The cosmic web environment of dark matter haloes:

for an enhanced understanding of structure formation

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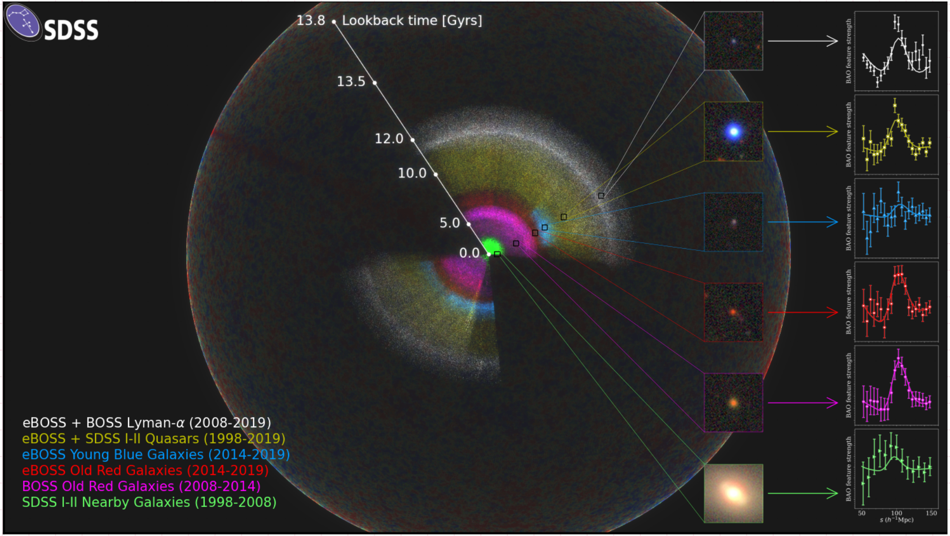
State of the Universe (SOTU) seminar

TIFR Mumbai

Introduction

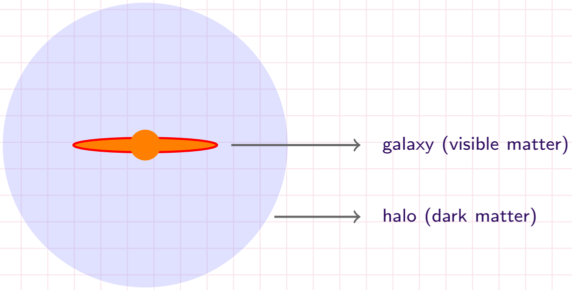


Reference: SDSS collaboration

Structure formation from gravitational instability

- growth of intitial density perturbation and expansion of its physical size until turn around.
- violent relaxation of dark matter and shocks in baryonic matter.
- cooling of gas results in galaxy formation
- galaxies form in a biased manner, depending on cooling, free fall and hubble time scales.
- properties of galaxy shaped by properties of the dark matter.
- Here, we will study the dark matter haloes and their correlation to the large scale environment.
- At these highly nonlinear scales and late times, we rely on N-body simulations.

(Mo, Bosch & White - Galaxy formation and Evolution)



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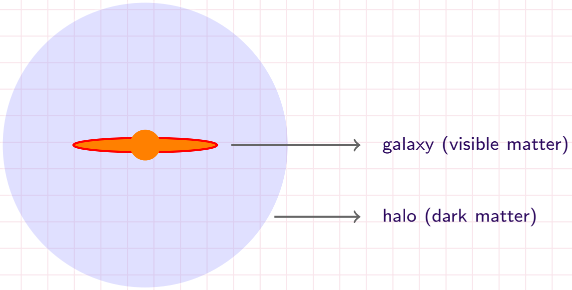
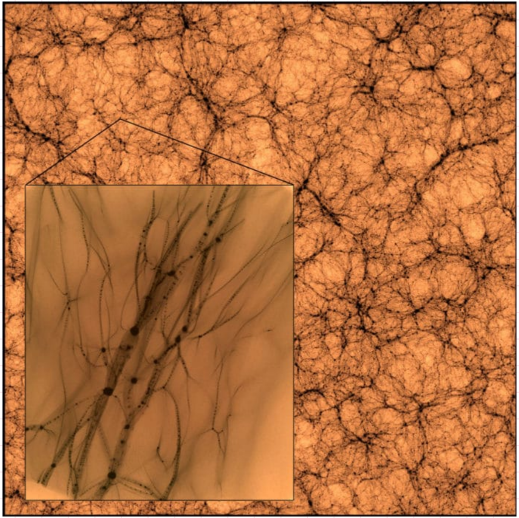


Mass and radius of a halo

- mass definitions - $M_{\rm vir}, M_{\rm 200b}$
- radius definitions - $R_{\rm vir}, R_{\rm 200b}$

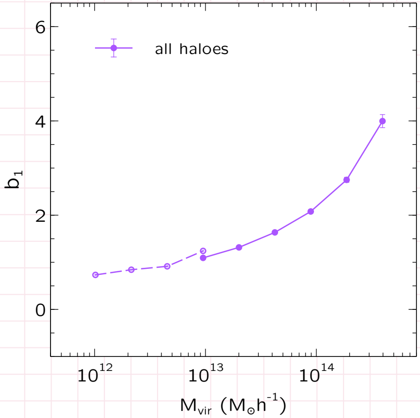
Dark matter halo bias

- halo number overdensity $\delta_h = n_h/\bar{n}_h - 1$
- dark matter overdensity $\delta_{\rm dm}$
- $\delta_h = b_1\ \delta_{\rm dm}$
- $b_1 = \left.\frac{P_{\rm hm}}{P_{\rm mm}}\right|_{k_{\rm small}}$
- In the peak background split approach, halo bias is the response of halo number density n_h to the presence of infinite wavelength perturbation δ_L



Release Images 2020-21: J. Wang; S. Bose/CfA

Halo *Secondary/Assembly* Bias

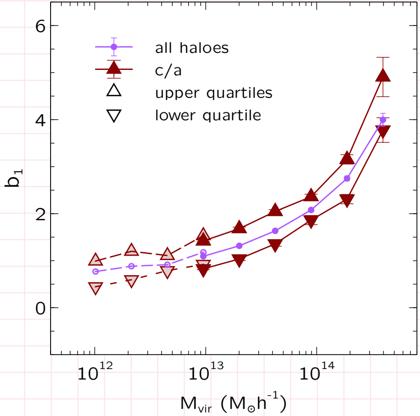


Mo & White (1996);
Sheth & Tormen (1999)

Large Scale Bias

- $b_1 = < \delta_h \delta_m > / < \delta_m \delta_m >$
- large scale density environment

Halo *Secondary/Assembly* Bias

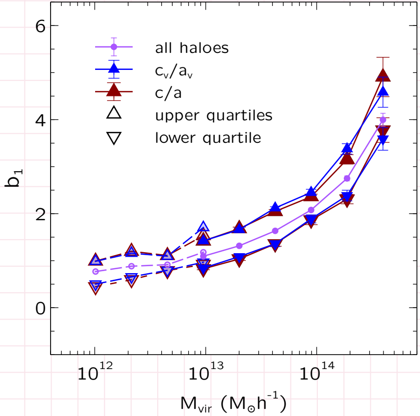


Faltenbacher & White (2010)

Shape Ratio c/a

- Shape Tensor $= \sum_{n \in \mathrm{halo}} x_{i,n} x_{j,n} / r_n^2$, where $x_{i,n}, x_{j,n}$ - particle position
- eigen values $a^2 > b^2 > c^2$ r_n - ellipsoidal distance from center.

Halo *Secondary/Assembly* Bias

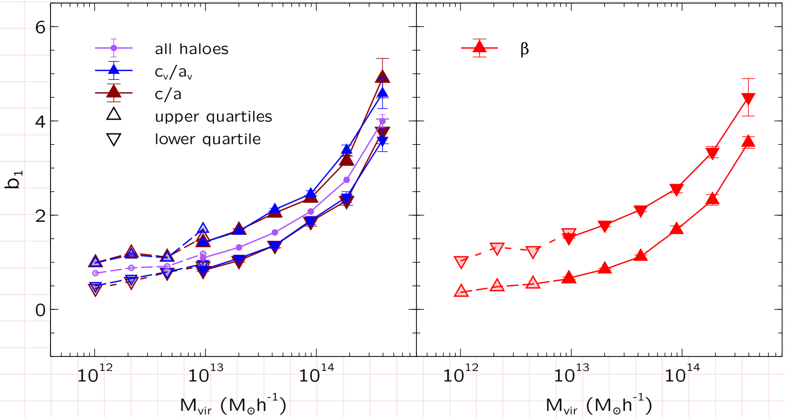


Faltenbacher & White (2010)

velocity asphericity ratio c_v/a_v

- Velocity ellipsoid $= < v_i v_j >$
- eigen values $a_v^2 > b_v^2 > c_v^2$

Halo *Secondary/Assembly* Bias

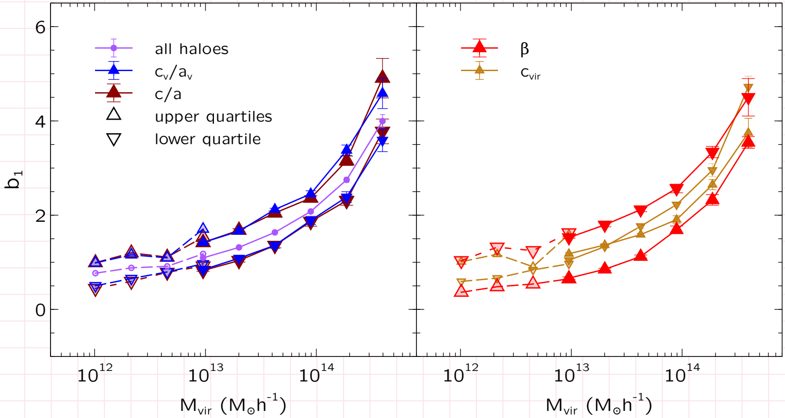


Faltenbacher & White (2010)
Borzyszkowski et al. (2017)

Velocity Anisotropy β

- $\beta = 1 - \sigma_t^2/2\sigma_r^2$
- σ_t/σ_r are tangential and radial dispersion

Halo *Secondary/Assembly* Bias

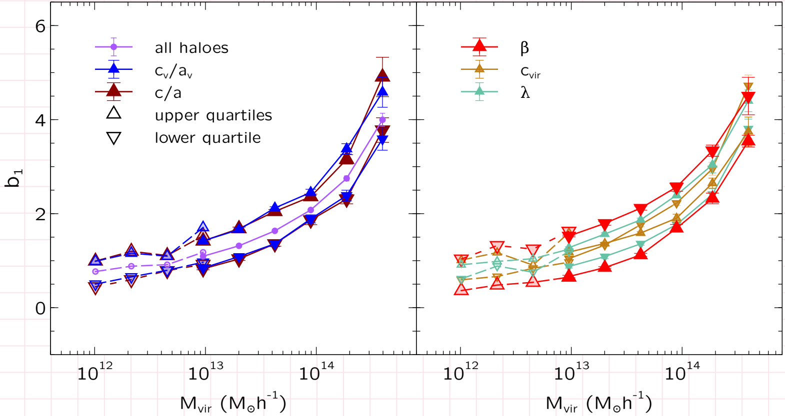


Faltenbacher & White (2010)
Borzyszkowski et al. (2017)
Wechsler et al. (2006)
Gao & White (2007)

Concentration

$c_{vir} = r_s/R_{vir}$ (alternate definition) $c_{200b} = r_s/R_{200b}$

Halo *Secondary/Assembly* Bias



Faltenbacher & White (2010)
Borzyszkowski et al. (2017)
Wechsler et al. (2006)
Gao & White (2007)
Bett et al. (2006)

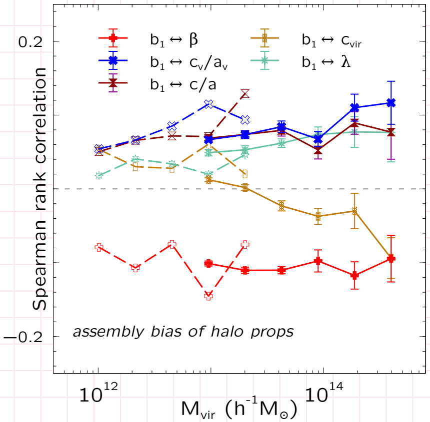
Spin λ

- $\lambda = J|E|^{0.5}/GM_{vir}^{2.5}$ (Peebles 1969) $\lambda = J_{\rm vir}/\sqrt{2}M_{\rm vir}R_{\rm vir}V_{\rm vir}$ Bullock et al.(2001)
- J ,E are the angular momentum and total energy, $V_{\rm vir}$ is the circular velocity.

Alternatively...

we can think of assembly bias as correlation between bias and halo properties

halo-by-halo bias Paranjape et al. (2018)



Spearman Rank Correlation

$$\gamma_{ab} = \frac{cov(R(a)R(b))}{\sigma_{R(a)}\sigma_{R(b)}}$$

where $R(a), R(b)$ are the ranks of a, b

Internal Properties $\longleftrightarrow$ Halo Bias

$\searrow$ $\nearrow$

Halo Environment

Internal Properties $\longleftrightarrow$ Halo Bias

$\searrow$ $\nearrow$

Halo Environment

- Two ways to characterize halo environment
    - overdensity of the halo environment
    - anisotropy of the halo environment

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  - anisotropy of the halo environment

Tidal Tensor $\partial_i \partial_j \psi_R(x)$ has eigen values $\lambda_1 > \lambda_2 > \lambda_3$

- overdensity - δ and tidal anisotropy - α
- $\delta = \lambda_1 + \lambda_2 + \lambda_3$
- $\alpha = (1+\delta)^{-1}\sqrt{[(\lambda_3 - \lambda_1)^2 + (\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2]/2}$ Paranjape et al. (2018)
- These definitions involve smoothing the tidal tensor at $4 \times R_{200b}$

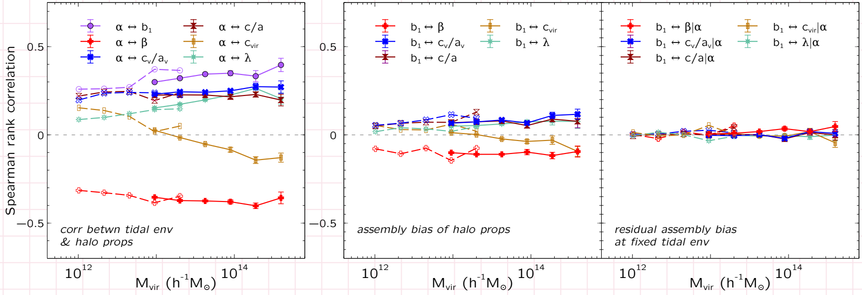
Assembly Bias at fixed Environment

For three gaussian variables a, b, c with correlation coefficients $\gamma_{ab}, \gamma_{bc}, \gamma_{ca}$

$$\gamma_{bc|a} = \gamma_{bc} - \gamma_{ba}\gamma_{ac} \tag{1}$$

b $\longrightarrow$ b_1 bias
a $\longrightarrow$ α tidal anisotropy
c $\longrightarrow$ c/a, c_v/a_v, β, λ, c_{200b} internal property of the halo

[Ramakrishnan et.al (2019)]



There is no residual assembly bias at fixed tidal anisotropy. $b_1 \leftrightarrow \alpha \leftrightarrow c$

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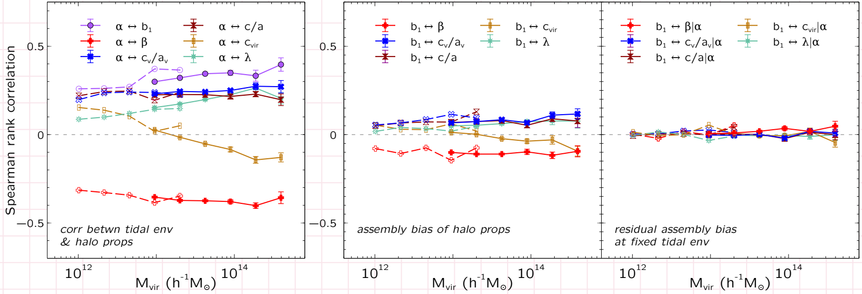

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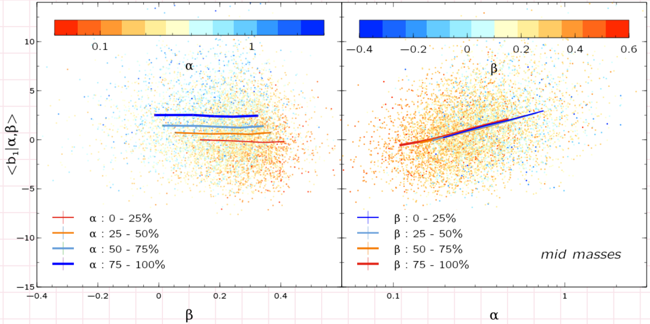
contrasting with previous findings in Paranjape et al. (2018)(p18)

- notion of tidal anisotropy, halo-by-halo bias, assembly bias using correlation coefficients introduced in p18/ notion of conditional correlation coefficient introduced here.
- halo assembly bias wrt to concentration and tidal anisotropy studied in p18/ halo assembly bias wrt 4 additional properties included here.
- tidal anisotropy as an intermediary in explaining halo assemby bias wrt all other properties is established first time here

Distribution for velocity anisotropy β



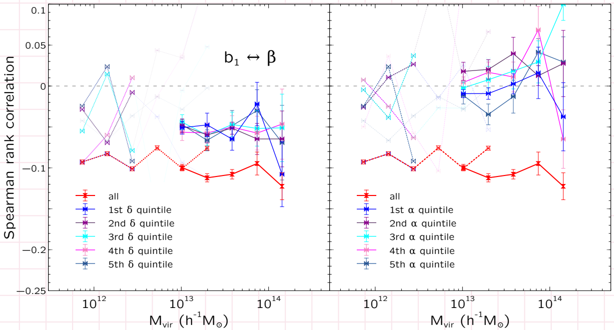
$p(\beta, b_1|\alpha) = p(\beta|\alpha)p(b_1|\alpha)$ conditional independence

$M_{\rm vir}/M_\odot h^{-1}$

mid masses - 6e12-1e13

Comparing the two Environment variables

- how does α compare with δ?



Han et al (2019)
Shi & Sheth (2018)

[Ramakrishnan et.al (2019)]

Summary and Next work

- In the previous work we see that the tidal anisotropy of the environment
  1. has the strongest dependence on bias among all the beyond mass halo properties.
  2. statistically explains all halo assembly bias wrt other properties.
- In the next work, we use the Separate Universe technique
  1. to calibrate $b(m, z, \alpha)$
  2. to calibrate $b(m, z, c)$

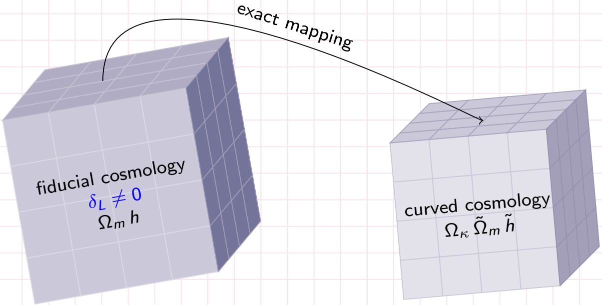
Peak background split approach to halo bias.

$$\frac{n_h(m|\delta_L)}{n_h(m|0)} = 1 + \sum_{N=1}^{\infty} \frac{b_N^L(m)}{N!}\delta_L^N \quad (2)$$

- in regular simulations - max wavelength of perturbations is limited by the box size, in other words $\delta_L = 0$
- Separate universe technique
  - offers a way to probe the infinite wavelength perturbation in a simulation.
  - it also gives the quadratic bias without any additional cost.

Separate Universe Technique

References: Tormen & Bertschinger (1996); Cole (1997); Baldauf et al. (2011); Li et al. (2016) ,Wagner et al. (2015), Lazeyras et al. (2016), Paranjape and Padmanabhan (2017)



Separate Universe Technique

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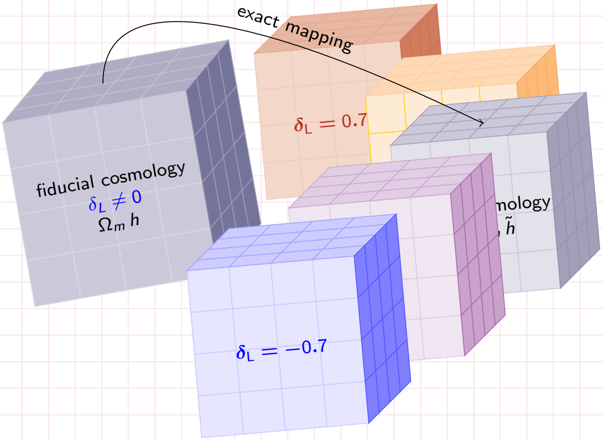

$\delta_L \in \{\pm0.7, \pm0.5, \pm0.4, \pm0.3, \pm0.2, \pm0.1, \pm0.07, \pm0.05, \pm0.02, \pm0.01, +0.15, +0.25, +0.35\}$

$$\frac{n(m \quad |\delta_L)}{n(m \quad |0)} = 1 + \sum_{N=1}^{\infty} b_N^L(m \quad)\delta_L^N$$

$$\frac{n(m,\alpha|\delta_L)}{n(m,\alpha|0)} = 1 + \sum_{N=1}^{\infty} b_N^L(m,\alpha)\delta_L^N$$

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$$= 1 + \sum_{N=1}^{4} b_N^L(m,\alpha)\delta_L^N + \mathcal{O}(\delta_L^5)$$

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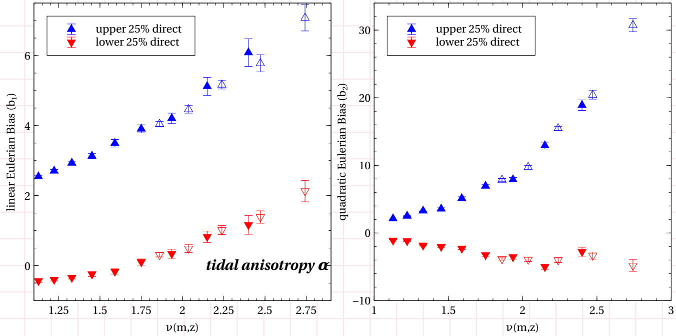

$$b_1 = 1 + b_1^L g(z)^{-1},$$
$$b_2 = b_2^L g(z)^{-2} + \frac{8}{21} b_1^L g(z)^{-1}\,.$$

Here $g(z) \equiv D(z)/D(0)$ and $D(z)$ is the linear theory growth factor of the fiducial cosmology.

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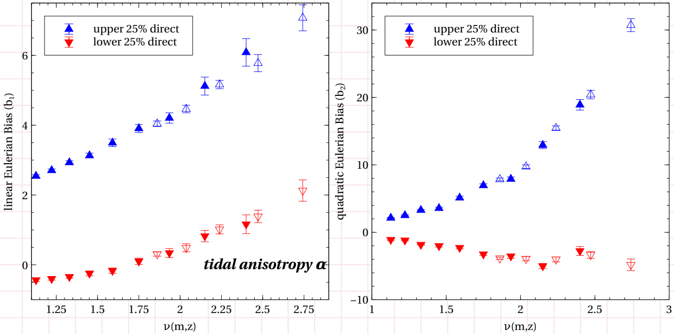

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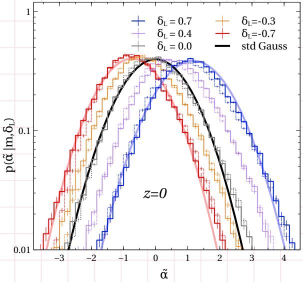
- arbitrary binning in m and α
- this can be avoided by expoliting nature of distribution of α

Lognormal Framework

also used in Paranjape and Padmanabhan (2017) for assembly bias wrt halo concentration. Tidal Anisotropy has near-Lognormal distribution

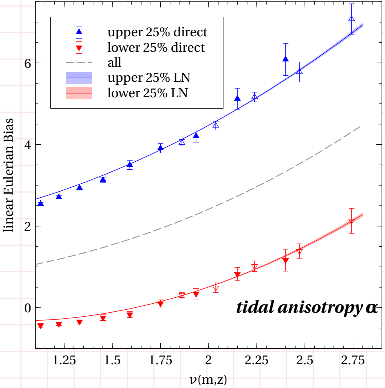
Standardizing α

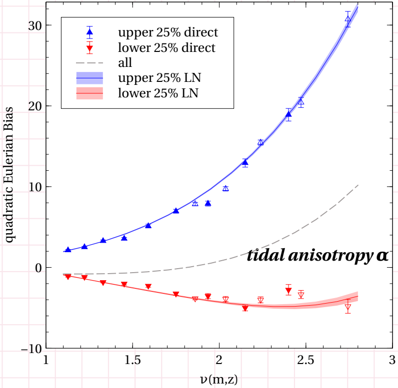
$$\tilde{\alpha} \equiv \frac{\ln \alpha - \mu_0}{\sigma_0}$$
$$\mu_0 \equiv \langle \ln \alpha | m, \delta_L = 0 \rangle$$
$$\sigma_0^2 \equiv \mathrm{Var}(\ln \alpha | m, \delta_L)$$



$$\frac{p(\tilde{\alpha}|m, \delta_L)}{p(\tilde{\alpha}|m, \delta_L = 0)} = \frac{n(m, \tilde{\alpha}|\delta_L)/n(m|\delta_L)}{n(m, \tilde{\alpha}|\delta_L = 0)/n(m|\delta_L = 0)}$$
$$\frac{\mathcal{N}(\mu, \sigma)}{\mathcal{N}(0, 1)} = (1 + \sum_{n=1}^{\infty} b_n^L(m, \alpha)\delta_L^n)/(1 + \sum_{n=1}^{\infty} b_n^L(m)\delta_L^n)$$

$$\mu(m, \delta) = \sum_{n=1}^{\infty} \frac{\mu_n^L(m)}{n!} \delta_L^n$$
$$\sigma^2(m, \delta_L) = 1 + \sum_{n=1}^{\infty} \Sigma_n^L(m)\delta_L^n$$





Lognormal Framework.

$$b_1^L(m,\,\tilde{\alpha}) = b_1^L(m) + \mu_1^L(m)H_1(\tilde{\alpha}) + \frac{1}{2}\Sigma_1^L(m)H_2(\tilde{\alpha}), \tag{3}$$

$$b_2^L(m,\,\tilde{\alpha}) = b_2^L(m) + \{\mu_2^L(m) + 2b_1^L(m)\mu_1^L(m)\}H_1(\tilde{\alpha}) + \tag{4}$$

$$\{\mu_1^L(m)^2 + b_1^L(m)\Sigma_1^L(m) + \frac{1}{2}\Sigma_2^L(m)\}H_2(\tilde{\alpha}) + \mu_1^L(m)\Sigma_1^L(m)H_3(\tilde{\alpha}) + \frac{1}{4}\Sigma_1^L(m)^2H_4(\tilde{\alpha}).$$

[Ramakrishnan & Paranjape (2020)]

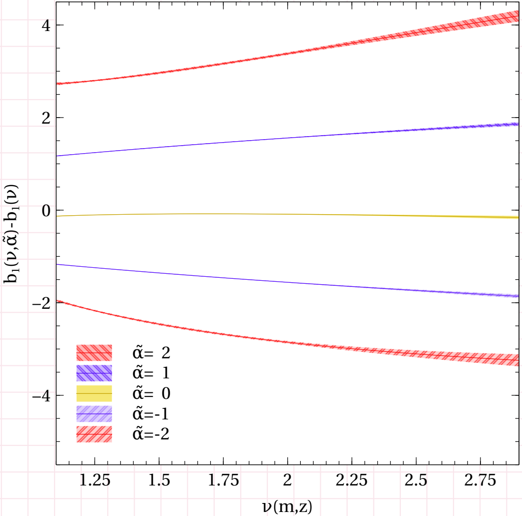
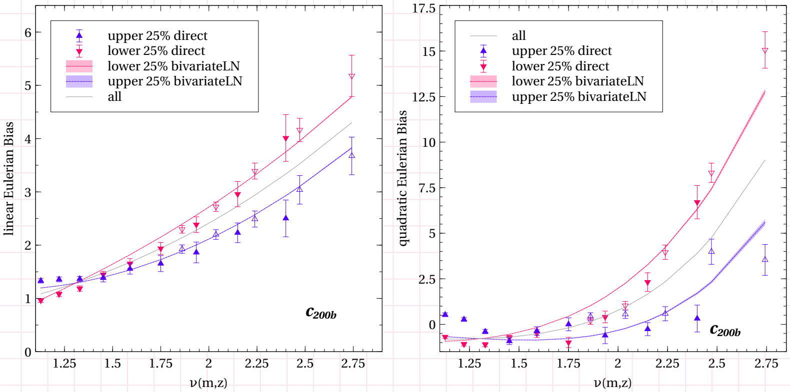


Figure: Our formalism provides b_1 and b_2 at fixed values of $\tilde{\alpha}$, not just in bins of $\tilde{\alpha}$



LN - Extended.

$$b_1(m,c) = b_1(m) + \mu_1(m)\rho(m)H_1(c) + \frac{1}{2}\Sigma_1(m)\rho^2(m)H_2(c). \quad (5)$$

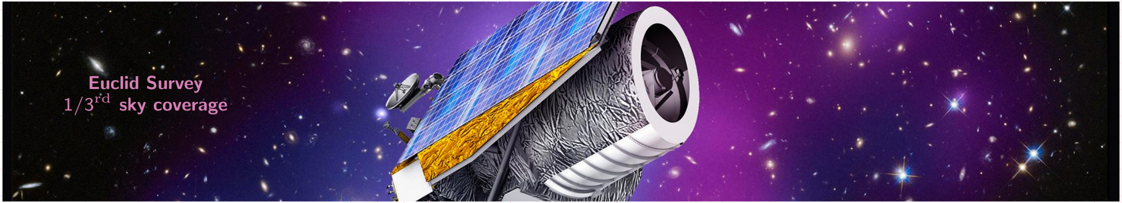
$$b_2(m,c,z) = b_2 + \{\mu_2^L + 2\mu_1^L(b_1 - 1) + \frac{8}{21}\mu_1^L\}\rho H_1(c)$$
$$+ \{(\mu_1^L)^2 + \Sigma_1^L(b_1 - 1) + \frac{1}{2}\Sigma_2^L + \frac{4}{21}\Sigma_1^L\}\rho^2 H_2(c) + \mu_1^L\Sigma_1^L\rho^3 H_3(c) + \frac{1}{4}(\Sigma_1^L)^2\rho^4 H_4(c) \quad (6)$$

Recap and Next project

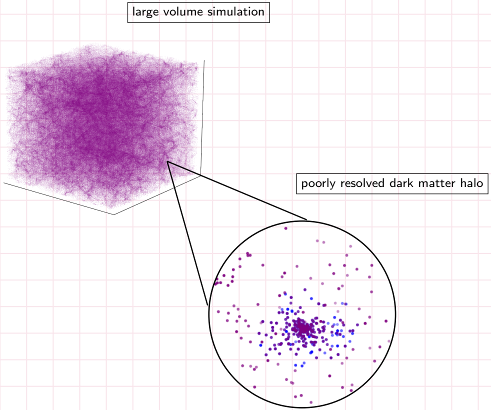
- In this project- calibrations for the assembly bias in b_1 and b_2
- In the next project- application for large volume surveys.

Future Large Sky surveys need large volume mocks

- to compare & connect theory with observations
- systematics and error assessment
- test beds for new cosmological probes
- N-body numerical techniques



Limitations of using N-body mocks



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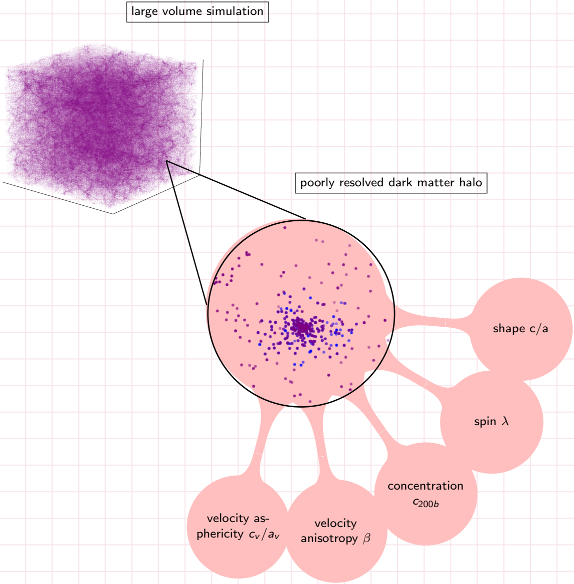
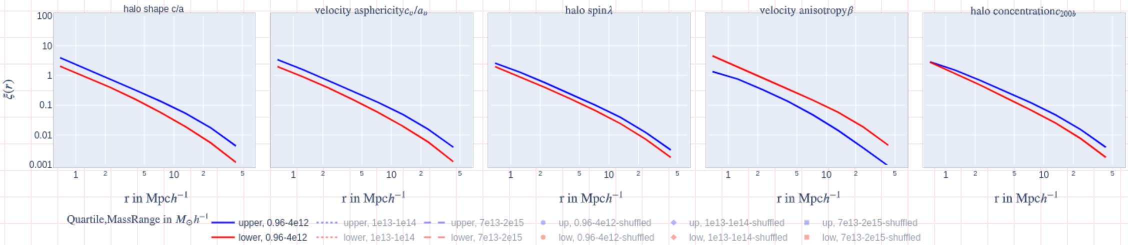
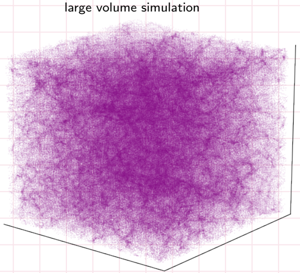


Limitation - Halo assembly bias signal is unresolved

- uncertainities in the 2-pt-correlation function of galaxy clusters
- At fixed halo mass the 2-pt function can be different based on the subsample of properties considered
- The large scale trends in these plots can be identified with the large scale assembly bias.
- Here, we look at the scale dependence of assembly bias.



Information in the cosmic web environment



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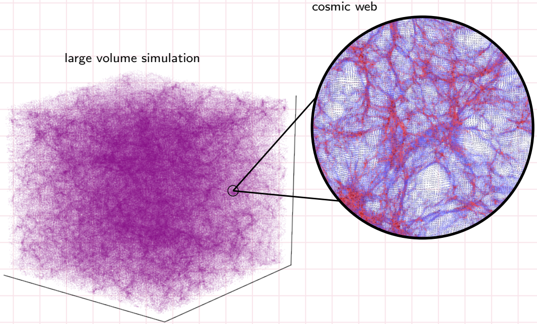

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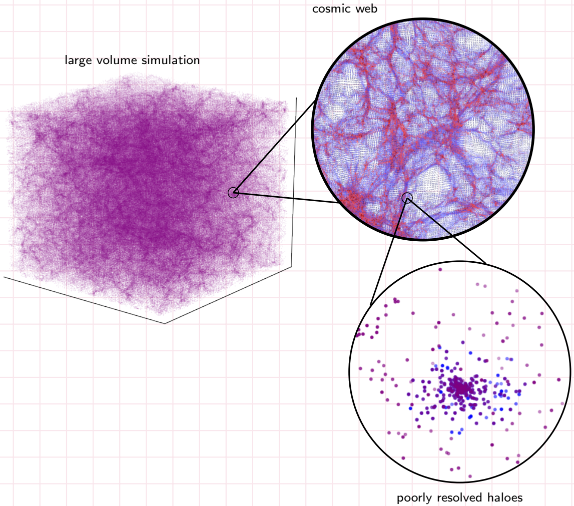
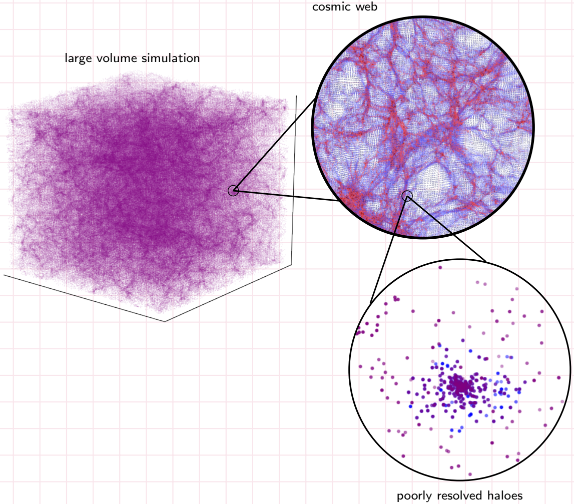


Information in the cosmic web environment

Each halo's environment can be described by the tidal anisotropy $\alpha = \sqrt{\frac{1}{2}\left[(\lambda_2 - \lambda_1)^2 + (\lambda_3 - \lambda_1)^2 + (\lambda_3 - \lambda_2)^2\right]} / (1 + \delta)$

$\lambda_1, \lambda_2, \lambda_3$ are the eigen values of the tidal tensor.



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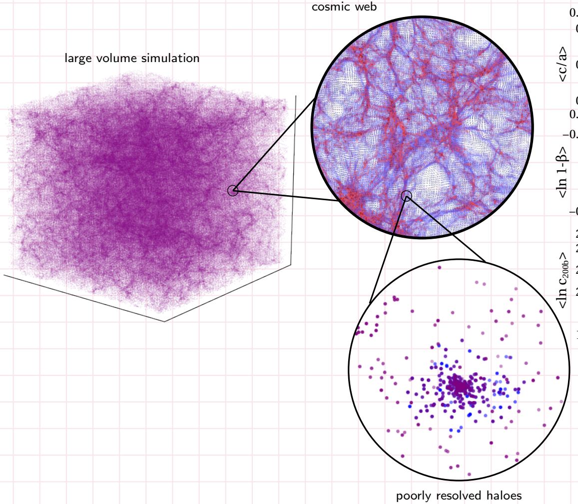
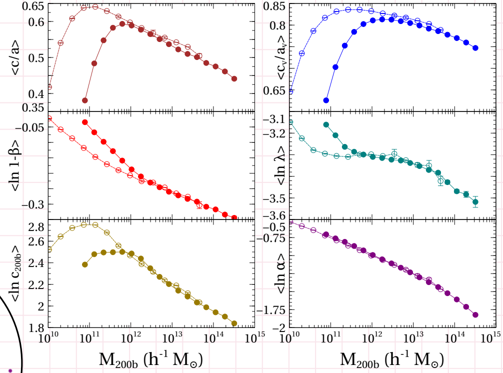


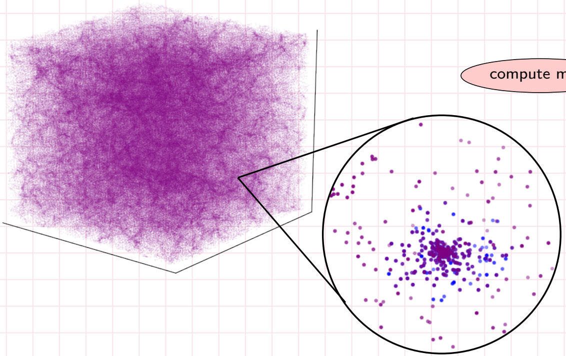


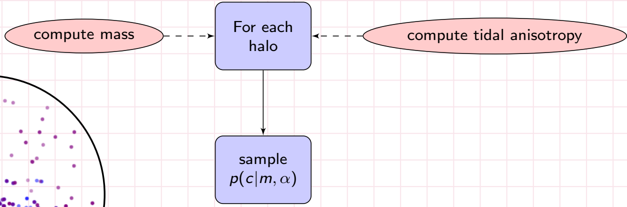
Creating Mock Catalogs :Algorithm

- In low resolution simulations where halo properties are not resolved, we can populate haloes with their internal properties by sampling $p(c|m, \alpha)$
- useful for large volume mock catalogs in large scale surveys

$$p(c|\alpha, m) = e^{-(c-\rho_c\alpha-\mu_c)^2/2\sigma_c^2(1-\rho_c^2)}$$

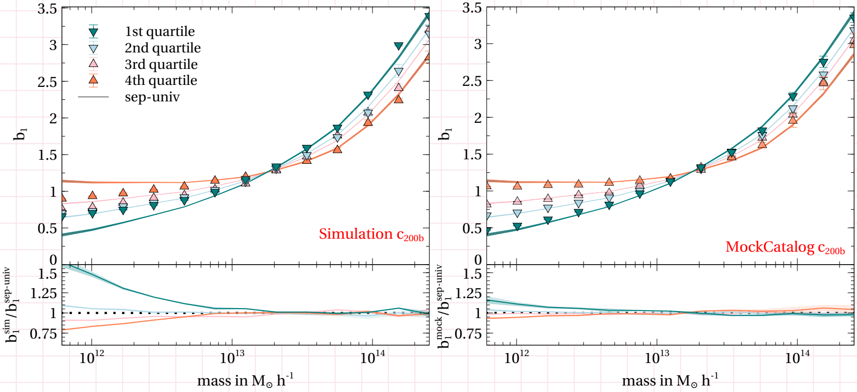
μ_c = mean of c , σ_c=standard deviation of c, ρ_c = pearson correlation between α and c.





Testing the algorithm

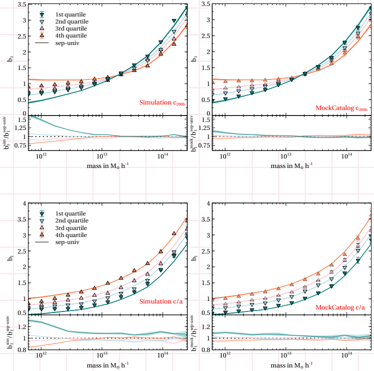
[Ramakrishnan et.al (2021)]

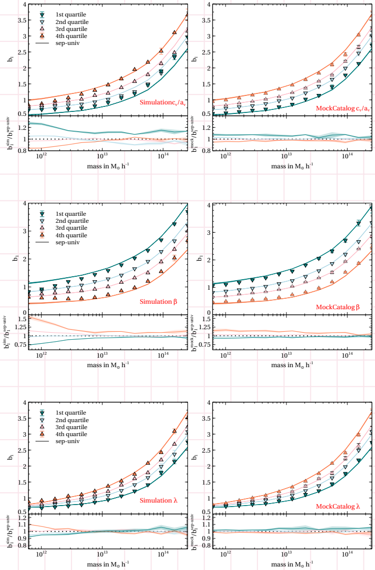


improves bias signal by percentages as large as 45% for the 30 particle haloes.

Testing the algorithm

[Ramakrishnan et.al (2021)]



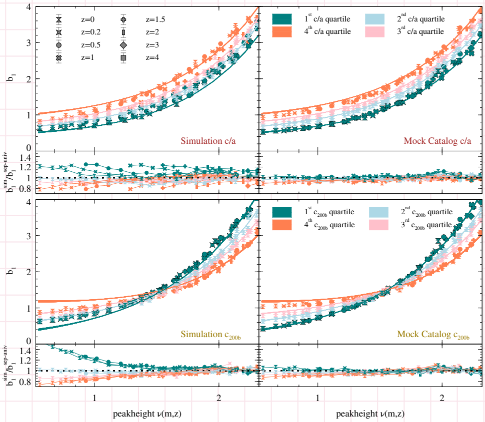


Note: We do not use any information about $c \leftrightarrow b_1$ for sampling the mocks but retrieve $c \leftrightarrow b_1$ naturally.

Algorithm can be easily extended to redshifts $z < 2$ and other cosmologies.

We span cosmologies 4σ around the Planck18 using

- 10 locally run large volume simulations
- 10 high-resolution cosmologies from CAMELS-SAMS [Perez et al 2022]



[Ramakrishnan & Velmani (2022)]

- redshift 0-4, and the planck cosmology is also included in addition to wmap cosmology in obtaining fits
- fits are still universal in peakheight
- 2-3 orders of magnitude gains in number density at large redshift z=2-4
- the gains in number density will be larger for larger Gpc sized boxes because steep fall in the mass function.

Recap

In this work we use the information in tidal anisotropy,

- to increase dynamic range and number density of simulation by an order of magnitude.
- useful for large volume simulations required for upcoming surveys like LSST, Euclid.
- Extendable to higher redshift $z < 2$ and other cosmologies.

Summary

- The tidal anisotropy around a halo is the primary driver of the secondary dependence of halo clustering wrt all other halo properties. This has been shown for in large scale halo bias as well as the two point correlation function.
- We verified the above for shape, velocity ellipsoid, velocity anisotropy, concentration and spin.
- We use the Separate Universe technique to calibrate dependence of linear and quadratic halo bias with tidal anisotropy and other halo properties.
- Tidal anisotropy is well resolved for haloes whose particle count is between 30-700, where the internal properties are not resolved. We develop an algorithm useful for large volume mocks that uses the tidal anisotropy and mass to assign mock halo properties to these unresolved haloes.

Future scope

- with our mocks calibrated upto redshift 4, we can think of making light cone catalogs to study the halo clustering and weak lensing profiles at large scales.
- Our calibration of $b(m, z, \alpha \text{ or } c)$ can be used to factor in the effect of large scale assembly bias in cluster scale analyses that use for e.g, the halo model approach.for eg, galaxy-galaxy lensing, SZ observations to test for effects on
  - cosmological parameter estimation
  - constrain the halo properties
  - or in the detection of the assembly bias signal

Halo-by-Halo Bias

$$\delta_h(x) = \frac{V}{N}\Sigma_n \delta_D(x - x_n) - 1$$
$$\tilde{\delta_h}(k \neq 0) = \frac{V}{N}\Sigma_n e^{ikx_n}$$

$$b =< \tilde{\delta_h}\tilde{\delta_m} > / < \tilde{\delta_m}\tilde{\delta_m} > \quad (7)$$
$$b = \Sigma_n \frac{V}{N} < e^{ikx_n}\tilde{\delta_m} > / < \tilde{\delta_m}\tilde{\delta_m} > \quad (8)$$
$$= \Sigma_n b_n \quad (9)$$

Back to main.