T invariance of Higgs interactions in the standard model

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In the standard model, the Cabibbo-Kobayashi-Maskawa matrix, which incorporates the time-reversal violation shown by the charged current weak interactions, originates from the Higgs-quark interactions. It will be shown that the complex Yukawa interactions of quarks with the neutral Higgs field nevertheless conserve T.

Outline

- Recall: quark mass term arising from Yukawa interaction with Higgs as a result of Higgs vacuum expectation value involves a complex mass matrix

- Diagonalized complex quark mass term, proportional to the physical Higgs-quark interaction term, possesses time-reversal invariance with a modified quark transformation

- Euclidean spacetime versions of the transformation and invariance exist

- A measure for quark functional integration can be constructed to be invariant under this euclidean transformation, in addition to strong and electromagnetic gauge transformations $\Rightarrow$ there is no QCD or QED anomaly in the time-reversal invariance

Mediator	Symmetry	Quark interactions
Photon	P, T	$\frac{2}{3}\bar{q}\gamma_\mu q - \frac{1}{3}\bar{\tilde{q}}\gamma_\mu \tilde{q}$
Gluons	P, T	$\bar{q}\frac{\lambda^i}{2}\gamma_\mu q + \bar{\tilde{q}}\frac{\lambda^i}{2}\gamma_\mu \tilde{q}$
W$^+$	$\not{P}$, T	$\bar{q}_L\gamma_\mu \tilde{q}_L$
W$^-$	$\not{P}$, T	$\bar{\tilde{q}}_L\gamma_\mu q_L$
Z	$\not{P}$, T	$\bar{q}_L\gamma_\mu q_L - \bar{\tilde{q}}_L\gamma_\mu \tilde{q}_L -$ $2\sin^2\theta_W(\frac{2}{3}\bar{q}\gamma_\mu q - \frac{1}{3}\bar{\tilde{q}}\gamma_\mu\tilde{q})$
Higgs	$\not{P}$, T?	$\bar{q}_L g q_R$+hc, $\bar{\tilde{q}}_L \tilde{g}\tilde{q}_R$+hc

- Mass terms $\bar{q}_L M q_R + \bar{\tilde{q}}_L \tilde{M}\tilde{q}_R$+hc with complex matrices from symmetry breaking.

- On diagonalization of $M = vg, \tilde{M} = v\tilde{g}$,

$$q_L \to A_L^{-1} q_L, \quad q_R \to A_R^{-1} q_R$$
$$\tilde{q}_L \to \tilde{A}_L^{-1} \tilde{q}_L, \quad \tilde{q}_R \to \tilde{A}_R^{-1} \tilde{q}_R,$$

  W-interactions pick up matrix $A_L \tilde{A}_L^{-1} \equiv C$ (CKM), may be complex$\Rightarrow \not{T}$.

- Diagonalized mass/Yukawa terms may be complex if $U(1)$ transformations not used $\bar{\psi}\exp(i\theta'\gamma_5)\psi \Rightarrow \not{P}?$, $\not{T}?$

Finding the time-reversal transformation

$$\begin{array}{c}\mathcal{T}\bar{\psi}(i\gamma^\mu D_\mu - m)\psi\mathcal{T}^{-1} = \\ \mathcal{T}\psi^\dagger\mathcal{T}^{-1}\gamma^{0*}(-i\gamma^{\mu*}\mathcal{T}D_\mu\mathcal{T}^{-1} - m)\mathcal{T}\psi\mathcal{T}^{-1} \\ = \psi^\dagger(-t)T^\dagger\gamma^{0*}(-i\gamma^{0*}(-)D_0(-t) \\ -i\gamma^{i*}D_i(-t) - m)T\psi(-t) \\ = \psi^\dagger(-t)\gamma^0(i\gamma^\mu D_\mu(-t) - m)\psi(-t)\end{array}$$

$\Rightarrow$

$$\begin{aligned} T^\dagger T &= 1 \\ -T^\dagger\gamma^{0*}\gamma^{i*}T &= \gamma^0\gamma^i \\ T^\dagger\gamma^{0*}T &= \gamma^0 \end{aligned}$$

Second equation $\Rightarrow$

$$-T^\dagger\gamma^{0*}TT^\dagger\gamma^{i*}T = \gamma^0\gamma^i$$

$\Rightarrow$

$$-T^\dagger\gamma^{i*}T = \gamma^i$$

In standard representation γ^2 purely imaginary, rest real:

$$\begin{aligned} T^\dagger\gamma^\mu T &= \gamma^\mu(\mu = 0,2) \\ T^\dagger\gamma^\mu T &= -\gamma^\mu(\mu = 1,3) \end{aligned}$$

Satisfied by $T = i\gamma^1\gamma^3$

For complex mass term, the situation changes:

$$\begin{gathered}\mathcal{T}\bar{\psi}(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi\mathcal{T}^{-1} = \\ \mathcal{T}\psi^\dagger\mathcal{T}^{-1}\gamma^{0*}(-i\gamma^{\mu*}\mathcal{T}D_\mu\mathcal{T}^{-1} - me^{-i\theta'\gamma_5^*})\mathcal{T}\psi\mathcal{T}^{-1} \\ = \psi^\dagger(-t)T^\dagger\gamma^{0*}(-i\gamma^{0*}(-)D_0(-t) \\ -i\gamma^{i*}D_i(-t) - me^{-i\theta'\gamma_5^*})T\psi(-t) \\ \stackrel{?}{=} \psi^\dagger(-t)\gamma^0(i\gamma^\mu D_\mu(-t) - me^{i\theta'\gamma_5})\psi(-t)\end{gathered}$$

$\Rightarrow$

$$\begin{aligned} T^\dagger T &= 1 \\ -T^\dagger\gamma^{0*}\gamma^{i*}T &= \gamma^0\gamma^i \\ T^\dagger\gamma^{0*}e^{-i\theta'\gamma_5^*}T &= \gamma^0 e^{i\theta'\gamma_5} \end{aligned}$$

Second equation $\Rightarrow$

$$-T^\dagger\gamma^{0*}e^{-i\theta'\gamma_5^*}\gamma^{i*}e^{-i\theta'\gamma_5^*}T = \gamma^0 e^{i\theta'\gamma_5}\gamma^i e^{i\theta'\gamma_5}$$

$\Rightarrow$

$$-T^\dagger\gamma^{i*}e^{-i\theta'\gamma_5^*}T = \gamma^i e^{i\theta'\gamma_5}$$

If γ^μ in standard representation, satisfied by

$$T = e^{i\theta'\gamma_5}T_{standard} = ie^{i\theta'\gamma_5}\gamma^1\gamma^3$$

To investigate anomalies, one goes to the Euclidean metric:

- *Linear* time-inversion instead of antilinear time-reversal
- $A_0(t) \to -A_0(-t), A_i(t) \to A_i(-t)$
- $\psi, \bar{\psi}$ independent
- γ_0 altered: all gamma matrices become antihermitian; can be made hermitian by absorbing i

Finding the time-reversal transformation in Euclidean space

$$\begin{aligned}
&\mathcal{T}\bar{\psi}(\gamma^\mu D_\mu - m)\psi\mathcal{T}^{-1} = \\
&\mathcal{T}\bar{\psi}\mathcal{T}^{-1}(\gamma^\mu \mathcal{T} D_\mu \mathcal{T}^{-1} - m)\mathcal{T}\psi\mathcal{T}^{-1} \\
&= \bar{\psi}(-t)\bar{T}(\gamma^0(-)D_0(-t) \\
&+\gamma^i D_i(-t) - m)T\psi(-t) \\
&= \bar{\psi}(-t)(\gamma^\mu D_\mu(-t) - m)\psi(-t)
\end{aligned}$$

The above condition is equivalent to

$$\begin{aligned}
-\bar{T}\gamma^0 T &= \gamma^0 \\
\bar{T}\gamma^i T &= \gamma^i \\
\bar{T}T &= 1
\end{aligned}$$

Satisfied by $\bar{T} = T = i\gamma^1\gamma^2\gamma^3$

For complex mass term, the situation changes:

$$
\begin{gathered}
\mathcal{T}\bar{\psi}(\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi\mathcal{T}^{-1} = \\
\mathcal{T}\bar{\psi}\mathcal{T}^{-1}(\gamma^\mu\mathcal{T}D_\mu\mathcal{T}^{-1} - me^{i\theta'\gamma_5})\mathcal{T}\psi\mathcal{T}^{-1} \\
= \bar{\psi}(-t)\bar{T}(\gamma^0(-)D_0(-t) \\
+\gamma^i D_i(-t) - me^{i\theta'\gamma_5})T\psi(-t) \\
\stackrel{?}{=} \bar{\psi}(-t)(\gamma^\mu D_\mu(-t) - me^{i\theta'\gamma_5})\psi(-t)
\end{gathered}
$$

$\Rightarrow$

$$
\begin{aligned}
-\bar{T}\gamma^0 T &= \gamma^0 \\
\bar{T}\gamma^i T &= \gamma^i \\
\bar{T}e^{i\theta'\gamma_5}T &= e^{i\theta'\gamma_5}
\end{aligned}
$$

Satisfied by

$$T = ie^{-i\theta'\gamma_5}\gamma^1\gamma^2\gamma^3$$

$$\bar{T} = ie^{i\theta'\gamma_5}\gamma^1\gamma^2\gamma^3$$

Construction of measure for fermion fields in Euclidean action

$$\int \bar{\psi}(\gamma^\mu D_\mu - m)\psi :$$

Use eigenfunctions ϕ_n of $i\gamma^\mu D_\mu$,

$$\psi = \sum_n a_n \phi_n, \quad \bar{\psi} = \sum_n \bar{a}_n \phi_n^\dagger,$$

$$Z = \int \mathcal{D}A \prod_n \int da_n \prod_n \int d\bar{a}_n e^{-S}.$$

For chiral transformation

$$\psi \to e^{i\alpha\gamma^5}\psi, \quad \bar{\psi} \to \bar{\psi} e^{i\alpha\gamma^5},$$

$$\begin{aligned} a_n &\to \sum_m \int \phi_n^\dagger e^{i\alpha\gamma^5} \phi_m a_m, \\ \bar{a}_n &\to \sum_m \bar{a}_m \int \phi_m^\dagger e^{i\alpha\gamma^5} \phi_n, \end{aligned}$$

$\Rightarrow$ nontrivial Jacobian calculable with regularization involving gauge invariant eigenvalues of $i\gamma^\mu D_\mu$: $e^{i\alpha F\tilde{F}/16\pi^2}$

$\Rightarrow$ breakdown of chiral symmetry even if $m = 0$: anomaly

Complex mass term:

$$\int \bar{\psi}(\gamma^\mu D_\mu - m e^{i\theta'\gamma^5})\psi :$$

$$\psi = e^{-i\beta\gamma^5/2} \sum_n a_n^\beta \phi_n, \quad \bar{\psi} = \sum_n \bar{a}_n^\beta \phi_n^\dagger e^{-i\beta\gamma^5/2}$$

$$Z = \int \mathcal{D}A \prod_n \int da_n^\beta \prod_n \int d\bar{a}_n^\beta e^{-S}$$

$\Rightarrow$ β-dependent family of measures

θ', β can be removed by chiral transformation at the expense of change in $\theta F\tilde{F}$ term

Effective parity and T violation parameter

$$\bar{\theta} = \theta - \theta' + \beta$$

Under chiral transformation,

$$\begin{aligned} a_n^\beta &\to \sum_m \int \phi_n^\dagger e^{i\alpha\gamma^5} \phi_m a_m^\beta, \\ \bar{a}_n^\beta &\to \sum_m \bar{a}_m^\beta \int \phi_m^\dagger e^{i\alpha\gamma^5} \phi_n. \end{aligned}$$

$\Rightarrow$ standard anomaly

Under time-inversion operation for gauge fields,

$$\begin{aligned}\phi_n(x_0,\vec{x}) &\rightarrow i\gamma^1\gamma^2\gamma^3\phi_n(-x_0,\vec{x}),\\ \phi_n^\dagger(x_0,\vec{x}) &\rightarrow \phi_n^\dagger(-x_0,\vec{x})i\gamma^1\gamma^2\gamma^3,\end{aligned}$$

so that

$$\begin{aligned}e^{-\frac{i\beta\gamma^5}{2}}\phi_n(x_0,\vec{x}) &\rightarrow ie^{-i\beta\gamma^5}\gamma^1\gamma^2\gamma^3 e^{-\frac{i\beta\gamma^5}{2}}\phi_n(-x_0,\vec{x}),\\ \phi_n^\dagger(x_0,\vec{x})e^{-\frac{i\beta\gamma^5}{2}} &\rightarrow \phi_n^\dagger(-x_0,\vec{x})e^{-\frac{i\beta\gamma^5}{2}}ie^{i\beta\gamma^5}\gamma^1\gamma^2\gamma^3.\end{aligned}$$

Consistency with T (with $a_n^\beta, \bar{a}_n^\beta$ unchanged)

$$\begin{aligned}&\Rightarrow \beta = \theta'\\ &\Rightarrow \bar{\theta} = \theta - \theta' + \beta = \theta.\end{aligned}$$

Measure with $\beta = 0$	**Measure with** $\beta = \theta'$
Agrees with standard measure for real mass term	Reduces to standard measure for real mass term
Gauge invariant	Gauge invariant
Anomaly standard	Anomaly standard
Measure symmetric under usual T only	Measure symmetric under actual T symmetry of action
$\bar{\theta} = \theta - \theta'$	$\bar{\theta} = \theta$