Observational indications of an apparent cosmic acceleration from the SNIa data

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The Standard Model of Cosmology : ΛCDM



The universe is dominated by dark energy, parametrized by the cosmological constant, Λ

First Evidence for Acceleration from high-z supernovae

"high redshift supernovae were found to be dimmer (15% in flux) than the low redshift supernovae (compared to what would be expected in a $\Lambda = 0$ universe)"

Results : Ω_M vs Ω_Λ from < 100 SNIa

The data favour a flat low-mass universe with Λ

$$\Omega_M = 0.28, \ \Omega_\Lambda = 0.72$$



Accelerating expansion of the universe



- Several alternative cosmological models have been proposed to explain observations, but most of them assume some forms of dark energy or abandon FRLW
- Large-scale peculiar motions are not widely taken into account and when they do their analysis is Newtionian
- No robust relativistic analysis of the peculiar velocity fields/effects

The tilted cosmological scenario can in principle explain the late-time cosmic acceleration <u>without</u> dark energy/ Λ , modified gravity, or abandoning the FRW models.

How the local universe looks like?



•
$$1 + z_{obs} = (1 + z_{cosm})(1 + \frac{v_{pec}}{c})$$



Colin, Mohayaee, Sarkar, Shafieloo., 2011, MNRAS, 414, 264-271

Claims for bulk flows inconsistent with ΛCDM

- Watkins et al., 2009 $(407 \pm 81 km/s)$ and Feldman et al., 2010 $(416 \pm 78 km/s)$ within a region of radius $r \approx 100 h^{-1}$ Mpc
- Colin et al. 2011 (260 km/s) at $r \approx 180 h^{-1}$ Mpc
- Macaulay et al., 2012 $(380^{+99}_{-132} km/s)$ at $r \approx 33 h^{-1}~{
 m Mpc}$
- Ma and Pan, 2013 $(290\pm 30 km/s)$ at $rpprox 58 h^{-1}$ Mpc
- Watkins et al., 2023 ($419\pm 36 km/s$) at $rpprox 200h^{-1}$ Mpc

Claims for "dark flows" : Kashlinsky et al., 2008 (600-1000 km/s) at $r \geqslant 300 h^{-1}$

They all approximately agree with the direction of the bulk flow (close to the CMB dipole) but not with the scale and the amplitude.

Tilted Cosmological Model

Tilted Model - Two Frames



Employ General Relativity

Observers O_1 in the CMB frame with 4-velocity u_a idealised observers

Observers O_2 in the tilted/matter frame with 4-velocity \tilde{v}_a real observers

$$\rightarrow \tilde{u}_a = u_a + \tilde{v}_a$$

The kinematics of the three 4-velocity fields

CMB reference frame

$$\nabla_b u_a = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab} - A_a u_b, \quad \Theta = 3H > 0$$

where $\Theta > 0$, σ_{ab} and ω_{ab} , A^a are the expansion, shear, vorticity and 4-acceleration of the idealised observer respectively.

Tilted frame

$$\nabla_b \tilde{u}_a = \frac{1}{3} \tilde{\Theta} \tilde{h}_{ab} + \tilde{\sigma}_{ab} + \tilde{\omega}_{ab} - \tilde{A}_a \tilde{u}_b, \quad \tilde{\Theta} > 0$$

Bulk peculiar flow

$$D_b \tilde{v}_a = \frac{1}{3} \tilde{\vartheta} \tilde{h}_{ab} + \tilde{\varsigma}_{ab} + \tilde{\varpi}_{ab}, \quad \tilde{\vartheta} = \tilde{D}^a \tilde{v}_a \gtrless 0$$

The dynamics in the two reference frames

In the case of non-relativistic peculiar motions ($\tilde{v}^a \ll 1$) and assuming a decelerating Einstein-de Sitter background, the linear relations of the dynamic quantities in the two reference frames are:

$$\tilde{\rho} = \rho, \quad \tilde{p} = p = 0, \quad \tilde{\pi}_{ab} = \pi_{ab} = 0, \quad \tilde{q}_a = q_a - \rho \tilde{v}_a$$

where ρ is the density, p is the pressure, π_{ab} the viscosity and q_a is the flux.

We set $\tilde{q}_a = 0$ in the bulk flow frame. Then, the peculiar flux in the CMB frame is: $q_a = \rho \tilde{v}_a \neq 0$, solely because of non-relativistic peculiar motions.

The peculiar flux contributes to the energy-momentum tensor and from the momentum conservation law, in the CMB frame, we get:

$$\rho A_a = -\dot{q}_a - 4Hq_a \neq 0$$

To linear order, the Raychaudhuri equations measured by the idealised and the bulk-flow observers differ:

$$3H^2\tilde{q} = \frac{1}{2}\rho, \qquad 3H^2q = \frac{1}{2}\rho - D^aA_a$$

where the dimensionless deceleration parameter is $q = -[1 + (\dot{H}/H^2)].$

The deceleration parameters measured in the two frames differ as well:

$$\tilde{q} = q + \frac{1}{3H^2} D^a A_a$$

The tilted cosmological model - Deceleration parameter II

In a perturbed Einstein-de Sitter universe (with p = 0 and $\Omega = 1$ in the background) the deceleration parameter measured by the real observers is:

- When $\lambda\gtrsim\lambda_H$, $\tilde{q}\to q$ and the peculiar motions effects fade away in the absence of peculiar flows
- On subhorizon scales ($\lambda \ll \lambda_H$), $\tilde{q} \neq q$ and the difference can be large depending on the bulk flow scale
- The difference crucially depends on the sign of $\tilde{\vartheta}$. For contracting bulk-flows ($\tilde{\vartheta} < 0$), $\tilde{q} < 0 \longrightarrow$ local apparent accelerated expansion for the real observers

Tsagas, 2011, DOI: 10.1103/PhysRevD.84.063503 Tsagas, Kadiltzoglou, 2015, DOI: 10.1103/PhysRevD.92.043515 Tsagas, 2021, Eur. Phys. J. C 81, 753

The tilted cosmological model - Transition scale

• When \tilde{q} becomes zero: $\frac{1}{9} \left(\frac{\lambda_H}{\lambda}\right)^2 \frac{|\tilde{\vartheta}|}{H} = q$, we get the "transition length":

$$\lambda_T = \sqrt{\frac{1}{9q} \frac{|\tilde{\vartheta}|}{H}} \lambda_H$$

• For contracting bulk flows, the deceleration parameters in the two frames are connected through : $\bar{\pi}_{\perp}$

$$\left| \tilde{q} = q \left[1 - \left(\frac{\lambda_T}{\lambda} \right)^2 \right] \right|$$



The tilted cosmological model - Parametrization of $\hat{\vartheta}$

- We assume that locally the bulk flow contracts $(ilde{artheta} < 0)$ and $q = rac{1}{2}$
- We consider a form of the local volume scalar $\tilde{\vartheta}$ in the tilted frame ^1

$$\tilde{\vartheta} = \tilde{\vartheta}(\lambda) = \frac{\lambda^2}{a + b\lambda^3}$$

• The scale dependent deceleration parameter in the tilted frame

$$\tilde{q} = \tilde{q}(\lambda) = \frac{1}{2} \left(1 - \frac{1}{a + b\lambda^3} \right)$$

¹K. Asvesta, L. Kazantzidis, L. Perivolaropoulos, C. Tsagas, 2022, DOI: 10.1093/mnras/stac922

Fit to the data

What are Supernovae of Type Ia ?







- SNIa lack of H,He and have strong Si line
- binary system of a C-O WD and a companion star or double WD
- They are exremely luminous. They can be as bright as their entire host galaxy
- almost uniform light-curves



Systematic uncertainties in SNIa distances

B-band light curves of a sample of SNe Ia with data taken from Hicken et al. (2009) and Stritzinger et al. (2011).

Corrections to the observed light-curves

- light curve shape correction ("Phillips relation")
- SNIa colour correction Corrected Distance modulus $\mu = m_B - M_B + \alpha X_1 - \beta C$

After corrections made, SNIa can be accurate distance indicators and used to cosmology via $\mu = 5 \log d_L(Mpc) + 25$

The Pantheon SNIa compilation

1048 Type Ia supernovae with redshift range $0.010 \leq z \leq 2.26$

JLA + additional SnIa from PanStarrs and HST (Scolnic et al. (2018) arXiv:1710.00845)

The SDSS stripe dominates for z > 0.07



Supernova fit

• The tilted redshift-dependent deceleration parameter:

$$\tilde{q}_{(z)} = \frac{1}{2} \left(1 - \frac{1}{a+b \, d_r^3(z)} \right), \quad d_r = \frac{H_0 \bar{\chi}(z)}{c} \tag{1}$$

where $\bar{\chi}(z)$ is the line of sight Einstein-de Sitter comoving distance

• The Hubble rate at any redshift connects with the deceleration parameter through:

$$\tilde{H}(z) = H_0 \exp\left[\int_0^z \left(\frac{1+\tilde{q}(u)}{1+u}\right) du\right]$$
(2)

• The luminosity distance of the SNIa (in Mpc):

$$\tilde{D}_L(z) = c \left(1+z\right) \int_0^z \frac{dz'}{\tilde{H}(z')} \tag{3}$$

• The theoretical apparent magnitude :

$$m_{th}(z) = \mathcal{M} + 5\log_{10}\tilde{D}_L(z), \quad \mathcal{M} = 5\log_{10}\left(\frac{c/H_0}{1Mpc}\right) + 25 \quad (4)$$

✓ Extract the best-fit parameters of the model

$$\chi^{2}_{min}(\mathcal{M}, \alpha, b) = (m_{obs,i}(z) - m_{th}(z)) \ C^{-1}_{ij} \ (m_{obs,j}(z) - m_{th}(z))$$
(5)

where $C = C_{stat} + C_{sys}$ is the total covariance matrix of the SNIa.

Minimization performed by employing a Bayesian Markov Chain Monte Carlo (MCMC) method. The likelihood function is $\mathcal{L} = \exp\left(-\frac{\chi^2_{min}}{2}\right)$.

Model	\mathcal{M}	a	b	Ω_{0m}	χ^2_{min}	χ^2_{red}
ΛCDM	23.809 ± 0.011	-	-	0.299 ± 0.022	1026.67	0.981
T-EdS	23.815 ± 0.013	0.521 ± 0.030	$6.66^{+5.49}_{-3.58}$	1.0	1026.76	0.982
T-EdS(a fixed)	23.808 ± 0.006	0.5	$8.47^{+3.72}_{-2.76}$	1.0	1027.05	0.982

Result: The tilted cosmological model performs equally well with $\Lambda {\rm CDM}$ $(\chi^2_{red}\approx 1$)

K. Asvesta, L. Kazantzidis, L. Perivolaropoulos, C. Tsagas, 2022, DOI: 10.1093/mnras/stac922

Evolutionary behaviour of \tilde{q}



✓ The transition scale is close to the one from the ΛCDM model, $z_T \sim 0.6$.

 $\checkmark~{\rm Local}$ apparent acceleration $\tilde{q}(z=0)\sim -0.45$

K. Asvesta, L. Kazantzidis, L. Perivolaropoulos, C. Tsagas, 2022, DOI: 10.1093/mnras/stac922

Condidence contours of best-fit parameters

Assuming and Einstein-de Sitter bulk flow



K. Asvesta, L. Kazantzidis, L. Perivolaropoulos, C. Tsagas, 2022, DOI: 10.1093/mnras/stac922

• Local apparent accelerated expansion

- Doppler-like apparent dipole in the deceleration/Hubble parameter
- The dipole should decay with redshift

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Evidence for dipoles in cosmological parameters

Potential signature of a quadrupolar hubble expansion in Pantheon+supernovae Jessica A. Cowell • 1.2.3* Suhail Dhawan • 1 and Hayley J. Macpherson 4.5 ¹Assitiate of Astronomy and Kavli Institute for Cosmology. University of Cambridge. Madingley Road, Cambridge. Advingence of Physics, University of Oxford Denys Wikinson Buildings, Keble Road, Oxford Oxford, Cambridge. Adving Denys Wikinson Buildings, Keble Road, Oxford Oxford, Cambridge. Adving Denys Wikinson Buildings, Keble Road, Oxford Oxford, Cambridge. Adving Denys Buildings, Chiba 2777-8581, Jacom Department of Physics, University of Oxford, Denys Witkinson Building, Keble Road, Knyl IPAU (NPJ), UTAS, The University of Takyo Kashhou, Chika 27-2523, dopan A Construction of American Academician and Theorematical Based on Chika 27-2523, dopan Kavli IPMU (WPI), UTIAS, The University of Tolyo, Kashiwa, Chiba 277-4583, Japan Appartment of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB 0004, UK 4 Kavli Institute for Commissional Physics, The University of Chicano, 5640 South Ellis Avenue, Chicano, 11-60572, 12 ⁴Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK ⁵Kavli Institute for Cosmological Physics, The University of Chicago, 3640 South Ellis Arenue, Chicago, 1L 00637, USA Accepted 2023 September 8. Received 2023 September 8: in original form 2023 February 3 ABSTRACT ABSTRACT The assumption of isotropy - that the Universe looks the same in all directions on large scales - is fundamental to cosmological model. It is therefore critical to empirically test in which reoince this assumption holds. Anisotropies in the standard The assumption of isotropy - that the Universe looks the same in all directions on large scales - is fundamental to empirically test in which regimes this assumption holds. Anisotropies in the late Universe Housewer the average to which these anisotropies in the late Universe Housewer the average to which these anisotropies in the late Universe Housewer the average to which these anisotropies in the cosmol expansion are expected due to non-linear structures in the late Universe. However, the extent to which these anisotropies miles are non-tradictively and development of the fully lested. We use general relativistic simulations to determine that the tradictively and development on narrameters are mandminular and directar and directar methods to determine that the impact our low-redshift observations remains to be fully tested. We use general relativistic simulations to determine the Nubble and deceleration parameters are quadrupolar and dipolar, respectively. We constrain the new Pantheon-i-sumernova commitation. In the rest frame of the creative microwave hack oronget expected anisotropics in the *Hubble* and deceleration parameters are quadrupolar and dipolar, respectively. We constrain the new Pantheon-supernova compilation. In the rest frame of the cosmic microwave background on ~2x dowintion from iconomy we constrain the actionary we for a new part of the cosmic microwave background on the rest frame of the multipoles simultaneously in the new Pantheon+supernova compilation. In the rest frame of the cosmic microwave background (CMB), including Peculiar velocity (PV) corrections, we find an $\sim 2\sigma$ deviation from isotropy. We constrain the eigenvalues of the constraint of the constrain (CMB), including peculiar velocity (PV) corrections, we find an $\sim 2\sigma$ deviation from isotropy. We constrain the eigenvalues of $2 \cos \sigma$ and $2 \cos \sigma$ a the quadrupole in the *number* parameter to be $\lambda_1 = 0.021 \pm 0.011$ and $\lambda_2 = 0.00 \pm 0.012$ and place a minimum of the final no significant dipole in the deceleration parameter with another as amplitude of 2.88 per cent. We find no significant dipole in the deceleration parameter, with all in the rest frame of the CMB without PV corrections, we find $a > 2\sigma$ positive amplitude with an In the rest frame of the Corto without r v corrections, we find a $\geq co$ positive anipulturalisotropies, the monopole of the Hubble parameter shifts hv only $n \ge c_0$ positive anipulturalises. Key words: cosmological parameters - distance coals of the second





Dipole in the Pantheon+ SNIa compilation

1701 Type Ia supernovae with redshift range $0.0008 \leq z \leq 2.26$

18 different surveys 727 SNIa at $z \leq 0.08$ (Dan Scolnic et al 2022 ApJ 938 113)

 $\begin{array}{l} {\sf Pantheon+ vs \ Pantheon \ SNIa} \\ 118 \ {\sf SNIa} \ {\rm at} \ z \leq 0.010 \end{array}$



Dipole in \tilde{q} in the Pantheon+ SNIa compilation - I

- We make a redshift cut in the Pantheon+ sample and we analyze SnIa with $z>=0.015\sim 60 {\rm Mpc.}$ In total, we have 1527 SNIa.
- The anisotropic deceleration parameter in the tilted frame becomes:

$$\tilde{q} = \tilde{q}_m(z) + q_d(\mathbf{n}_{SN} \cdot \mathbf{n}_{dip}) \mathcal{F}_{dip}$$
(6)

$$\begin{split} \tilde{q}_m(z) &= \frac{1}{2} \left(1 - \frac{1}{a+b\left(2 - \frac{2}{\sqrt{1+z}}\right)^3} \right) \text{ is the monopole,} \\ q_d \text{ is the amplitude of the dipole,} \end{split}$$

 \mathbf{n}_{SN} is a vector pointing to the location of the SNIa, \mathbf{n}_{dip} a vector pointing to the location of the dipole and \mathcal{F}_{dip} describes the scale-dependence of the dipole.

• We examine a form of the function of the dipole \mathcal{F}_{dip} which is constant, $\mathcal{F}_{dip} = 1$ and $\mathcal{F}_{dip}(z) = \exp\left(\frac{-z}{S}\right)$

Dipole in \tilde{q} in the Pantheon+ SNIa compilation - II

• The tilted redshift-dependent deceleration parameter:

$$\tilde{q}_{(z)} = \frac{1}{2} \left(1 - \frac{1}{a+b \, d_r^3(z)} \right) + q_d (\mathbf{n}_{SN} \cdot \mathbf{n}_{dip}) \mathcal{F}_{dip} \tag{7}$$

• The Hubble rate at any redshift connects with the deceleration parameter through:

$$\tilde{H}(z) = H_0 \exp\left[\int_0^z \left(\frac{1+\tilde{q}(u)}{1+u}\right) du\right]$$
(8)

• The luminosity distance of the SNIa (in Mpc):

$$\tilde{D}_L(z) = c \left(1+z\right) \int_0^z \frac{dz'}{\tilde{H}(z')} \tag{9}$$

• The theoretical distance modulus :

$$\mu_{th}(z) = 5 \log_{10} \tilde{D}_L(z) + 25 \tag{10}$$

• The minimization function:

$$\chi_{SN}^2 = (\mu_{th}(z) - \mu_{SN})C_{SN}^{-1}(\mu_{th}(z) - \mu_{SN})$$
(11)

K. Asvesta et al., in preparation

To study the evolution of the dipole parameters with redshift, we employ a redshift tomographic method dividing the dataset (1527 SNIa) into 4 equal number redshift bins.

- We fix the monopole parameters, (a, b) to the best-fit ones from the Pantheon + SNIa, in the applied *z*-cut sample: $\mathbf{a} = 0.512^{+0.018}_{-0.015}$ and $\mathbf{b} = 8.3^{+4.3}_{-3.1}$
- The first redshift bin contains 359 SNIa with $0.015 \leq z_{hel} < 0.034$ and mean redshift 0.024
- The second redshift bin contains 394 SNIa with $0.034 \leq z_{hel} < 0.195$ and mean redshift 0.10
- The third redshift bin contains 381 SNIa with $0.195 \leq z_{hel} < 0.34$ and mean redshift 0.265
- The fourth redshift bin contains 393 SNIa with $0.34 \leq z_{hel} < 2.3$ and mean redshift 0.575

K. Asvesta et al., in preparation

Spatial Distribution of the subsamples



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Tomographic analysis - Dipole fit in three redshift frames

We constrain the dipole in all three redshift frames, namely the heliocentric (Hel), the CMB (CMB) and the Hubble Diagram frame (HD).

The direction of the dipole coincides with the CMB dipole.

Frame	Bin	q_{mono}	F(z,S)	q_{dip}	S
Hel	1	-0.4764	1	$-2.32_{-0.48}^{+0.51}$	-
Hel	1	-0.4764	$e^{-z/S}$	$-2.39^{+0.53}_{-0.58}$	unconstrained
CMB	1	-0.4764	1	2.33 ± 0.47	-
CMB	1	-0.4764	$e^{-z/S}$	$2.43^{+0.69}_{-0.62}$	unconstrained
HD	1	-0.4764	1	1.27 ± 0.47	
HD	1	-0.4764	$e^{-z/S}$	$1.11_{-0.57}^{+0.62}$	unconstrained
Hel	2	-0.463	1	- 0.159 ±0.084	-
CMB	2	-0.463	1	0.096 ± 0.086	-
HD	2	-0.463	1	0.057 ± 0.085	-
Hel	3	-0.329	1	0.058 ±0.047	-
CMB	3	-0.329	1	$0.115_{-0.049}^{+0.045}$	-
HD	3	-0.329	1	0.108 ± 0.047	-
Hel	4	0.032	1	0.069 ±0.027	-
CMB	4	0.032	1	0.089 ± 0.028	-
HD	4	0.032	1	0.089 ± 0.028	-

The posterior distributions of the full sample ($z \ge 0.015$) when the tilted anisotropic deceleration parameter, in the heliocentric frame, is: $\tilde{q} = -0.35 + q_d(\mathbf{n}_{SN} \cdot \mathbf{n}_{CMB}) \exp\left(\frac{-z}{S}\right)$



K. Asvesta et al., in preparation

Dipole in the full sample - II

The posterior distributions of the full sample ($z \ge 0.015$) when the tilted anisotropic deceleration parameter, in the heliocentric frame, is: $\tilde{q} = -0.35 + q_d(\mathbf{n}_{SN} \cdot \mathbf{n}_{dip})$



2M++ Bulk flow: Jonathan Carrick et al. 2015, MNRAS, 450, 317

K. Asvesta et al., in preparation

Other dipoles on the sky



FIG. 1: Directions of anisotropy in the Universe in the Galactic (l, b) coordinates with the galactic center in the middle, as inferred from Table 1. Directions from the literature are shown with different markers or ellipses (data points and their 1 σ uncertainties) with text labels.

Data Point	(<i>l</i> , <i>b</i>)	color in Fig. 1	Ref.
Galaxy cluster	$(280^{\circ} \pm 35^{\circ}, -15^{\circ} \pm 20^{\circ})$	tan	[99]
NVSS	$(248^{\circ} \pm 12.5^{\circ}, 44^{\circ} \pm 8^{\circ})$	orange	[100]
TGSS	$(247^{\circ} \pm 14.6^{\circ}, 52^{\circ} \pm 8^{\circ})$	light-blue	[101]
Dipole in the cosmological parameters	$(48.8^{\circ}_{-14.4^{\circ}}, -5.6^{\circ}_{-17.4^{\circ}})$	grey	[80]
CatWISE dipole	(238.2°, 28.8°)	green	[102]
CMB kinematic dipole	$(280^{\circ}, 42^{\circ})$	red x	[103]
CMB dipole	(263.99°, 48.26°)	red x	[104]
CMB quadrupole	$(224.2^{\circ}, 69.2^{\circ})$	red x	[65]
CMB octopole	$(239^{\circ}, 64.3^{\circ})$	red x	[65]
Planck-VA (Variance Asymmetry)	$(212^{\circ}, -13^{\circ})$	red >	[105]
Planck-DM (Dipole Modulation)	$(227^{\circ}, -15^{\circ})$	red >	[105]
Planck-PA (Power Asymmetry)	$(218^{\circ}, -21^{\circ})$	red >	[105]
WMAP9-VA	$(219^{\circ}, -24^{\circ})$	black >	[105]
WMAP5-DM	$(224^{\circ}, -22^{\circ})$	black >	[105]
WMAP9-PA	$(227^{\circ}, -27^{\circ})$	black >	[105]
Great Attractor	$(307^{\circ}, 9^{\circ})$	brown >	[106]
SNe Ia	$(310.6^{\circ} \pm 18.2^{\circ}, -13.0^{\circ} \pm 11.1^{\circ})$	blue	[107]
Dark flow	$(290^{\circ} \pm 20^{\circ}, 30^{\circ} \pm 15^{\circ})$	dark-violet	[42]
Bulk flow	$(282^{\circ} \pm 11^{\circ}, 6^{\circ} \pm 6^{\circ})$	light-red	[108]

TABLE I: Directions of anisotropy in the Universe as inferred from several data sets, the locations of the data points are shown in Fig. 1.

P.K. Aluri et al., 2023, DOI: 10.1088/1361-6382/acbefc

Thank you for your attention!

Back-up slides

"Cosmology is the search for two numbers. The Hubble parameter H_0 and the deceleration parameter q_0 " - Allan R. Sandage

• $H = \frac{\dot{a}}{a}$ • $q = -\frac{\ddot{a}a}{a^2}$ (q > 0: deceleration, q < 0: acceleration)

The deceleration parameters measured in the Hubble and tilted frames are:

$$q = -\left(1 + \frac{3\dot{\Theta}}{\Theta^2}\right) \quad \text{and} \quad \tilde{q} = -\left(1 + \frac{3\tilde{\Theta}'}{\tilde{\Theta}^2}\right)$$
(12)

$$\tilde{q} = q + \frac{\tilde{\vartheta}'}{3\dot{H}} \left(1 + \frac{1}{2}\Omega \right)$$
 to linear order (13)

In the absence of peculiar flows ($\tilde{\vartheta}' = 0$), $\tilde{q} \to q$

$$\frac{\tilde{\vartheta}'}{\dot{H}} = \frac{4}{3} \left[1 + \frac{1}{6} \left(\frac{\lambda_H}{\lambda} \right)^2 \right] \frac{\tilde{\vartheta}}{H}$$
(14)