \cdot \Delta t_{\mathrm{try}}) \implies \epsilon(\Delta t_{\mathrm{try}}|\mu_{\mathrm{try}}) = \sum_i^{N_D} \left[f_{1,\mathrm{rec}}(t_i) - f_{1,\mathrm{rec}}(t_{i+1})\right]^2$$

$$\epsilon(\Delta t_{\mathrm{try}}|\mu_{\mathrm{try}}) = \sum_i H(t_i)^2 - 2\mu_{\mathrm{try}} \left(\sum_i H(t_i) H(t_i - \Delta t_{\mathrm{try}})\right) + \mu_{\mathrm{try}}^2 (\ldots) + \ldots$$

where $H(t_i) \equiv F(t_{i+1}) - F(t_i)$ **(derivative of the joint light curve)**



- $\epsilon(\Delta t_{\mathrm{try}})$ is dominated by ACF of derivative of the joint light curve.
- For red power spectrum: $\mathrm{ACF}(H;\Delta t_{\mathrm{try}})$ is more reliable than $\mathrm{ACF}(H;\Delta t_{\mathrm{try}})$.
- Fluctuation curve performs better than $\mathrm{ACF}(H;\Delta t_{\mathrm{try}})$ for noisy data.

SB, Sohn, Shafieloo, Liao and Treu 2022, in preparation.

Summary: Unresolved QSO

- Main challenge: the intrinsic time variability of QSO flux is unknown.
- We show that it is possible.
  Without any assumption about intrinsic QSO light curve or additional information.
- Tested on simulated data; Time Delay Challenge 1 (LSST-like).
- Works well on existing data quality (e.g. SDSS J1226-0006).
- Can detect quad systems.
- Can be used on the light curve data from ZTF, will become more important in the era of LSST.

- Analysing large number of simulations:
  - Uncertainty, Selection criteria.
  - Effect of different noise level, cadence, microlensing etc.
  - Favoured observation strategy.
- Improve the metric for measuring fluctuation and selection criteria.
- Validation on all COSMOGRAIL systems blindly.
- Searching lensed QSO's in exiting database: **ZTF**.
- Applying reconstruction method on unresolved SNe light curves (classification with CNN).

  [Park, **SB**, Shafieloo+, (2022), in progress]

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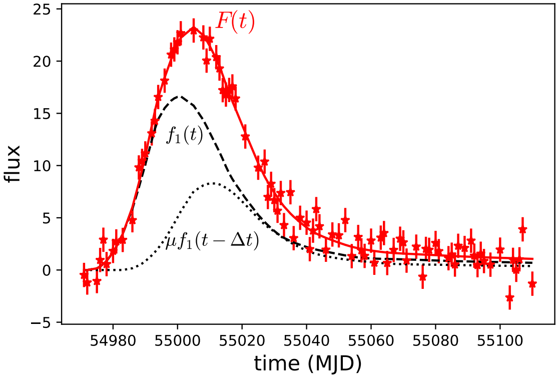
  [Park, **SB**, Shafieloo+, (2022), in progress]

Thank You!!!

Outline

3. Appendix: Unresolved SNe

4. Appendix: Unresolved QSO



- Joint Light curve of a strong lensed SN in a filter:

$$F(t) = \sum_{i=1}^{N_I} a_i \, \mathcal{F}(t - t_i)$$

- **For simplicity we consider only two images**

$$F(t) = f_1(t) + \mu \, f_1(t - \Delta t) \; ,$$

$$f_1(t) \equiv a_1 \, \mathcal{F}(t - t_1)$$

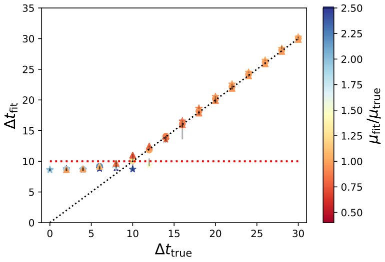
(Observed lightcurve of the 1st image)

$$\mu = \mathbf{a_2/a_1} \; , \quad \mathbf{\Delta t = t_2 - t_1}$$

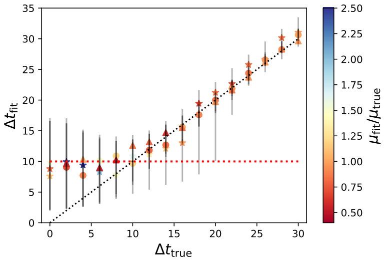
magnification and time delay relative to the 1st image

0.5 and 10.0% noise levels

0.5%

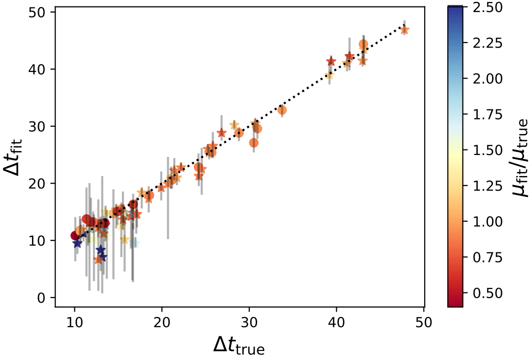


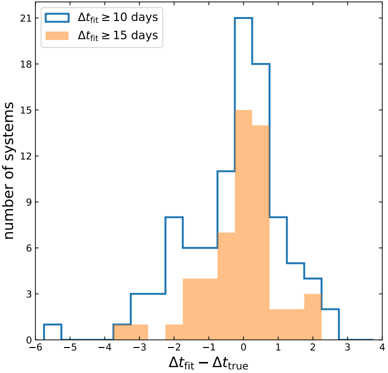
10.0%



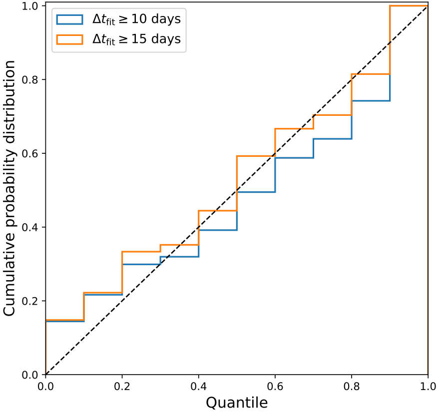
- For $\Delta t_{\rm true} \geq 10$ days: $\Delta t_{\rm fit} \approx \Delta t_{\rm true}$ **Good match!!!**
- For $\Delta t_{\rm true} < 10$ days: $\Delta t_{\rm fit} \lesssim 10$ days. **Not confident!!!**

Unresolved SNe (crossing): larger, noisier set Goldstein et al 1809.10147





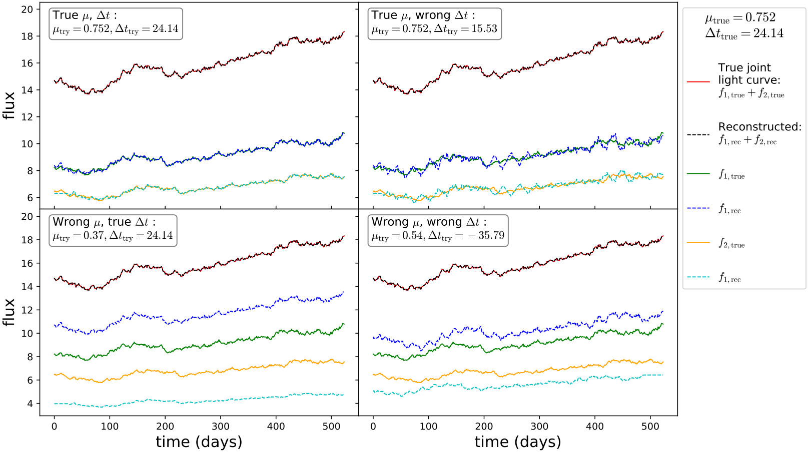
Unresolved SNe (crossing): Combined set



SB, Kim, Linder and Shafieloo, ApJ 910 (2021) 65

Outline

Image reconstruction: High quality data (marginal noise)

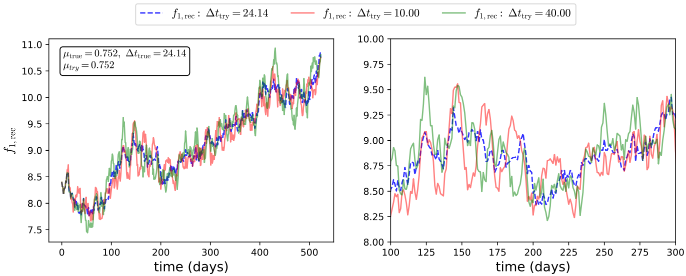


- Any choice of $\{\mu_{\rm try}, \Delta t_{\rm try}\}$ is allowed $\Longrightarrow$ exact reconstruction of the joint light curve.
- Mathematical degeneracy; see also [Geiger & Schneider 1996].

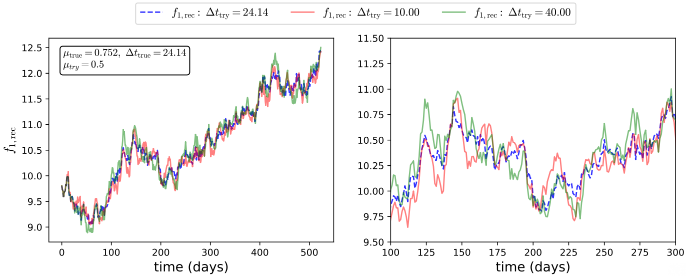
Image reconstruction

- One can reconstruct the images $(f_1(t))$ for any choice of $(\mu, \Delta t)$, **BUT**.

- $\mu_{\rm try} = \mu_{\rm true}$

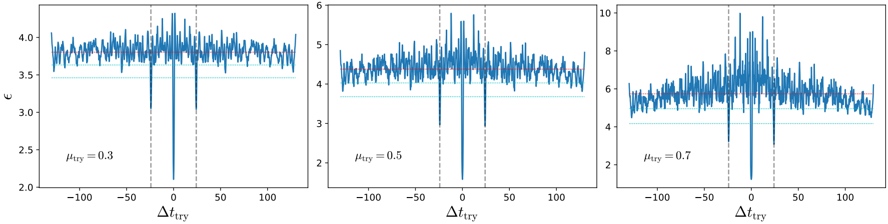


- $\mu_{\rm try} \neq \mu_{\rm true}$



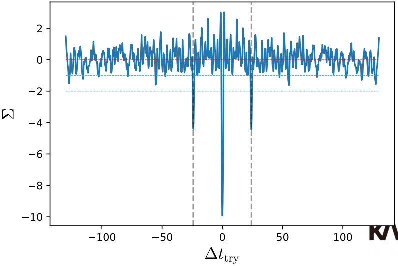
- **More fluctuation in $f_{1,\rm rec}$ for wrong Δt.**

- $$\epsilon(\Delta t_{\rm try}) = \sum_{i}^{N_D} \left(f_{1,\rm rec}(t_i) - f_{1,\rm rec}(t_{i+1})\right)^2$$



- Symmetric around $\Delta t_{\rm try} = 0$.
- Global minima at $\Delta t_{\rm try} = 0$: unlensed solution.
- At $\Delta t_{\rm try} = \pm\Delta t_{\rm true}$: **two prominent secondary minima.**
  Lensed systems can be identified.
- Roughly valid for different $\mu_{\rm try}$'s.
- All four observations are robust. Mathematical proof under progress in a separate work.
  SB, Sohn, Shafieloo+ (2022), in preparation

- $$\Sigma(\Delta t_{\rm try}) = \frac{\epsilon(\Delta t_{\rm try}) - \langle \epsilon(\Delta t_{\rm try}) \rangle}{\sigma_\epsilon}$$



Quantifying fluctuation

- $\epsilon(\Delta t_{\rm try}) = \sum_i^{N_D} (f_{1,\rm rec}(t_i) - f_{1,\rm rec}(t_{i+1}))^2$

  - Symmetric around $\Delta t_{\rm try} = 0$.
  - Global minima at $\Delta t_{\rm try} = 0$: unlensed solution.
  - At $\Delta t_{\rm try} = \pm \Delta t_{\rm true}$: **two prominent secondary minima.**
    Lensed systems can be identified.
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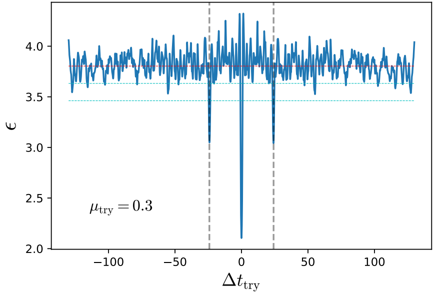
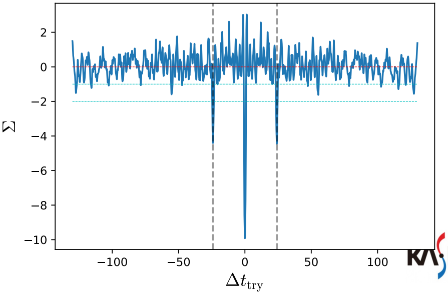


- $\Sigma(\Delta t_{\rm try}) = \frac{\epsilon(\Delta t_{\rm try}) - \langle \epsilon(\Delta t_{\rm try}) \rangle}{\sigma_\epsilon}$



KASI

Summary of results with ZTF like noise (Known set)

System No.	True time delay Δt_{true} in days	Estimated time delay Δt_{est} in days
3	46.7	46.70 ± 2.33
7	15.9	16.05 ± 0.80
1 (*)	55.37	57.35 ± 2.87
2 (*)	39.1	42.80 ± 2.14
5 (*)	117.7	118.45 ± 5.92
6 (*)	24.14	22.60 ± 1.13
4 (**)	97.19	99.15 ± 4.96

- Known set: 2 confirmed detection + 5 highly probable identification out of 10 lensed system.

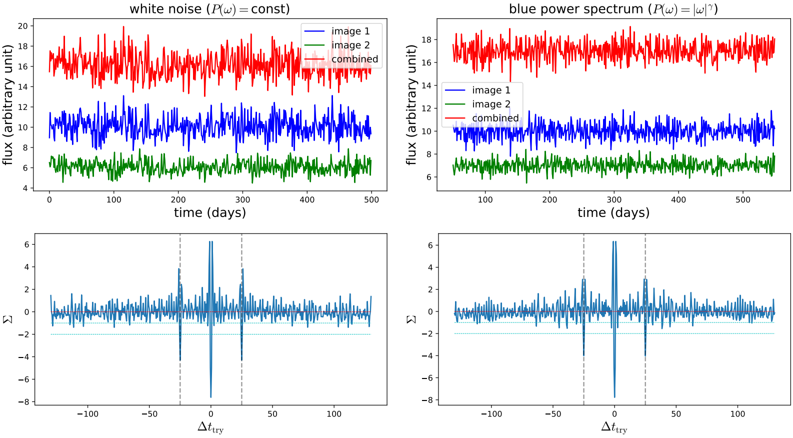
Summary of results with ZTF like noise (blind set)

System No.	Estimated time delay $\Delta t_{\rm est}$ in days	True time delay $\Delta t_{\rm true}$ in days
11	24.45 ± 1.22	24.14
7 (*)	96.55 ± 4.83	97.19
9 (*)	122.10 ± 6.11	117.7
3 (**)	40.10 ± 2.01	39.1
13 (**)	15.80 ± 0.79	15.9

- Blind set: 1 confirmed detection + 4 highly probable identification out of 10 lensed system.
- Combining known and blind set: 12 lensed system identified out of 20, one probable false positive case.
- **Precision** $= 92.3\%$. **Recall** $= 60\%$.

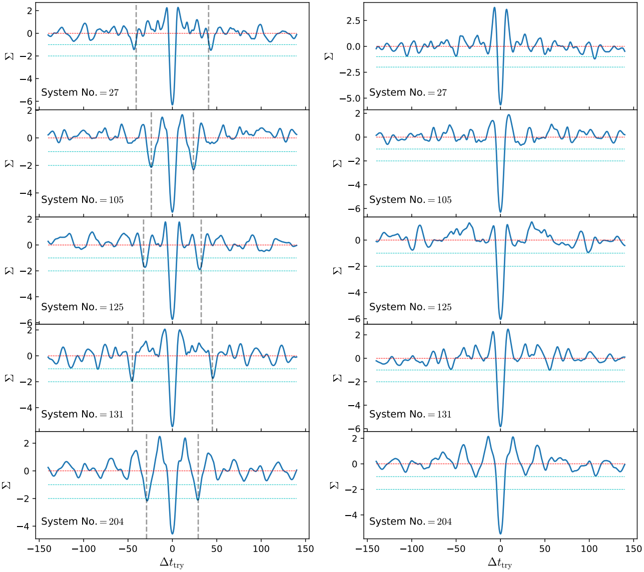
Works for other types of power spectra

- **Power spectrum :** $P(\omega) = \langle \widehat{f}(\omega)\widehat{f}(\omega)^* \rangle$
- All the above examples $\Longrightarrow$ red power spectrum type $(P(\omega) = |\omega|^{-\gamma},\ \gamma > 0)$



- **Method works independent of power spectrum.**

More examples from TDC 1 : rung 0



Iterative Smoothing

- $F_n(t) = F_{n-1}(t) + \frac{1}{N(t)} \sum_i^{N_D} \frac{(F_{\text{obs}}(t_i) - F_{n-1}(t_i))}{\sigma^2_{\text{obs}}(t_i)} \times \exp\left\{ \left[-\frac{(t-t_i)^2}{2\delta^2} \right] \right\}$

- $N(t) = \sum_i^{N_D} \left(\frac{1}{\sigma^2_{\text{obs}}(t_i)} \right) \times \exp\left\{ \left[-\frac{(t-t_i)^2}{2\delta^2} \right] \right\}$

- Two parameters: smoothing scale (δ) and number of iteration.