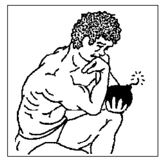
Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

The Enigma of Entropy

Deepak Dhar, Professor
Department of Theoretical Physics
Tata Institute of Fundamental Research
Homi Bhabha Road
Mumbai 400005, India
Email: ddhar@theory.tifr.res.in

One of the most mysterious laws of Nature is the second law of thermodynamics. There are several equivalent formulations of this law. For our present discussion, it is enough to take the Clausius formulation that says that for an isolated system evolving in time, the entropy cannot decrease. Many of you have encountered this in your BSc textbooks already. Let me explain my reasons for calling it enigmatic. I do not have any answers. I just want to say why I think that it is an interesting question.

Originally published in Crescendo,*TIFR Student Society's Magazine,* 2012.

Let me explain myself using a parable. A foreign scientist is visiting a laboratory in China. In the daytime, he discusses his work with his hosts, and in the evenings, he has to find some place to eat. Unfortunately, he knows no Chinese.

So, he goes to a restaurant, sits down at a table, and the waiter brings him the menu card. It has a very long menu. All kinds of dishes are listed, about one thousand in all, and the price of each in understandable numerals. Knowing what he is willing to spend, the man selects one of the entries in the correct price range at random,

Keywords
Entropy, second law of thermodynamics.

and points to it. Eventually, the waiter brings him the food, he eats and leaves. Next day, it is the same story. The food is satisfactory and the man has no complaints.

The food item chosen defines the 'microstate' of the system at the time.

But, after a few days, he discovers a new restaurant in the neighbourhood. It is the same price range, and the same quality of food. He tells his friends that he prefers this restaurant to the other. The friend knows how he places his order, and wants to know what is special about the second restaurant. The man says that there the menu is bigger, ten thousand entries, and he likes to have more choices.

Of course, one would think that the man was quite foolish. What difference does it make that the menu has one thousand or one million entries; after all, at one time, you are only going to eat one meal! The important point to emphasize is that thermodynamical systems, even those with no brains at all, do exactly this all the time.

As a paranthetical remark, let me add here that the description of a system of say gas molecules in a box 'having a wish to' increase their entropy is quite inappropriate, even though in textbooks one often encounters such expressions as "the electron wants to go to the lowest energy state". The electron has no brains, and in no effective sense could this be true. Also, if everybody wanted to lower their energy, where would the extra energy go? In my parable, this was done deliberately, as a rhetorical device. In a more serious discussion, one should ignore this deliberate anthropomorphic flavour added to the story. The amazing fact remains that 'having more choices' is such an important fundamental principle of Nature.

A physical system tends to evolve to a state of higher entropy.

Consider a known mass of a dilute gas, say hydrogen, kept in a box at a fixed temperature. Then, given this specification of the macroscopic system, we are able to

Entropy seems to be not a function of what is, but of what could have been.

determine other properties of the system, like the pressure or the viscosity of the gas, etc. However, given the macroscopic parameters characterizing the system, we cannot really tell the precise position and velocity of each individual molecule: there are very many 'microstates' corresponding to a fixed macrostate. According to a famous equation of Boltzmann, the entropy S is equal to $k \log \Omega$, where Ω is the number of microstates available to the system. So, the entropy is not determined by the current state (the actual microstate), but by what other states it could have been in. It seems to be not a function of what is, *but of what could have been*!

Often one explains the notion of entropy by saying that it is a measure of disorder. The system is prepared in a macrostate, but it could have been in any of the microstates of the system. When the system is observed after some time, we can only say that each microstate occurs with probability $1/\Omega$. To any such probability distribution, in which microstate i occurs with probability p_i, we can define a quantity, the Shannon entropy $S_{\text{Sh}} = \sum_i p_i \log p_i$. But this measures the uncertainty the observer feels, given the limited information. It seems to be a measure of disorder in the head of the observer, not in the system.

Continuing our story, when our hero is asked to explain what the point of having more choices is, he says, "Well, my boss in the lab is a very nosey person. I think he wants to know exactly what I eat, even though it is none of his business. I chose the second restaurant, just to spite him." This explanation seems not really convincing. In fact, most of the time, the boss (the external observer) does not want to know the details of what the person ate, and maybe the hero is not really so full of negative emotions, like spitefulness. One is reminded of Einstein's words: "Subtle is the Lord, but malicious He is not." For the more agnostic readers, 'Lord' in the above quotation may be replaced by 'Nature'.

Entropy as a measure of lack of information about the system.

In fact, for systems that undergo deterministic evolution, the microstate at a later time is fully determined by the present, and there is no need to invoke any probability ideas. In this scenario, the hero of our story has a simpler method of making the selection: he selects the item with the right price that comes next in the menu to what he chose on the previous day. Here he has no choice whatsoever. In that case, why does he still 'prefer' the second restaurant over the first?

Deterministic evolution also tends to increase entropy.

That is the mystery.