Homework 2: DIS and QCD

4 November 2019

Problem 1: $e^-(k)\mu^-(p) \to e^-(k')\mu^-(p')$ in lab frame

Using the expressions derived in class, prove that

$$L^e_{\mu\nu} L^{m,\mu\nu} = 8EE' \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right)$$

Problem 2: $e^-(k)p(p) \to e^-(k')X$ in lab frame

Using the expressions for $L^e_{\mu\nu}$ and $H^{\mu\nu}$ derived in class, prove that

$$L^e_{\mu\nu} H^{\mu\nu} = 4EE' \left(H_2 \cos^2\frac{\theta}{2} - 2H_1 \sin^2\frac{\theta}{2} \right)$$

Problem 3: $e^+e^- \to q\bar{q}g$

Complete the calculation for the process $e^+(p_1)e^-(p_2') \to \bar{q}(p_3)g(p_4)q(p_5)$ where besides a quark q and an anti-quark $\bar{q}$ a gluon g is produced through a virtual photon.

1. Write down the amplitudes corresponding to the Feynman diagrams (see appendix for a selection of Feynman rules). Show that for each diagram n the amplitude $i\mathcal{M}_n$ can be written as contraction of a leptonic with a hadronic matrix element

$$i\mathcal{M}_n = \frac{g_s Q_e Q_q}{s} \mathcal{L}_\mu \mathcal{H}^\mu_n \,.$$

2. Calculate the four different terms from the hadronic tensor $\mathcal{H}^{*\mu}_i \mathcal{H}^\nu_j$ up to the trace expression (where $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$).

3. We showed in class that the colour factor factorises out so that $\mathcal{H}$ can be written as

$$H^{\mu\nu} = \sum_{s_1,s_3,\lambda} \mathcal{H}^{*\mu}\mathcal{H}^\nu = |C|^2 R^{\mu\nu} \,,$$

where the colour amplitude square $|\mathcal{C}|^2 = \sum \mathrm{Tr}(\mathrm{t^a t^a}) = \mathrm{C_F}$ contains only fully contracted generators t^a and the tensor $R^{\mu\nu}$ is built from the hadronic Lorentz part of the amplitude. We can further write down

$$R^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) K$$

To calculate the value of K, we can use $R_{\mu\nu} g^{\mu\nu} = \left(g^\mu_\mu - \frac{q^\mu q_\mu}{q^2}\right) K = 3K$, i.e. one needs only to calculate R^μ_μ. Simplify the above trace expressions and calculate the contribution to R^μ_μ resulting from all the terms.

4. Sum up all contributions to R^μ_μ in order to write down the differential cross-section for the production. Use kinematic invariants (i.e. x_i's defined in class) which are suitable for the phase space integration.

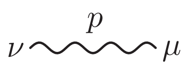
5. Using the expression for 3-body phase space

$$\mathrm{d}R_3 = \frac{s}{16} \frac{1}{(2\pi)^3} \mathrm{d}x_1 \mathrm{d}x_2\,.$$

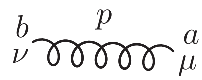
show that the cross section for N_F quarks can be written as

$$\sigma = \frac{s}{32(2\pi)^3} \int \mathrm{d}x_1 \mathrm{d}x_3 C_F N_F \left(\frac{g_s Q_e Q_q}{s}\right)^2 \left[\frac{8}{3} \frac{x_1^2 + x_3^2}{(x_1 - 1)(x_3 - 1)}\right]\,.$$

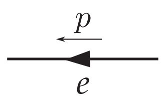
Appendix A: Selected Feynman Rules in the massless limit



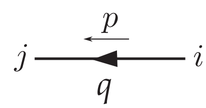
$$= \frac{-i}{p^2+i\varepsilon} g^{\mu\nu} ,$$



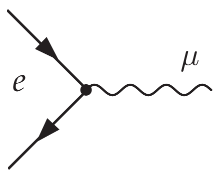
$$= \frac{-i}{p^2+i\varepsilon}\delta_{ab} g^{\mu\nu} ,$$



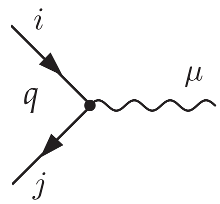
$$= \frac{i\not{p}}{p^2+i\varepsilon} ,$$



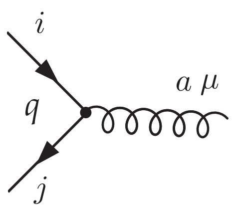
$$= \frac{i\not{p}}{p^2+i\varepsilon}\delta^j{}_i ,$$



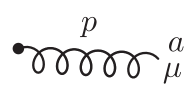
$$= -iQ_e\gamma_\mu ,$$



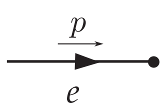
$$= -iQ_q\gamma_\mu\delta^j{}_i ,$$



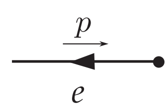
$$= -ig_s\gamma_\mu(T^a)^j{}_i ,$$



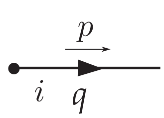
$$= [\varepsilon^*(p,\lambda)]^\mu_a .$$



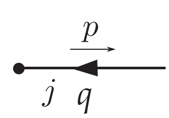
$$= u_e(p,s_b) ,$$



$$= \overline{v}_e(p,s_a) ,$$



$$= [\overline{u}_q(p,s_2)]_i ,$$



$$= [v_q(p,s_1)]^j .$$

Appendix B: Completeness relations in the massless limit

$$\sum_s [v_q(p,s)]^j[\overline{v}_q(p,s)]_i = \not{p}\delta^j{}_i , \quad \sum_s v_e(p,s)\overline{v}_e(p,s) = \not{p} ,$$

$$\sum_s [u_q(p,s)]^j[\overline{u}_q(p,s)]_i = \not{p}\delta^j{}_i , \quad \sum_s u_e(p,s)\overline{u}_e(p,s) = \not{p} ,$$

$$\sum_\lambda [\varepsilon(p,\lambda)]^\mu_a[\varepsilon^*(p,\lambda)]^\nu_b = -g^{\mu\nu}\delta_{ab} .$$