Conformality Lost

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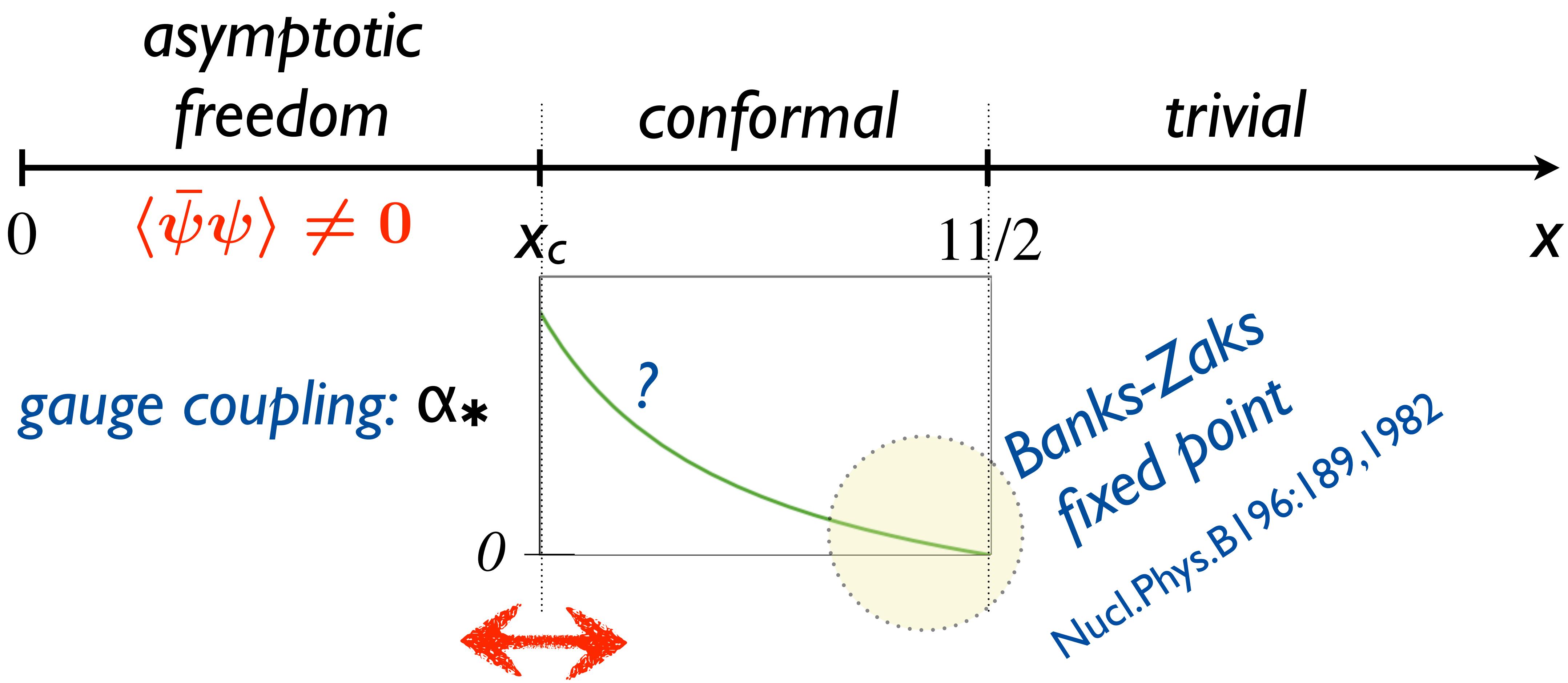
Motivation: QCD at LARGE N_c and N_f

Colors Flavors

Define **$x = N_f/N_c$**, treat as a continuous variable



What is the nature of this transition?

How does the IR scale appear as conformality is lost?

Outline:

I. A mechanism for vanishing conformal invariance

II. The Berezinskii-Kosterlitz-Thouless (BKT) transition

III. A quantum mechanics model: the $1/r^2$ potential

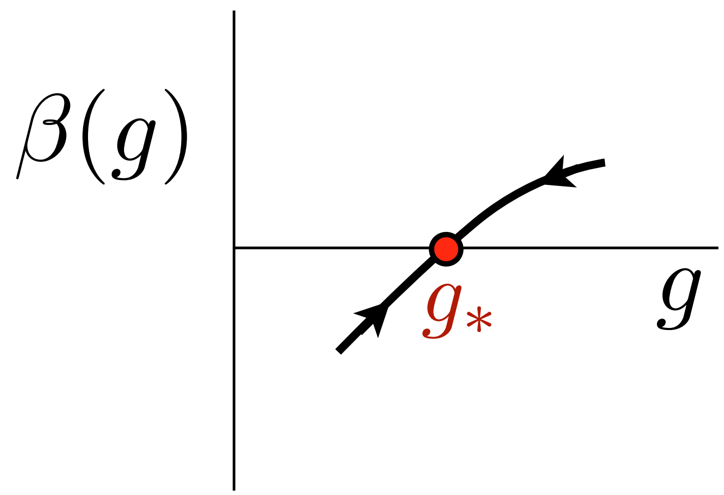
IV. AdS/CFT

V. Relativistic model: defect Yang-Mills

VI. QCD with many flavors? A partner theory QCD* with a nontrivial UV fixed point?

A theory with an infrared conformal fixed point at $g = g_*$ has a zero in the beta function:

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = \frac{\partial g}{\partial t}$$



Suppose the theory has another parameter κ such that the fixed point at $g = g_*$ vanishes for $\kappa > \kappa_*$

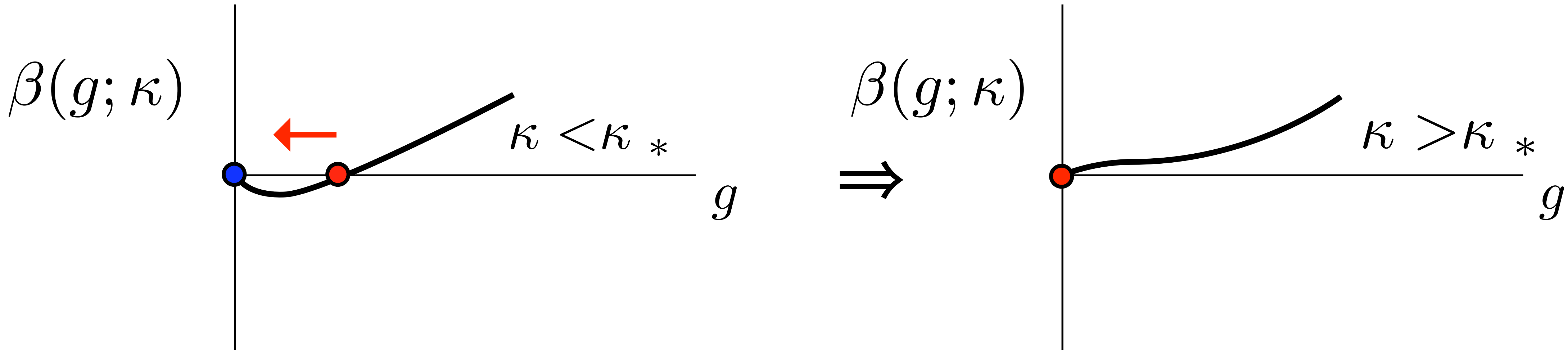
<u>*Example*</u>: *supersymmetric QCD is conformal for $3/2 \leq N_f/N_c \leq 3$*

"κ" = N_f/N_c, "κ_*" = 3/2, 3

How is conformality lost?

Three ways to lose an infrared fixed point:

#1: Fixed point runs to zero:



Example: Supersymmetric QCD at large N_c and N_f

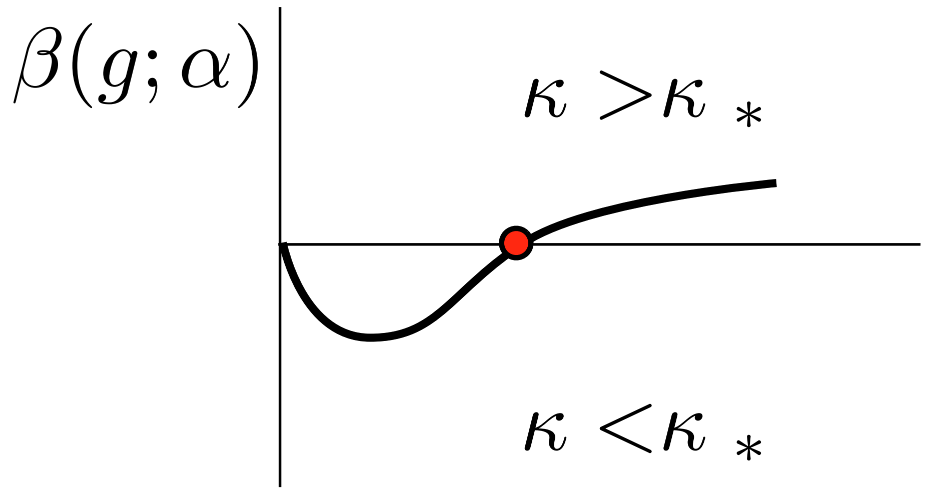
→ $\kappa = N_f/N_c$, $\kappa_* = 3$

$N_f/N_c \lesssim 3 \Rightarrow$ weak coupling Banks-Zaks conformal fixed point

$N_f/N_c \gtrsim 3 \Rightarrow$ trivial QED-like "free electric" theory

$$F_E \sim \frac{g^2}{r^2 \ln(r\Lambda_{\rm UV})}$$

#2: Fixed point runs off to infinity:



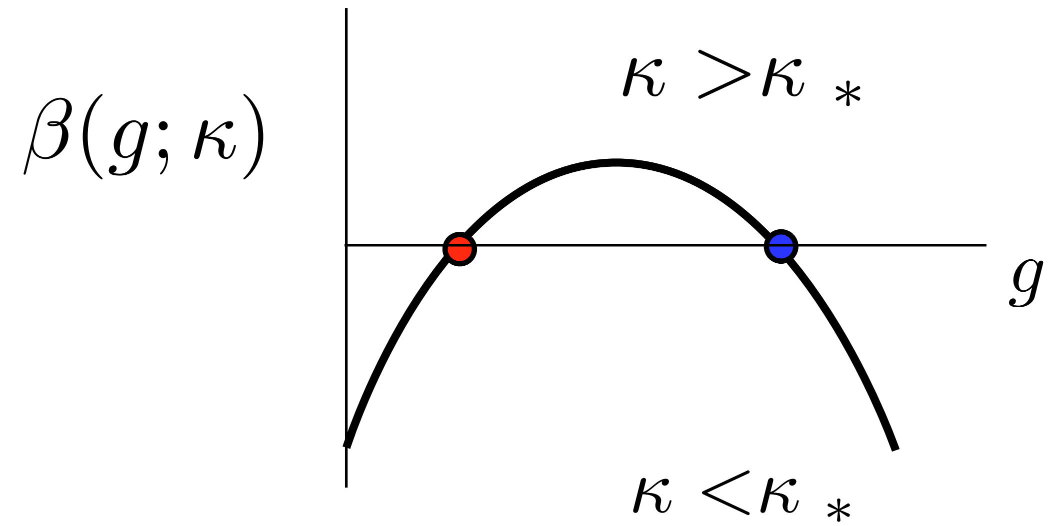
Possible example? SQCD again → $\kappa = N_f/N_c$, $\kappa_* = 3/2$

For $\kappa \leq \kappa_*$ get "free magnetic phase" [Seiberg]

➤ electric theory dual to a QED-like magnetic theory:

$$F_E \sim \frac{g^2 \ln(r\,\Lambda_{UV})}{r^2} \qquad F_M \sim \frac{g_M^2}{r^2 \ln(r\,\Lambda_{UV})} \qquad g_M \sim 1/g$$

#3: UV and IR fixed points annihilate:



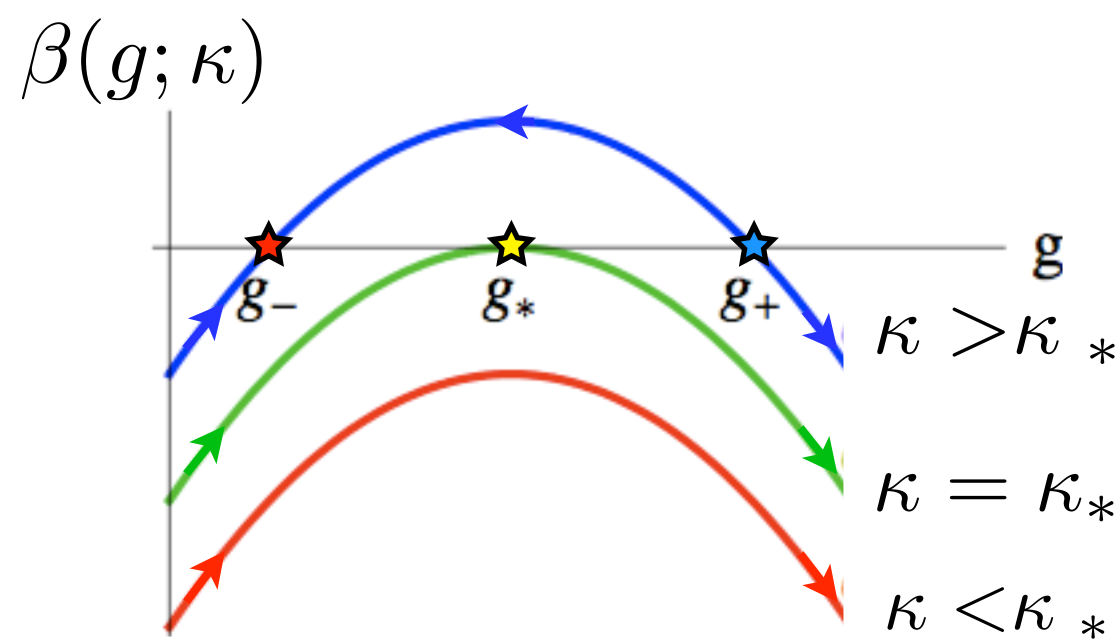
A toy model: $\beta(g;\kappa) = (\kappa - \kappa_*) - (g - g_*)^2$

$\kappa \geq \kappa_*$: $g_\pm = g_* \pm \sqrt{\kappa - \kappa_*}$

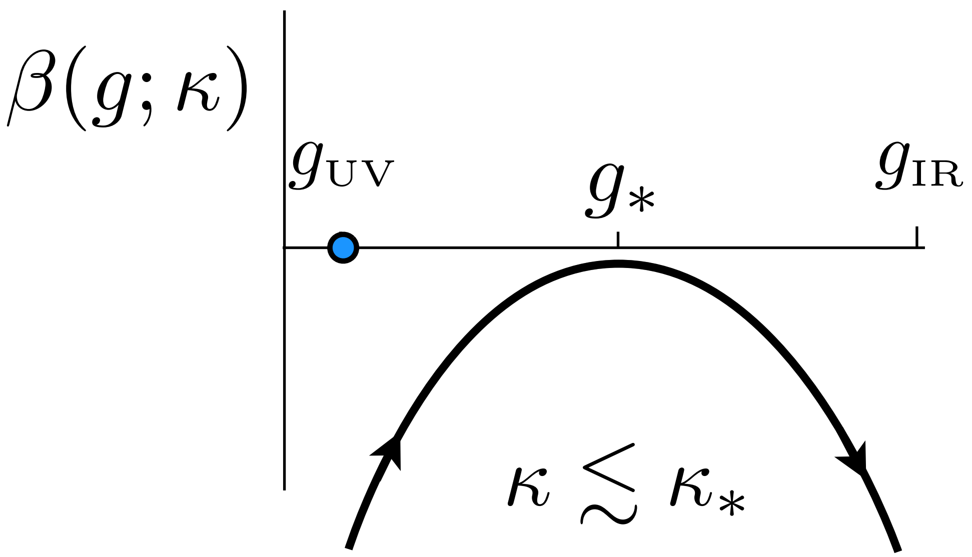
UV, IR fixed points

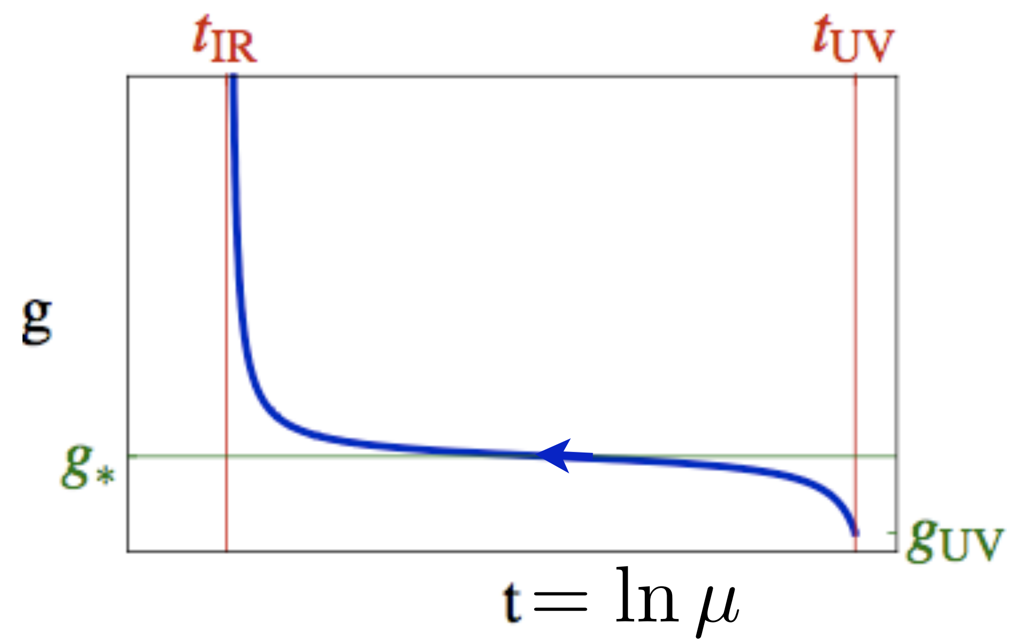
$\kappa = \kappa_*$ fixed points merge

$\kappa < \kappa_*$ conformality lost



What happens close to the transition on the nonconformal side?





i. Start: $g = g_{UV} < g_*$ in the UV

ii. g grows, **stalling** near g_*

iii. g strong at scale Λ_{IR}

$$\Lambda_{\rm IR} \simeq \Lambda_{\rm UV}\, e^{-\int \frac{dg}{\beta(g)}}$$

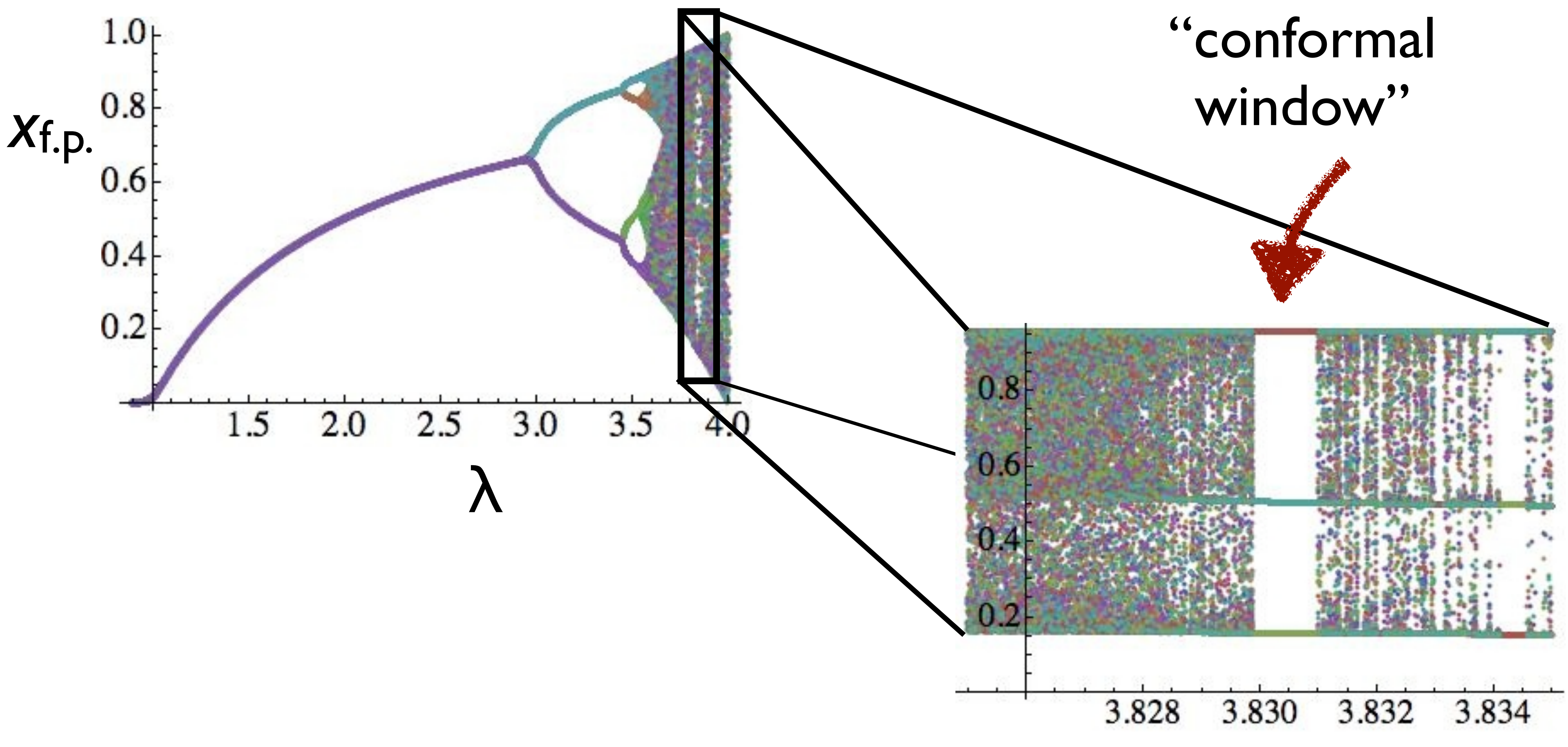
$$= \Lambda_{\rm UV}\, e^{-\frac{\pi}{\sqrt{|\kappa-\kappa_*|}}}$$

(Not like 2nd order phase transition: $\Lambda_{\rm IR} \simeq \Lambda_{\rm UV}\sqrt{|\kappa-\kappa_*|}$)

Aside:

Analogue to "intermittency" in chaotic systems

Iterative maps: $x_{n+1} = f(x_n)$, $f(x) = \lambda\, x(1-x)$



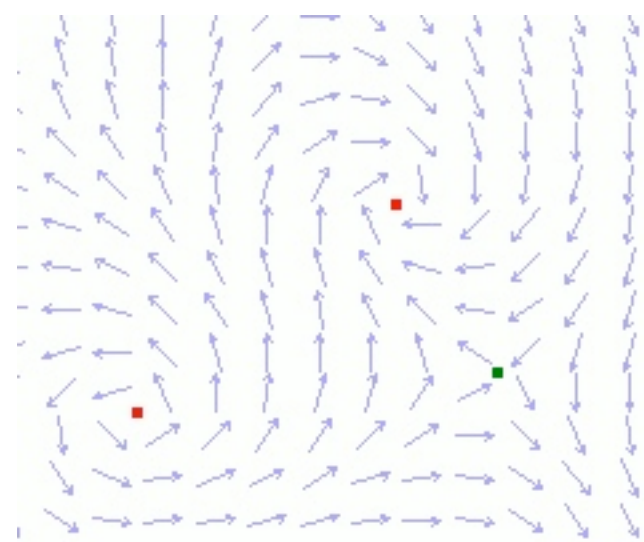
Find 3-pt orbit at $\lambda'_c \approx 3.829$, lost at $\lambda_c \approx 3.831$

$$\Lambda_{\rm IR} \quad \simeq \quad \Lambda_{\rm UV}\, e^{-\frac{\pi}{\sqrt{|\kappa-\kappa_*|}}}$$

Scaling behavior of toy model is reminiscent of the Berezinskii-Kosterlitz-Thouless (BKT) transition (an "infinite order" phase transition)

BKT: a classical phase transition in the 2-d XY-model

Vortices in XY model
box size R, vortex core size a:



$$E = E_0 \ln R/a\ , \qquad S = 2\ln R/a$$

$$F = E - TS = \boxed{(E_0 - 2T)} \ln R/a$$

Vortices condense for $T > T_c = E_0/2$;
can show correlation length forms:

$$\xi \simeq a\, e^{b/\sqrt{T-T_c}}$$

Classical XY model BKT transition = zero temperature quantum transition in Sine-Gordon model:

$$\mathcal{L} = \frac{T}{2}(\nabla\phi)^2 - 2z\cos\phi$$

New variables:

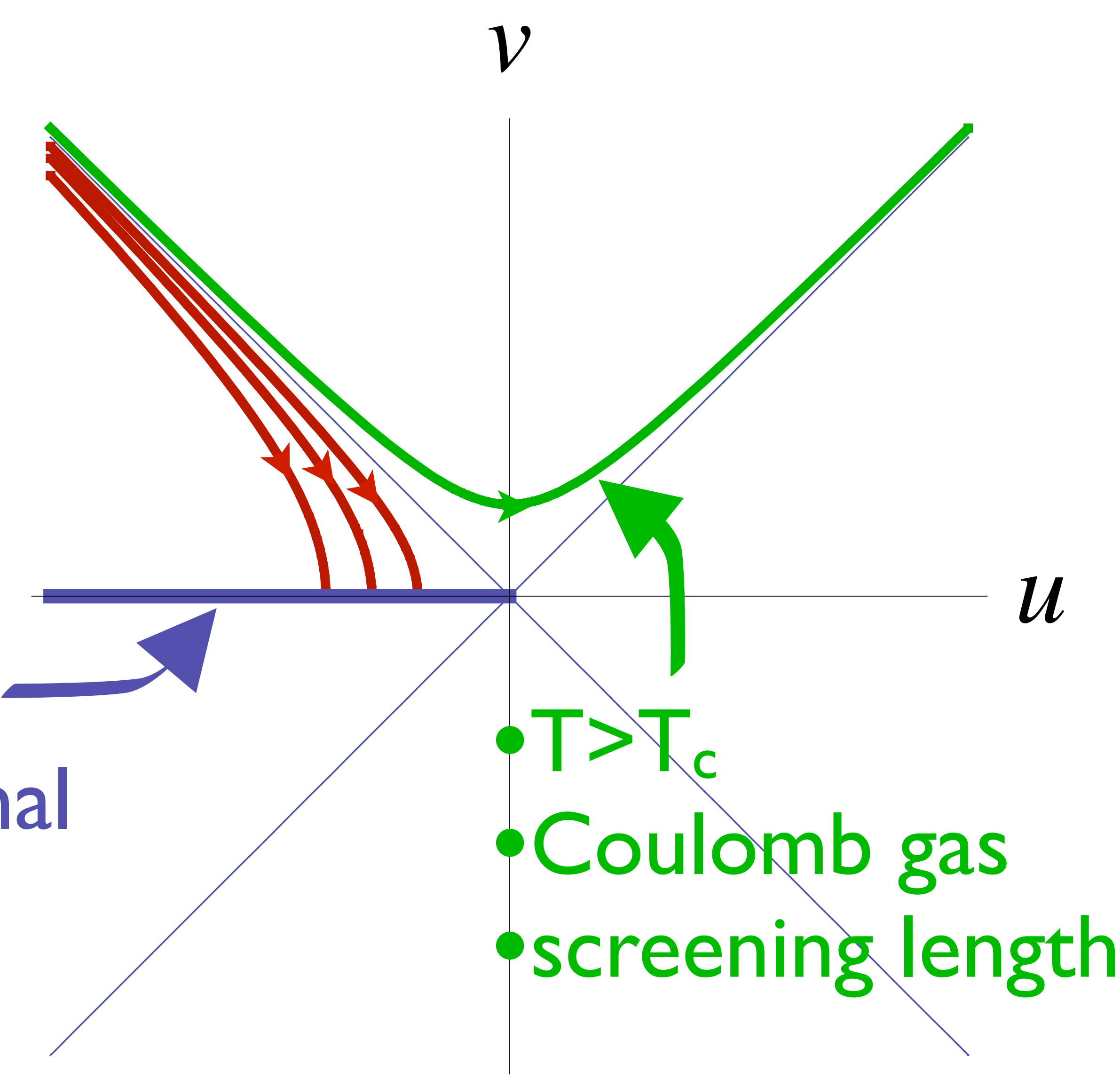
$$u = 1 - \frac{1}{8\pi T}\ , \qquad v = \frac{2z}{T\Lambda^2}$$

Perturbative β-functions:

$$\beta_u = -2v^2\ , \qquad \beta_v = -2uv$$

~ Λ = UV cutoff at vortex core

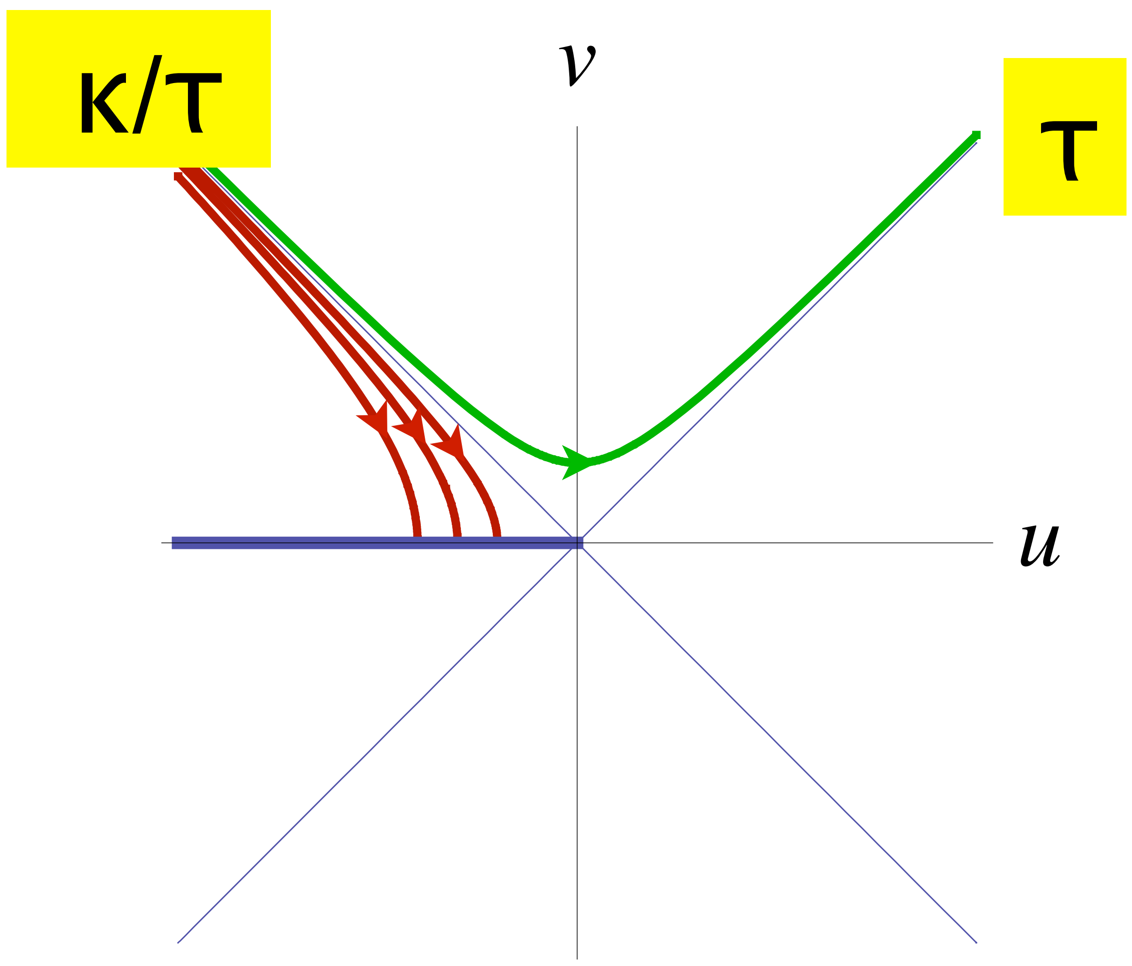
~ Dimensionful quantities in units of XY model interaction strength



- $T<T_c$
- bound vortices
- trivially conformal

$$u = 1 - \frac{1}{8\pi T} \,, \qquad v = \frac{2z}{T\Lambda^2}$$

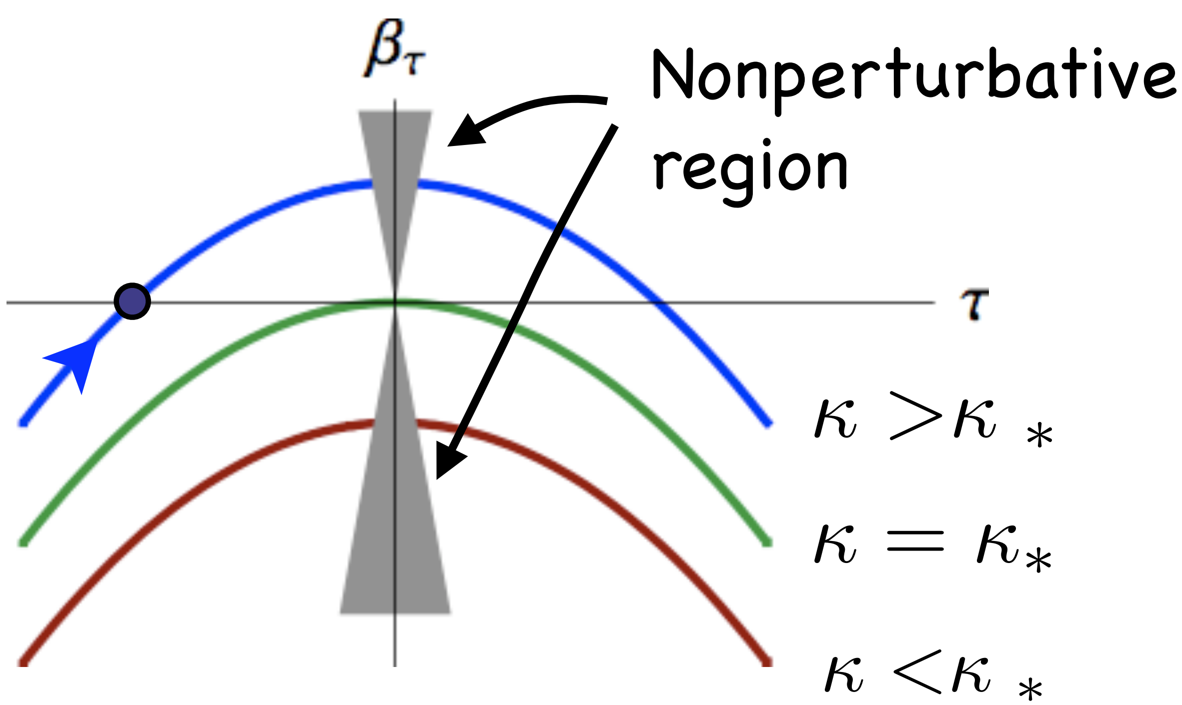
$$\beta_u = -2v^2 \,, \qquad \beta_v = -2uv$$



Newer variables:

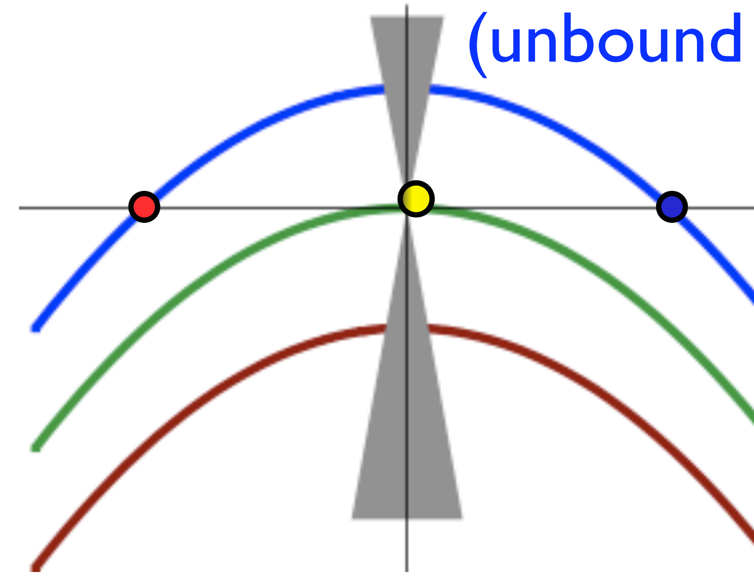
$$\tau = (u+v) \,, \qquad \kappa = (u^2 - v^2)$$

$$\beta_\tau = \kappa - \tau^2 \,, \qquad \beta_\kappa = 0$$



$$\kappa_* = 0$$

Correlation length in BKT transition:



For small negative κ, assume τ small & positive in UV

τ blows up in RG time

$$t = \int \frac{d\tau}{\beta(\tau)} = -\frac{\pi}{2\sqrt{-\kappa}}$$

...giving rise to an IR scale (like Λ_{QCD}) which sets the scale for the finite correlation length for α<0:

$$\xi_{\rm BKT} \sim \frac{1}{\Lambda} e^{\frac{\pi}{2\sqrt{-\alpha}}}$$

So far:

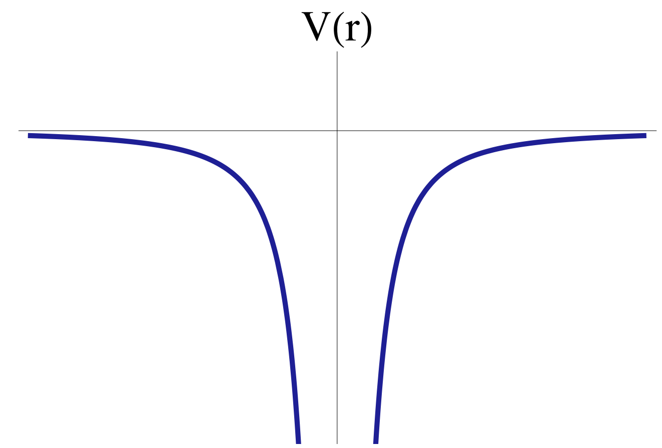
- BKT transition = loss of conformality via fixed point merger
- Mechanism of fixed point merger in general gives rise to "BKT scaling":

$$\Lambda_{\rm IR} \quad \simeq \quad \Lambda_{\rm UV} e^{-\frac{\pi}{\sqrt{|\kappa-\kappa_*|}}}$$

Next: other examples:

- QM with $1/r^2$ potential
- AdS/CFT
- Defect Yang-Mills
- QCD with many flavors

Example: QM in d-dimensions with $1/r^2$ potential



$$\left[-\nabla^2 + V(r) - k^2\right]\psi = 0\ , \qquad V(r) = \frac{\kappa}{r^2}$$

k=0 solutions: $\psi = c_- r^{\nu_-} + c_+ r^{\nu_+}$

$$\nu_\pm - \left(\frac{d-2}{2}\right) \pm \sqrt{\kappa - \kappa_*} \qquad \kappa_* = -\left(\frac{d-2}{2}\right)^2$$

- valid for $\kappa_* < \kappa < (\kappa_*+1)$
  - $\kappa < \kappa_*$: $\nu_\pm$ complex, no ground state
  - $\kappa = \kappa_*$: $\nu_+ = \nu_-$
  - $\kappa > (\kappa_*+1)$: r^{ν_-} too singular to normalize

$$\left[-\nabla^2 + V(r) - k^2\right]\psi = 0\ , \qquad V(r) = \frac{\kappa}{r^2}$$

k=0 solutions: $\psi = c_- r^{\nu_-} + c_+ r^{\nu_+}$

$$\nu_\pm - \left(\frac{d-2}{2}\right) \pm \sqrt{\kappa - \kappa_*} \qquad \kappa_* = -\left(\frac{d-2}{2}\right)^2$$

- c_+ =0 **<u>or</u>** c_-=0 are scale invariant solutions
- If $c_+ \neq 0$, $\psi \to c_+ r^{\nu_+}$ for large r ($\nu_+ > \nu_-$)
- to make sense of BC at r=0, introduce δ-function:

$$V(r) = \frac{\kappa}{r^2} - g\delta^{(d)}(r)$$

- r^{ν_+} corresponds to IR fixed point of g
- r^{ν_-} corresponds to unstable UV fixed point of g

RG treatment of $1/r^2$ potential: *I. Perturbative*

$\kappa_* \equiv -(d-2)^2/4$ so work in $d=2+\varepsilon$

$$S = \int dt\, d^d\mathbf{x} \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla\psi|^2}{2m} + \frac{g\pi}{4}\psi^\dagger\psi^\dagger\psi\psi \right)$$

δ-function

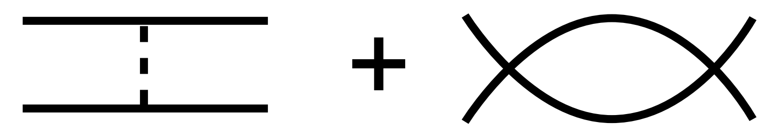
$$- \int dt\, d^d\mathbf{x}\, d^d\mathbf{y}\, \psi^\dagger(t,\mathbf{x})\psi^\dagger(t,\mathbf{y}) \frac{\kappa}{|\mathbf{x}-\mathbf{y}|^2} \psi(t,\mathbf{y})\psi(t,\mathbf{x})$$

propagator: $\dfrac{i}{\omega - \mathbf{p}^2/2m}$

contact vertex: $i\pi g \mu^{-\epsilon}$

"meson exchange": $\dfrac{2\pi i \kappa}{\epsilon}\dfrac{1}{|\mathbf{q}|^\epsilon}$

Find g runs:



$$\beta(g;\kappa) = \mu\frac{\partial g}{\partial \mu} = \left(\kappa + \frac{\epsilon^2}{4}\right) - (g-\epsilon)^2$$

Same as toy model! $\kappa_* = -\varepsilon^2/4$, $g_* = \varepsilon$

$\kappa > \kappa_*$: conformal

$\kappa = \kappa_*$: critical

$\kappa < \kappa_*$: g blows up in IR

$$B \sim \left(\frac{\Lambda_{\mathrm{IR}}^2}{m}\right) \sim \left(\frac{\Lambda_{\mathrm{UV}}^2}{m}\right) e^{-2\pi/\sqrt{\kappa_* - \kappa}}$$

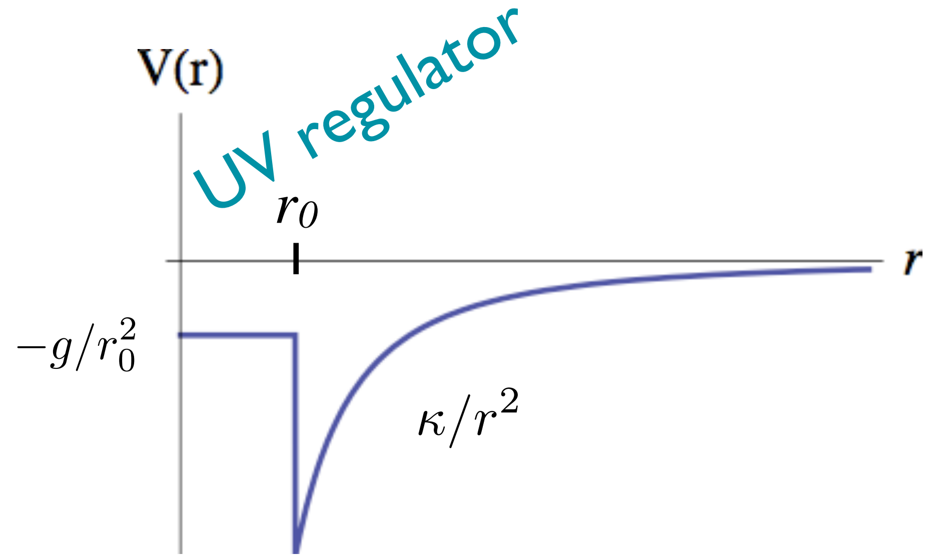
BKT scaling

bound state energy

RG treatment of $1/r^2$ potential: II. *Non-perturbative*

regulate with square well:

$$V(r) = \begin{cases} \kappa/r^2 & r > r_0 \\ -g/r_0^2 & r > r_0 \end{cases}$$



E=0 solution for r>r₀: $\psi = c_- r^{\nu_-} + c_+ r^{\nu_+}$

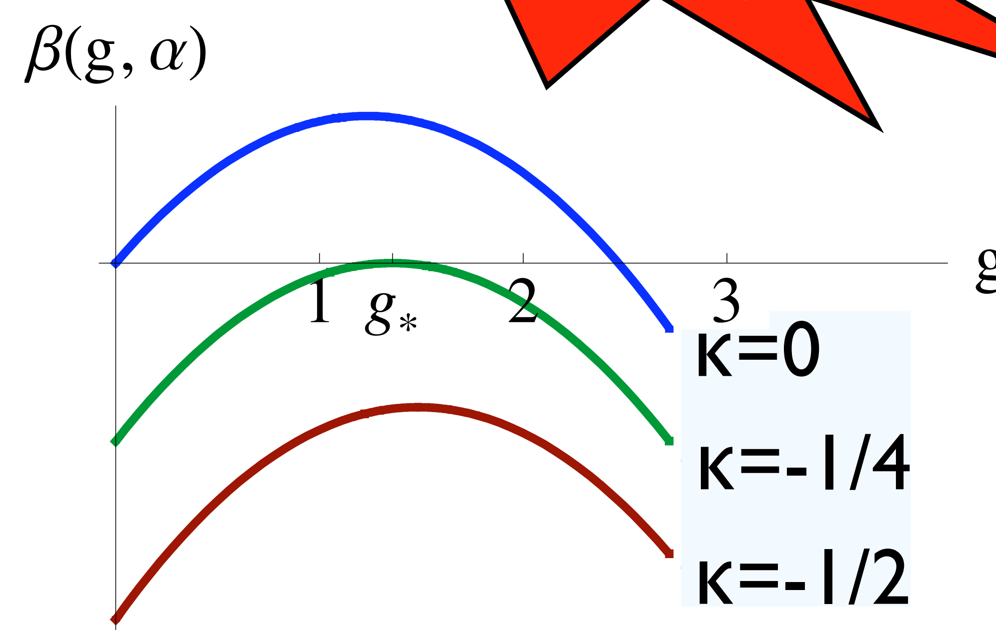
Solve for c_+/c_- (a physical dimensionful quantity) and require invariance: $d(c_+/c_-)/dr_0 = 0$:

Qualitatively same as pert. result

Find <u>exact</u> β-function for g. Eg, for d=3:

$$\beta = \frac{2\sqrt{g}\left(\kappa + \sqrt{g}\cot\sqrt{g} - g\cot^2\sqrt{g}\right)}{-\cot\sqrt{g} + \sqrt{g}\csc^2\sqrt{g}}$$

$\kappa_* = -\tfrac{1}{4}$, $g_* \approx 1.36$



Aside: Even better to define a modified coupling constant

$$\gamma = \left(\frac{\sqrt{g}\, J_{d/2}(\sqrt{g})}{J_{d/2-1}(\sqrt{g})} \right)$$

Condition $d(c_+/c_-)/dr_0$ yields exact β-function in d-dimensions:

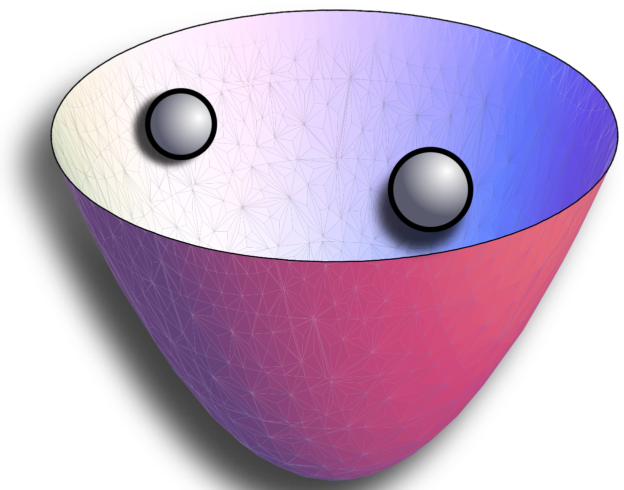
$$\beta_\gamma = \frac{\partial \gamma}{\partial t} = (\kappa - \kappa_*) - (\gamma - \gamma_*)^2, \quad \gamma_* = \frac{d-2}{2}$$

- Toy model is exact!
- γ is a periodic function of g, γ=±∞ equivalent
- Aside: Limit cycle behavior for $\kappa < \kappa_*$: describes "Efimov states" for trapped atoms at Feschbach resonance

Conformal phases: measure correlations, not β-functions! Look at operator scaling dimensions:

From Nishida & Son, 2007:

- Replace $V(r_1-r_2) \rightarrow V(r_1-r_2) + \frac{1}{2}\omega^2|r_1^2+r_2^2|$
- Compute 2-particle ground state energy E_0
- Operator dimension of ψψ is $\Delta_{\psi\psi} = E_0/\omega$



2-particle wave-function at $|r_1-r_2|=0$

As the two conformal theories merge when $\kappa \rightarrow \kappa_*$, operator dimensions in the two CFTs merge

For $1/r^2$ potential -- find for the two conformal theories:

[ψψ]: $$\Delta_\pm = (d+\nu_\pm) = \left(\frac{d+2}{2}\right) \pm \sqrt{\kappa-\kappa_*}$$

"+" = UV fixed point
"−" = IR fixed point

Note: $(\Delta_+ + \Delta_-) = (d+2)$: scaling dimension of nonrelativistic spacetime.

Analog in AdS/CFT:

AdS: $$ds^2 = \frac{1}{z^2}\left(dz^2 + \sum_{i=1}^{d} dx_i^2\right)$$

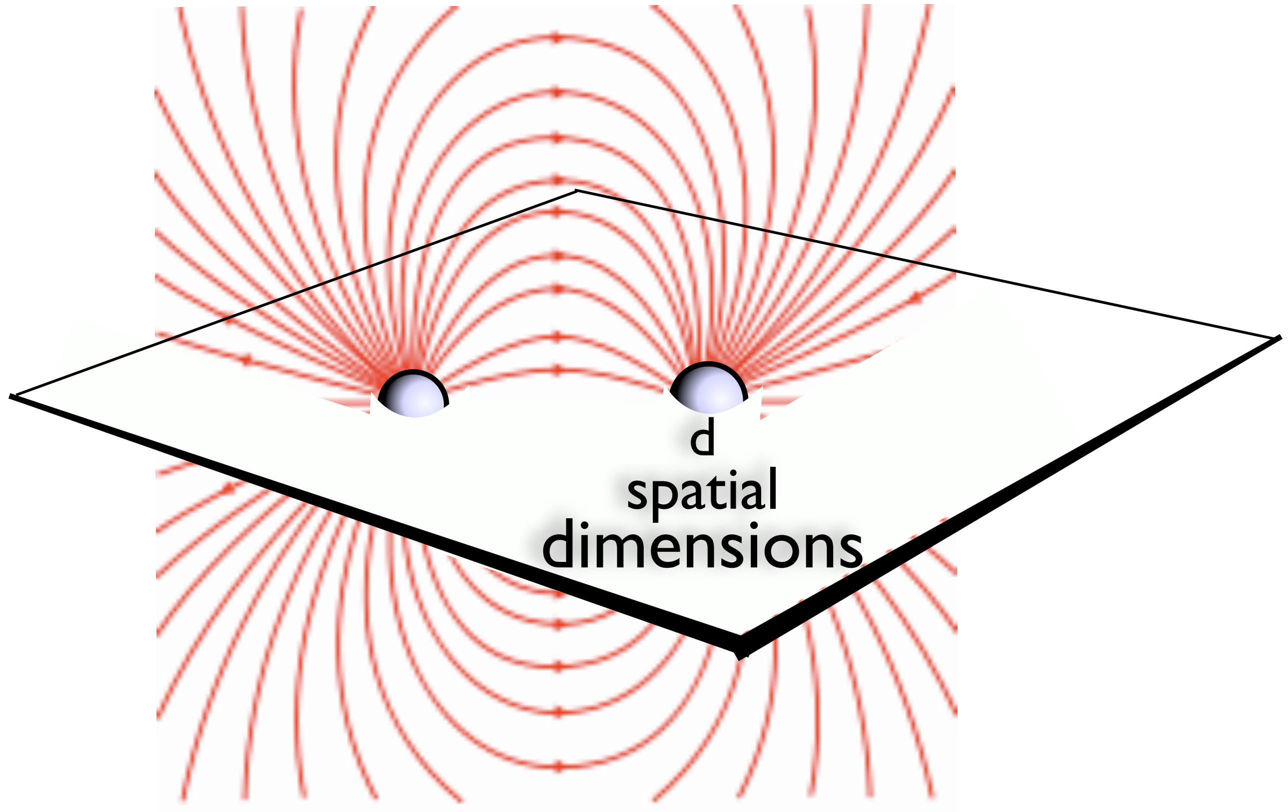
Massive scalar in the bulk
two solutions to eq. of motion:

$$\varphi = c_+ z^{\Delta_+} + c_- z^{\Delta_-}$$

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{m^2 + \left(\frac{d}{2}\right)^2} \equiv \frac{d}{2} \pm \sqrt{m^2 - m_*^2}$$

AdS	**QM**
• $(\Delta_+ + \Delta_-) = d =$ spacetime dim of CFT • when $m^2 = m_*^2 = -d^2/4$, $\Delta_\pm = d/2$ • Instability (no AdS or CFT) for $m^2 < m_*^2$ (B-F bound)	• $(\Delta^+_{\psi\psi} + \Delta^-_{\psi\psi}) = (d+2) =$ conformal wt. of nonrelativistic d-space+time • $\kappa = \kappa_* = -(d-2)^2/4 \Rightarrow \Delta_\pm = (d+2)/2$ • Conformality lost for $\kappa < \kappa_*$

A relativistic example: defect Yang-Mills theory

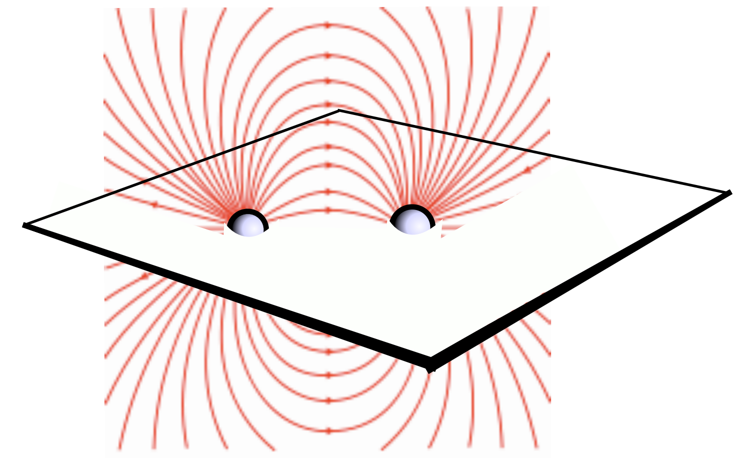


Charged relativistic fermions on a d-dimensional defect
+ 4D conformal gauge theory (eg, N=4 SYM)

$$S = \int d^{d+1}x \; i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu}F^{a,\mu\nu}$$

g doesn't run

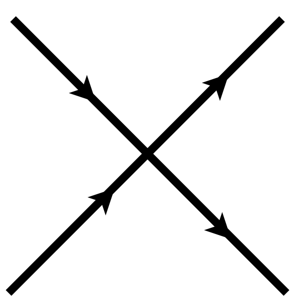
g doesn't run by construction



Expect a phase transition as a function of g:

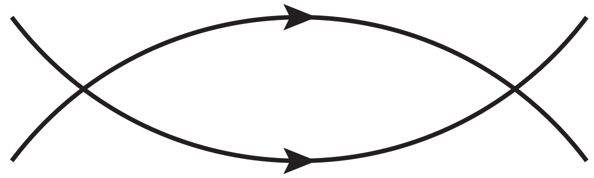
$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0 & g < g_* \\ \Lambda_{\rm IR}^d & g > g_* \end{cases}$$

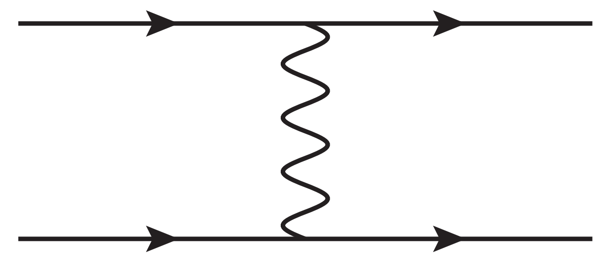
Add a contact interaction to the theory (as in QM & AdS/CFT examples!) and study its running:

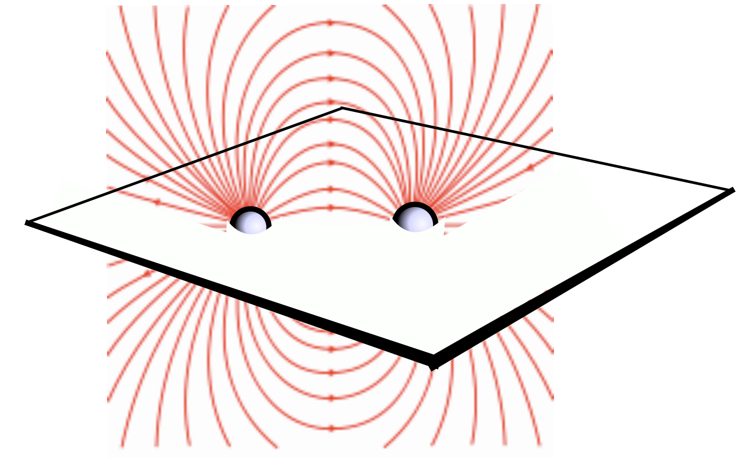


$$\Delta S = \int d^{d+1}x \, \left(-\frac{c}{2}(\bar{\psi}\gamma_\mu T_a \psi)^2\right)$$

Phase transition is in perturbative regime for d=1+ε (spatial dimensions of "defect"): compute β-function β(c):







1/ε pole for d=(1+ε)

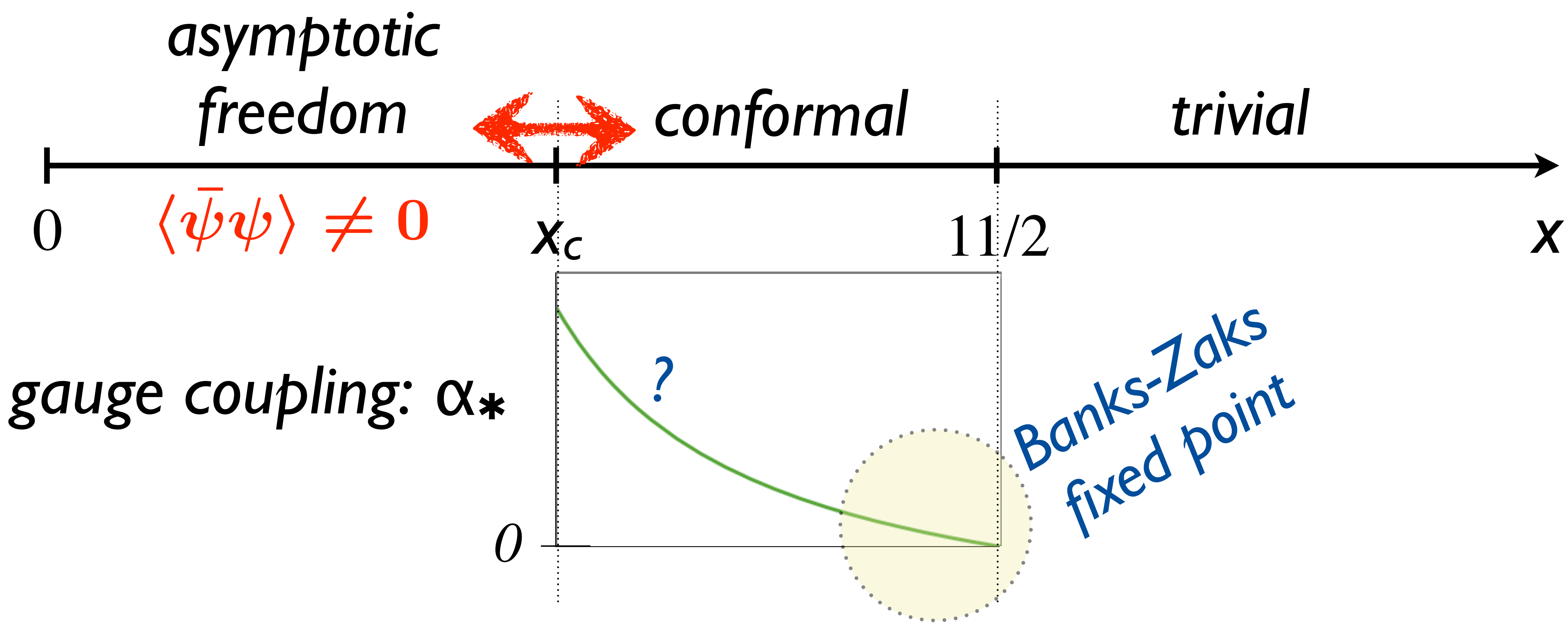
$$\beta(c) = -\frac{g^2}{2\pi} - \epsilon c - \frac{N_c}{2\pi}c^2$$
$$= \frac{1}{2\pi}\left(\frac{\pi^2\epsilon^2}{N_c} - g^2\right) - \frac{N_c}{2\pi}\left(c - \frac{\epsilon\pi}{N_c}\right)^2$$

- Find BKT transition at $g^2 = g_*^2 = (\varepsilon\pi)^2/N_c$

  $\Lambda_{IR} \sim \Lambda_{UV} \exp[-\pi/\sqrt{(g^2-g_*^2)}]$

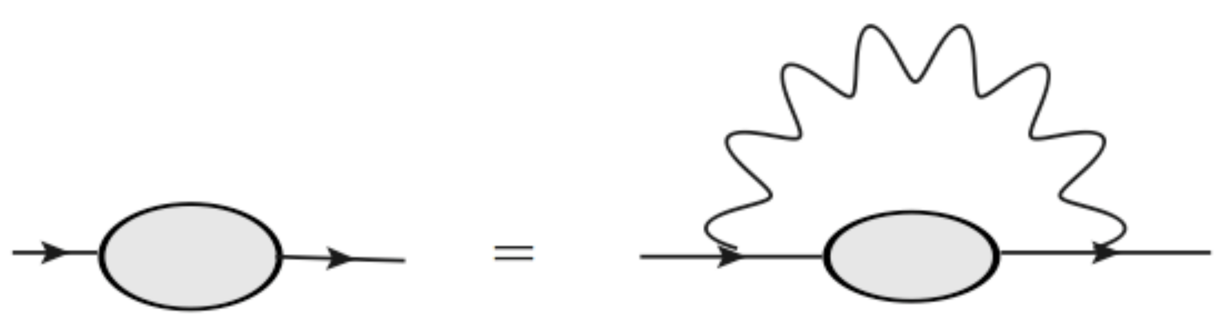
- Schwinger-Dyson gap eq (rainbow approx) gives qualitatively same results

Back to QCD at LARGE N_c and N_f:



Transition at $x=x_c$?

Schwinger-Dyson (rainbow approximation):

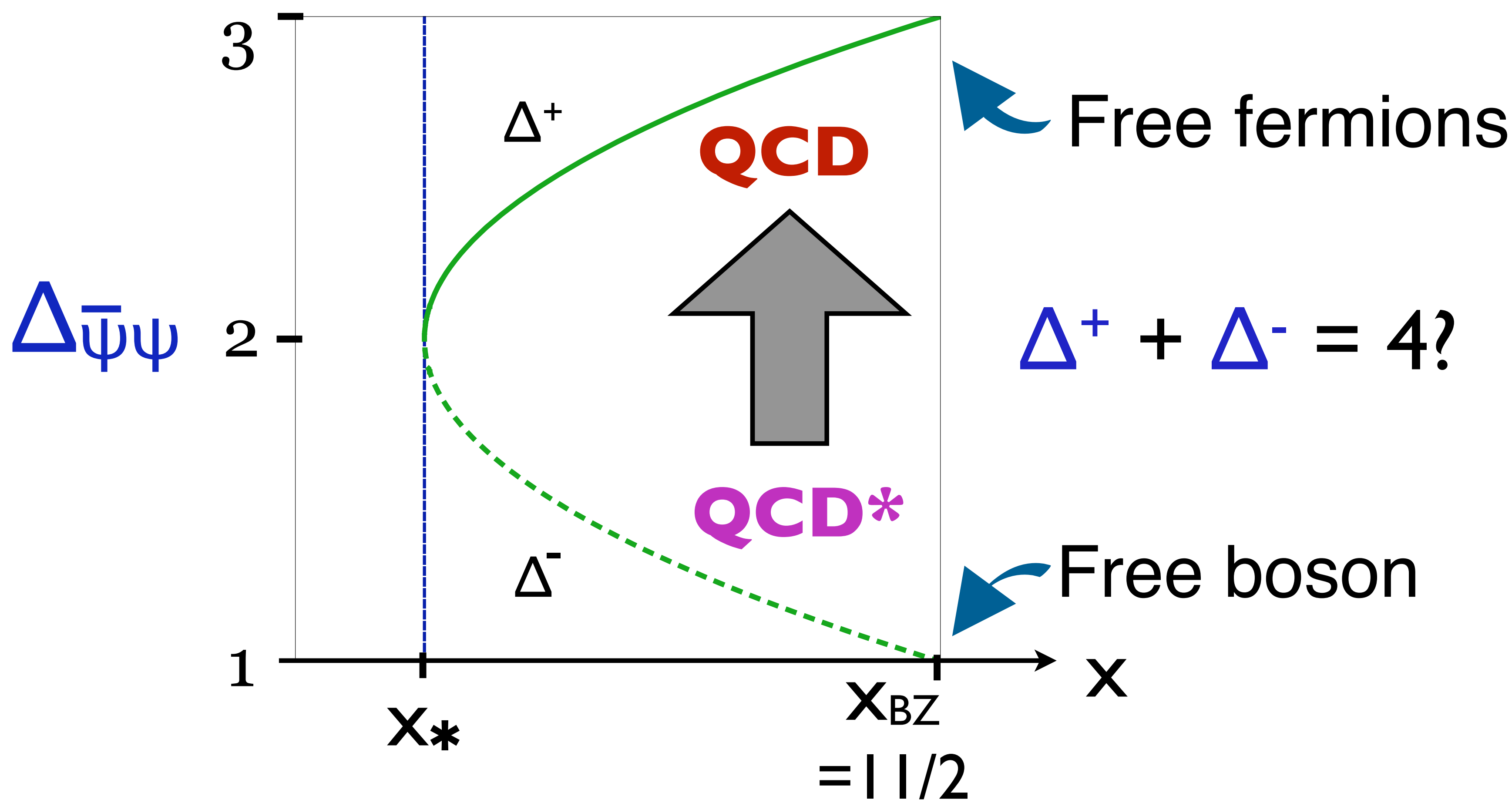


Miransky 1985

Appelquist, Terning, Wijerwardhana 1996

Found: BKT scaling for $<\bar\psi\psi>$...not rigorous, but qualitatively correct?

Conjecture: loss of conformality for QCD at x_c is of BKT type, due to fixed point merger.



Near Banks-Zaks (IR) fixed point:

QCD:

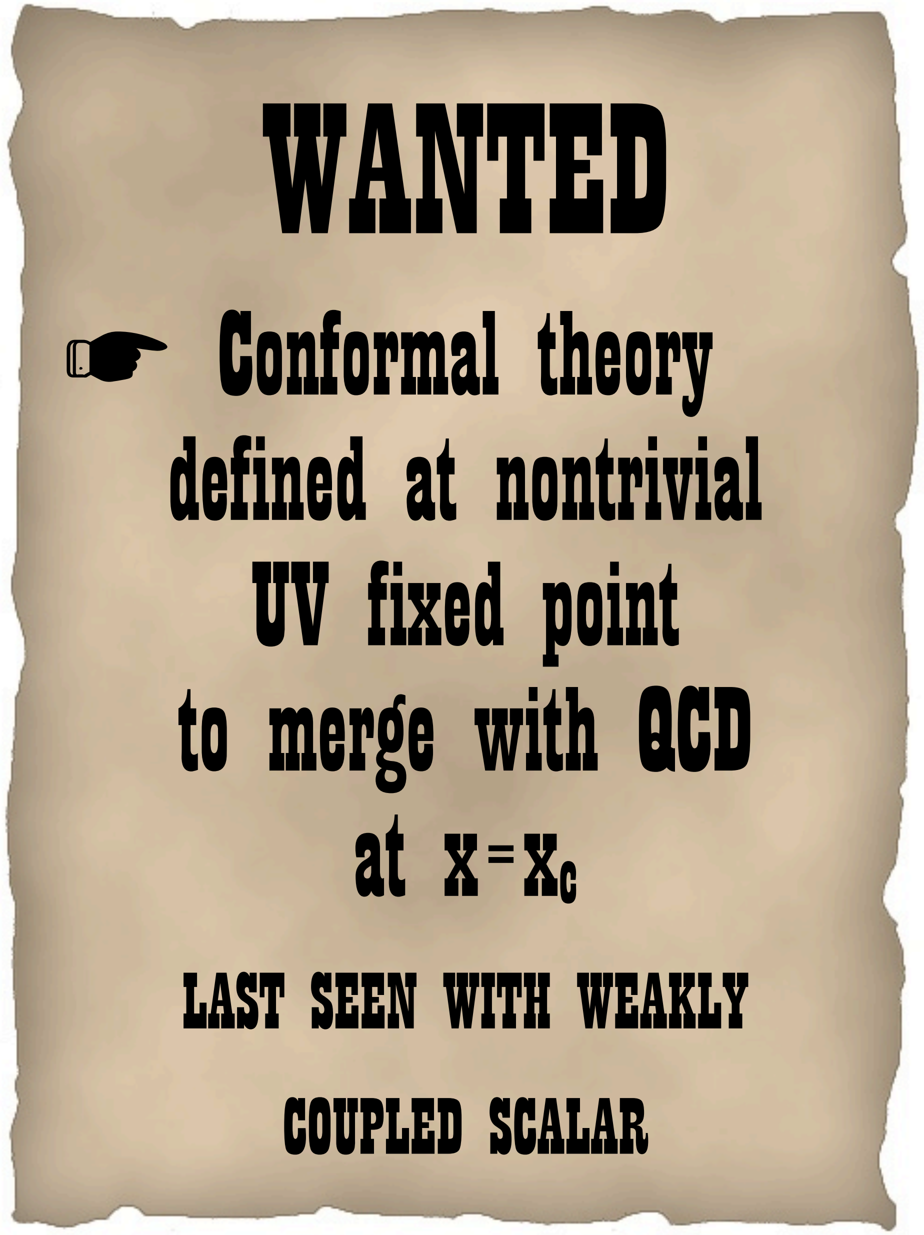
$$\Delta^+_{\psi\bar{\psi}} = 3 - \#\, g^2 N_c$$

(almost free quarks)

Partner theory QCD*:

$$\Delta^-_{\psi\bar{\psi}} = d - \Delta^+_{\psi\bar{\psi}} = 1 + \#\, g^2 N_c$$

(almost free scalar?)



Haven't found a perturbative example with:

(i) weakly coupled scalar;

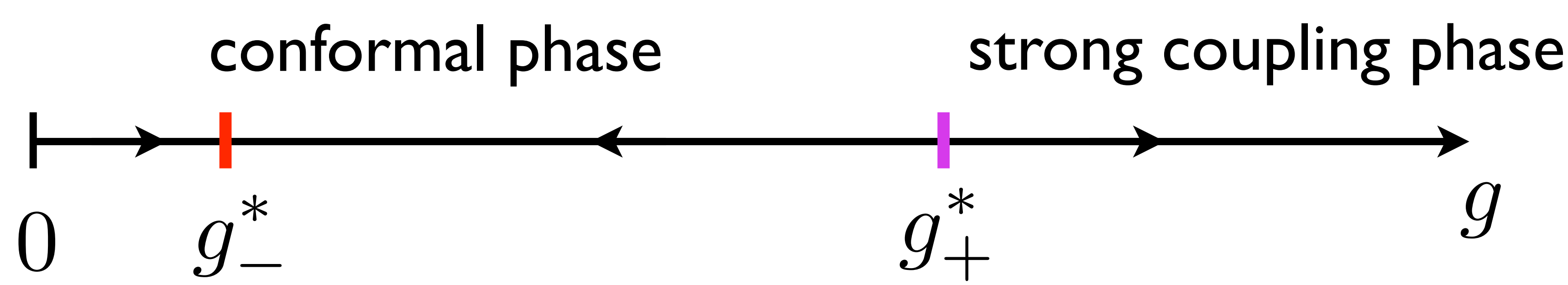
(ii) full $SU(N_f)xSU(N_f)$ chiral symmetry

(iii) Matching anomalies

Look for it on the lattice?

One place to start: strong/weak transition for QCD with N_f in conformal window?

(A. Hasenfratz)



QCD* possibly at $g_+{}^*$?

Conclusions:

I. Fixed point annihilation appears to be a generic mechanism for the loss of conformality

II. Leads to similar scaling as in the BKT transition: $\Lambda_{IR} \sim \Lambda_{UV}\, e[-\pi/\sqrt{(-\kappa-\kappa_*)}]$

III. Both relativistic & non-relativistic examples

IV. Analog in AdS/CFT; implications for AdS below the Breitenlohner-Freedman bound?

V. Implications for QCD with many flavors? Is there a pair of conformal QCD theories? What is QCD*?

VI. Finding QCD* should be on field theory / lattice QCD "to-do" list.


E Paradiso ab Angelo pelluntur. Genes. III.