Homework 1: Lagrangians and Group Theory

4 November 2019

Problem 1: Lagrangians in Classical Mechanics

1. Write the general lagrangian for a single particle in cyclical co-ordinates in 2D and show that the angular momentum is a conserved quantity if the potential depends only on the radial co-ordinate.

2. Re-write the lagrangian in Cartesian co-ordinates. What does the conserved angular momentum correspond to?

Problem 2: Lagrangian for a complex scalar field

The Lagrangian for a complex scalar field ϕ is given by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2}M^2 \phi^* \phi \tag{1}$$

1. Rewrite ϕ in terms of component fields $\phi_1 + i\phi_2$. What does this Lagrangian correspond to? Write it in terms of column matrix

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

2. Check that the Lagrangian in (1) is invariant under the transformation $\phi \to e^{i\alpha}\phi$.

3. What does this transformation correspond to in terms of Φ?

Problem 3: Nœther Currents for scalar fields

1. For the transformation of the complex field given by $\phi \to e^{i\alpha}\phi$, assuming α is small, find the transformations of the components ϕ_1 and ϕ_2

2. Calculate j^μ for the complex scalar as well as for the two real scalar system using definition in equation (1.12) in the notes.

Problem 4: Degrees of freedom

Find the number of independent entries in $R \subset \mathrm{SO}(N)$

Problem 5: SO(3) Group

Check that

1. the rotation matrices are recovered by exponentiating the so(3) generators.
2. They follow the commutation relation $[\tau_i, \tau_j] = i\epsilon_{ijk}\tau_k$

Problem 6: SU(N)

Show that the number of independent entries in U is $N^2 - 1$. This means one needs $N^2 - 1$ generators to get all possible elements.