Primordial Black Holes and Stochastic Inflation beyond slow roll

State of the Universe (SOTU) Seminar @ TIFR, Mumbai

Swagat Saurav Mishra

Postdoctoral Research Fellow

Centre for Astronomy and Particle Theory (CAPT)
School of Physics and Astronomy, University of Nottingham.

With **Edmund J. Copeland** and **Anne M. Green**

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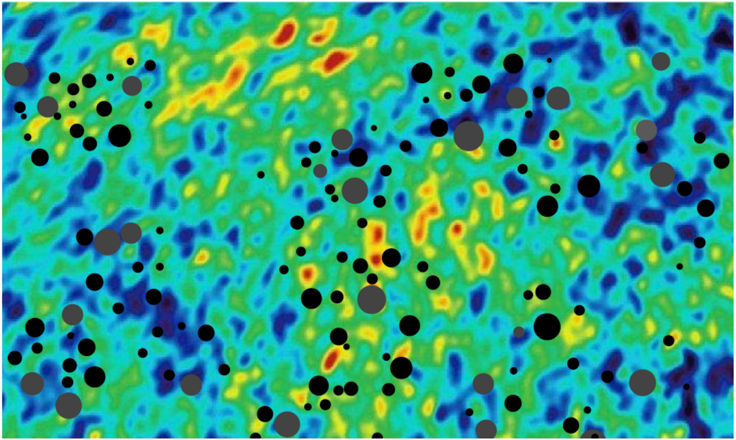
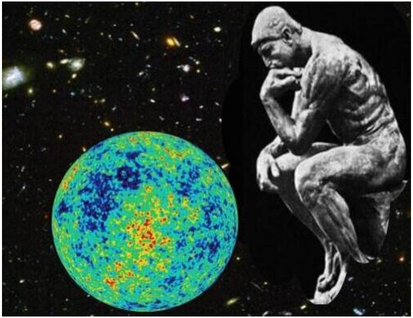
Primordial Black Holes (PBHs)



Candidates for Dark Matter, Hawking Radiation, Baryogenesis, Reheating, seeds of SMBHs etc.

Extremely interesting rich phenomenology!

CMB $\longrightarrow$ LSS



- **Adiabatic** $\zeta(\vec{x})$
- Almost **scale-invariant**

$$\boxed{\mathcal{P}_\zeta = A_S \left(\frac{k}{k_*}\right)^{n_S}}$$

$$A_S \simeq 2 \times 10^{-9}\,,\ n_S \simeq -0.035$$

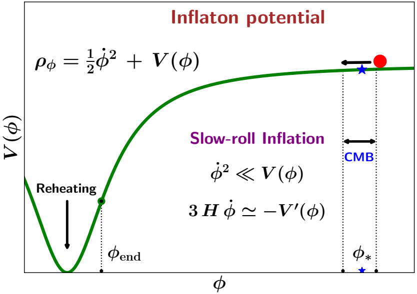
- Nearly **Gaussian**

$$\boxed{P[\zeta] = \mathcal{B} \exp\left[\frac{-\zeta^2}{2\sigma^2}\left(1 + f_{\rm NL}\,\zeta + ...\right)\right]}$$

$\rightarrow$ **LSS, CMB $\Rightarrow$ Large-scale tiny quantum fluctuations**

$\rightarrow$ **PBHs, $\text{GW}^{(2)}$s $\Rightarrow$ Small-scale larger fluctuations ?**

Source of Inflation: A Scalar Field



Density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

Pressure $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_\phi ,$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)\left(\rho_\phi + 3\,p_\phi\right)$$

Condition for Inflation

$$\boxed{\dot{\phi}^2 < V(\phi)}$$

Motion of the scalar field is governed by

$$\boxed{\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0}$$

$\dot{\phi}^2 \ll V(\phi) \Rightarrow$ **Nearly Exponential expansion** $a \sim e^{Ht}$ at the background level.

Full System during Inflation

System = Gravity $(g_{\mu\nu})$ + **Scalar Field** (ϕ)

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left(\frac{m_p^2}{2}\,R - \frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} - V(\phi) + \ldots\right)$$

During Inflation, the metric takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\left(e^{2\Psi(t,\vec{x})}\,\delta_{ij} + h_{ij}(t,\vec{x})\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

In particular, **two light fields** are guaranteed to exist –

1. **Comoving Curvature Perturbation**

$$-\zeta(t,\vec{x}) = \Psi + \frac{1}{\sqrt{2\,\epsilon_H}}\frac{\delta\phi}{m_p}$$

(Later becomes **density and temperature fluctuations**)

2. **Tensor Perturbation** (Transverse, traceless $h_{ij}(t,\vec{x})$ – **relic Gravitational Waves**)

Power-spectra: Linear Perturbation Theory

During slow-roll (**SR**) inflation, $\boxed{\epsilon_H,\ |\eta_H| \ll 1}$

where $$\boxed{\epsilon_H = -\frac{\dot{H}}{H^2}\,, \quad \eta_H = \epsilon_H - \frac{1}{2}\frac{\mathrm{d}\ln\epsilon_H}{\mathrm{d}N}}$$

Primordial Scalar power-spectrum at least at **large scales** –

$$\boxed{\mathcal{P}_\zeta = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2\frac{1}{\epsilon_H} = A_S\left(\frac{k}{k_*}\right)^{n_S}}$$

From CMB observations,

$$A_s = 2.1\times 10^{-9}$$

Physical wavelength $\boxed{\lambda_p = \frac{a}{k}}$

CMB pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$

Scalar spectral index

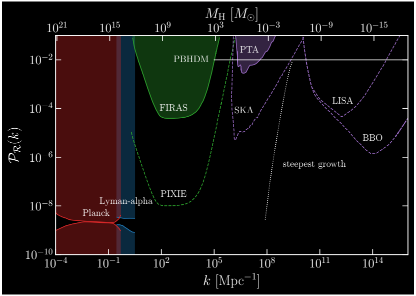
$\boxed{n_S \simeq -0.035}$ **Small red tilt**

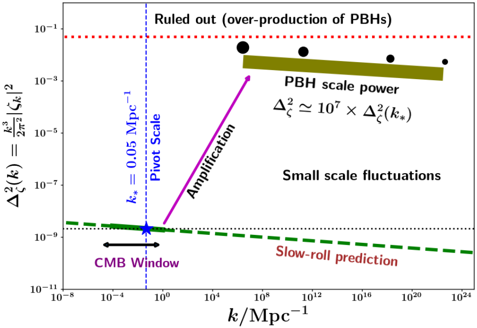
⇒ **Tiny fluctuations** that are **nearly scale-invariant**

What we know from Observations

CMB probes scales $\boldsymbol{k} \in [\mathbf{0.0005},\, \mathbf{0.5}]\ \mathrm{Mpc}^{-1} \Rightarrow \boldsymbol{\Delta N \simeq 7}$

Small-scale power spectrum is not constrained!

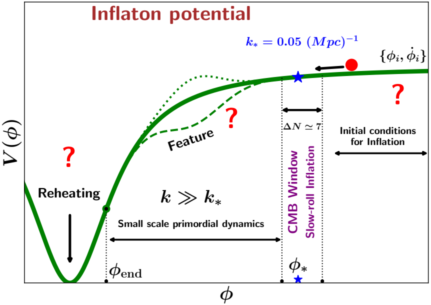


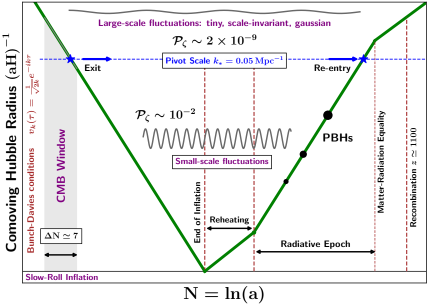


Possibility of enhancement of small-scale fluctuations!

**Green and Kavanagh, J. Phys. G 48 (2021) 4, 043001

⇒ **Scope for non-trivial small-scale dynamics**





$$\textbf{CMB scales}: P_\zeta \sim k^{-0.035} \quad \text{(Slightly red – tilted)}\,;\ \eta_H \simeq -0.018$$

$$\textbf{Small-scale growth}: P_\zeta \sim k^{n_s\,(\leq 4)} \quad \text{(Blue – tilted)}\,;\ \eta_H \geq 3/2$$

**Byrnes et. al JCAP 06(2019) 028

Large Quantum Fluctuations

1. **Breakdown of scale-invariance at small-scales**

$$\boxed{\epsilon_H = -\frac{\mathrm{d}\ln H}{\mathrm{d}N}\,, \quad \boldsymbol{\eta_H = \epsilon_H - \frac{1}{2}\frac{\mathrm{d}\ln\epsilon_H}{\mathrm{d}N}}} \quad ; \quad \mathrm{N} = \ln(\mathrm{a})$$

2. **Breakdown of Gaussian nature of primordial fluctuations**

For $\zeta \gg 1$

$$\boxed{P[\zeta] \neq \mathcal{B}\exp\left[\frac{-\zeta^2}{2\int_{k_1}^{k_2}\mathrm{d}\ln k\,\mathcal{P}_\zeta(k)}\left(1 + f_{\mathrm{NL}}\,\zeta + g_{\mathrm{NL}}\,\zeta^2 + \ldots\right)\right]}$$

****Celoria et. al JCAP 06 (2021) 051**

Breakdown of Scale-invariance via feature

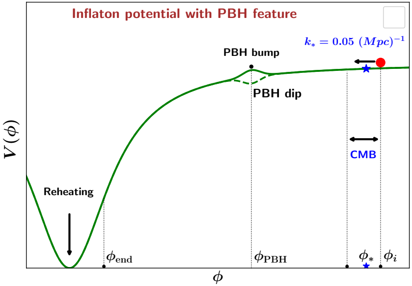
A feature: **an inflection point** or **a local bump/dip** at **low scales** slows down the inflaton

$\Rightarrow$ **Breaking of scale invariance!!**

At small scales $\boxed{\epsilon_H \ll 1\,, \quad \eta_H \gtrsim 3}$ **Violation of slow-roll**

Criteria for PBH from single field Inflation–

1. Large scales satisfying with CMB constraints.
2. Intermediate scale feature to enhance power for PBH formation.
3. Successful Reheating mechanism.



**Motohashi, Hu PRD 96(2017) 6, Cole et. al arXiv:2304.01997

Computing Power spectrum

$$\boxed{\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}\left|\zeta_k\right|^2}_{k<<aH}$$

Mukhanov-Sasaki variable $\boxed{\boldsymbol{v_k = z \times \zeta_k}}$; $z = am_p\sqrt{2\epsilon_H}$

satisfies the **MS** equation:

$$\boxed{\frac{\mathrm{d}^2 v_k}{\mathrm{d}N^2} + (1-\epsilon_H)\frac{\mathrm{d}v_k}{\mathrm{d}N} + \left[\left(\frac{k}{aH}\right)^2 + \boldsymbol{M_{\mathrm{eff}}^2}(N)\right]v_k = 0}$$

where the **effective mass term** is

$$\boxed{\boldsymbol{M_{\mathrm{eff}}^2} = -(aH)^2\left[2 + 2\epsilon_H + 2\epsilon_H^2 \boldsymbol{- 3\eta_H + \eta_H^2} - 3\epsilon_H\boldsymbol{\eta_H - \frac{\mathrm{d}\eta_H}{\mathrm{d}N}}\right]}$$

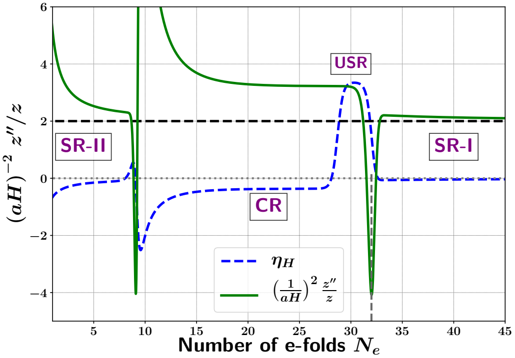
Background dynamics dependent and complicated!

Typical Inflationary Dynamics

SR-I (CMB scale) $\longrightarrow$ **USR** $\longrightarrow$ **CR** $\longrightarrow$ **SR-II**

$$\boxed{\boldsymbol{\eta_H :} \quad \boldsymbol{\eta_1} \longrightarrow \boldsymbol{\eta_2} \longrightarrow \boldsymbol{\eta_3} \longrightarrow \boldsymbol{\eta_4}}$$ Wands Duality

Background



Reason for duality

For $\epsilon_H \ll 1$,

$$\frac{M_{\text{eff}}^2}{(aH)^2} \simeq 2 - 3\eta_H + \eta_H^2 - \frac{d\eta_H}{dN}$$

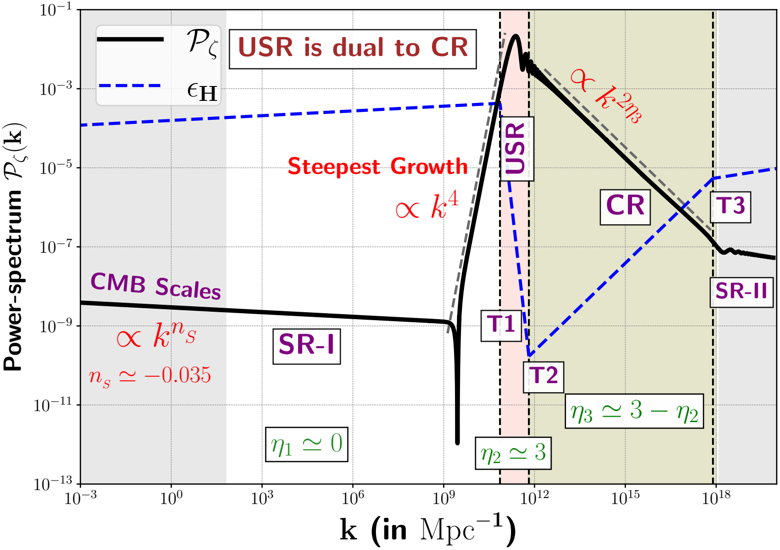
Assuming

$$\boxed{\eta_H = \frac{3}{2} + C \tanh\left[C\left(N_e - \tilde{N}_e\right)\right]}$$

$$\nu^2 \equiv \frac{M_{\text{eff}}^2}{(aH)^2} + \frac{1}{4} \simeq \text{const.}$$

**SSM, Sahni JCAP 04(2020) 007, **Karam et. al JCAP 03(2023) 013

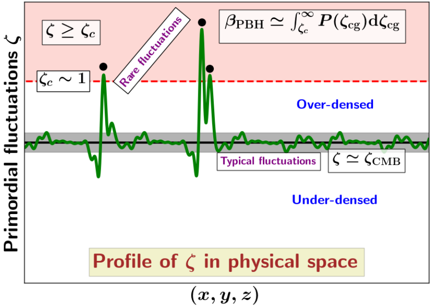
Typical Power-spectrum

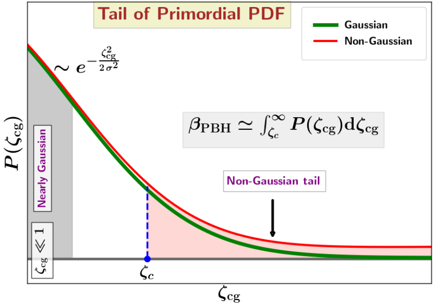


Statistics of Primordial Fluctuations

Is the Primordial PDF $P[\zeta]$ Gaussian or Non-Gaussian?

Non-Gaussian for $\zeta \gg 1$ in general





PBHs from Rare Peaks: Sensitive to the tail of PDF

Non-Perturbative Methods for full PDF

Approach - I

Classical Non-linear δN formalism

Approach - II

Semi-classical Approximation

Approach - III

Stochastic Inflation

Stochastic Inflation: Effective IR description

Coarse-grained description $\phi = \boldsymbol{\Phi} + \varphi \quad , \quad \pi_\phi = \boldsymbol{\Pi} + \pi$

Langevin Equations (Non-linear)

$$\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}N} = D_{\boldsymbol{\Phi}} + \boldsymbol{\xi}_\phi \, ; \quad \frac{\mathrm{d}\boldsymbol{\Pi}}{\mathrm{d}N} = D_{\boldsymbol{\Pi}} + \boldsymbol{\xi}_\pi$$

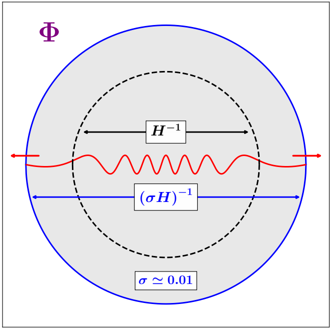
$$\frac{\mathrm{d}F_{\mathrm{cg}}}{\mathrm{d}N} = \mathbf{Drift}_{\mathrm{cl}} + \mathbf{Diffusion}_{\mathrm{Q}}$$

Gaussian White noise statistics

$$\langle \boldsymbol{\xi}_i(N)\, \boldsymbol{\xi}_j(N') \rangle = \Sigma_{ij}(N)\, \delta_D(N - N')$$

Noise Matrix elements

$$\boldsymbol{\Sigma_{ij}}(N) = (1 - \epsilon_H) \frac{k^3}{2\pi^2} \phi_{i_k}(N) \phi^*_{j_k}(N) \Big|_{k=\sigma a H}$$



Coarse-graining scale

$k = \boldsymbol{\sigma}\, aH \, , \quad \sigma \ll 1$

**A. A. Starobinsky (1986)

PDF from first-passage time analysis

$$\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}N} = D_{\boldsymbol{\Phi}} + \boldsymbol{\xi}_\phi \, ; \qquad \frac{\mathrm{d}\boldsymbol{\Pi}}{\mathrm{d}N} = D_{\boldsymbol{\Pi}} + \boldsymbol{\xi}_\pi$$

First-passage no. of e-folds $\mathcal{N}$

and **PDF** $P(\mathcal{N})$

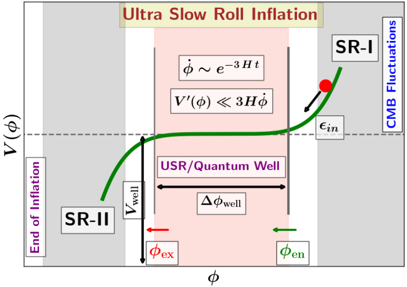
Subject to boundary conditions

1. **Reflecting boundary** at $\boldsymbol{\Phi} = \phi_{\rm en}$:

$$\frac{\partial}{\partial \boldsymbol{\Phi}} P(\mathcal{N}) \Big|_{\boldsymbol{\Phi}=\phi_{\rm en}} = 0$$

2. **Absorbing boundary** at $\boldsymbol{\Phi} = \phi_{\rm ex}$:

$$P(\mathcal{N}) \Big|_{\boldsymbol{\Phi}=\phi_{\rm ex}} = \delta_D(\mathcal{N})$$



- Numerical Simulations
- **Fokker-Planck Equation** (for analytical treatment)

Langevin $\longrightarrow$ Fokker-Planck Equation

PDF of first-passage number of e-foldings $\mathcal{N}$: **Adjoint FPE**

$$\frac{\partial P}{\partial \mathcal{N}} = \left[D_{\mathbf{\Phi}} \frac{\partial}{\partial \mathbf{\Phi}} + D_{\mathbf{\Pi}} \frac{\partial}{\partial \mathbf{\Pi}} + \frac{1}{2} \mathbf{\Sigma}_{\phi\phi} \frac{\partial^2}{\partial \mathbf{\Phi}^2} + \mathbf{\Sigma}_{\phi\pi} \frac{\partial^2}{\partial \mathbf{\Phi} \partial \mathbf{\Pi}} + \frac{1}{2} \mathbf{\Sigma}_{\pi\pi} \frac{\partial^2}{\partial \mathbf{\Pi}^2} \right] P(\mathcal{N})$$

$$P(\mathcal{N}) \equiv P_{\mathbf{\Phi},\mathbf{\Pi}}(\mathcal{N})$$

Stochastic $\delta\mathcal{N}$ Formalism

Statistics of $\mathcal{N}$ → Statistics of $\zeta_{\rm cg}$: $P[\mathcal{N}] \longrightarrow P[\zeta_{\rm cg}]$

$$\boxed{\zeta_{\rm cg} \equiv \zeta(\mathbf{\Phi}) = \mathcal{N} - \langle \mathcal{N}(\mathbf{\Phi}) \rangle} \, ; \quad \langle \mathcal{N}(\mathbf{\Phi}) \rangle = \int_0^\infty \mathcal{N} \, P(\mathcal{N}) \, \mathrm{d}\mathcal{N}$$

Abundance of PBHs $\boxed{\beta \sim \int_{\zeta_c}^\infty P(\zeta_{\rm cg}) \, \mathrm{d}\zeta_{\rm cg}}$

**Pattison et. al JCAP 04 (2021) 080

Quasi de Sitter approximation

Mode functions $\{\phi_k, \pi_k\} \longrightarrow \mathrm{dS}$

$$\boxed{\boldsymbol{\Sigma}_{\boldsymbol{\phi\phi}} \simeq \left(\frac{H}{2\pi}\right)^2 \,, \quad \boldsymbol{\Sigma}_{\boldsymbol{\phi\pi}}, \, \boldsymbol{\Sigma}_{\boldsymbol{\pi\pi}} \ll \boldsymbol{\Sigma}_{\boldsymbol{\phi\phi}}}$$

The Langevin equations become

$$\boxed{\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}N} = D_{\boldsymbol{\Phi}} + \frac{H}{2\pi}\,\boldsymbol{\xi}\,; \qquad \frac{\mathrm{d}\boldsymbol{\Pi}}{\mathrm{d}N} = D_{\boldsymbol{\Pi}}}$$

with single **Gaussian white noise** $\boldsymbol{\xi}$ satisfying

$$\langle \boldsymbol{\xi}(N) \rangle = 0 \,, \quad \text{and} \quad \langle \boldsymbol{\xi}(N)\boldsymbol{\xi}(N') \rangle = \delta_D\left(N - N'\right)$$

Adj. Fokker-Planck Equation becomes

$$\frac{\partial P(\mathcal{N})}{\partial \mathcal{N}} = \left[\frac{H^2}{8\pi^2}\frac{\partial^2}{\partial \boldsymbol{\Phi}^2} + D_{\boldsymbol{\Phi}}\frac{\partial}{\partial \boldsymbol{\Phi}} + D_{\boldsymbol{\Pi}}\frac{\partial}{\partial \boldsymbol{\Pi}}\right] P(\mathcal{N})$$

PDF for flat Quantum Well: Pure diffusion

$$V(\boldsymbol{\Phi}) = V_0 \,, \quad H^2 \simeq \frac{V_0}{3m_p^2}$$

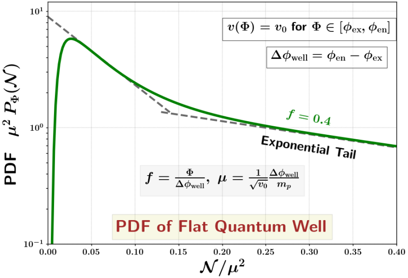
Leading to

PDF $$\boxed{P(\mathcal{N}) = \sum_{n=0}^{\infty} A_n(\boldsymbol{\Phi})\, \mathrm{e}^{-\Lambda_n \mathcal{N}}}$$

with $$\Lambda_n = (2n+1)^2 \frac{\pi^2}{4} \frac{1}{\mu^2}$$

$$A_n = (2n+1)\frac{\pi}{\mu^2} \sin\left[(2n+1)\frac{\pi}{2}\left(\frac{\boldsymbol{\Phi}}{\Delta\boldsymbol{\Phi}}\right)\right]$$

Control Parameter : $$\boxed{\mu = 2\sqrt{2}\pi \frac{\Delta\phi_{\rm well}}{H}}$$



Exponential Tail

Highly Non-Gaussian!!

****Pattison et. al JCAP 10(2017) 046; Ezquiaga et. al. JCAP 03(2020) 029**

Additional Complications

- General form of the feature

$$V(\phi) = V_0 \pm \frac{1}{2}\, m^2\, \phi^2 \pm \frac{\mu}{2}\, \phi^3 + \frac{\lambda}{4}\, \phi^4 \pm ...$$

- When inflaton **drift** is included

$$\frac{\partial}{\partial \mathcal{N}} P(\mathcal{N}) = \left[\frac{\boldsymbol{\Sigma}_{\phi\phi}}{2} \frac{\partial^2}{\partial \mathbf{\Phi}^2} + \left(D_{\mathbf{\Phi}} \frac{\partial}{\partial \mathbf{\Phi}} + D_{\mathbf{\Pi}} \frac{\partial}{\partial \mathbf{\Pi}} \right) \right] P(\mathcal{N})$$

- **Beyond the de Sitter mode functions for noise**

$$\frac{\partial P}{\partial \mathcal{N}} = \left[D_{\mathbf{\Phi}} \frac{\partial}{\partial \mathbf{\Phi}} + D_{\mathbf{\Pi}} \frac{\partial}{\partial \mathbf{\Pi}} + \frac{\boldsymbol{\Sigma}_{\phi\phi}}{2} \frac{\partial^2}{\partial \mathbf{\Phi}^2} + \boldsymbol{\Sigma}_{\phi\pi} \frac{\partial^2}{\partial \mathbf{\Phi} \partial \mathbf{\Pi}} + \frac{\boldsymbol{\Sigma}_{\pi\pi}}{2} \frac{\partial^2}{\partial \mathbf{\Pi}^2} \right] P(\mathcal{N})$$

Recently concluded work

SSM, Edmund J. Copeland and Anne M. Green,

"*Primordial black holes and stochastic inflation beyond slow roll: I - Noise Matrix Elements*"

[arXiv:2303:17375]

Computing Noise Matrix Elements

$$\boldsymbol{\Sigma_{ij}}(N) = (1-\epsilon_H)\frac{k^3}{2\pi^2}\phi^*_{i_k}(N)\phi_{j_k}(N)\Big|_{k=\sigma aH}\,; \qquad \phi_{i_k} \equiv \{\phi_k, \pi_k\}$$

$$\phi_k(N) = \frac{v_k(N)}{a}\,, \quad \pi_k(N) = \frac{\mathrm{d}\phi_k}{\mathrm{d}N}$$

Mukhanov-Sasaki variable $\boldsymbol{v_k}$ in **spatially-flat gauge**

$$\boxed{\frac{\mathrm{d}^2 v_k}{\mathrm{d}N^2} + (1-\epsilon_H)\frac{\mathrm{d}v_k}{\mathrm{d}N} + \left[\left(\frac{k}{aH}\right)^2 + M^2_{\mathrm{eff}}\right] v_k = 0}$$

where the **effective mass term** is

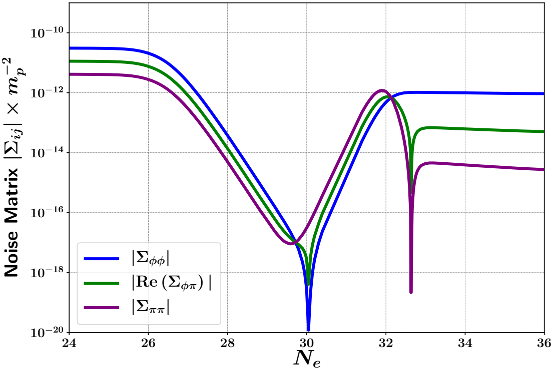
$$\boxed{M^2_{\mathrm{eff}}(aH)^{-2} = 2 + 2\epsilon_H + 2\epsilon_H^2 \boldsymbol{- 3\eta_H + \eta_H^2} - 3\epsilon_H\boldsymbol{\eta_H - \frac{\mathrm{d}\eta_H}{\mathrm{d}N}}}$$

Background dynamics dependent and complicated

Numerical Noise Matrix Elements

Potential with a **tiny Gaussian bump/dip feature**

$$V(\phi) = V_0 \frac{\phi^2}{\phi^2 + M^2} \left[1 \pm A \exp\left(-\frac{1}{2} \left(\frac{\phi - \phi_0}{\Delta\phi} \right)^2 \right) \right]$$



Σ_{ij} evolves and swaps hierarchy! ****Mishra et. al JCAP 04(2020) 007**

Analytical appprox: Sharp transitions

Assume $|\epsilon_H| \ll |\boldsymbol{\eta_H}|$ and $\epsilon_H \ll 1$ (**qdS** approx.)

$$\Rightarrow \boxed{-\boldsymbol{M^2_{\text{eff}}}(aH)^{-2} \simeq 2 - 3\boldsymbol{\eta_H} + \boldsymbol{\eta_H} - \frac{1}{aH}\boldsymbol{\eta_H}'}$$

And $\boldsymbol{\eta_H} \rightarrow$ combination of **Step functions**

$$\boxed{\boldsymbol{\eta_H}(\tau) = \eta_1 + (\eta_2 - \eta_1)\ \Theta(\tau - \tau_1)}$$

For which

$$\boxed{-\boldsymbol{M^2_{\text{eff}}}(aH)^{-2} \simeq \boldsymbol{\mathcal{A}}\,\tau\,\delta_D(\tau - \tau_1) + \left(\nu_1^2 - \frac{1}{4}\right) + \left(\nu_2^2 - \nu_1^2\right)\ \Theta(\tau - \tau_1)}$$

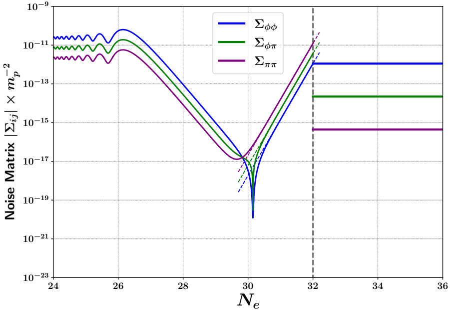
Where the **strength of transition** is $\boxed{\boldsymbol{\mathcal{A}} = \eta_2 - \eta_1}$ and

Order of Hankel $\boxed{\nu_{1,2}^2 = \left(\frac{3}{2} - \eta_{1,2}\right)^2}$

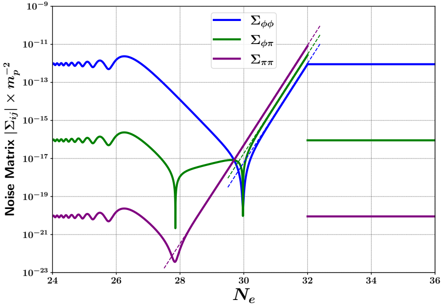
Results from Analytical Techniques

$$\boxed{\boldsymbol{\eta_H}(\tau) = \eta_1 + (\eta_2 - \eta_1)\ \Theta(\tau - \tau_1)}\,, \qquad \text{Conformal time } \tau = \frac{-1}{aH}$$

$$\eta_1 \simeq -0.02\,; \qquad \eta_2 \simeq 3.3$$



Reproduces numerical results



dS approximation

Primary Conclusions

1. During **SR-I** phase, $\Sigma_{\phi\phi}^{\rm SR} \simeq \left(\frac{H}{2\pi}\right)^2$

$$\boxed{\boldsymbol{\Sigma_{\phi\phi}} : |\boldsymbol{\Sigma_{\phi\pi}}| : \boldsymbol{\Sigma_{\pi\pi}} \simeq 1 : \left|\nu_1 - \frac{3}{2}\right| : \left(\nu_1 - \frac{3}{2}\right)^2}$$

2. Immediately after the transition, $\Sigma_{ij} \propto e^{-2\mathcal{A}N}$, and

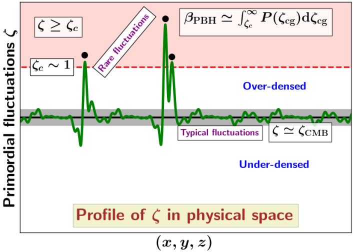
$$\boxed{\boldsymbol{\Sigma_{\phi\phi}} : |\boldsymbol{\Sigma_{\phi\pi}}| : \boldsymbol{\Sigma_{\pi\pi}} \simeq 1 : \mathcal{A} : \mathcal{A}^2}$$

3. During **CR** phase, $\Sigma_{\phi\phi}^{\rm CR} \simeq 2^{2(\nu_2-\nu_1)} \left[\frac{\Gamma(\nu_2)}{\Gamma(\nu_1)}\right]^2 \sigma^{2(\nu_1-\nu_2)} \Sigma_{\phi\phi}^{\rm SR}$

$$\boxed{\boldsymbol{\Sigma_{\phi\phi}} : |\boldsymbol{\Sigma_{\phi\pi}}| : \boldsymbol{\Sigma_{\pi\pi}} \simeq 1 : \left|\nu_2 - \frac{3}{2}\right| : \left(\nu_2 - \frac{3}{2}\right)^2}$$

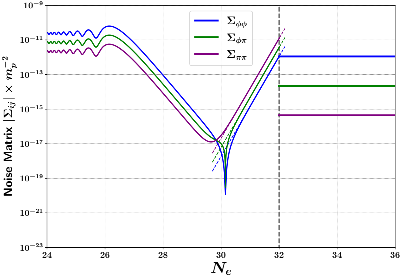
$\Rightarrow$ **Strongest diffusion during Constant-Roll epoch!**

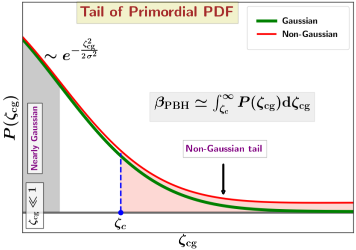
What is the nature of PDF $P[\zeta]$? **Work in Progress**



PBHs $\longrightarrow$ Large QFs

$\Rightarrow$ Sensitive to the tail





Stochastic Inflation formalism

(Fokker-Planck Equation)

Noise-Matrix Elements

$$\frac{\partial P}{\partial \mathcal{N}} = \left[D_{\Phi} \frac{\partial}{\partial \Phi} + D_{\Pi} \frac{\partial}{\partial \Pi} + \frac{1}{2} \Sigma_{\phi\phi} \frac{\partial^2}{\partial \Phi^2} + \Sigma_{\phi\pi} \frac{\partial^2}{\partial \Phi \partial \Pi} + \frac{1}{2} \Sigma_{\pi\pi} \frac{\partial^2}{\partial \Pi^2} \right] P(\mathcal{N})$$

Caveats

1. Mode functions evolved in a fixed (deterministic background). **Figueroa *et. al* 2021
2. Computed in spatially-flat gauge. **Pattison *et. al* 2019
3. Only a single transition was considered analytically (duality).
4. Both $\mathbf{\Phi}$ and $\mathbf{\Pi}$ were treated stochastically. **Tomberg 2022
5. $\beta_{\rm PBH}$ in terms of ζ rather than δ. **Tada, Vennin 2020

Questions & Comments are most welcome.

This is my second SOTU talk! Looking forward to visiting TIFR in-person in the near future..