Domination of Black Hole Accretion in Brane Cosmology

A. S. Majumdar

S. N. Bose Natl. Centre, Calcutta

Plan

$\ast$ Introduction to the RS(II) model

$\ast$ FRW Cosmology on the Brane

$\ast$ Primordial Black Holes on the Brane

$\ast$ Formation and Evaporation of PBH's

$\ast$ Accretion from the radiation bath in the Early Universe

$\ast$ Domination of Accretion over Evaporation: Long-lived Black Holes

Introduction to the RS(II) model

$*$ 3-brane embedded in the 5-d bulk (AdS_5).

$*$ Non-compact (large) extra dimension.

$*$ 3-brane with **positive tension** λ and **negative cosmological constant** Λ.

$*$ Matter (Standard Model) confined to the 3-brane. Gravity propagates also in the bulk.

$*$ Massless 4-d graviton is the zero mode of the 5-d wave equation. Bulk Curvature "binds" the graviton to the brane. Production of K-K modes is suppressed.

$*$ Einstein tensor:

$$G_{AB} = R_{AB} - \frac{1}{2} R g_{AB} = \kappa^2 T_{AB}$$

$*$ Effective 4-d Einstein tensor:

$$G_{\mu\nu} = \frac{8\pi}{M_4^2} \tau_{\mu\nu} + \kappa^4 \Pi_{\mu\nu} - E_{\mu\nu}$$

FRW Cosmology on the Brane

Isotropic and homogeneous space

Perfect fluid EM tensor: Equation of state: $p = w\rho$

Modified Friedmann Equation:

$$H^2 = \frac{8\pi}{3M_4^2}\Big(\rho + \frac{\rho^2}{2\lambda} + \rho_{KK}\Big) + \frac{\Lambda_4}{3} - \frac{k}{a^2}$$

Brane tension $\lambda = \frac{3M_5^6}{4\pi M_4^2}$

Bulk Cosm. Const. $\Lambda_5 = -\frac{3M_5^3}{4\pi l}$ (l: AdS curvature radius)

$\lambda_4 = 3(\frac{M_5^6}{M_4^4} - \frac{1}{l^2})$. Set $\lambda_4 = 0$. Relate l with ($l_4 \equiv M_4^{-1}$) through λ:

$$\lambda^{-1/4} = \Big(\frac{4\pi}{3}\Big)^{1/4}\Big(\frac{l}{l_4}\Big)^{1/2} l_4$$

Gravitational potential: $V(r) = \frac{2m}{M_4^2 r}(1 + \frac{2l^2}{3r^2})$

Consistent with current gravity experiments in the range $r \geq 0.2$mm. Implies $l \leq 10^{31} l_4$.

Cosmological Solutions for the Scale Factor and Energy Density

$$\rho = \frac{3M_4^2}{32\pi t(t+t_c)}$$

$$a = a_0\Big[\frac{t(t+t_c)}{t_0(t_0+t_c)}\Big]^{1/4}$$

$t_c \equiv l/2$ is the **transition time**.

For $t < t_c$ (or $\rho >> \lambda$): Non-conventional **high energy regime**:

$a = a_0(\frac{t}{t_0})^{1/4}$; $\quad \rho = \frac{3M_4^2}{32\pi t_c t}$

Radiation temperature: $\rho = \frac{\pi^2 g T^4}{30}$

Modified temperature-time relation: $T_c(l_{\rm max}) \approx 10^3 \mathrm{GeV}$

For $t >> t_c$ (or $\rho \leq \lambda$): **Standard low energy regime**:

$a = a_0(\frac{t}{(t_0 t_c)^{1/2}})^{1/2}$; $\; \rho = \frac{3M_4^2}{32\pi t^2}$

Primordial Black Holes on the Brane

Formation:

Inflation $\rightarrow$

Quantum Fluctuations of the Scalar field $\rightarrow$

Density Perturbations in the early universe $\rightarrow$

Gravitational Collapse of horizon-sized overdensities

Density inhomogeneities $\rightarrow$

Gravitational Waves $\rightarrow$

CMBR anisotropies.

COBE results: Upper limit on the inflationary energy scale $\rightarrow$

Lower limit on the **Horizon Mass**:

$$M_H > 2 \times 10^6 \Big(\frac{l_4}{l}\Big)^{1/3} M_4$$

Formation of a Black Hole with mass $M_0 \sim M_H$ (size $r_0 << l$) at time t_0

$$\frac{t_0}{t_4} \simeq \frac{1}{4}\Big(\frac{M_0}{M_4}\Big)^{1/2}\Big(\frac{l}{l_4}\Big)^{1/2}$$

Braneworld Black Holes

Geometry: 5d Schwarzschild Solution

$$dS_5^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_3^2$$

$$f(r) = 1 - r_0^2/r^2$$

$d\Omega_3^2$ is the volume element of a 3-sphere. Hoop conjecture: The black hole will have no way of distinguishing between the AdS dimension and the others near the horizon.

Near the BH (event) horizon the induced metric on the brane is

$$dS_4^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2$$

Different from the usual 4d Schwarzschild soln. where,

$$f(r) = 1 - r_0/r$$

Evaporation of Black Holes

Hawking process on the brane is essentially identical to the standard case.

Liddle *et al*, PRD **66** (2002)

BH radius, area and temperature:

$$r_{BH} = \left(\frac{8}{3\pi}\right)^{1/2}\left(\frac{l}{l_4}\right)^{1/2}\left(\frac{M}{M_4}\right)^{1/2} l_4; \quad A_5 = 2\pi^2 r_{BH}^3$$

$$T_{BH} = \frac{1}{2\pi r_{BH}}$$

contrasted with the usual 4d result $T_{BH} = M_4^2/8\pi M$.

BH radiates energy into the brane and the bulk. Only gravitational radiation can propagate in the bulk.

No. of particles of species j emitted in D-dimensional spacetime by a black hole of temperature T_{BH} in time interval dt with momentum in the interval $(\mathbf{k}, \mathbf{k} + d\mathbf{k})$

$$dN_j = \sigma_j(k) \frac{dt}{\exp(\omega/T_{BH} \pm 1)} \frac{d^{D-1}k}{(2\pi)^{D-1}}$$

Hawking evaporation ....contd.

Rate of energy loss by D-dimensional evaporation:

$$\frac{dM}{dt} = -\sum_j \int \sigma_j(k) \frac{\omega}{\exp(\omega/T_{BH} \pm 1)} \frac{d^{D-1}k}{(2\pi)^{D-1}}$$

In the high frequency limit ($\omega >> T_{BH}$), the cross-section

$$\sigma \approx \frac{A_{\text{eff}}}{4} \equiv \frac{\Omega_{D-2} r_{\text{eff},D}^{D-2}}{4}$$

$r_{\text{eff},D}$: effective radius of black body emission

$$r_{\text{eff},D} = \left(\frac{D-1}{2}\right)^{1/(D-3)} \left(\frac{D-1}{D-3}\right)^{1/2} r_{BH}$$

Total emitted power

$$\frac{dM}{dt} \approx -g_{\text{brane}} \bar{\sigma}_4 A_{\text{eff},4} T_{BH}^4 - g_{\text{bulk}} \bar{\sigma}_5 A_{\text{eff},5} T_{BH}^5$$

$\bar{\sigma}_D$: D-dimensional Stefan-Boltzmann constant

$$\bar{\sigma}_D = \frac{\Omega_{D-2}}{4(2\pi)^{D-1}} \Gamma(D) \zeta(D)$$

Since, $g_{\text{brane}} >> g_{\text{bulk}}$, evaporation into the bulk is a subdominant effect even for very low mass black holes.

Accretion from the Radiation bath

Black Holes **NOT** immersed in a vacuum

Zeldovich, Novikov,'66
Hawking, Carr,'74

Radiation (fluid) in the early universe with temperature T_R. Energy density of radiation ρ_R in the high energy phase

$\rho_R = \frac{\pi^2}{30} g_{\text{brane}} T_R^4 \qquad T_R \sim t^{-1/4}$

$$\rho_R = \frac{3M_4^2}{32\pi t_c t}$$

Capture of radiation by black holes $\rightarrow$ Mass gain through accretion. Rate:

$$\left(\frac{dM}{dt}\right)_{\text{acr}} \propto \rho_R A_{\text{eff},4}$$

Evolution of a primordial Black Hole

The processes of both **accretion** and **evaporation** act on the BH concurrently. Net energy change:

$$\dot{M} = 4\pi r_{BH}^2(-g_{brane}\sigma T_{BH}^4 + \rho_R)$$

Braneworld expansion $\rightarrow$

$$\dot{M} = -\frac{AM_4^2}{Mt_c} + \frac{BM}{t}$$

$$A \simeq \frac{3}{(16)^3\pi} \qquad\qquad B \simeq \frac{2}{\pi}$$

Solution for formation mass M_0

$$M(t) = \left[\left(M_0^2 - \frac{2AM_4^2}{2B-1}\frac{t_0}{t_c}\right)\left(\frac{t}{t_0}\right)^{2B} + \frac{2AM_4^2}{2B-1}\frac{t}{t_c}\right]^{1/2}$$

Extremum for $M(t)$ exists only if $(M_0/M_4)^2 < (2A/2B-1)(t_0/t_c)$. This inequality implies violation of lower bound on horizon (or BH) mass. Thus **accretion dominates** over evaporation in the early braneworld.

Evolution of primordial Black Holes in Standard Cosmology

Hawking evaporation: $T_{BH} \propto M^{-1}$

Accretion: $\dot{M} \propto \rho_R$, where $\rho_R \propto t^{-2}$

Net change of mass:

$$\dot{M} = -\frac{\mathcal{A} M_4^4}{M^2} + \frac{\mathcal{B} M^2}{M_4^2 t^2}$$

No analytical solution for the BH equation.
Population of black holes in the early universe.
Numerical integration of the coupled system of the BH eqn., radiation density eqn., and hubble expansion eqn. indicates prolongation of BH lifetimes.

ASM, P.DasGupta, R.P.Saxena

IJMP '95

$\{\{t_{\text{end}} \geq t_{\text{EW}} \rightarrow$ Baryogenesis through primordial black holes$\}\}$

Cosmology with Black Holes

A **population** of primordial black holes with an initial **number density** $n_{BH}(t_0)$ and total **energy fraction** β exchange energy with the surrounding radiation by accretion and evaporation. $\rho_T(t_0) = \rho_R(t_0) + \rho_{BH}(t_0)$
$\rho_{BH}(t_0) = \beta\rho_T(t_0) = M_0 n_{BH}(t_0)$
Evolution of energy components:

$$\begin{aligned}\dot{M} &= -\frac{AM_4^2}{Mt_c} + \frac{BM}{t} \\ \frac{d}{dt}\Big(\rho_R(t)a^4(t)\Big) &= -\dot{M}(t)n_{BH}(t)a(t)\end{aligned}$$

BH's grow monotonically for a while

$$\frac{M(t)}{M_0} \simeq \Big(\frac{t}{t_0}\Big)^B$$

During this period $n_{BH}(t) \propto a(t)^{-3}$. Since $a(t) \propto t^{1/4}$, thus $(n_{BH}(t)/n_{BH}(t_0)) = (t_0/t)^{3/4}$.

The condition for the universe to remain radiation dominated (i.e., $\rho_{BH}(t) < \rho_R(t)$) at any instant t is

$$\beta < \frac{(t_0/t)^{B+1/4}}{1 + (t_0/t)^{B+1/4}}$$

Radiation domination up to t_c

Demand: Evolution stays radiation dominated up to the time brane effects are important $\rightarrow$

$$\frac{\beta}{1-\beta} < \left(\frac{t_0}{t_c}\right)^{B+1/4}$$

Growth of BH's should be limited to $(M/M_4) < (3\pi/4)(t/t_4)$ for the evaporation law to be valid ($r_{BH} < l$). Upper bound on the initial mass:

$$\frac{M_0}{M_4} < \left(\frac{3\pi}{4(2\sqrt{2})^B}\right)^{\frac{2}{2-B}} \frac{t_c}{t_4}$$

Dilution of radiation density with expansion $\rightarrow$ Critical mass M_{max} is reached before evaporation starts dominating from the transition time t_t onwards. **Lifetime** of BHs:

$$\frac{t_{end}}{t_4} \simeq \frac{4}{A}(2\sqrt{2})^B \left(\frac{M_0}{M_4}\right)^{2-B} \frac{t_c}{t_4} \left(\frac{t_t^2}{t_c t_4}\right)^B$$

Effect of accretion in lifetime enhancement:

$$\frac{t_{end}(M,5D)}{t_{end}(M_0,5D)} \simeq \left(\frac{M_4}{M_0}\right)^B \left(\frac{t_t^2}{t_c t_4}\right)^B$$

Effect pronounced for small BH's.

Examples of BH lifetime enhancement

Numerical integration of the coupled system of the BH equation, radiation density equation, and the hubble expansion equation is required to determine the evolution exactly. Nevertheless:

$*$ For $(l/l_4) \simeq 10^{30}$, BHs with $M_0 = 10^8 M_4 \simeq 10^3$g survive up to the **present era** if $t_t \sim t_c$.

$*$ For $(l/l_4) \sim 10^{20}$, black holes formed with $M_0 = 10^{15} M_4 \simeq 10^{10}$g, will have $M_{max} \simeq 10^{15}$g, and will **evaporate now** completing their life cycle as $5D$ black holes.

$*$ For $(l/l_4) \sim 10^{20}$, BHs with $M_0 \leq M_4$ survive up to the era of **nucleosynthesis** and beyond if $t_t \geq 10^5 t_c$.

Matter Domination before t_c

Initial energy fraction β is such that the the accretion process causes $\rho_{BH}(t)$ to exceed $\rho_R(t)$ at a time $t < t_c$. ($\beta << O(1)$): **Black Hole Dominated Evolution**

BH mass grows up to $M_{\rm max}$ before $(t_{heq}) \approx t_t$: era of matter-radiation equality in the high-energy phase is reached. Then evaporation takes over the dynamics

BH equation in the evaporating regime ($t > t_t$):

$$M(t) = \left[M^2(t_t) - \frac{2AM_4^2}{t_c}(t - t_t)\right]^{1/2}$$

Accretion induced **enhancement of BH lifetime**:

$$\frac{t_{end}(M,5D)}{t_{end}(M_0,5D)} = \left(\frac{M_{max}}{M_0}\right)^2 \approx \left(\frac{1-\beta}{\beta}\right)^{\frac{8B}{4B+1}}$$

Effect **independent of initial BH mass**. E.g. BH with $M_0 \simeq 10^{12}$g has $t_{\rm end} > t_{\rm now}$ for $l/l_4 \simeq 10^{20}$, and $\beta = 10^{-3}$.

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Summary:

∗ **Primordial Black Holes** on the Brane **(RS-II)**

∗ **Modified** Hawking **evaporation**: $T_{BH} \propto M^{-1/2}$

∗ **Accretion** from radiation bath: $\dot{M} \propto \rho_R$

∗ Accretion dominates over evaporation in the early high energy brane phase. → Net **mass growth** of black holes.

∗ Lifetime of black holes calculated in radiation dominated and matter dominated evolutions.

∗ Significant **enhancement** of black hole **life-times**

∗ Average time and mass of formation taken. **Spectrum** needs to be considered.

∗ Cosmological consequences: **baryogenesis ?**, **dark matter ?**, **gamma rays ?**, **neutrino background ?**

∗ Astrophysical ramifications **? supermassive black holes ??**