Orientifolds of type IIA strings on Calabi-Yau manifolds

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Motivation

M-theory compactifications on 7 dim manifolds of G_2-holonomy give rise to a four-dimensional theories with $\mathcal{N} = 1$ supersymmetry. Joyce has constructed a large class of such manifolds as $\mathbb{Z}_2$ orbifolds of $CY^3 \times S^1$ by an anti-holomorphic involution of the CY^3 and inversion of the S^1.

Case (i) When there are no fixed points, one obtains a smooth manifold.

Case (ii) When there is a fixed point set Σ, one obtains a singular manifold. Joyce: The singularity can been smoothed out when $b_1(\Sigma) > 0$.

Harvey-Moore; Partouche-Pioline

Do these M-theory compactifications admit type IIA descriptions?

Other possibilities: F-theory on Calabi-Yau fourfolds, Heterotic string on Calabi-Yau threefolds.

Sen: It is useful to think of M-theory compactifications as type IIA compactifications with a non-constant dilaton, i.e., with a spatially varying string coupling.

What does this mean for the case of M-theory compactifications on Joyce manifolds?

Kachru-McGreevy: For cases for which the Joyce manifold is an orbifold of the seven-torus, there are special points in the moduli space where the M-theory compactification is a type IIA orientifold.

Our goal

- Find orientifold duals to M-theory compactifications on Joyce manifolds (especially, Case (ii)).

$$X = (M \times S^1)/\mathbb{Z}_2 \qquad (M = CY^3)$$

- Obtain an exact CFT description when M admits a Gepner model description.

An example of Case (ii): The Fermat quintic is given by the hypersurface

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \quad ,$$

in $\mathbb{CP}^4$(z_i are homogeneous coordinates of $\mathbb{CP}^4$).

Consider the anti-holomorphic involution

$$\sigma : \qquad z_i \to \bar{z}_i \quad , \quad i = 1, \ldots, 5 \quad ,$$

whose fixed-point Σ is an $\mathbb{RP}^3$, which is a special Lagrangian(sL) submanifold of the Fermat quintic. Becker-Becker-Strominger

Since $b_1(\mathbb{RP}^3) = 0$, the singularity of X, which is locally of the form $\Sigma \times \mathbb{C}^2/\mathbb{Z}_2$, cannot be resolved.

Σ is actually one in a family of $5^4 = 625$ sL submanifolds of the Fermat quintic, all of whom are $\mathbb{RP}^3$'s. They are all fixed-points of the anti-holomorphic involutions:

$$z_i \to \alpha^{n_i} \ \bar{z}_i \quad , \ \alpha^5 = 1 \ .$$

Plan of talk

- Obtain the type IIA orientifold dual.
- Discuss the orientifold projection in CFT
- Constructing Crosscap states
- A surprise and its resolution

Obtaining the orientifold dual

M-theory compactification on $M \times S^1$ is dual to type IIA compactification on M with the size of the S^1 identified with the string coupling.

The Joyce manifold X is the $\mathbb{Z}_2$ orbifold:

$$X = (M \times S^1)/\sigma \cdot \mathcal{I}_1$$

σ: anti-holomorphic involution of M
$\mathcal{I}_1$: inversion of S^1.

If we can identify the action of $\mathcal{I}_1$ on the type IIA side, then we can obtain the required orientifold group for the type IIA dual of M-theory on X.

But, $\mathcal{I}_1$ is not a symmetry of M-theory and thus cannot quite be identified with a symmetry on the type IIA side

However, the inversion of an even number of coordinates is a symmetry of M-theory.

In the example of the quintic that we considered, Σ is the base the SYZ T^3 fibration of the quintic and σ inverts the fibre. Thus, $\sigma \cdot \mathcal{I}_1$ corresponds to the simultaneous inversion of four circles – three from the SYZ fibre and one from the S^1.

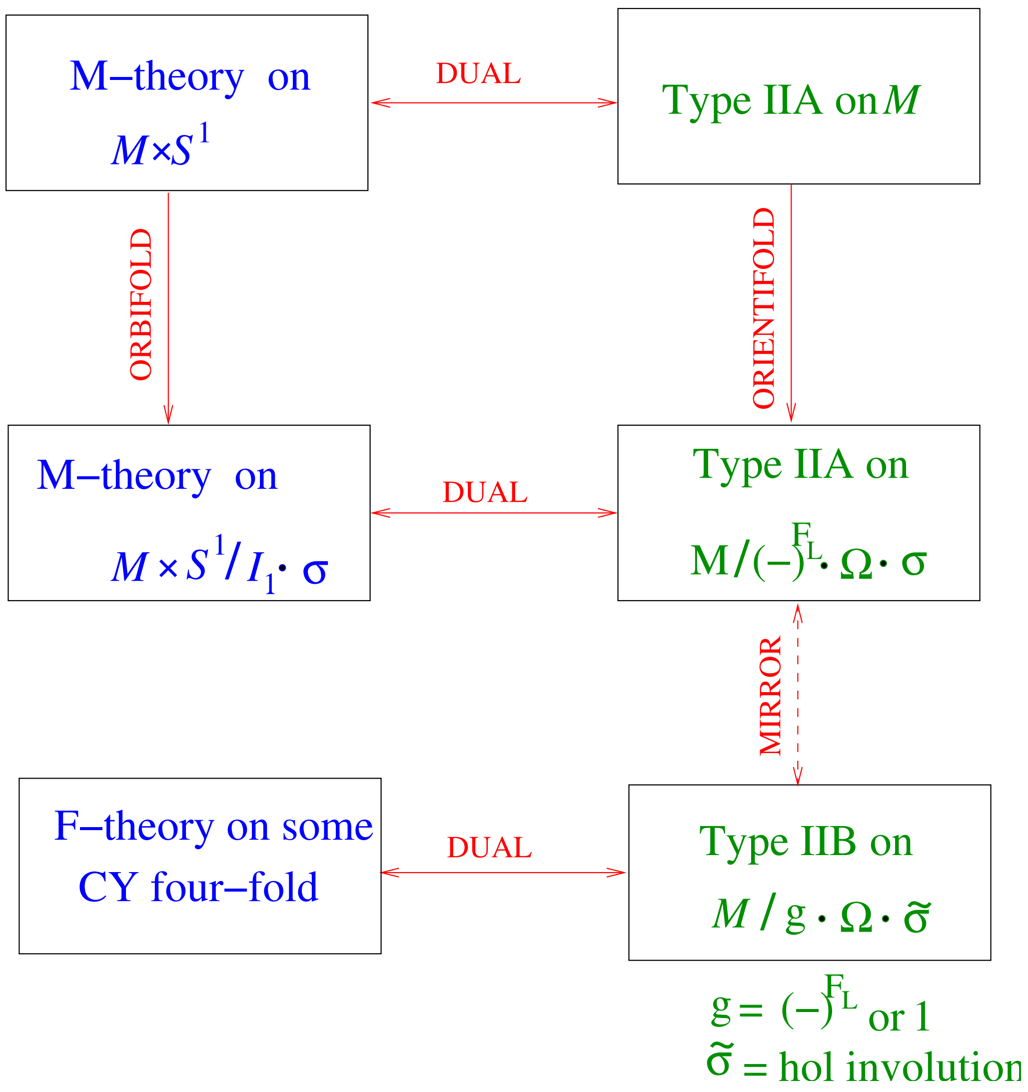
This leads us to two possible actions on the IIA side:

$$[\sigma \cdot \Omega] \qquad or \qquad [(-)^{F_L} \cdot \sigma \cdot \Omega]$$

Ω: worldsheet parity, F_L: spacetime fermion number.

It turns out only the second choice preserves $\mathcal{N} = 1$ supersymmetry. This is easily understood by studying the action on the vertex operators involving the Ramond sector. Thus, we obtain the following picture.

A pictorial depiction: (Possible heterotic duals not shown)



Relevant Spectra in M-theory

Let $h_{1,2}$ and $h_{1,1}$ be the Dolbeaut numbers of a CY^3 M. The spectrum of M-theory on $M \times S^1$ has $\mathcal{N} = 2$ supersymmetry in $d = 4$ and consists of:

- ★ the $\mathcal{N} = 2$ supergravity multiplet,
- ★ $h_{1,1}(M)$ abelian vector multiplets,
- ★ $h_{2,1}(M) + 1$ hypermultiplets.

The spectrum for a smooth Joyce manifold X with Betti numbers b_3 and b_2 has $\mathcal{N} = 1$ supersymmetry in $d = 4$ and consists of:

- ★ the $\mathcal{N} = 1$ supergravity multiplet,
- ★ $b_2(X) = h^+_{1,1}(M)$ abelian vector multiplets,
- ★ $b_3(X) = h_{2,1}(M) + h^-_{1,1}(M) + 1$ chiral multiplets.

Howe-Papadopoulos; Vafa-Witten

$h^+_{1,1}(M)[h^-_{1,1}(M)]$ are the number of Kahler moduli that are even[odd] under σ.

Returning to the example

- $h_{2,1} = 101$; $h^{+}_{1,1} = 0$; $h^{-}_{1,1} = 1$
- Two fixed points of the form $\Sigma \times \mathbb{R}^{3,1}$
- Each singularity is locally of the form $\mathbb{R}^4/\mathbb{Z}_2$, i.e., it is an A_1 singularity – expect $U(1) \times U(1)$ enhanced gauge symmetry in M-theory.
- In the type IIA dual, we expect an $O6$-plane with the SO-projection.
- The RR-charge will be equal the $\mathbb{R}^3/\mathbb{Z}_2$ orientifold plane in flat space. Based on this, we add 4 $D6$-branes wrapping $\Sigma \times \mathbb{R}^{3,1}$ implying a $SO(4)$ gauge symmetry.

Can these expectations be realised in CFT?

Imposing the Orientifold Projection

Let $\widetilde{\Omega} = (-)^{F_L} \cdot \sigma \cdot \Omega$. This the the orientifolding $\mathbb{Z}_2$. Under its action, the states of the original type II theory fall into three representations:

- **Real representations:**[$\epsilon = +1$] These have eigenvalue $+1$ and survive orientifold projection.

- **Pseudo-real representations:**[$\epsilon = -1$] These have eigenvalue -1. and are projected out.

- **Complex representations:**[$\epsilon = 0$] Under $\widetilde{\Omega}$, a state gets mapped to another one. In such cases, one linear combination is projected out.

In our example, it is easy to see that states that arise from the (c, c) and (a, a) rings are in the complex representation while those that arise from the (a, c) and (c, a) have $\epsilon = \pm 1$.

The orientifold projection = KB projection

The presence of orientifold planes leads to unoriented strings and hence unoriented surfaces. At 'one-loop', this adds a Klein bottle to the torus.

The Klein bottle amplitude has two "channels" related by the modular tranformation:

Direct Channel		Transverse Channel
$K(q) = \mathrm{Tr}\left(\widetilde{\Omega}\ q^{H_{cl}}\right)$		$\tilde{K}(\tilde{q}) = \langle C\vert\tilde{q}^{H_{cl}}\vert C\rangle$
$= \sum_i \epsilon_i\ \chi_i(q)$	$\overset{S}{\longleftrightarrow}$	$= \sum_j {\Gamma_i}^2\ \chi_i(\tilde{q})$
$\epsilon_i = 0, \pm 1$		$\vert C\rangle =$ crosscap state

We assume (for simplicity) that all states have multiplicity of one.

Thus, the direct channel amplitude encodes the orientifold projection.

Orientifolding in CFT: The PSS Ansatz

In the CFT of unoriented strings, one first constructs a crosscap state whose direct channel amplitude encodes the required projection. One general class of solutions has been provided by Pradisi-Sagnotti-Stanev.

$$|C\rangle = \sum_i \Gamma_i \, |C:i\rangle\rangle = \sum_i \frac{P_{0i}}{\sqrt{S_{0i}}} \, |C:i\rangle\rangle$$

where $|C:i\rangle\rangle$ are the Ishibashi basis for crosscap states and $P \equiv \sqrt{T} S T^2 S \sqrt{T}$. This plays the analogue of the S-matrix in Cardy's ansatz for the boundary states.

The matrices $Y_{ij}^k \equiv \sum_m \frac{S_{mi} P_{mj} P_m^k}{S_{m0}}$ plays a role analogous to the fusion matrix for boundary states. They satisfy the fusion algebra

$$Y_i \, Y_j = N_{ij}{}^k \, Y_k \ .$$

$Y_{00}^k = \epsilon_k$ determines the KB projection.

An application: $\mathcal{N}=2$ minimal models

States are labelled by (L, M, S) with $L = 0, \ldots, k$, $M = 0, \ldots (2k+3) \bmod (2k+4)$, $S = 0, 1, 2, 3 \bmod 4$ and $L + M + S = \mathit{even}$.

The S-matrix and P-matrix are given by

$$
\begin{aligned}
S_{LMS}^{\tilde{L}\tilde{M}\tilde{S}} &\propto \sin(L, \tilde{L})_k \; e^{\frac{i\pi M\tilde{M}}{k+2}} \; e^{\frac{-i\pi S\tilde{S}}{2}} \\
P_{LMS}^{\tilde{L}\tilde{M}\tilde{S}} &\propto \Big(\sin \frac{1}{2}(L, \tilde{L})_k \; e^{\frac{i\pi M\tilde{M}}{(2k+4)}} \; e^{\frac{-i\pi S\tilde{S}}{4}} \; \delta^{(2)}_{M+\tilde{M}+k} \; \delta^{(2)}_{S+\tilde{S}} \\
&\qquad + \sin \frac{1}{2}(k - L, \tilde{L})_k \; e^{\frac{i\pi(M+k+2)\tilde{M}}{(2k+4)}} \; e^{\frac{-i\pi(S+2)\tilde{S}}{4}} \; \delta^{(2)}_{M+\tilde{M}} \; \delta^{(2)}_{S+\tilde{S}} \Big)
\end{aligned}
$$

$(L, \tilde{L})_k = \pi(L+1)(\tilde{L}+1)/(k+2)$.

- S even: NS-sector and S odd: R-sector
- Delta function in P-matrix implies that only NSNS (or RR)states alone appear in the PSS crosscap state.

The Gepner model

• Tensor copies of $\mathcal{N} = 2$ minimal models(MM) such that total central charge is 9. For the quintic – tensor five copies of $k = 3$ MM.

• Tensor NS states with NS states and R with R from each component minimal model.

• Project onto states with total (including spacetime sector) $U(1)$ charge an odd integer.

Crosscap states in the Gepner model

• Take the tensor product of crosscap states in the individual minimal model.

• Implement the Gepner projection on this crosscap state.

This is a natural guess for the crosscap state in the Gepner model. But this cannot be the crosscap that realises the type IIA orientifold!

The problem

• The PSS crosscap state has only contributions from the NSNS sector. This implies that its Ramond charge is zero.

• The direct channel KB amplitude is not supersymmetric.

And its resolution

Consider the two crosscap states in a single MM

$$|C : NSNS\rangle \equiv P^{LMS}_{000}\ |C : LMS\rangle\rangle$$
$$|C : RR\rangle \equiv P^{LMS}_{011}\ |C : LMS\rangle\rangle$$

The first one is the PSS crosscap state while the second one is the PSS crosscap state associated with the simple current that is related to spacetime supersymmetry. It contains only RR Ishibashi states.

Then, we propose that the correct crosscap state takes the form

$$|C\rangle_{\text{Gepner}} = \mathcal{P}\left(\prod_{i=1}^{r}|C_i : NSNS\rangle + \prod_{i=1}^{r}|C_i : RR\rangle\right)$$

$\mathcal{P}$ imposes the $U(1)$ charge projection of Gepner.

Some consistency checks

- Now the crosscap state clearly carries RR charge.

- It has all the terms to provide a supersymmetric KB amplitude.

- For the quintic, in fact, we find a full family of 625 distinct crosscap states in agreement with the 625 anti-holomorphic involutions.

- More detailed checks[KB projection, tadpole cancellation] being carried out in specific examples will be discussed in the paper to appear soon.