The observer-dependence of the Hubble parameter: a covariant perspective

Jessica Santiago

In collaboration with Roy Maartens, Chris Clarkson, Basheer Kalbouneh and Christian Marinoni

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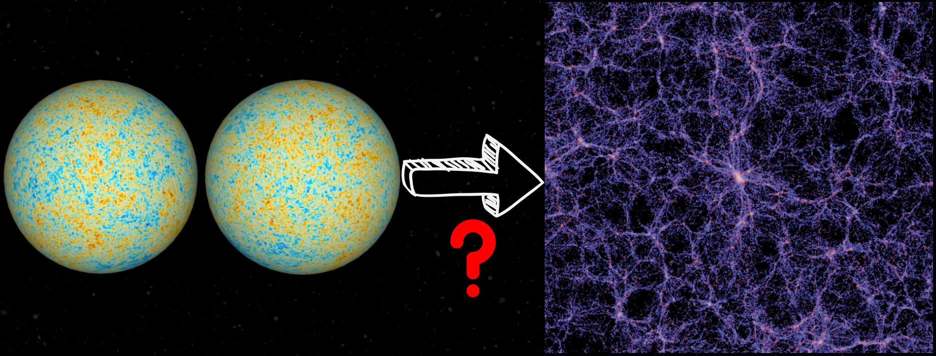
Why do we care?

A universe with a highly homogeneous and isotropic beginning necessarily keeps that way along its evolution?



This is an *open question* which, together with current tensions challenging the standard model, highlights the importance of **testing the cosmological principle**.

Evidence for anisotropies in the Hubble parameter

PHYSICAL REVIEW D **107**, 023507 (2023)

Multipole expansion of the local expansion rate

Basheer Kalbouneh,* Christian Marinoni,† and Julien Bel‡
Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France

Hints of FLRW Breakdown from Supernovae

Chethan Krishnan,[1,*] Roya Mohayaee,[2,†] Eoin Ó Colgáin,[3,4,‡] M. M. Sheikh-Jabbari,[5,§] and Lu Yin[3,4,¶]

Astronomy & Astrophysics manuscript no. Migkas_etal_21 ©ESO 2021
2021-03-26

Cosmological implications of the anisotropy of ten galaxy cluster scaling relations

K. Migkas[1], F. Pacaud[1], G. Schellenberger[2], J. Erler[1,3], N. T. Nguyen-Dang[4], T. H. Reiprich[1], M. E. Ramos-Ceja[5] and L. Lovisari[2,6]

PAPER

A new way to test the Cosmological Principle: measuring our peculiar velocity and the large-scale anisotropy independently

Tobias Nadolny[1], Ruth Durrer[1], Martin Kunz[1] and Hamsa Padmanabhan[1]

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arXiv > astro-ph > arXiv:2212.13569

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 27 Dec 2022]

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae

Jessica A. Cowell, Suhail Dhawan, Hayley J. Macpherson

And many others...

Evidence for a dipole in the deceleration parameter

Physics of the Dark Universe
Volume 40, May 2023, 101224

Testing ΛCDM cosmology in a binned universe: Anomalies in the deceleration parameter

Erick Pastén, Víctor H. Cárdenas

A&A 631, L13 (2019)

Letter to the Editor

Evidence for anisotropy of cosmic acceleration*

Jacques Colin[1], Roya Mohayaee[1], Mohamed Rameez[2] and Subir Sarkar[3]

arXiv > astro-ph > arXiv:2212.13569

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 27 Dec 2022]

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae

Jessica A. Cowell, Suhail Dhawan, Hayley J. Macpherson

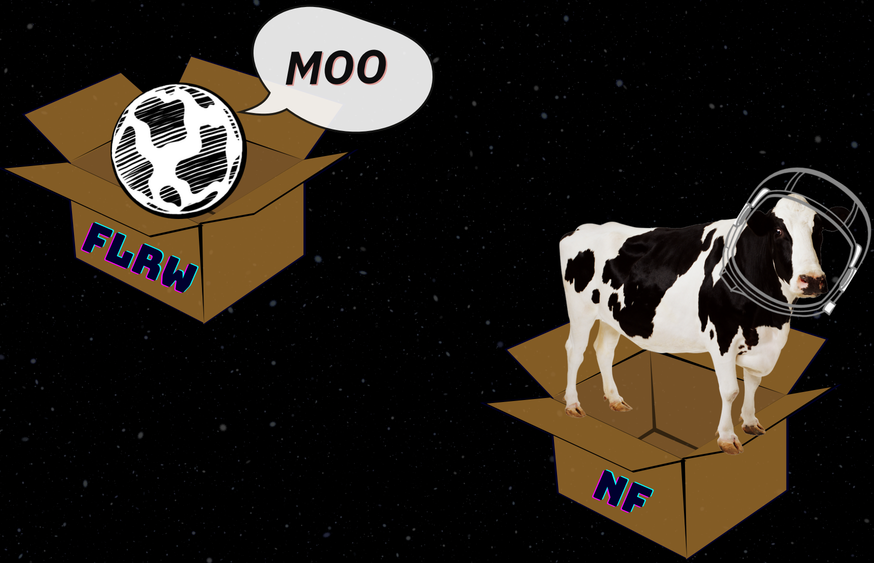
Do supernovae indicate an accelerating universe?

Roya Mohayaee, Mohamed Rameez & Subir Sarkar

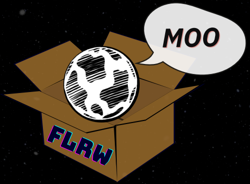
The European Physical Journal Special Topics **230**, 2067–2076 (2021) | Cite this article

More analysis required...

Two different theoretical approaches



FLRW



In the FLRW case, we have:

$$1 + z = \frac{a(t)}{a(t_0)} = z(t) \ .$$

By making use of the photon travelled distance, we obtain a redshift-distance relation given by:

$$z(d) = H_0 d + \frac{2 + q_0}{2} d^2 + \cdots \ .$$

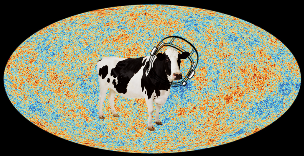
where the Hubble and deceleration parameters are defined as:

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \qquad q(t) = -\frac{a\,\ddot{a}(t)}{\dot{a}^2(t)} = -\left(1 + \frac{\dot{H}}{H^2}\right) \ .$$

Drawbacks

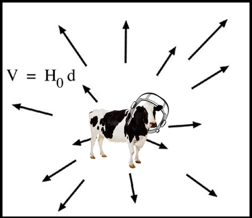
- Cannot account for possible spatial anisotropies
- Doesn't allow directional dependence
- Cannot handle the influence of local effects into data

Cosmological principle vs. frames in cosmology



CMB frame

The observers with a 4-velocity u_{CMB} such that they see no dipole in the CMB;



The Hubble observer

Set of observers with 4-velocity $u_\mu = (-1, 0, 0, 0)$ in a FLRW metric;

Matter frame

Assuming the dust model for the matter in the Universe, the frame that shares the matter 4-velocity u^a_m satisfies:

$$J^a_{\mathrm{m}} = \rho_{\mathrm{m}}\, u^a_{\mathrm{m}} \quad \text{and} \quad T^{ab}_{\mathrm{m}} = \rho_{\mathrm{m}}\, u^a_{\mathrm{m}} u^b_{\mathrm{m}}\,.$$

Cosmological principle vs. frames in cosmology

FLRW models obey the Cosmological Principle exactly: *exactly isotropic and homogeneous* in the unique frame defined by $\bar{u}^a$ which is normal to the 3d surfaces of homogeneity and isotropy.

In FLRW models: $\bar{u}^a_{\rm cmb} = \bar{u}^a_{\rm m} = \bar{u}^a .$

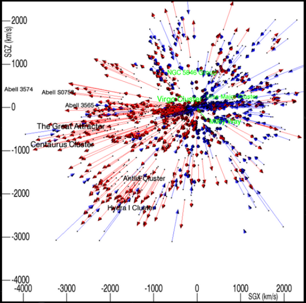
Perturbed FLRW models 'statistically' obey the Cosmological Principle: statistically homogeneous and isotropic. The matter and CMB 4-velocities must agree up to small perturbations, i.e.

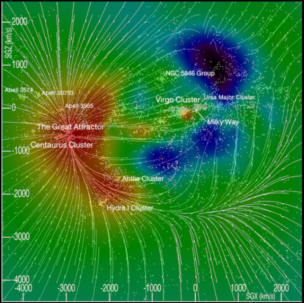
In perturbed FLRW models: $u^a_{\rm cmb} = \bar{u}^a + \delta u^a_{\rm cmb} \,, \quad u^a_{\rm m} = \bar{u}^a + \delta u^a_{\rm m} \,.$

General cases: Do not obey the cosmological principle and

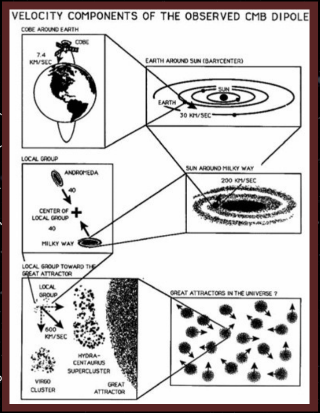
$$u^a_{\rm cmb} \neq u^a_{\rm m} .$$

A disclosure note on the matter frame

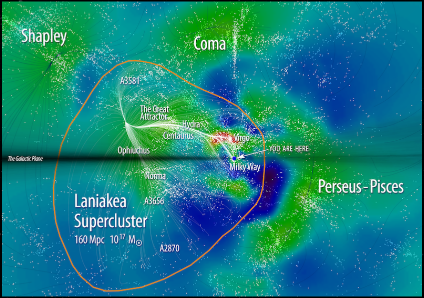




Hélène M. Courtois et al 2013 AJ 146 69



George F. Smoot Rev. Mod. Phys. 79, 1349 (2007)



In our work

Our aim is to investigate the Hubble parameter in a **general spacetime**.
Therefore we **do not** assume the Cosmological Principle.

Our formalism focuses on the **measurement of Hubble parameter** by using supernovae, galaxies, and other forms of matter.

Therefore,

the Hubble parameter in this work is deffined in terms of the matter fluid flow.

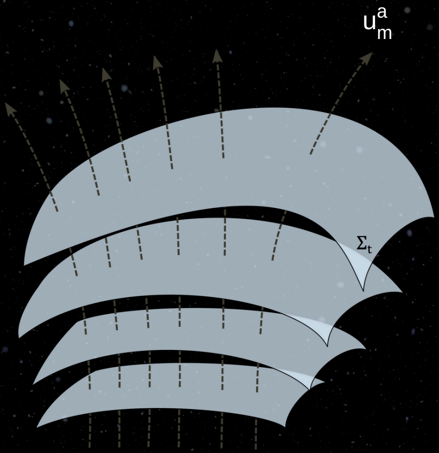
Congruence of observers

We need to deffine a congruence of matter observers with four-velocity u_m^a .

This allows us to realize a 3+1 split, where the metric on each surface is given by:

$$h_{ab} = g_{ab} + u_a u_b \ ,$$

where h_{ab} is also known as the projection operator.

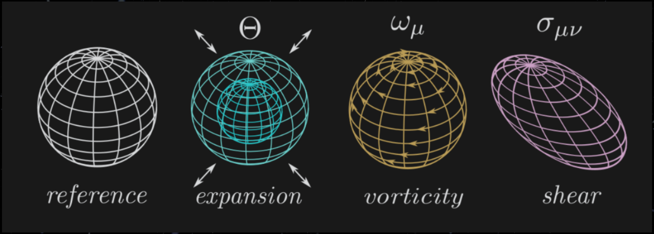


Congruence of observers

How the matter congruence varies along all 4 space-time directions can be used to infer the dynamical properties of the universe:

$$\nabla^b u^a_{\mathrm{m}} = \frac{1}{3}\Theta_{\mathrm{m}} h^{ab}_{\mathrm{m}} + \sigma^{ab}_{\mathrm{m}} \equiv \Theta^{ab}_{\mathrm{m}} \,,$$

where Θ_{m} and σ^{ab}_{m} are the *expansion* and *shear*, respectively, while Θ^{ab}_{m} represents the **expansion tensor**. Note we are assuming a vorticity free geodesic fluid.

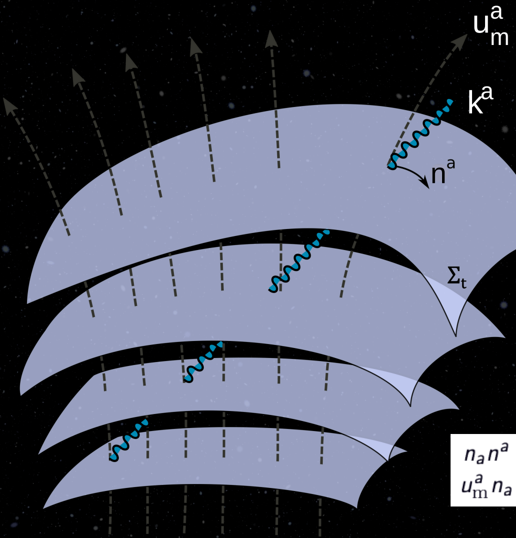


Rezzolla & Zanotti (2013)

Null formalism

$$k^a = E(u^a_{\mathrm{m}} - n^a)$$

$$k^a \nabla_a k_b = 0$$



$$K^a = -u^a_{\mathrm{m}} + n^a$$

$$K^a \nabla_a K_b = -K_b \, \mathbb{H}$$

$$E = -k_a u^a_{\mathrm{m}}, \quad n_a u^a_{\mathrm{m}} = 0$$

$$n_a n^a = 1$$

$$u^a_{\mathrm{m}} n_a = 0$$

Null formalism

The Hubble and deceleration parameters for the null case can be obtained via the redshift-distance relation:

$$z(d,n) = \mathbb{H}_{\rm o} d + \frac{1}{2}\mathbb{X}_{\rm o}\, z^2 + (d)\ ,$$

where the null Hubble parameter is given by

$$\mathbb{H} = K_a K_b \Theta_{\rm m}^{ab} = \frac{\Theta_{\rm m}}{3} + \sigma_{\rm m}^{ab} n_a n_b\ ,$$

while

$$\mathbb{X} = K_a K_b K_c \nabla^a \nabla^b u_{\rm m}^c$$

relates to the standard deceleration parameter as:

$$\text{FLRW:} \qquad \mathbb{X}_{\rm o} = \langle \mathbb{X} \rangle_{\rm o} = \left(-\dot{H} + 2H^2 \right)_0 = (q_0 + 3) H_0^2\ .$$

This motivates a covariant deceleration parameter given by*:

$$\mathbb{Q}_o = \frac{\mathbb{X}_{\rm o}}{\mathbb{H}_{\rm o}^2} - 3 \quad \Rightarrow \quad \mathbb{Q}_o = \left[\left(\frac{{\rm d}z}{{\rm d}d} \right)^{-2} \frac{{\rm d}^2 z}{{\rm d}d^2} \right]_{\rm o} - 3\ .$$

*Clarkson 1999, Umeh 2013, Heinesen 2020

Null formalism

By expanding the relation $\mathbb{X} = K_a K_b K_c \nabla^a \nabla^b u^c_{\rm m}$ and taking the traces, we obtain:

$$\mathbb{X} \doteq \langle \mathbb{X} \rangle + \mathbb{X}_a\, n^a + \mathbb{X}_{ab}\, n^{\langle a} n^{b \rangle} + \mathbb{X}_{abc}\, n^{\langle a} n^b n^{c \rangle}$$

where the multipoles are:

$$\begin{aligned}
\langle \mathbb{X} \rangle &\doteq -\frac{1}{3}\dot{\Theta}_{\rm m} + \frac{2}{9}\Theta^2_{\rm m} + 2\sigma^{\rm m}_{ab}\sigma^{ab}_{\rm m}\,,\\
\mathbb{X}^a &\doteq \frac{1}{3}h^{ab}_{\rm m}\,\nabla_b\Theta_{\rm m} + \frac{2}{5}h^{ab}_{\rm m}\,h^{cd}_{\rm m}\,\nabla_c\sigma^{\rm m}_{db}\,,\\
\mathbb{X}^{ab} &\doteq -\dot{\sigma}^{\langle ab \rangle}_{\rm m} + \frac{4}{3}\Theta_{\rm m}\sigma^{ab}_{\rm m} + 2\sigma^{\langle a}_{{\rm m}\,c}\,\sigma^{b \rangle c}_{\rm m}\,,\\
\mathbb{X}^{abc} &\doteq \nabla^{\langle a}\sigma^{bc \rangle}_{\rm m}\,.
\end{aligned}$$

Now we have a dipole, quadrupole and octopole terms.
Note that in the limit of $\sigma_{\rm m} \to 0$, we recover the FLRW result.

Null formalism

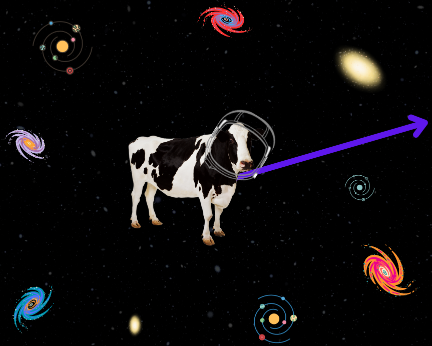
Advantages:

- No need to pre-deffine a metric;
- Allows for the presence of spatial anisotropies;
- Allows directional dependence;
- Can account for the influence of local effects into data;
- Directly connected to observations.

Drawbacks:

- More parameters to fit $\longrightarrow$ need for more data.

What an observer ***boosted with respect to the matter frame*** will measure?

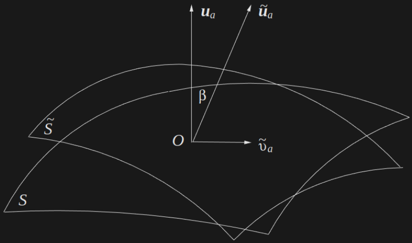


What happens under a boost?

Another observer $\tilde{\mathrm{O}}$, passing by an event o, with 4-velocity

$$\tilde{u}^a \doteq \gamma\left(u^a_{\mathrm{m}} + v^a\right) \doteq u^a_{\mathrm{m}} + v^a + O(v^2)$$

where $\gamma \doteq \left(1 - v^2\right)^{-1/2}$ and $u^a_{\mathrm{m}} v_a \doteq 0$.



Since K^a is observer-independent, we have:

$$K^a \doteq \left(-\tilde{u}^a + \tilde{n}^a\right) \doteq \left(-u^a_{\mathrm{m}} + n^a\right).$$

What happens under a boost?

- The matter expansion tensor

$$\Theta_{\mathrm{m}}^{ab} = \frac{\Theta_{\mathrm{m}}}{3}\, h_{\mathrm{m}}^{ab} + \sigma_{\mathrm{m}}^{ab}$$

is intrinsic to the physical matter flow – remains unaffected by a change of observer.

- The **measured** Hubble parameter depends on who is measuring it:

$$\begin{aligned} \tilde{\mathbb{H}}(\tilde{n}) &\mathrel{\hat{=}} \Theta_{\mathrm{m}}^{ab}\, \tilde{K}_a \tilde{K}_b \mathrel{\hat{=}} \Theta_{\mathrm{m}}^{ab} \left(\gamma^2 v^a v^b - 2\gamma v^a \tilde{n}^b + \tilde{n}^a \tilde{n}^b\right) \\ &\mathrel{\hat{=}} \langle\tilde{\mathbb{H}}\rangle + \tilde{\mathbb{H}}_a\, \tilde{n}^a + \tilde{\mathbb{H}}_{ab}\, \tilde{n}^a \tilde{n}^b\,, \end{aligned}$$

where the boosted multipoles are

$$\begin{aligned} \langle\tilde{\mathbb{H}}\rangle &\mathrel{\hat{=}} \gamma^2 \left[\left(1 + \frac{v^2}{3}\right)\langle\mathbb{H}\rangle + \sigma_{\mathrm{m}}^{ab}\, v_a v_b\right]\,, \\ \tilde{\mathbb{H}}^a &\mathrel{\hat{=}} -2\gamma\left(\langle\mathbb{H}\rangle\, v^a + \sigma_{\mathrm{m}}^{ab}\, v_b\right)\,, \\ \tilde{\mathbb{H}}^{ab} &\mathrel{\hat{=}} \sigma_{\mathrm{m}}^{ab} + \langle\mathbb{H}\rangle\, v^{\langle a} v^{b\rangle}\,. \end{aligned}$$

Example: perturbed FLRW

- In the Newtonian (or longitudinal) gauge, the metric at first order in scalar perturbations is

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Phi)\mathrm{d}\boldsymbol{x}^2\,,$$

where the background coordinates are $x^\mu = (t, x^i)$ and Φ is the gravitational potential.
The 4-velocities of gauge observers and matter are:

$$\tilde{u}^\mu = (1-\Phi, 0)\,, \quad u^\mu_{\mathrm{m}} = \left(1-\Phi, -v^i\right) \ \text{ with } \ v^i = -\frac{\mathrm{d}x^i}{\mathrm{d}t}\,.$$

- Here v^i is a perturbative peculiar velocity $\to$ velocities of boosted observers relative to the matter.
- Gauge observers are accelerating: $\tilde{u}^\nu \nabla_\nu \tilde{u}^\mu = a^{-2}(0, \partial^i \Phi)$.
- Matter 4-acceleration vanishes: $u^\nu_{\mathrm{m}} \nabla_\nu u^\mu_{\mathrm{m}} = 0 \implies \dot{v}^i + Hv^i = a^{-2}\partial^i\Phi$.

Example: perturbed FLRW

The projection tensor into the matter rest-space and the matter expansion tensor are

$$h^{\mathrm{m}}_{\mu\nu} = a^2 \begin{bmatrix} 0 & v_j \\ v_i & (1-2\Phi)\delta_{ij} \end{bmatrix}, \quad \Theta^{\mathrm{m}}_{\mu\nu} = a^2 \begin{bmatrix} 0 & Hv_j \\ Hv_i & \delta_{ij}\left(H - 3H\Phi + \dot{\Phi}\right) - \partial_i v_j \end{bmatrix}.$$

From these equations we find the volume expansion and shear rates for matter:

$$\begin{aligned} \Theta_{\mathrm{m}} &= 3H + \delta\Theta_{\mathrm{m}} = 3H\left(1 - \frac{1}{3H}\partial_i v^i - \frac{\dot{\Phi}}{H} - \Phi\right) \approx 3H - \partial_i v^i, \\ \sigma^{\mathrm{m}}_{ij} &= -a^2 \partial_{\langle i} v_{j\rangle} = -a^2\left(\partial_i v_j - \frac{1}{3}\partial_k v^k \delta_{ij}\right), \quad \sigma^{\mathrm{m}}_{0\mu} = 0\,. \end{aligned}$$

Example: perturbed FLRW

In perturbed FLRW, the monopole and quadrupole measured by the matter observers are:

$$\langle \mathbb{H} \rangle = \frac{1}{3}\Theta_{\mathrm{m}} = H - \frac{1}{3}\partial_i v^i\,, \qquad \mathbb{H}_{ij} = \sigma^{\mathrm{m}}_{ij} = -a^2 \partial_{\langle i} v_{j\rangle}\,.$$

The gauge observer measures a boosted Hubble parameter given by

$$\begin{aligned} \tilde{\mathbb{H}}(\tilde{n}) &= \Theta^{\mathrm{m}}_{\mu\nu}\, \tilde{K}^{\mu} \tilde{K}^{\nu} \\ &= -2\Theta^{\mathrm{m}}_{0i}\, \tilde{u}^0\, \tilde{n}^i + \Theta^{\mathrm{m}}_{ij}\, \tilde{n}^i \tilde{n}^j \quad \text{where} \quad \tilde{n}^{\mu} = (0, \tilde{n}^i) \\ &= \frac{1}{3}\Theta_{\mathrm{m}} - 2H v_i\, \tilde{n}^i + \sigma^{\mathrm{m}}_{ij}\, \tilde{n}^i \tilde{n}^j\,, \end{aligned}$$

Example: perturbed FLRW

- The boosted observer measures a **monopole and quadrupole** that *reduce to the physical Hubble monopole and quadrupole at leading order in v_o.*
- **The boost produces a dipole** induced by the matter expansion and shear, i.e. $-2\big(\gamma\,\Theta^{ab}_{\mathrm{m}}\,v_b\big)_\mathrm{o}$.
- *No higher-order multipoles appear.*
- At leading order in v_o:

$$\tilde{\mathbb{H}}(\tilde{n}) \mathrel{\hat{=}} \mathbb{H}(\tilde{n}) + \tilde{\mathbb{H}}_a\,\tilde{n}^a + O(v^2)$$

  where $\tilde{\mathbb{H}}^a \mathrel{\hat{=}} -2\big(\langle\mathbb{H}\rangle\,v^a + \sigma^{ab}_{\mathrm{m}}\,v_b\big) + O(v^2)$.
- If $|\sigma^{ab}_{\mathrm{m}}|_\mathrm{o} \ll \langle\mathbb{H}\rangle_\mathrm{o}$, then the dipole is directed almost opposite to the direction of v^a, with magnitude of $\approx 2v_\mathrm{o}\,\langle\mathbb{H}\rangle_\mathrm{o}$.

Extracting the Hubble parameter from cosmic distances

The relation between the matter-frame redshift and the lightray affine parameter λ, ($k^a = \mathrm{d}x^a/\mathrm{d}\lambda$) is:

$$\frac{\mathrm{d}z}{\mathrm{d}\lambda} = k^a \nabla_a (1+z) = \frac{1}{(u_{\mathrm{m}}^b k_b)_o} k_a k_b \nabla^a u_{\mathrm{m}}^b = E_o (1+z)^2 \,\mathbb{H}\,.$$

It then follows that the Hubble parameter at event o ($z=0$) is

$$\mathbb{H}_{\mathrm{o}} = \frac{1}{E_{\mathrm{o}}} \frac{\mathrm{d}z}{\mathrm{d}\lambda}\bigg|_{\mathrm{o}} \tag{1}$$

Using the boost relations:

$$\tilde{\mathbb{H}}_{\mathrm{o}}(\tilde{n}) = \mathit{\Gamma}_{\mathrm{o}}^{-2} \mathbb{H}_{\mathrm{o}}(n)\,, \quad \tilde{E}_{\mathrm{o}} = \mathit{\Gamma}_{\mathrm{o}} E_{\mathrm{o}}\,, \quad 1+\tilde{z} = \mathit{\Gamma}_{\mathrm{o}}^{-1}(1+z) \text{ and } \tilde{\lambda} = \lambda\,,$$

we have:

$$\tilde{\mathbb{H}}_{\mathrm{o}} = \frac{1}{\tilde{E}_{\mathrm{o}}} \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{\lambda}}\bigg|_{\mathrm{o}}\,.$$

Therefore, relation (1) is invariant under boost transformations

Extracting the Hubble parameter from cosmic distances

- The affine parameter λ is not observable.

By the Equivalence Principle any distance d defined via lightrays at o, must reduce to the Minkowski distance near o:

$$\begin{aligned}
\text{Minkowski:}\quad k^\mu &= \frac{\Delta x^\mu}{\Delta\lambda} = \frac{1}{\lambda}\big(-d, dn^i\big) = \frac{d}{\lambda}\big(-1, n^i\big) \\
\text{and}\quad k^\mu &= E_\mathrm{o}\big(-1, n^i\big) \quad \Rightarrow \quad d = E_\mathrm{o}\lambda\,,
\end{aligned}$$

With this, we obtain:

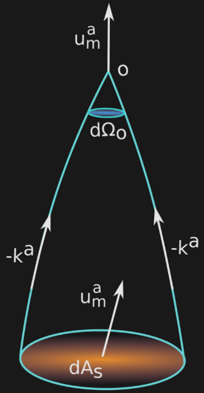
$$\boxed{d_P = E_\mathrm{o}\lambda \quad \text{in Minkowski}\,.}$$

$$\boxed{d = E_\mathrm{o}\lambda + O(\lambda^2) \quad \text{in the matter frame.}}$$

which gives us :

$$\boxed{\mathbb{H}_\mathrm{o} = \left.\frac{\mathrm{d}z}{\mathrm{d}d}\right|_\mathrm{o} \qquad \text{in the matter frame.}}$$

Covariant cosmic distance measures



The observer area distance is defined by

$$\boxed{\mathrm{d}A_{\rm s} = d_A^2\,\mathrm{d}\Omega_{\rm o} \quad \text{in the matter frame,}}$$

Peforming a boost $\quad u^a_{\rm m}\big|_{\rm o} \to \tilde{u}^a_{\rm o}$

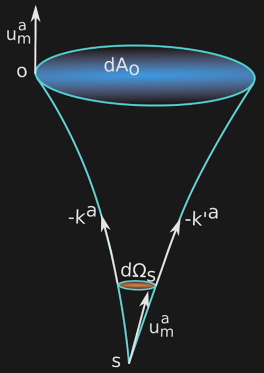
we have that

$$\mathrm{d}\tilde{A}_{\rm s} = \mathrm{d}A_{\rm s}\,, \quad \mathrm{d}\tilde{\Omega}_{\rm o} = \Gamma_{\rm o}^{-2}\,\mathrm{d}\Omega_{\rm o}\,.$$

which gives us:

$$\boxed{\tilde{d}_A(\tilde{z},\tilde{n}) = \Gamma_{\rm o}(n)\,d_A(z,n) = \gamma_{\rm o}\big(1+v_a n^a\big)_{\rm o}\,d_A(z,n)\,.}$$

Covariant cosmic distance measures



The luminosity distance is defined by

$$\boxed{d_L^2 = \frac{L_s/4\pi}{F_o} \quad \text{in the matter frame.}}$$

Peforming a boost $u^a_m\big|_o \to \tilde{u}^a_o$

we have that

$$d\tilde{A}_o = dA_o\,, \quad \tilde{L}_s = L_s\,, \quad \tilde{F}_o = \Gamma_o^2\, F_o\,.$$

which gives us:

$$\boxed{\tilde{d}_L(\tilde{z},\tilde{n}) = \Gamma_o(n)^{-1}\, d_L(z,n) = \gamma_o^{-1}\big(1+v_a n^a\big)_o^{-1}\, d_L(z,n)\,.}$$

So which distances should we use?

So the transformation under a boost for d_A and d_L are given by:

$$\tilde{d} = \Gamma_o^{\alpha}\, d \quad \text{where} \quad \alpha = (1, -1) \;\; \text{for} \;\; d = \big(d_A\,, d_L\big)\,.$$

which implies the following transformation for the Hubble parameter:

$$\left.\frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}}\right|_o = \Gamma_o^{-1-\alpha}\left.\frac{\mathrm{d}z}{\mathrm{d}d}\right|_o = \Gamma_o^{-1-\alpha}\,\Gamma_o^2\,\tilde{\mathbb{H}}_o \quad \Rightarrow \quad \tilde{\mathbb{H}}_o = \Gamma_o^{\alpha-1}\left.\frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}}\right|_o\,.$$

Therefore, the boosted Hubble parameter in terms of d_A and d_L will be:

$$\tilde{\mathbb{H}}_o = \left.\frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}_A}\right|_o = \Gamma_o^{-2}\left.\frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}_L}\right|_o\,.$$

CONCLUSION

The relation

$$\mathbb{H}_o = \left.\frac{\mathrm{d}z}{\mathrm{d}d}\right|_o \quad \xrightarrow[u^a_{\mathrm{m}}|_o \,\to\, \tilde{u}^a_o]{} \quad \tilde{\mathbb{H}}_o = \left.\frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}}\right|_o$$

is preserved under a boost for the area distance, but NOT for the luminosity distance.

Boosted observers need to use a corrected luminosity distance

If a moving observer wants to measure the Hubble parameter via the luminosity distance, the options are:

Use the correct boosted relation: $\boxed{\tilde{\mathbb{H}}_{\rm o} = \Gamma_{\rm o}^{-2} \left.\frac{{\rm d}\tilde{z}}{{\rm d}\tilde{d}_L}\right|_{\rm o} .}$

Use a luminosity distance modified by the appropriate redshift factor in order to achieve consistency:

$$d_{L*} = (1+z)^{-2} d_L \qquad \xrightarrow[u^a_{\rm m}|_{\rm o} \to \tilde{u}^a_{\rm o}]{} \qquad \tilde{d}_{L*} = \Gamma_{\rm o}\, d_{L*}$$

In this case, the Hubble parameter for a boosted observer will be given by:

$$\boxed{\mathbb{H}_{\rm o} = \left.\frac{{\rm d}z}{{\rm d}d_{L*}}\right|_{\rm o} \quad \text{implies} \quad \tilde{\mathbb{H}}_{\rm o} = \left.\frac{{\rm d}\tilde{z}}{{\rm d}\tilde{d}_{L*}}\right|_{\rm o} \quad \text{where} \quad d_{L*} = \frac{d_L}{(1+z)^2} .}$$

Boosted observers need to use a corrected luminosity distance

If the moving observer uses the unmodified luminosity distance, they will derive an *incorrect* boosted Hubble parameter

$$\tilde{\mathbb{H}}_{\mathrm{o}}^{\times}(\tilde{n}) = \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{d}_L}(\tilde{n})\bigg|_{\mathrm{o}} = \tilde{\mathbb{H}}_{\mathrm{o}}(\tilde{n})\,\tilde{\Gamma}_{\mathrm{o}}(\tilde{n})^{-2}\,,$$

Which can be rewritten as:

$$\tilde{\mathbb{H}}^{\times}(\tilde{n}) \doteq \langle\mathbb{H}\rangle - \frac{6}{5}\sigma_{\mathrm{m}}^{ab}\,v_b\,\tilde{n}_a + \sigma_{\mathrm{m}}^{ab}\,\tilde{n}_a\tilde{n}_b + 2v^{\langle a}\,\sigma_{\mathrm{m}}^{bc\rangle}\,\tilde{n}_a\tilde{n}_b\tilde{n}_c + O(v^2)\,.$$

An observer moving relative to the matter who wrongly uses the luminosity distance predicts an incorrect dipole in the Hubble parameter:

- the dominant correct dipole term $-2\langle\mathbb{H}\rangle\,v^a$ disappears;
- the $\sigma_{\mathrm{m}}^{ab}v_b$ dipole term has the wrong factor;
- a spurious octupole is predicted.

Conclusions and remarks

We define the Hubble parameter in a covariant form, observable on the past lightcone.

$$\mathbb{H} = K_a K_b \Theta^{ab}_{\rm m} = \frac{\Theta_{\rm m}}{3} + \sigma^{ab}_{\rm m} n_a n_b \ ,$$

Its monopole reduces to the standard H parameter in a FLRW spacetime, and a quadrupole, which is generated by shear anisotropy.

To leading order, an observer moving relative to the matter frame will measure this monopole and quadrupole, but will also detect a dipole, generated by Doppler and aberration effects.

$$\langle \tilde{\mathbb{H}} \rangle \triangleq \gamma^2 \left[\left(1 + \frac{v^2}{3}\right) \langle \mathbb{H} \rangle + \sigma^{ab}_{\rm m} \, v_a v_b \right] \ ,$$

$$\tilde{\mathbb{H}}^a \triangleq -2\gamma \left(\langle \mathbb{H} \rangle \, v^a + \sigma^{ab}_{\rm m} \, v_b \right) ,$$

$$\tilde{\mathbb{H}}^{ab} \triangleq \sigma^{ab}_{\rm m} + \langle \mathbb{H} \rangle \, v^{\langle a} v^{b \rangle} \ .$$

Higher-order Hubble multipoles in the moving observer's frame are a signal of systematics or a breakdown in the dust model.

Conclusions and remarks

When using the relation between the Hubble parameter and luminosity distance, a moving observer should correct the luminosity distance by a redshift factor.

$$\boxed{d_{L*} = \frac{d_L}{(1+z)^2}.}$$

Otherwise an incorrect dipole and a spurious octupole are detected.

$$\tilde{\mathbb{H}}^{\times}(\tilde{n}) \doteq \langle \mathbb{H} \rangle - \frac{6}{5}\sigma_{\mathrm{m}}^{ab}\, v_b\, \tilde{n}_a + \sigma_{\mathrm{m}}^{ab}\, \tilde{n}_a \tilde{n}_b + 2v^{\langle a}\, \sigma_{\mathrm{m}}^{bc\rangle}\, \tilde{n}_a \tilde{n}_b \tilde{n}_c + O(v^2)\,.$$

In perturbed FLRW models, this error leads to a false prediction of *no dipole.*


THANKS!

Boosted deceleration parameter

The boost transformation of $\mathbb{X}_{\mathrm{o}}$ is given by $\tilde{\mathbb{X}}_{\mathrm{o}} = \Gamma(n)^{-3}\mathbb{X}_{\mathrm{o}}$. Using (i) $\tilde{\Gamma}(\tilde{n}) = \gamma(1+\tilde{v}_a\tilde{n}^a) = \Gamma(n)^{-1}$ (ii) $\gamma\tilde{v}_a\tilde{n}^a = -v_a\tilde{n}^a$ and (iii) symmetrizing the tracefree tensors in terms of $\tilde{h}_{ab}$, we obtain:

$$\langle\tilde{\mathbb{X}}\rangle = \gamma^3(1+v^2)\langle\mathbb{X}\rangle - \left(2\gamma^3 - \frac{\gamma}{3}\right)(\mathbb{X}_a v^a) + 2\gamma^3(\mathbb{X}_{ab}v^a v^b) - 2\gamma^3(\mathbb{X}_{abc}v^a v^b v^c)\ ;$$

$$\begin{aligned}\tilde{\mathbb{X}}_a =& -3\gamma^2\left(1+\frac{v^2}{5}\right)\langle\mathbb{X}\rangle v_a + \gamma^2\left(1+\frac{v^2}{5}\right)\mathbb{X}_a + \frac{12}{5}\gamma^2(\mathbb{X}_b v^b)v_a \\ &- \frac{12}{5}\gamma^2(\mathbb{X}_{ab}v^b) - \gamma^2\left(1+\frac{v^2}{5}\right)(\mathbb{X}_{bc}v^b v^c)v_a + \frac{18}{5}\gamma^3(\mathbb{X}_{abc}v^a v^b)\ ;\end{aligned}$$

$$\tilde{\mathbb{X}}_{ab} = 3\gamma\langle\mathbb{X}\rangle v_a v_b - \gamma(\mathbb{X}_c v^c)v_a v_b - 2\gamma\,\mathbb{X}_b v_a + \gamma\mathbb{X}_{ab} + 2\gamma(\mathbb{X}_{ac}v^c v_b) - 3\gamma(\mathbb{X}_{abc}v^c);$$

$$\tilde{\mathbb{X}}_{abc} = -\langle\mathbb{X}\rangle\, v_a v_b v_c + \mathbb{X}_c\, v_a v_b - \mathbb{X}_{ab}\, v_c + \mathbb{X}_{abc},$$

where $\langle\mathbb{X}\rangle$, $\mathbb{X}_a$, $\mathbb{X}_{ab}$ and $\mathbb{X}_{abc}$ are the matter frame multipoles.

Measurements on spheres of constant redshift

Spheres of constant distance: In $\mathbb{H}_o = \frac{\mathrm{d}z}{\mathrm{d}d}\Big|_o$, the Hubble constant is defined as the slope of the $z(d)$ relation at the observer ($d = 0$).

Spheres of constant redshift: It is more usual to consider the Hubble diagram as a distance-redshift relation. In this case, we extract not $\mathbb{H}_o$ but its inverse:

$$\tilde{d} = \frac{1}{\tilde{\mathbb{H}}_o}(\tilde{z} - \tilde{z}_o) + O(\tilde{z}^2) \quad \Rightarrow \quad \frac{1}{\tilde{\mathbb{H}}_o} = \frac{\mathrm{d}\tilde{d}}{\mathrm{d}\tilde{z}}\Big|_o \quad \text{where} \quad \tilde{d} = \tilde{d}_A \,,\ \tilde{d}_{L*} \,.$$

The multipole expansion then gives:

$$\frac{1}{\tilde{\mathbb{H}}} \doteq \left(\langle \tilde{\mathbb{H}} \rangle + \tilde{\mathbb{H}}_a \, \tilde{n}^a + \tilde{\mathbb{H}}_{ab} \, \tilde{n}^{\langle a} \tilde{n}^{b \rangle} \right)^{-1} \doteq \sum_{\ell=0}^{\infty} \tilde{\mathbb{I}}_{a_1 a_2 \cdots a_\ell} \, \tilde{n}^{\langle a_1} \tilde{n}^{a_2} \dots \tilde{n}^{a_\ell \rangle} \,.$$

- Infinite number of multipoles in $\tilde{\mathbb{H}}_o^{-1}$ and $\mathbb{H}_o^{-1}$.

Measurements on spheres of constant redshift

- Assuming $|\sigma_{\mathrm{m}}^{ab}|_{\mathrm{o}} \ll \langle \mathbb{H} \rangle_{\mathrm{o}}$ and $|v^a|_{\mathrm{o}} \gg |\sigma_{\mathrm{m}}^{ab}|_{\mathrm{o}}$, then we can neglect higher order terms $O(3) \equiv O(\hat{\sigma}^2, v^3, \hat{\sigma} v^2)|_{\mathrm{o}}$ (where $\hat{\sigma}_{\mathrm{m}}^{ab} = \sigma_{\mathrm{m}}^{ab} / \langle \mathbb{H} \rangle$):

$$\tilde{\mathbb{H}}^{-1} \triangleq \langle \mathbb{H} \rangle^{-1} \Big[1 + v^2 + \Big(2\, v^a + \frac{2}{5} \hat{\sigma}_{\mathrm{m}}^{ab}\, v_b \Big) \tilde{n}_a + \Big(3\, v^{\langle a} v^{b\rangle} - \hat{\sigma}_{\mathrm{m}}^{ab} \Big) \tilde{n}_a \tilde{n}_b - 4\, v^{\langle a}\, \hat{\sigma}_{\mathrm{m}}^{bc\rangle}\, \tilde{n}_a \tilde{n}_b \tilde{n}_c \Big] + O(3)\,.$$

- At leading order in v: presence of a dipole, quadrupole and **octupole**.
- Contrast with $\tilde{\mathbb{H}}_{\mathrm{o}}$ (only dipole).

In the matter frame, this reduces to

$$\mathbb{H}^{-1} \triangleq \langle \mathbb{H} \rangle^{-1} \Big[1 - \hat{\sigma}_{\mathrm{m}}^{ab}\, n_a n_b \Big] + O(3)\,.$$

Dipole and octupole vanish at leading order.

Example: LTB spacetimes

• Relaxing the isotropy and homogeneity assumptions on FLRW, the simplest model we can get is the Lemaître-Tolman-Bondi (LTB) spacetime:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A_{\|}^2(t,r)\,\mathrm{d}r^2 + A_{\perp}^2(t,r)\,\mathrm{d}\Omega^2\,.$$

• $u_{\mathrm{m}}^{\mu} = \delta_{\mathrm{m}}^{\mu}$.
• Isotropic about the worldline $r = 0 \implies \sigma = 0$ for observers at the center.
• The radial direction is a preferred direction for the observer.
• There is an expansion rate $H_{\|}$ along r^{μ} and an expansion rate $H_{\perp}$ in the screen space orthogonal to it. The average expansion rate and shear are:

$$\langle \mathbb{H} \rangle = \frac{1}{3}\left(H_{\|} + 2H_{\perp}\right)\,, \quad \sigma_{\mathrm{m}}^{\mu\nu} = \frac{1}{3}\left(H_{\|} - H_{\perp}\right)\left(2e^{\mu}e^{\nu} - S^{\mu\nu}\right)\,.$$

Example: LTB spacetimes

The covariant matter-frame Hubble parameter is then given by:

$$\mathbb{H}(n) \mathrel{\hat{=}} \frac{1}{3}\left(H_{\parallel} + 2H_{\perp}\right) + \left(H_{\parallel} - H_{\perp}\right) e_{\langle\mu} e_{\nu\rangle} n^{\mu} n^{\nu} \, .$$

For the boosted observer, we get:

$$\langle \tilde{\mathbb{H}} \rangle \mathrel{\hat{=}} \gamma^2 \left[\frac{1}{3}\left(1 + \frac{1}{3} v^2\right)\left(H_{\parallel} + 2H_{\perp}\right) + \left(H_{\parallel} - H_{\perp}\right) e_{\langle\mu} e_{\nu\rangle} v^{\mu} v^{\nu} \right] ,$$

$$\tilde{\mathbb{H}}_{\mu} \mathrel{\hat{=}} -2\gamma \left[\frac{1}{3}\left(H_{\parallel} + 2H_{\perp}\right) v_{\mu} + \left(H_{\parallel} - H_{\perp}\right) e_{\langle\mu} e_{\nu\rangle} v^{\nu} \right] ,$$

$$\tilde{\mathbb{H}}_{\mu\nu} \mathrel{\hat{=}} \left(H_{\parallel} - H_{\perp}\right) e_{\langle\mu} e_{\nu\rangle} + \frac{1}{3}\left(H_{\parallel} + 2H_{\perp}\right) v_{\langle\mu} v_{\nu\rangle} \, .$$