Hadron Spectrum from Lattice Calculations

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Hadron Spectrum Collaboration

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• Introduction
• Baryon excitation spectrum in quenched and full QCD
• Lattices for spectrum calculations
• Identifying the continuum quantum numbers: meson spectrum
• Electromagnetic properties of excited states
• Conclusions
Resonance Spectrum of QCD

• Why is it important?
  – What are the key degrees of freedom describing the bound states?
  – What is the role of the gluon in the spectrum – search for exotics?
  – What is the origin of confinement, describing 99% of observed matter?
  – If QCD is correct and we understand it, expt. data must confront ab initio calculations

• NSAC Performance Measures
  • “Complete the combined analysis of available data on single $\pi$, $\eta$, and $K$ photo-production of nucleon resonances…” (HP3:2009)
  • “Measure the electromagnetic excitations of low-lying baryon states (<2 GeV) and their transition form factors…” (HP12)
  • “First results on the search for exotic mesons using photon beams will be completed” (HP15)
Spectroscopy - I

- **Nucleon Spectroscopy**: Quark model masses and amplitudes – states classified by isospin, parity and spin.

- Are states Missing, because our pictures are not expressed in correct degrees of freedom?
- Do they just not couple to probes?

*Capstick and Roberts, PRD58 (1998) 074011*
Exotics – I

• Exotic Mesons are those whose values of $J^{PC}$ are in accessible to quark model
  – Multi-quark states: $q\bar{q}q\bar{q}$
  – Hybrids with excitations of the flux-tube
• Study of hybrids: revealing gluonic and flux-tube degrees of freedom of QCD.
Lattice QCD: Hybrids and GlueX - I

- GlueX aims to photoproduce hybrid mesons in Hall D at JLab.
- Lattice QCD has a crucial role in both predicting the spectrum and in computing the production rates

\[ \pi_1(1600) \] in pion production at BNL

No evidence in photoproduction at CLAS

Important goal for LQCD
Low-lying Hadron Spectrum

\[ C(t) = \sum_{\vec{x}} \langle 0 | N(\vec{x}, t) \bar{N}(0) | 0 \rangle = \sum_{n, \vec{x}} \langle 0 | e^{ip\cdot x} N(0) e^{-ip\cdot x} | n \rangle \langle n | \bar{N}(0) | 0 \rangle \]
\[ = | \langle n | N(0) | 0 \rangle |^2 e^{-E_n t} = \sum_n A_n e^{-E_n t} \]

Control over:
- Quark-mass dependence
- Continuum extrapolation
- Finite-volume effects (pions, resonances)

Durr et al., BMW Collaboration
Science 2008
Variational Method

• Extracting excited-state energies described in C. Michael, NPB 259, 58 (1985) and Luscher and Wolff, NPB 339, 222 (1990)

• Can be viewed as exploiting the variational method

• Given $N \times N$ correlator matrix $C_{\alpha\beta} = \langle 0 | \mathcal{O}_\alpha(t) \mathcal{O}_\beta(0) | 0 \rangle$, one defines the $N$ principal correlators $\lambda_i(t, t_0)$ as the eigenvalues of

$$C^{-1/2}(t_0) C(t) C^{-1/2}(t_0)$$

• Principal effective masses defined from correlators plateau to lowest-lying energies

$$\lambda_i(t, t_0) \rightarrow e^{-E_i(t-t_0)} \left( 1 + O(e^{-\Delta E(t-t_0)}) \right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states
Variational Method - II

- Spectrum on lattice looks different – states at rest classified by isospin, parity and representation under cubic group

| Illustration | Name          | Explicit form \(|\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \bar{\psi}_{C\gamma} \) |
|--------------|---------------|--------------------------------------------------------------------------------------------------|
| ![Single-site](image) | single-site   | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \bar{\psi}_{C\gamma} \) |
| ![Singly-displaced](image) | singly-displaced | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \left( \bar{D}_{j}^{(p)} \bar{\psi} \right)_{C\gamma} \) |
| ![Doubly-displaced-I](image) | doubly-displaced-I | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \left( \bar{D}_{j}^{(p)} \bar{\psi} \right)_{B\gamma} \) |
| ![Doubly-displaced-L](image) | doubly-displaced-L | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \left( \bar{D}_{j}^{(p)} \bar{\psi} \right)_{C\gamma} \) |
| ![Triply-displaced-T](image) | triply-displaced-T | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \left( \bar{D}_{j}^{(p)} \bar{\psi} \right)_{A\gamma} \) |
| ![Triply-displaced-O](image) | triply-displaced-O | \(\phi_{ABC} \varepsilon_{abc} \bar{\psi}_{A\sigma} \bar{\psi}_{B\sigma} \left( \bar{D}_{j}^{(p)} \bar{\psi} \right)_{C\gamma} \) |

Extension to \(qqq \bar{q}q\)

![Lattice PWA](image)
Low-lying Baryon Spectrum
The nucleon spectrum as seen on the lattice!

• **Challenges/opportunities:**
  – Compute excited energies
  – Compute decays

signal-to-noise \( \sim e^{-\left(m_H - \frac{3}{2}m_\pi \right)t} \)

Anisotropic: \( a_t < a_s: \exp \left(- m a_t t \right) \)
Resonance Spectrum - Quenched

- Demonstration of our ability to extract nucleon resonance spectrum
- Hints of patterns seen in experimental spectrum
- Methodology central to remainder of project
- Do not recover ordering of $P_{11}$ and $S_{11}$

$m_\pi = 490\text{MeV}$
Resonance Spectrum – Nf=2


- First identification of spin-5/2 state in LQCD

Little evidence for multi-particle states
Roper Resonance - I

Roper (1440): lightest positive parity excitation of the nucleon – lighter than the N(1535) negative-parity excitation. Hard to reconcile with constituent quark model.

Two quenched calculations observe light Roper

Mahbub et al., arXiv:0910:2789
Roper from Amplitude analysis

Reaction model developed to analyse pion-nucleon reaction data to $W = 2$ GeV, and pion production data from Jlab. Analytic continuation method to extract parameters of nucleon resonances within EBAC dynamical coupled-channel model.

Single bare state in $P_{11}$ channel gives rise to three poles: two around the Roper $N^*(1440)$, and the other around the $N^*(1710)$.

Juelich-DCC; Roper generated dynamically

Challenges

- Lattices with two light and strange quark
- Identification of spin
- Seeking two-particle states in spectrum of energies – *region where states unstable.*
Anisotropic Clover Generation - I

- “Clover” Anisotropic lattices $a_t < a_s$: major gauge generation program under INCITE and discretionary time at ORNL designed for spectroscopy

Challenge: setting scale and strange-quark mass

$$s_X = \left(\frac{9}{4}\right)[2m_K^2 - m_\pi^2]/m_X^2$$

Express physics in (dimensionless) $(l,s)$ coordinates

Anisotropic Clover – II

<table>
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<tr>
<th>$L_s$(fm)</th>
<th>2.45fm</th>
<th>2.95fm</th>
<th>3.93fm</th>
<th>4.91fm</th>
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<td>$m_\pi$(MeV)</td>
<td>$20^3 \times 128$</td>
<td>$24^3 \times 128$</td>
<td>$32^3 \times 256$</td>
<td>$40^3 \times 256$</td>
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<td>11k, Tenn<a href="7.4">22M</a></td>
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<td>11k, INCITE<a href="3.4">390M</a></td>
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</tbody>
</table>

Two volumes
Discovering the continuum quantum numbers: low-lying meson spectrum
Identification of Spin - I

• We have seen lattice does not respect symmetries of continuum: *cubic symmetry for states at rest*

**Problem:** requires data at several Lattice spacings – density of states in each irrep large.

**Solution:** exploit known continuum behavior of overlaps

• Construct interpolating operators of *definite* (continuum) JM: $O^{JM}$

\[
\langle 0 \, | \, O^{JM} \, | \, J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}
\]

• Use projection formula to find subduction under irrep. of cubic group

\[
O^{[J]}_{\lambda, \lambda}(t, \bar{x}) = \frac{d_{\Lambda}}{g_{O_{h}^{D}}} \sum_{R \in O_{h}^{D}} D^{(\Lambda)*}_{\lambda, \lambda}(R) U_{R} O^{J,M}(t, \bar{x}) U_{R}^{\dagger}
\]

\[
= \sum_{M} S^{J,M}_{\lambda, \lambda} O^{J,M}
\]
Identification of Meson Spins

Overlap of state onto subduced operators

\[ \langle 0 \mid O^{J,M} \mid J', M' \rangle = Z_J \delta_{J,J'} \delta_{M,M'} \]

\[ \langle 0 \mid O^{J}_{\Lambda,\lambda} \mid J', M' \rangle = S^{J,M'}_{\Lambda,\lambda} Z_J \delta_{J,J'} \]

Common across irreps., up to \( O(a) \)

\[ N_f = 3 \]
\[ N_f = 3 \text{ Spectrum} \]

Dudek et al. (HadSpec Collab), in preparation

Exotic quantum numbers

\[ 16^3 \quad 20^3 \]
Nf = 2 + 1 Spectrum

Spectrum of light isovector mesons: $m_{\pi^*} = 520$ MeV
Whence the multi-hadrons?

Non-interacting two-particle energies: volume-dependent for P-wave

Quark bilinears insensitive to multihadron states
Low-lying Exotic Spectrum

High-precision calculation of mesons spectrum, and those with exotic quantum numbers

HadSpec Collaboration (J. Dudek et al.), preliminary
Multi-hadron States and Strong Decays

See also talk of Nilmani
Multi-hadron Operators

Need “all-to-all”

Usual methods give “point-to-all”
In QCD, even $\rho$ is unstable under strong interactions – resonance in $\pi$-$\pi$ scattering (quenched QCD not a theory – won’t discuss).

Spectral function continuous; finite volume yields discrete set of energy eigenvalues

Momenta quantised: known set of free-energy eigenvalues

$$E_n = 2\sqrt{m_\pi^2 + \left(\frac{2n\pi}{L}\right)^2}$$
Strong Decays - II

- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding $E$.

Breit-Wigner fit
CP-PACS, arXiv:0708.3705

QCDSF, 2008

Ulf Meissner et al.
EM Properties of Excited States

Example: Single-pion photoproduction

Radiative transition amplitudes

Example: Photoproduction at GlueX

Axial-vector Couplings?

Radiative transition amplitude
Lattice QCD computes the transition between isolated states

\[
N_2 | V_\mu | N_1 \rangle_{\mu(q)} = \overline{u}_{N_2}(p') \left[ F_1(q^2) \left( \gamma_\mu - \frac{q_\mu}{q^2} - (M_{N_2} - M_{N_1}) \gamma_5 \phi \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq \cdot x},
\]
Isovector Form Factor

J.D. Bratt et al (LHPC), arXiv:0810.1933

Euclidean lattice: form factors in space-like region

Extension to higher $Q^2$
EM Properties of Delta

Alexandrou et al., PRD79, 014509 (2009)

Electric form factor

\[ Q_{s_{\perp}}^\Delta \equiv c \int d^2\vec{b} \left( b_x^2 - b_y^2 \right) \rho_{T s_{\perp}}^\Delta (\vec{b}) . \]
Nucleon Radiative Transition - I

$N_f=0$ exploratory: $P_{11} \rightarrow$ Nucleon transition


Proton-$P_{11}$  720 MeV Pion  Neutron-$P_{11}$
Spectrum and Properties of Mesons in LQCD

Initial studies in charmonium


Use of variational method, and the optimized meson operators, to compute radiative transitions between excited states and exotics.

Considerable phenomenology developed from the results - supports non-relativistic models and limits possibilities for form of excited glue.

Radiative width of hybrid comparable to conventional meson – important for GlueX

HP15
Conclusions

• Lattice calculations evolving from studies of properties of ground-state hadrons to those of resonances
  • Lattices with correct spectrum of flavors
    – Variational method to precisely determine energies
    – Identification of spin both for mesons and for baryons
    – New correlator construction methods: many operators, high precision
• Properties of lowest-lying resonances studied
  – Delta form factor and charge distribution
  – “Roper” transition form factor
  – Radiative transitions between mesons
• Challenges:
  – Identifying the multiparticle states
  – Entering regime of strong decays
  – Transition Form Factors at higher $Q^2$
  – Mapping to Chiral Perturbation Theory
N-Δ Transition Form Factor - I

- Transition between lowest lying $I=3/2, J=3/2$ ($Δ$), and $I=1/2, J=1/2$ (N)
- Comparison between different lattice calculations and expt.
  - Milder $Q^2$ dependence than experiment but
  - Quark masses corresponding to pion masses around 350 MeV
  - $Q^2$ range up to around 2 GeV²

*Alexandrou et al, arXiv:0710.4621*
N-$\Delta$ Transition Form Factor - II

$R_{EM} \rightarrow +1$

Alexandrou et al, arXiv:0710.4621

Deformation in nucleon or delta
Delta Form Factors

Pascalutsa, Vanderhaeghen (2004) Thomas, Young (…)

Comparison of LQCD, EFT + expt: lattice QCD can vary quark masses
Roper Resonance

- Bayesian statistics and constrained curve fitting
- Used simple three-quark operator

Dong et al., PLB605, 137 (2005)

Axial-vector Charges

• The axial-vector charges $g_A^{N_1 N_2}$ can provide additional insight into hadron structure
• Recent calculation of axial-vector charges of two lowest-lying $\frac{1}{2}^-$ states, associated with N(1535) and N(1650).

Takahashi, Kunihiro, arXiv:0801.4707

Consistent with NR quark model
Roper Resonance

- Bayesian statistics and constrained curve fitting
- Used simple three-quark operator

Dong et al., PLB605, 137 (2005)

Correlation functions: Distillation

- Use the new “distillation” method.
- Observe
  \[ L^{(J)} \equiv (1 - \frac{\kappa}{n} \Delta)^n = \sum_i f(\lambda_i) u^{(i)} \otimes v^{*(i)} \]
- Truncate sum at sufficient \( i \) to capture relevant physics modes – we use 64: set “weights” \( f \) to be unity
- Meson correlation function
  \[ C_M(t, t') = \langle 0 \mid \bar{d}(t') \Gamma^B(t') u(t') \bar{u}(t) \Gamma^A(t) d(t) \mid 0 \rangle \]
- Decompose using “distillation” operator as

M. Peardon et al., arXiv:0905.2160

where

\[ \Phi_{\alpha\beta}^{A,ij} = v^{*(i)}(t) [\Gamma^A(t) \gamma_5]_{\alpha\beta} v^{(j)}(t') \]
\[ \tau_{\alpha\beta}^{ij}(t, t') = v^{*(i)}(t') M^{-1}_{\alpha\beta}(t', t) v^{(j)}(t). \]
Distillation Results

ρ Variational Analysis

Errors < 3%

Nucleon Variational Analysis

I=2 pi-pi

Overall momentum 0
Basis: pairs of back-to-back operators at momentum $\rho$

Thomas Jefferson National Accelerator Facility