Towards QCD thermodynamics using exact chiral symmetry on lattice

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**In collaboration with Debasish Banerjee and Sayantan Sharma, TIFR, Mumbai*

arXiv : 0803.3925, to appear in Phys. Rev. D, & in preparation.

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Introduction : Why Exact Chiral Symmetry ?

Overlap and Domain Wall Fermions

Our Results

Summary

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- Quest for Quark-Gluon Plasma : Heavy Ion Collisions at SPS, RHIC and LHC.
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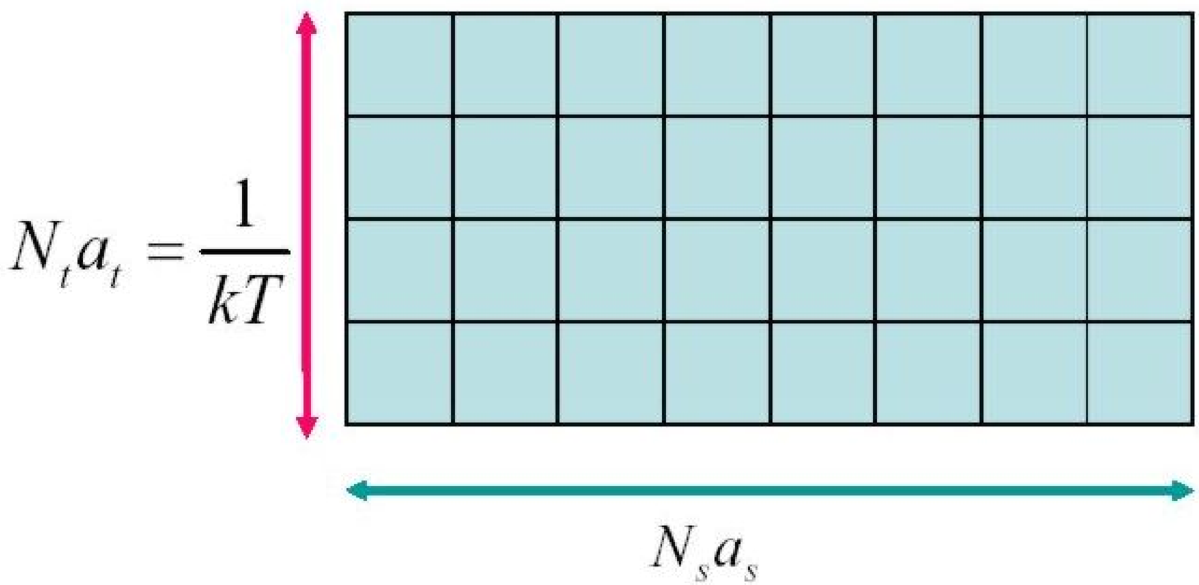
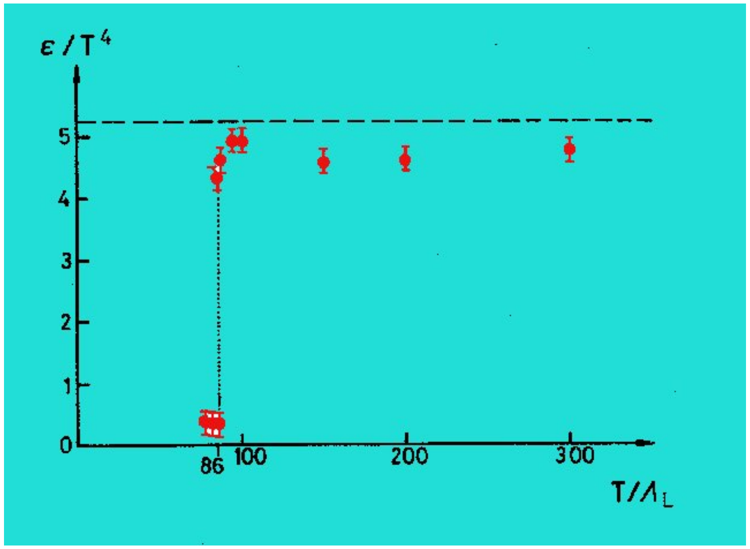


- Completely parameter-free : Λ_{QCD} and quark masses from hadron spectrum.

EoS of QGP

- First results from Bielefeld :



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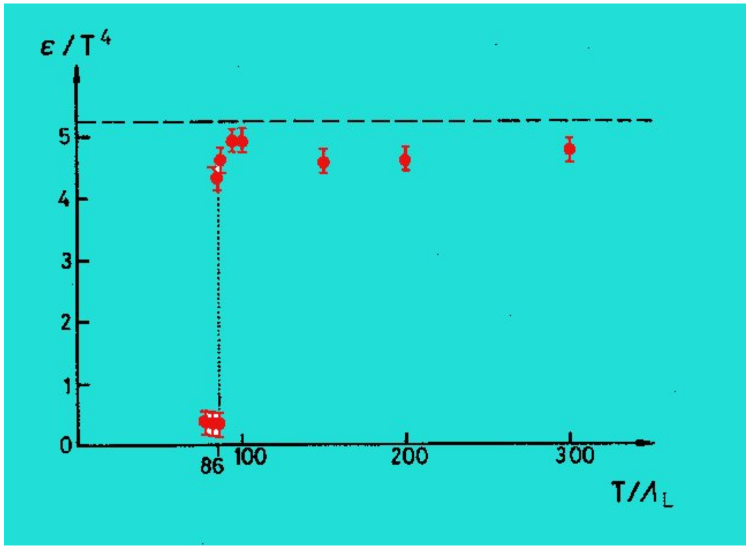

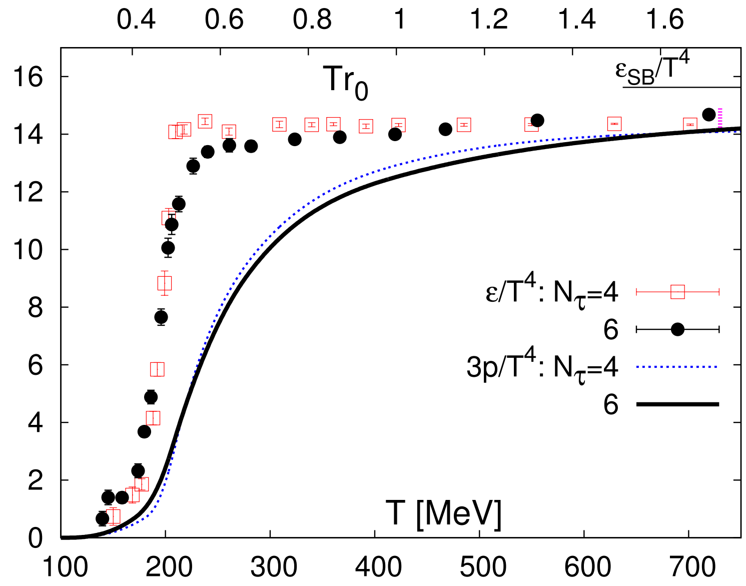

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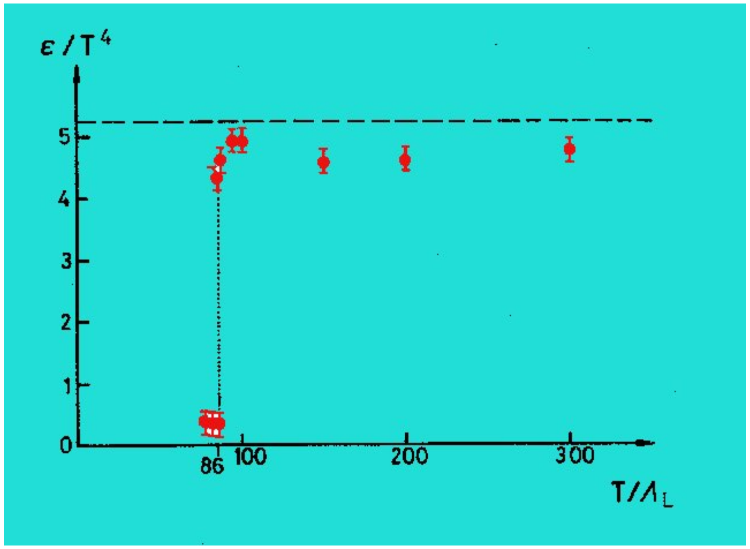

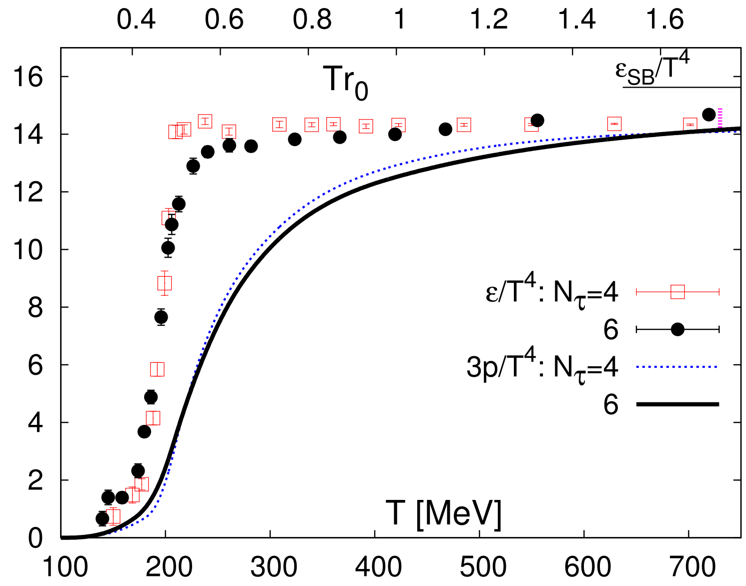

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- Recent results for EoS : N_t=6, Smaller quark masses. Small differences for N_t = 4 & 6; $\epsilon(T_c) \sim 6T_c^4$ still.

Baryon-Strangeness Correlation

♣Correlation between quantum numbers K and L can be studied through the ratio $C_{(KL)/L} = \frac{\langle KL \rangle - \langle K \rangle \langle L \rangle}{\langle L^2 \rangle - \langle L \rangle^2}$.

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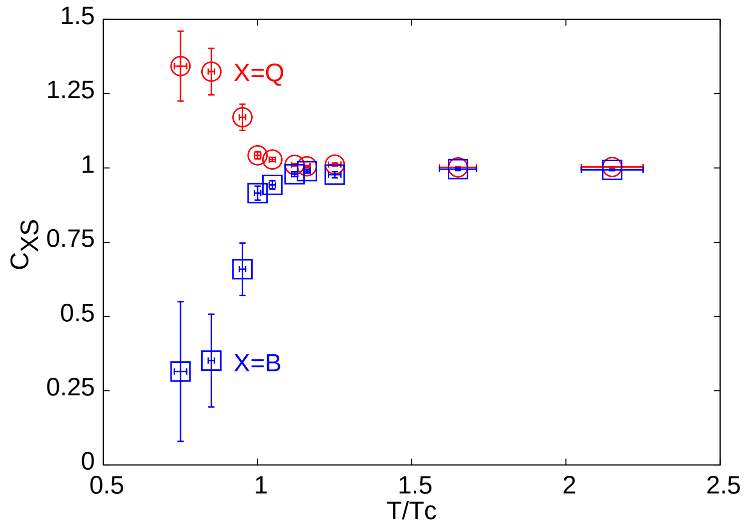

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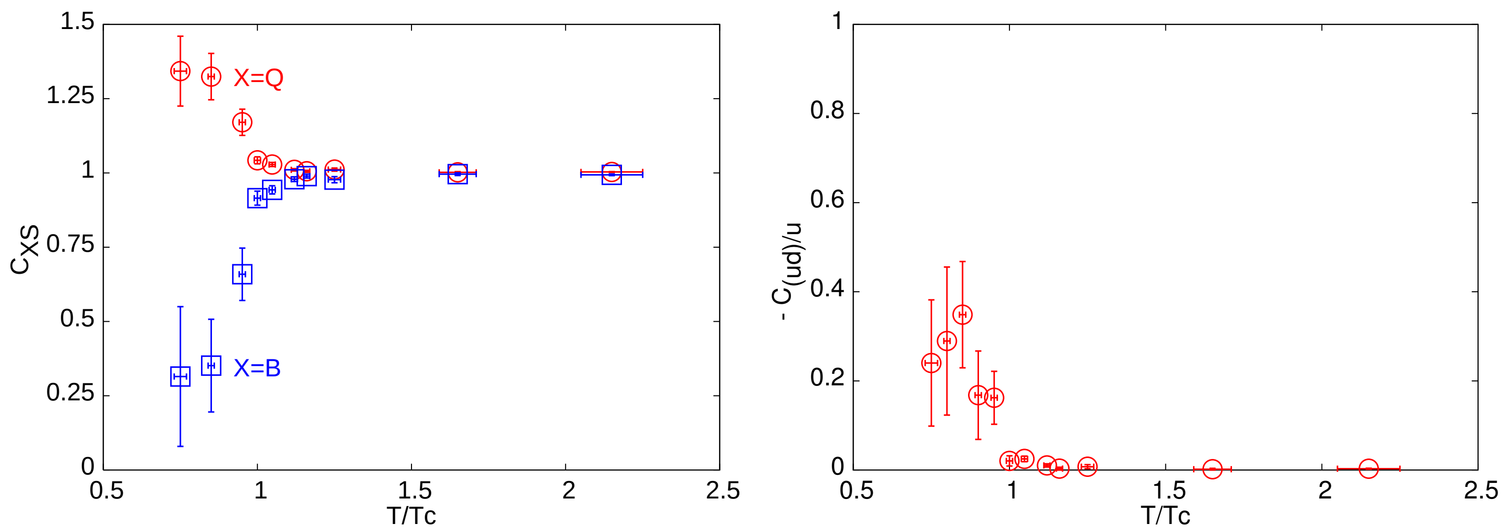

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- Is π really different in QGP ? or are there "artifacts" of lattice formulation dominating it ?

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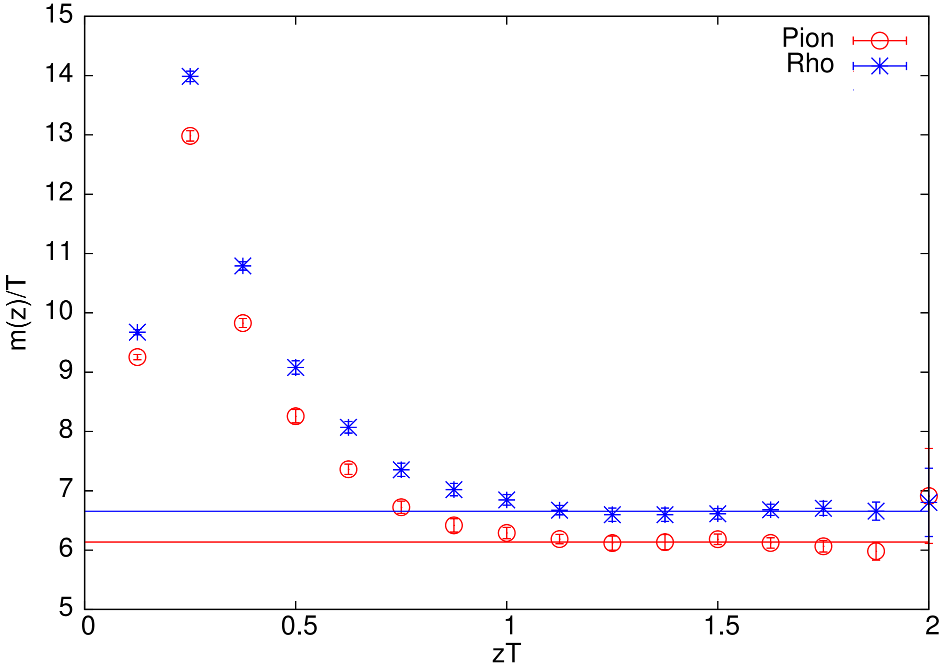
- Type of quarks ? Fermions on lattice have a well-known "No-Go" theorem due to Nielsen-Ninomiya : Popular choices
  - Wilson Fermions – Break *all* chiral symmetries.
  - Kogut-Susskind Fermions – Have some chiral symmetry *but* break flavour symmetry.
  - Overlap Fermions – *both* correct chiral and flavour symmetry on lattice.
  - Domain Wall Fermions – *small* violations of chiral symmetry [$\sim \exp(-L_5)$] with exact flavour symmetry on lattice.

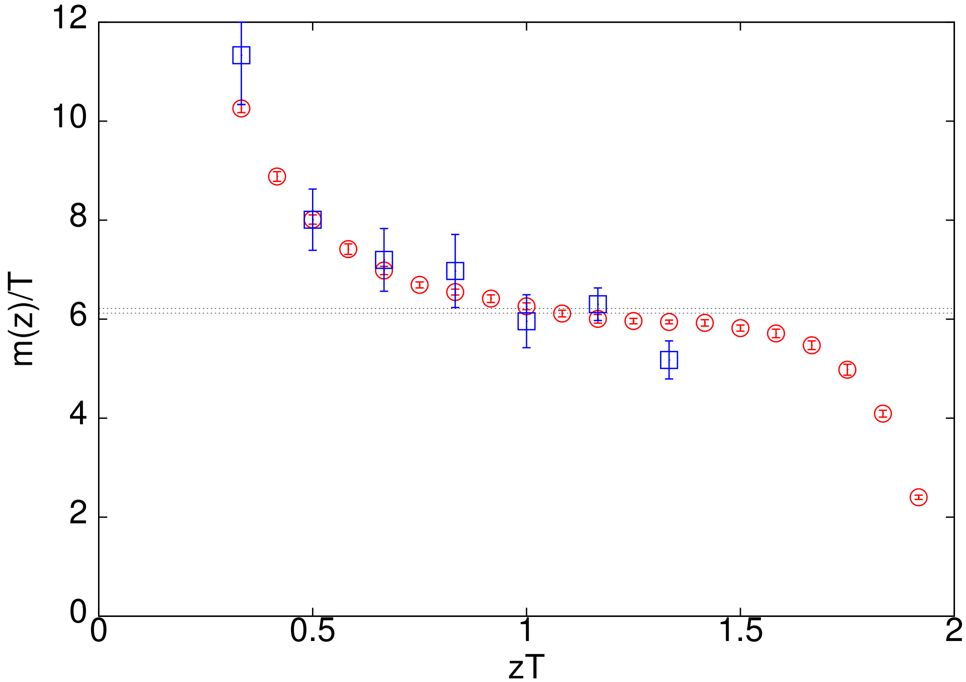
Overlap Compared with Staggered Fermions

♣ Local masses $[\sim \ln(C(r)/C(r+1)]$ show nice plateau behaviour for pi & rho for Overlap (left) unlike staggered (right) fermions.

Gavai, Gupta, Lacaze PRD 2008

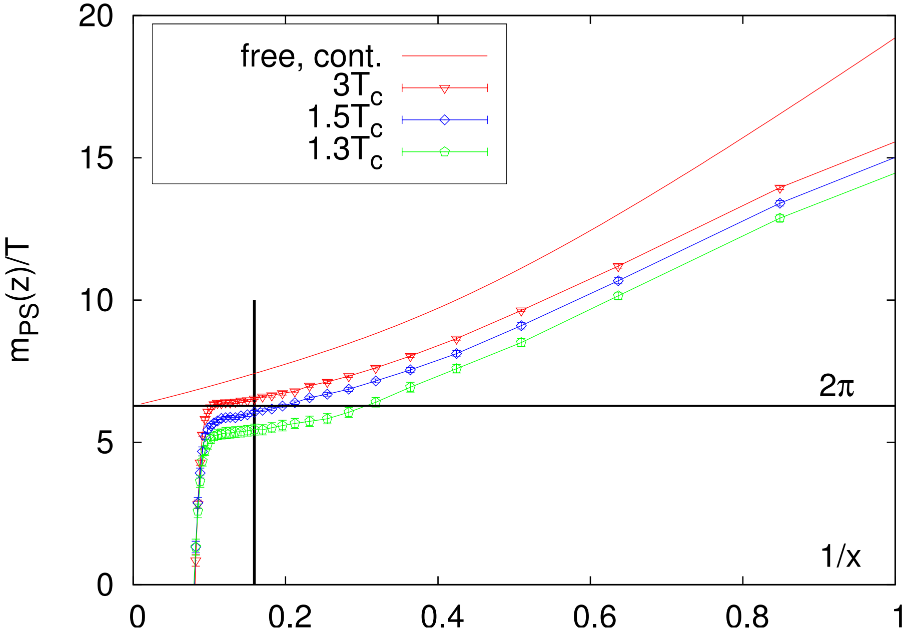
Gavai, Gupta PRD 2002

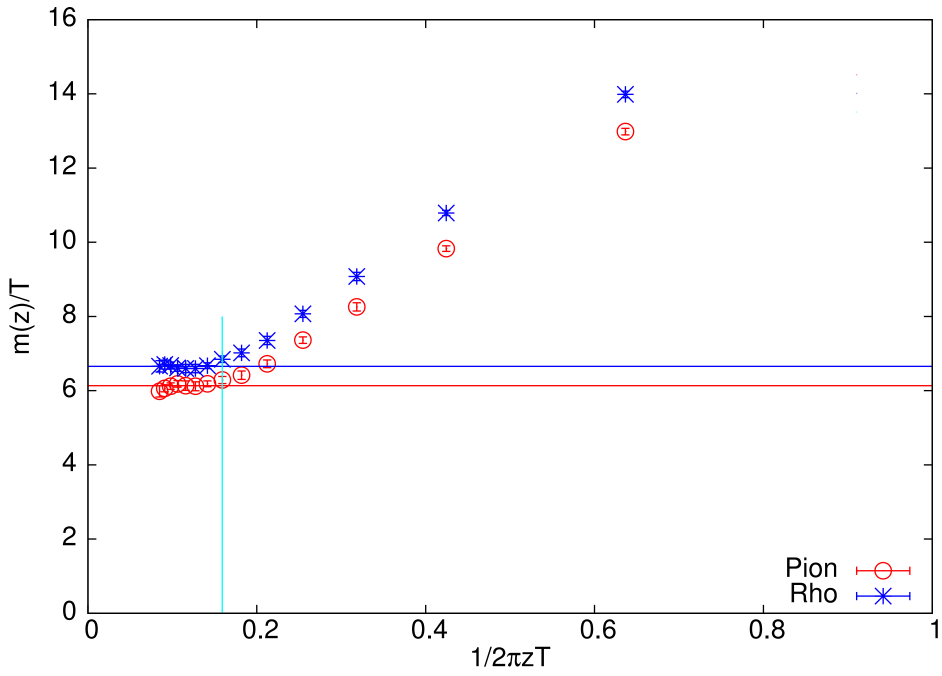




Overlap Compared with Wilson Fermions

♣ Wilson Fermions (Figure from PoS Lattice 2005, 164. (Bielefeld Group). Nice plateau behaviour for Overlap fermions (Gavai, Gupta, Lacaze PRD 2008).





Screening Masses Compared

♣ The pionic screening length shows significant a^2 corrections for staggered (left) unlike Overlap (right) fermions.

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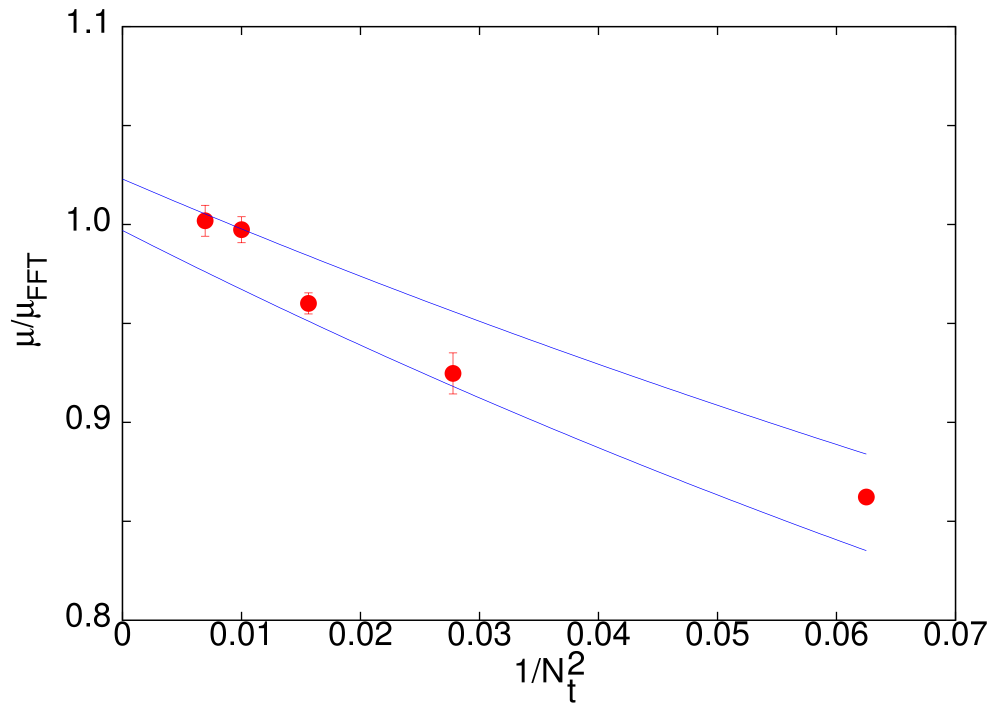

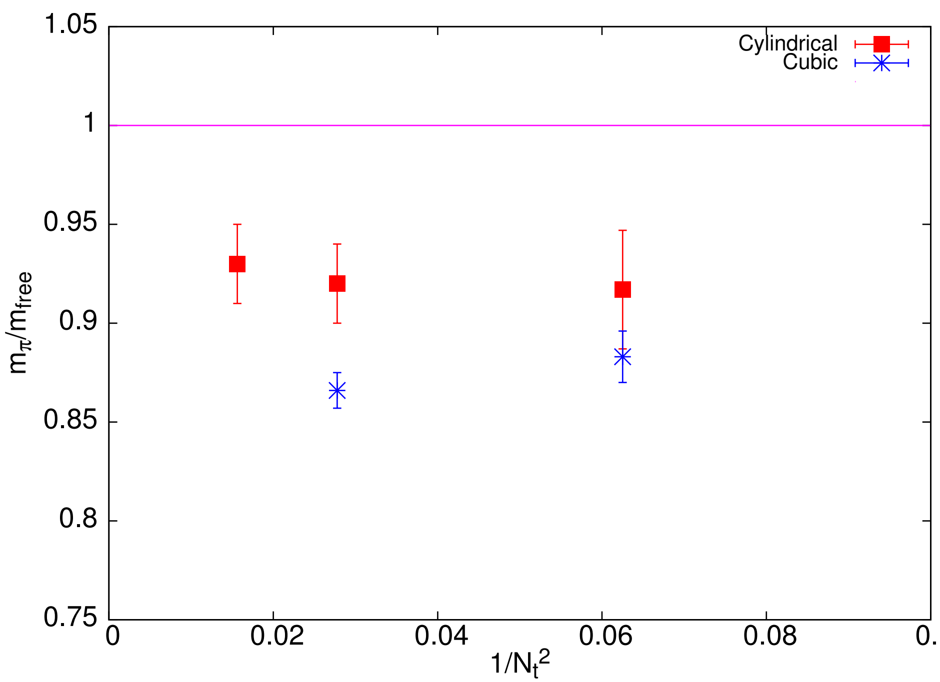

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♠ Another fundamental aspect – Critical Point in T-μ_B plane; based on symmetries and models.

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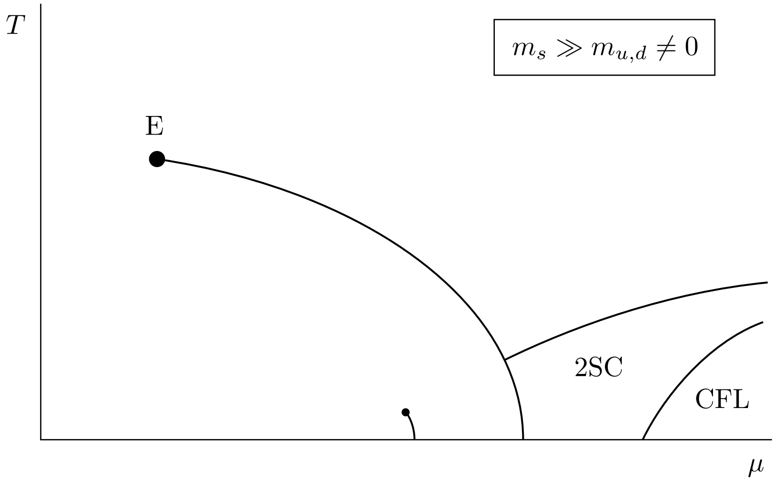

From Rajagopal-Wilczek Review

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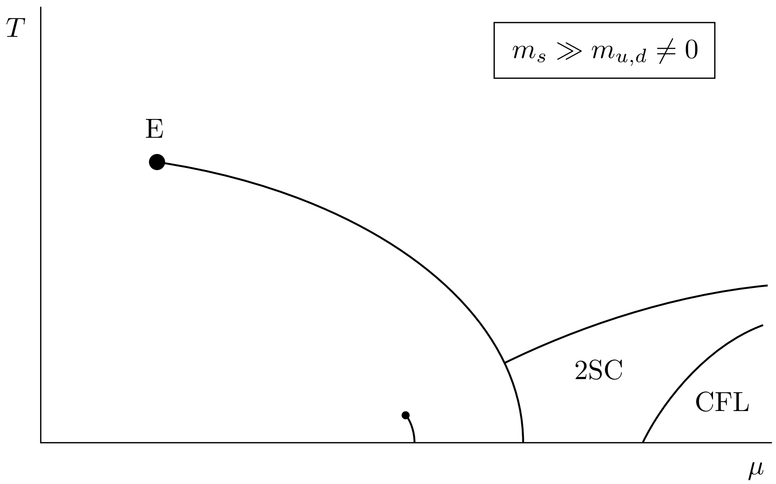

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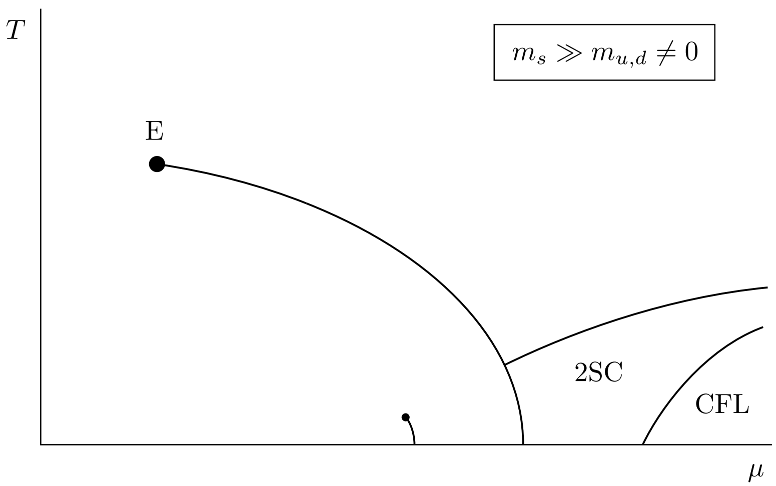

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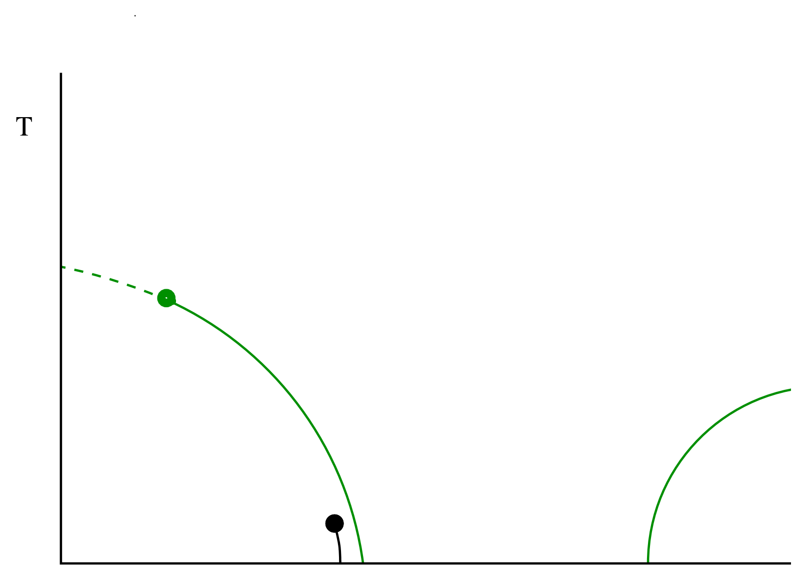

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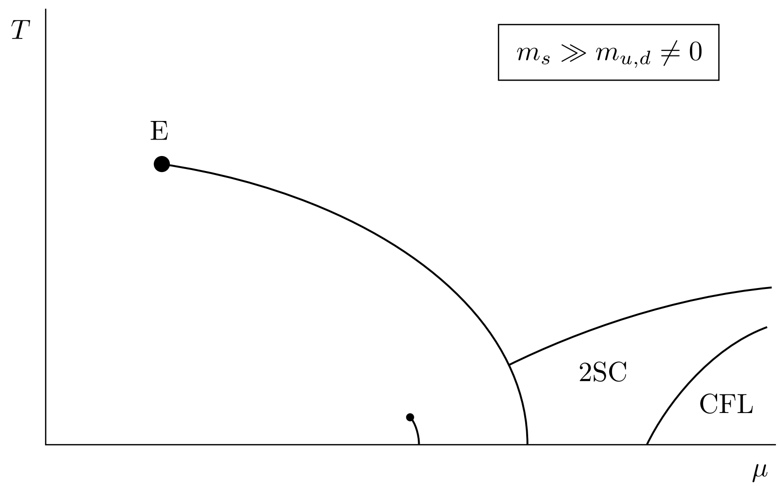

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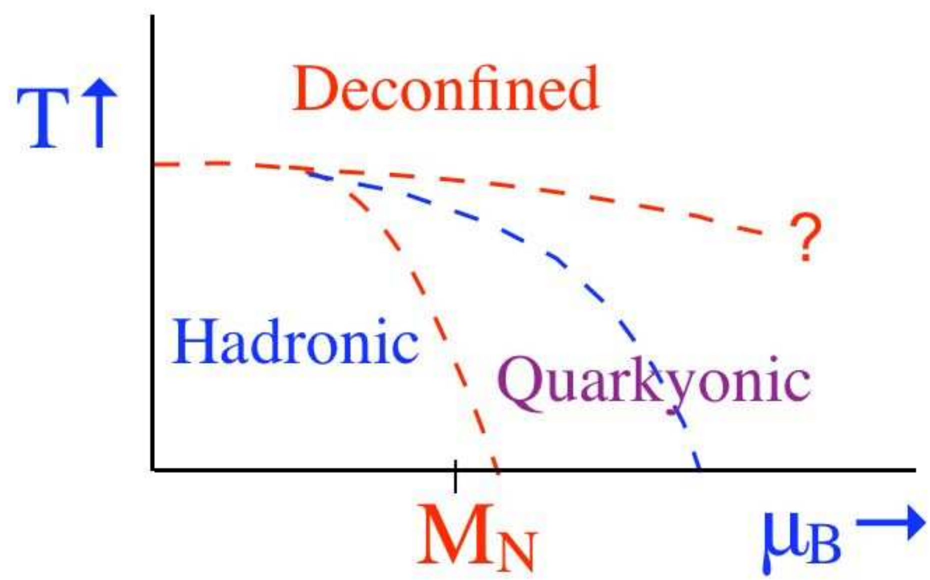

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$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D + D\gamma_5 - \frac{a}{2}D\gamma_5 D - \frac{a}{2}D\gamma_5 D\Big]_{xy} \psi_y = 0 \qquad (1)$$

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♠ Overlap fermions, and Domain Wall fermions in the limit of large fifth dimension satisfy this relation.

Overlap-Dirac Operator

♠ Neuberger (PLB 1998) proposed the overlap-Dirac operator :

$$aD = 1 + A(A^\dagger A)^{-1/2} = 1 + \gamma_5 \, \text{sign}(\gamma_5 A) \qquad \text{with} \quad A = aD_w, \qquad (2)$$

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$$aD_w = \frac{1}{2}\{\gamma_\mu(\partial^*_\mu + \partial_\mu) - a\partial^*_\mu\partial_\mu\} + M, \tag{3}$$

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♠ quark with a mass : $D(ma) = ma + (1 - ma/2)D$

Domain Wall Fermions

♠ Proposed by Kaplan (PLB 1992), a convenient form for Domain Wall fermion action (Shamir, NPB, 1993) is:

$$S_F = \sum_{s,s'=1}^{N_5} \sum_{x,x'} \bar{\psi}(x,s) D_{dw}(x,s;x',s') \psi(x',s') \ , \tag{4}$$

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$$D_{dw}(x,s;x',s') = [a_5 D_w + 1]\delta_{s,s'} - [P_- \delta_{s,s'-1} + P_+ \delta_{s,s+1'}] \ , \qquad (5)$$

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♠ Only light modes attached to the wall(s) are physical. Divide out heavy modes by having the $D_{dw}(am)/D_{dw}(am=1)$ as the effective Domain Wall operator in $\mathcal{Z}$.

♡ As outlined in Chiu, hep-lat/0303008, one can integrate out the fermionic fields in the fifth direction to rewrite the above ratio as

$$[(1+am) - (1-am)\gamma_5 \tanh(\frac{N_5}{2}\ln T)] \ , \tag{6}$$

with $T = (1 + a_5\gamma_5 D_w P_+)^{-1}(1 - a_5\gamma_5 D_w P_-)$.

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♡ Taking the limit $N_5 \to \infty$ for $a_5 = 1$, one obtains sign function of log T, proving that the DWF satisfy the Ginsparg-Wilson relation in this limit.

♡ Taking the limit $a_5 \to 0$ such that $L_5 = a_5 N_5 =$ constant, one can show $N_5 \ln T \to L_5\gamma_5 D_{dw}$. Further, for $L_5 \to \infty$, DWF reduce to the overlap fermions.

♡ We use this form in our numerical work.

Introducing Chemical Potential

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- Note $\gamma_5 D_w(a\mu)$ is no longer hermitian, requiring an extension of the sign function : For complex $\lambda = (x + iy)$, sign(λ) = sign (x).
- Gattringer-Liptak, PRD 2007, showed numerically that this has no μ^2 divergences for the free case (U =1) and with M = 1.

- We show this to be true analytically and for all M as well. Furthermore, this holds for all functions such that $K(a\mu) \cdot L(a\mu) = 1$. (Banerjee, Gavai, Sharma, PRD 2008)

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$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D(a\mu) + D(a\mu)\gamma_5 - \frac{a}{2} D(0)\gamma_5 D(a\mu) - \frac{a}{2} D(a\mu)\gamma_5 D(0) \Big]_{xy} \psi_y \ , \tag{7}$$

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which is not sufficient to make $\delta S = 0$. True for both Overlap and Domain Wall fermions and any K,L.

Our Results

- We investigated thermodynamics of free overlap and domain wall fermions with an aim to examine the continuum limit analytically and numerically.

- Analytic efforts to prove absence of μ^2-divergences for general K and L. Numerical results to tune the irrelevant parameter M to obtain small deviations from continuum limit on coarse lattices.

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- Energy density and pressure can be obtained from $\ln Z = \ln\ \det\ D_{ov}$ by taking T and V, or equivalently a_4 and a, partial derivatives.

- Dirac operator is diagonal in momentum space. Use its eigenvalues to compute $\mathcal{Z}$.

- Easy to show that $\epsilon = 3P$ for all a and a_4.

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- Hiding p_i-dependence in terms of known functions g, d and f, the energy density on an $N^3 \times N_T$ lattice is found to be

$$
\begin{aligned}
\epsilon a^4 &= \frac{2}{N^3 N_T} \sum_{p_i, n} \left[(g + \cos\omega_n) + \sqrt{d + 2g\cos\omega_n} \right] \\
&\times \left[\frac{(1 - \cos\omega_n)}{d + 2g\cos\omega_n} + \frac{\sin^2\omega_n (g + \cos\omega_n)}{(d + 2g\cos\omega_n)(f + \sin^2\omega_n)} \right] . \qquad (9)
\end{aligned}
$$

where ω_n are the Matsubara frequencies.

- I will show results for ϵ/ϵ_{SB} which is also P/P_{SB}.
- Hiding p_i-dependence in terms of known functions g, d and f, the energy density on an $N^3 \times N_T$ lattice is found to be

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where ω_n are the Matsubara frequencies.

- Can be evaluated using the standard contour technique or numerically.
- Continuum limit of the contour result shown to be ϵ_{SB}.

Numerical Evaluation

♣ Zero temperature contribution : as $N_T \to \infty$, ω sum becomes integral which we estimated numerically.

♣ Continuum limit by holding $\zeta = N/N_T = LT$ fixed and increasing N_T.

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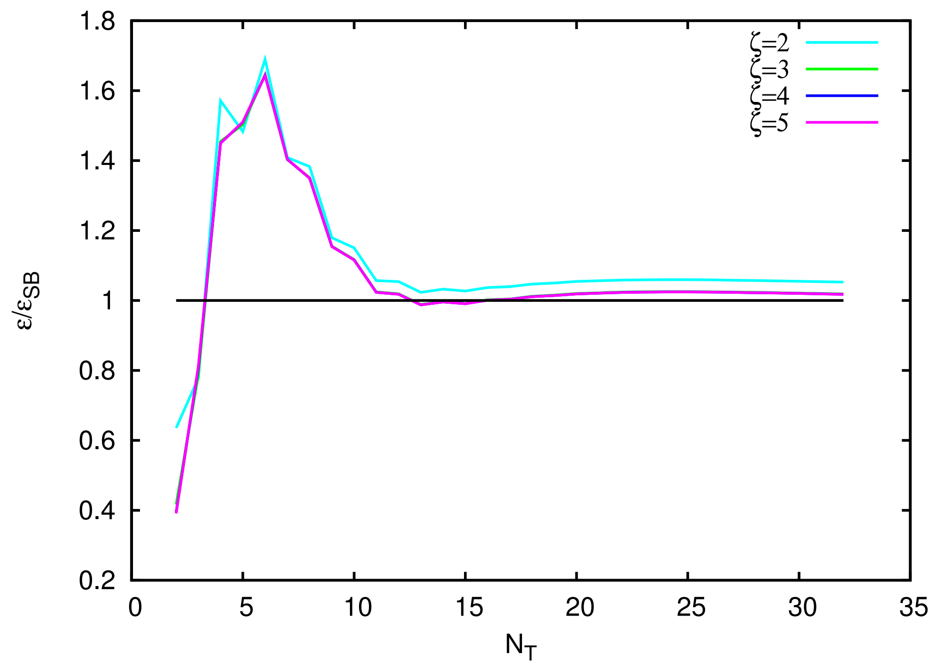

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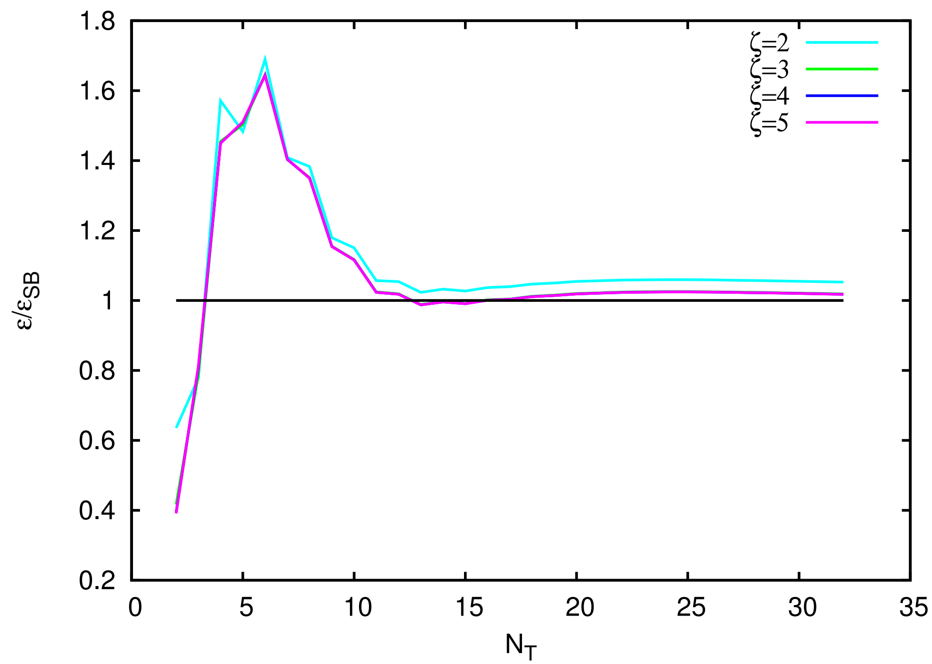
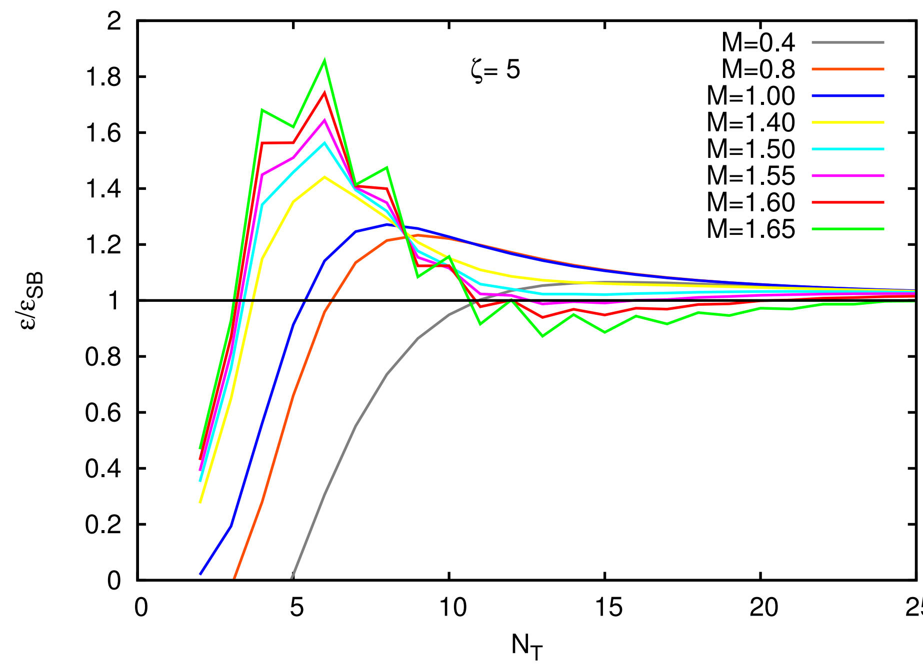
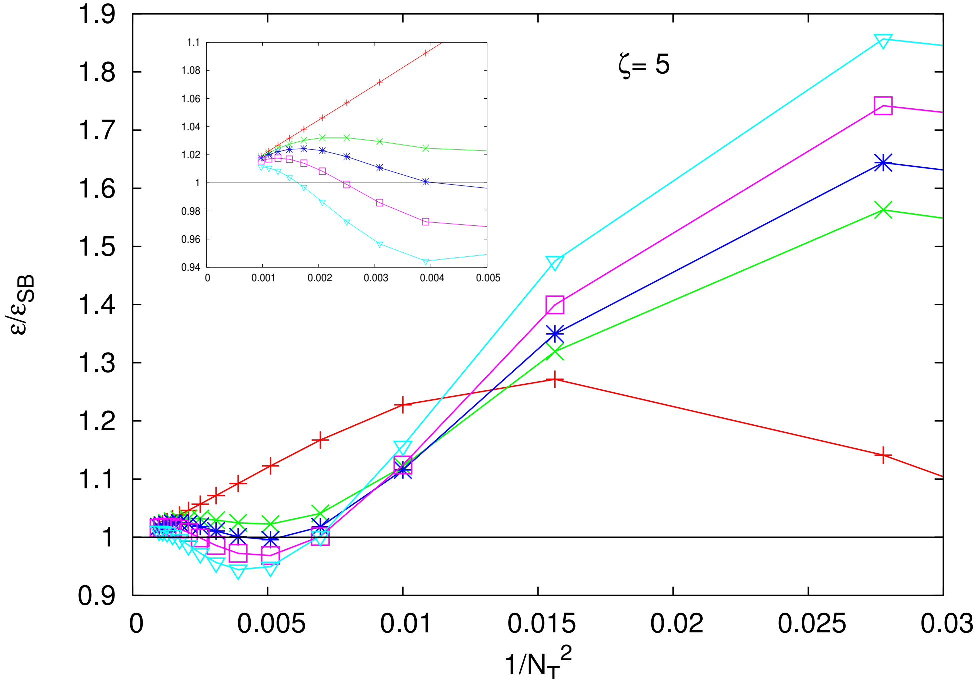




Approach to SB-Limit



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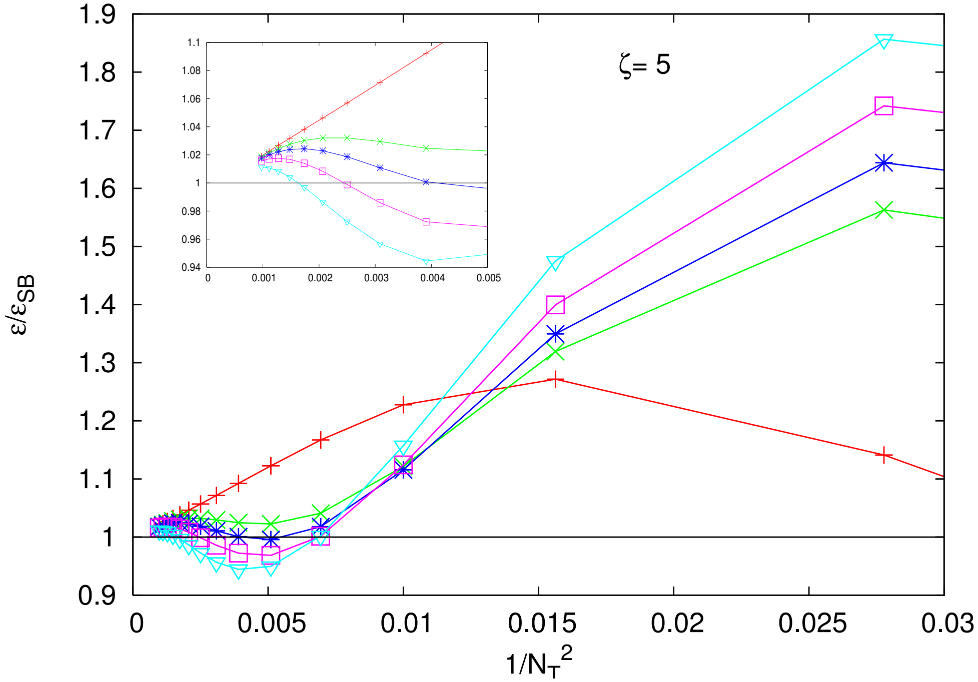
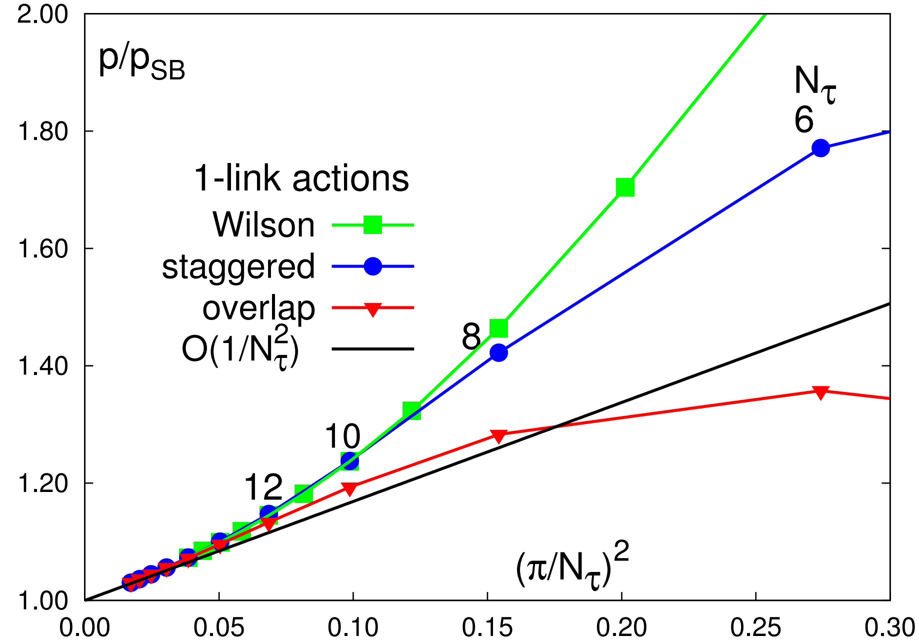


Banerjee, Gavai & Sharma , arXiv:0803.3925



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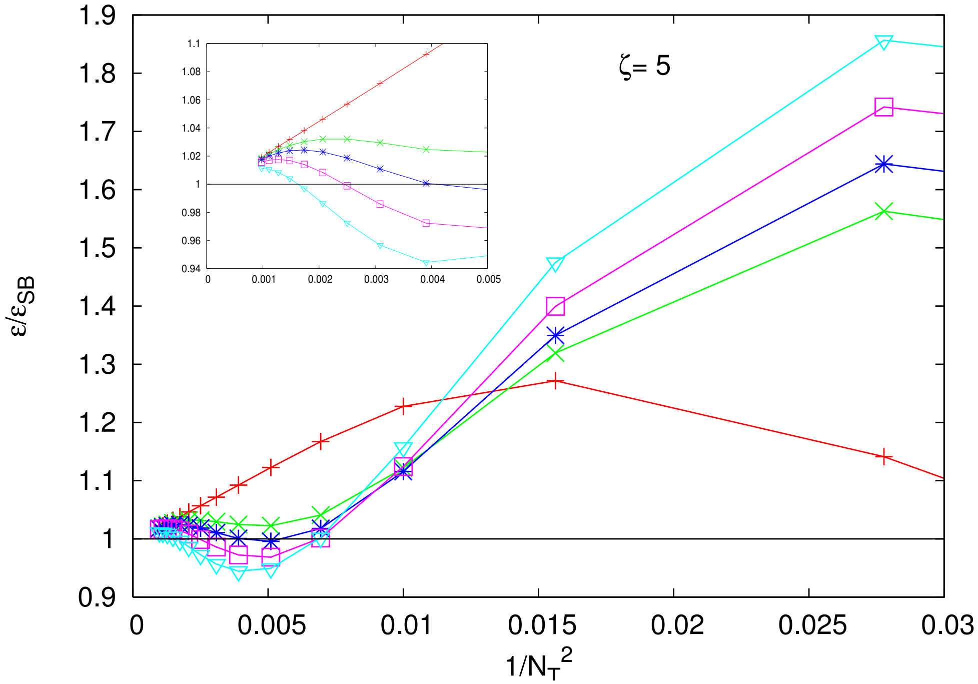

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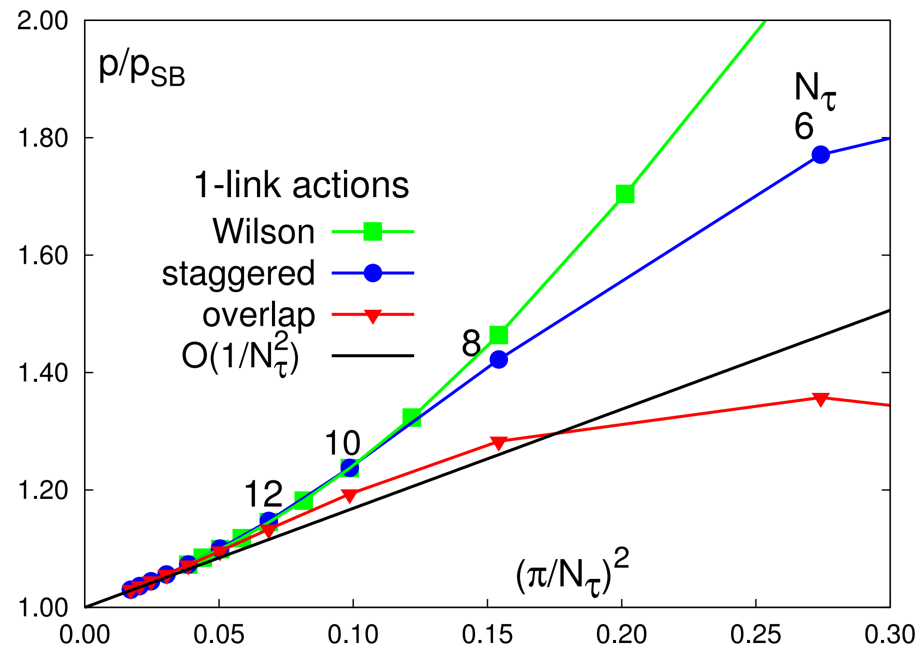

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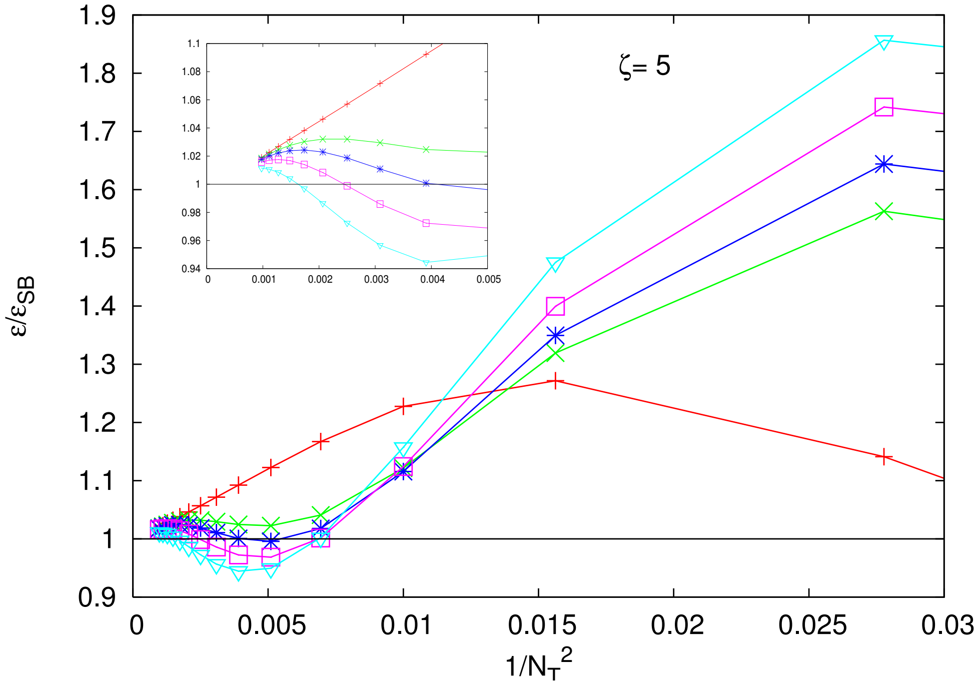

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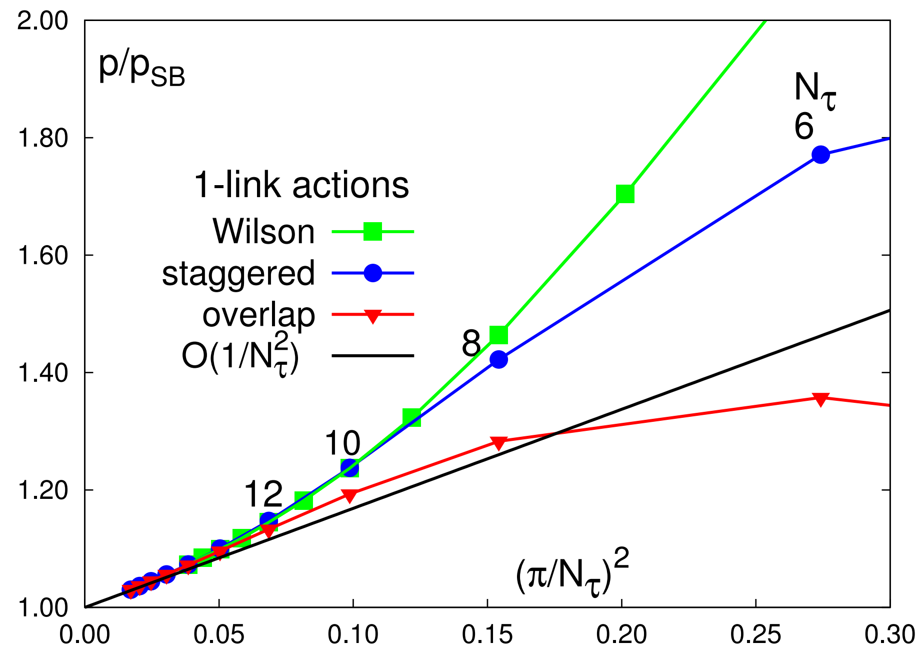

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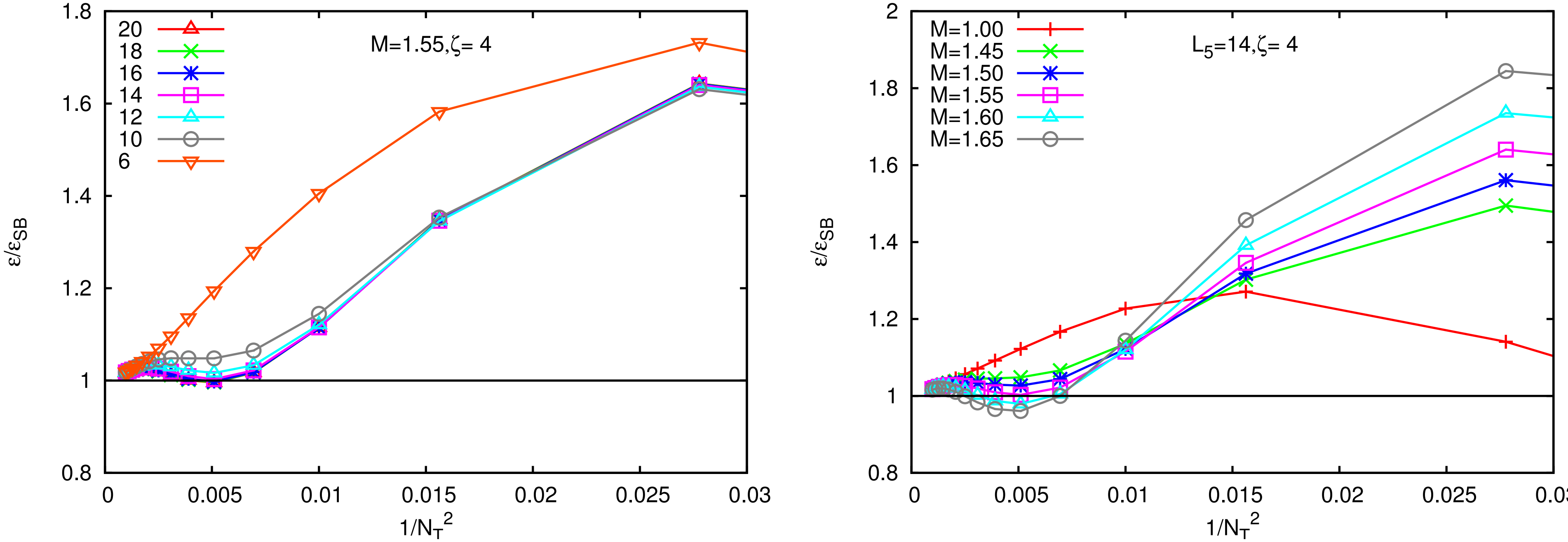
Domain Wall Fermions



Rajiv V. Gavai and Sayantan Sharma, in preparation.

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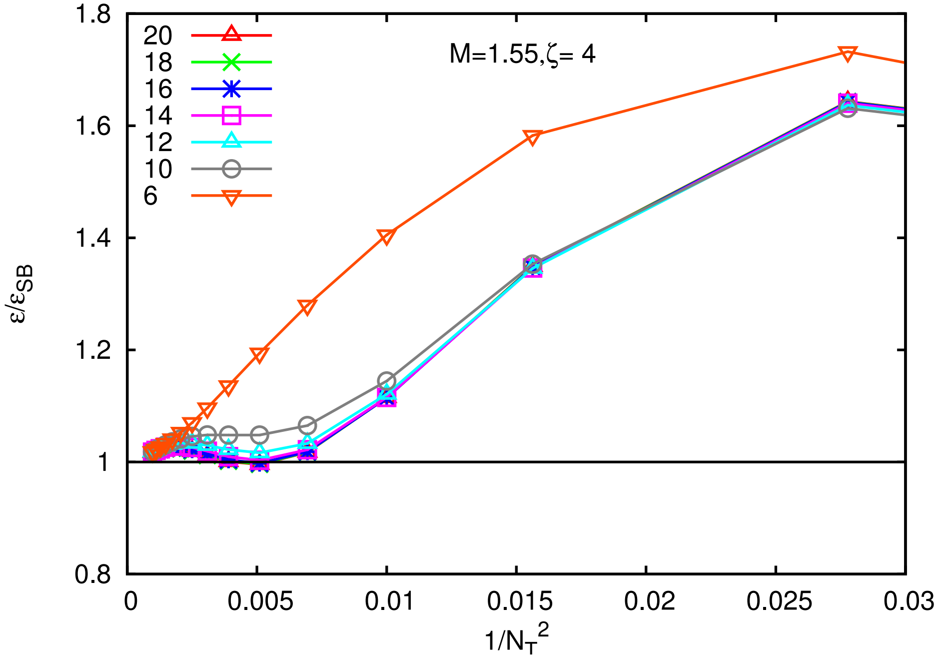

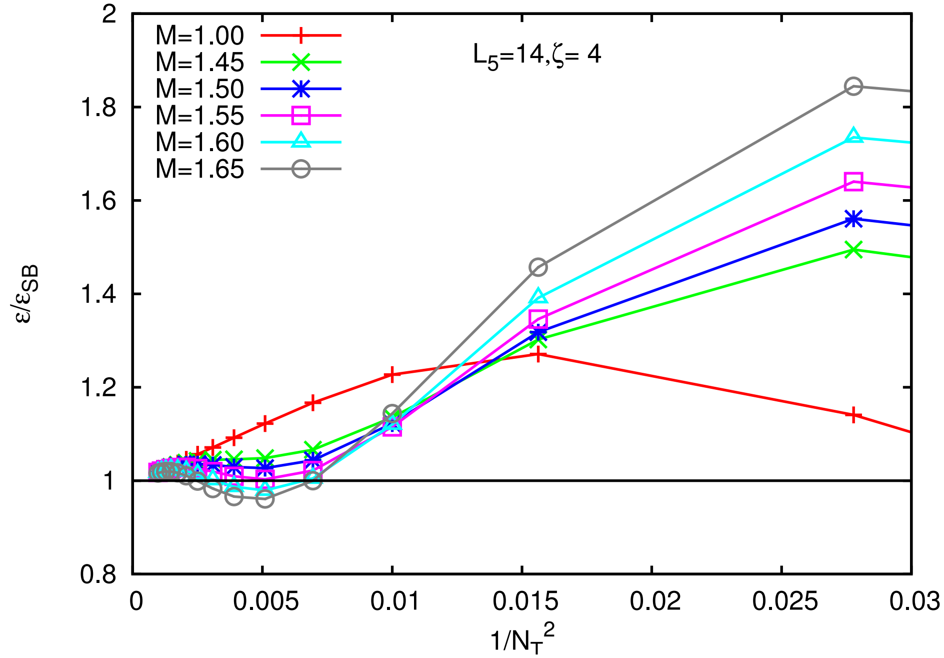

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◇ $L_5 \geq 14$ seems to be large enough to get L_5-independent results.

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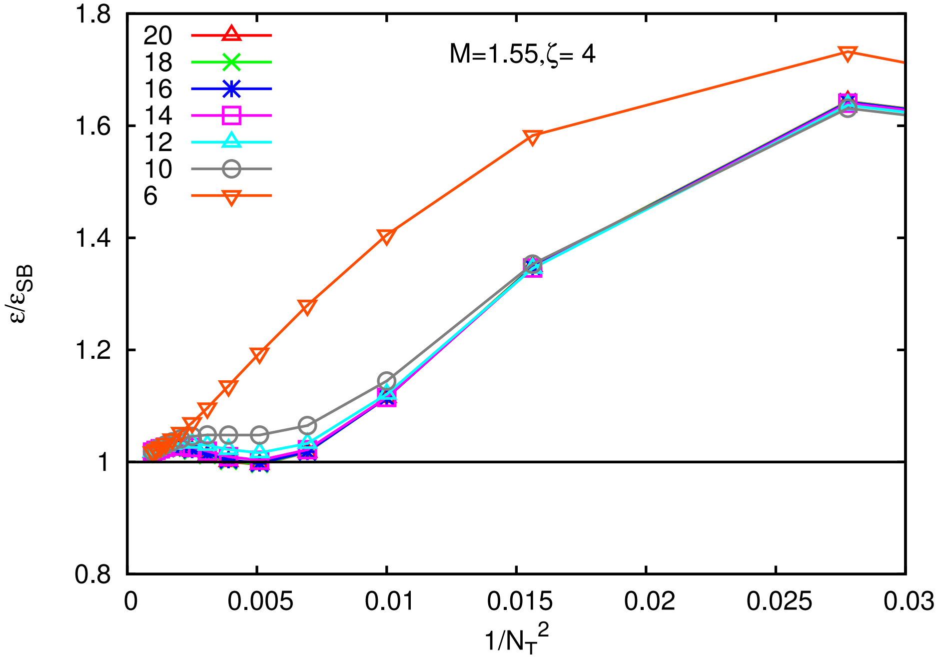

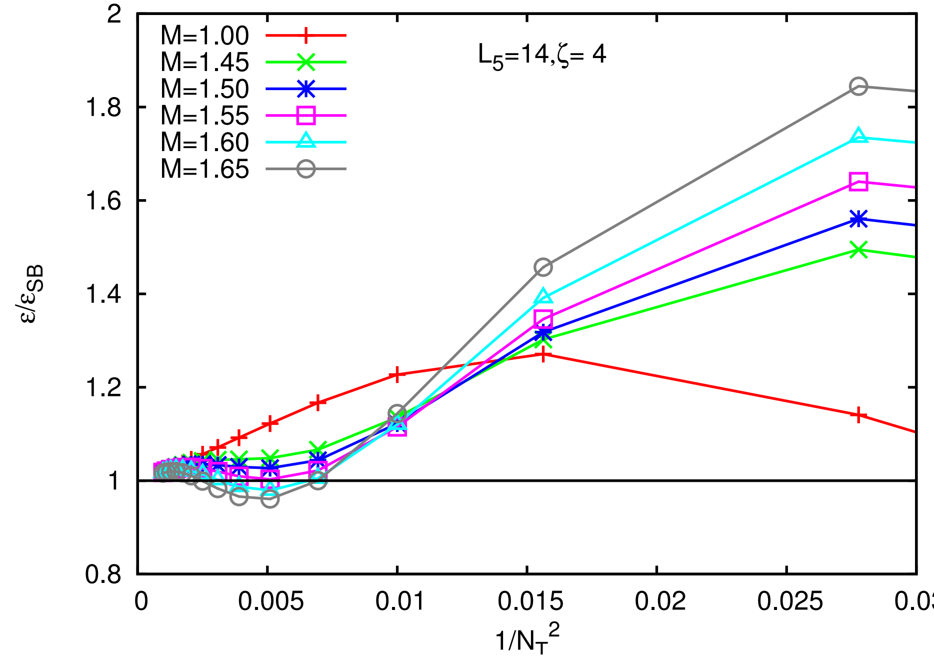

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◇ $L_5 \geq 14$ seems to be large enough to get L_5-independent results.

◇ Optimal range again seems to be $1.50 \leq M \leq 1.60$; $M = 1.9$ used by Chen et al. (PRD 2001) in their study of order paraemters of FTQCD.

$T = 0$, $\mu \neq 0$

- Doing the contour integral, the energy density turns out to be :

$$\epsilon a^4 = (\pi N^3)^{-1} \sum_{p_j} \Big[2\pi \mathrm{Res}\ F(R,\omega) \Theta \left(K(a\mu) - L(a\mu) - 2\sqrt{f} \right) + \int_{-\pi}^{\pi} F(R,\omega) d\omega - \int_{-\pi}^{\pi} F(1,\omega) d\omega \Big].$$

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- Generalization to $T \neq 0$ and $\mu \neq 0$ case straightforward. One merely needs two different contours depending on pole locations and value of θ.

Numerical Evaluation

$\diamondsuit$ Two Observables : $\Delta\epsilon(\mu, T) = \epsilon(\mu, T) - \epsilon(0, T)$ and Susceptibility, $\sim \partial^2 \ln \mathcal{Z}/\partial\mu^2$.

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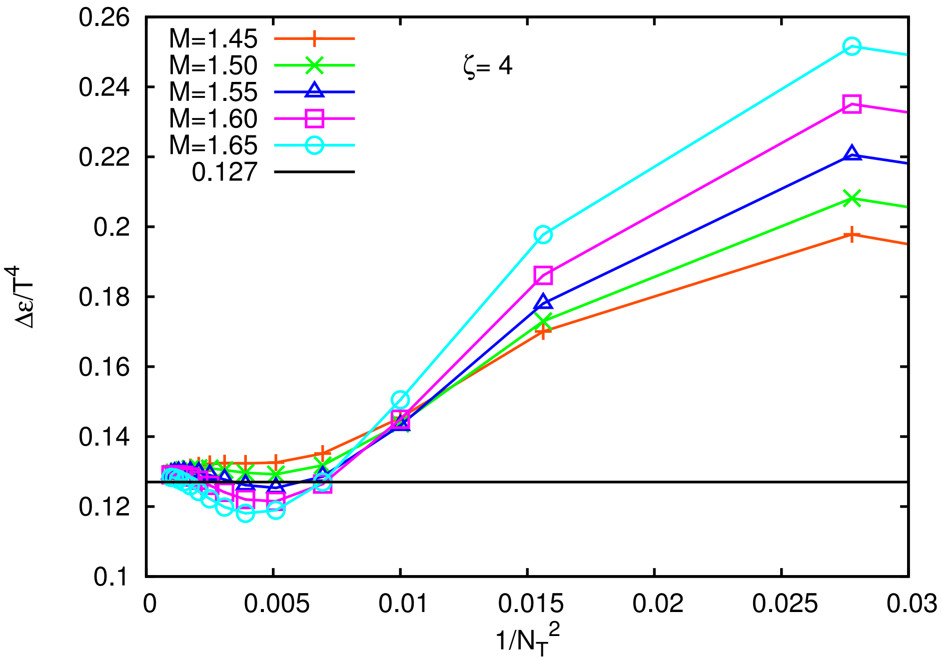
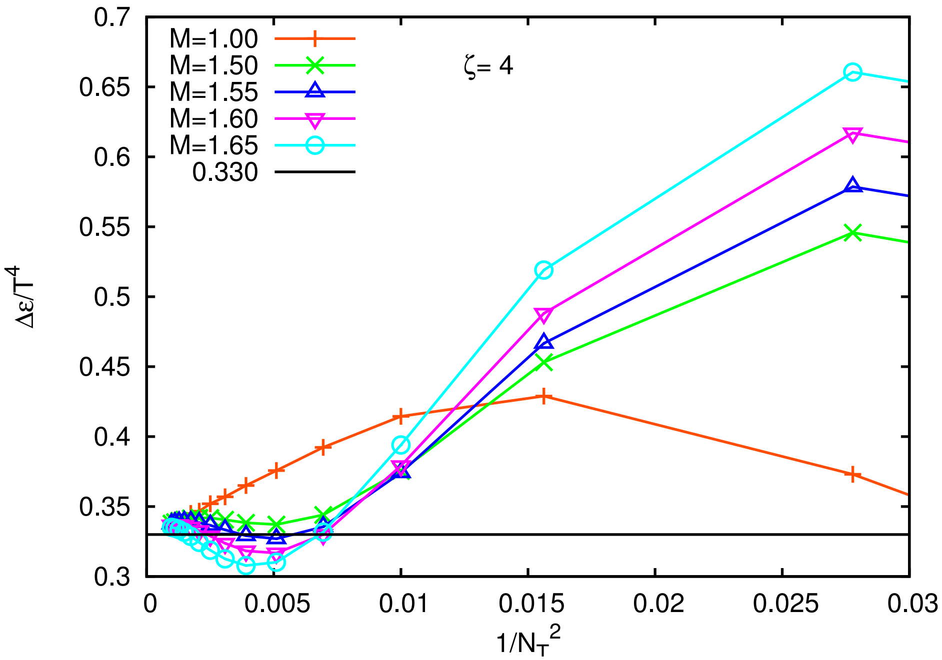
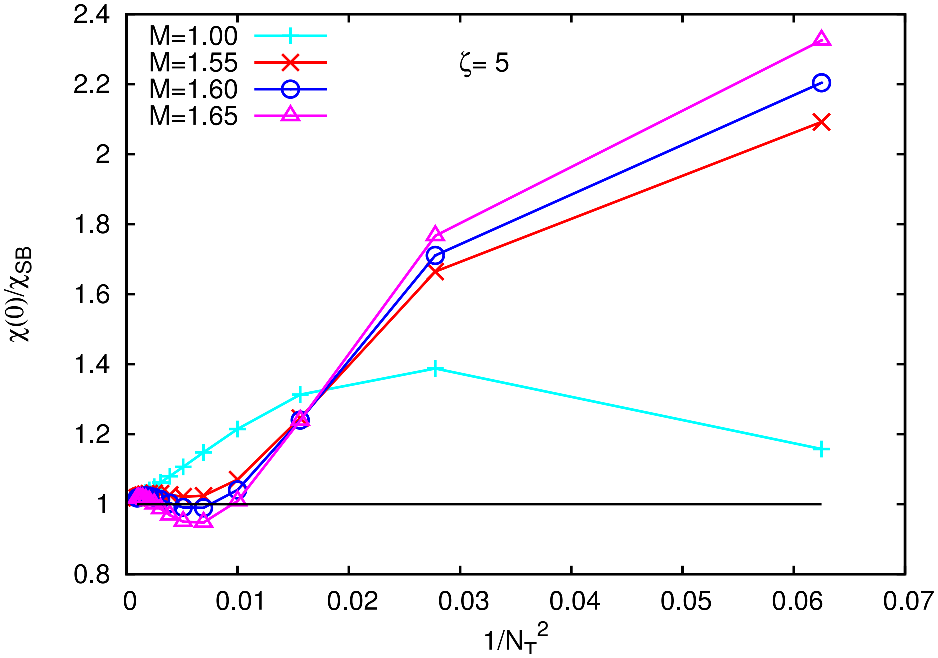
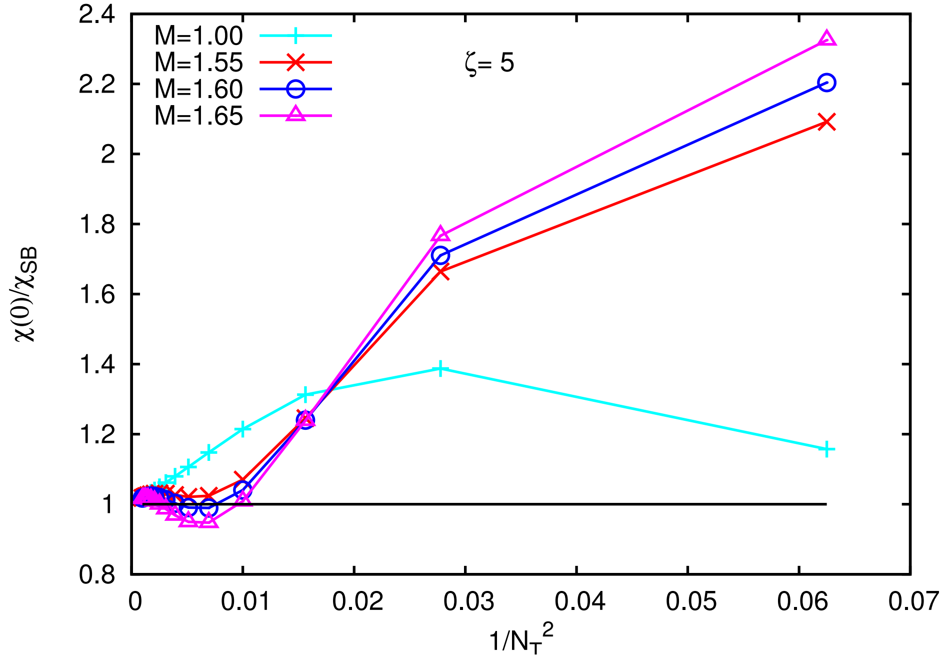
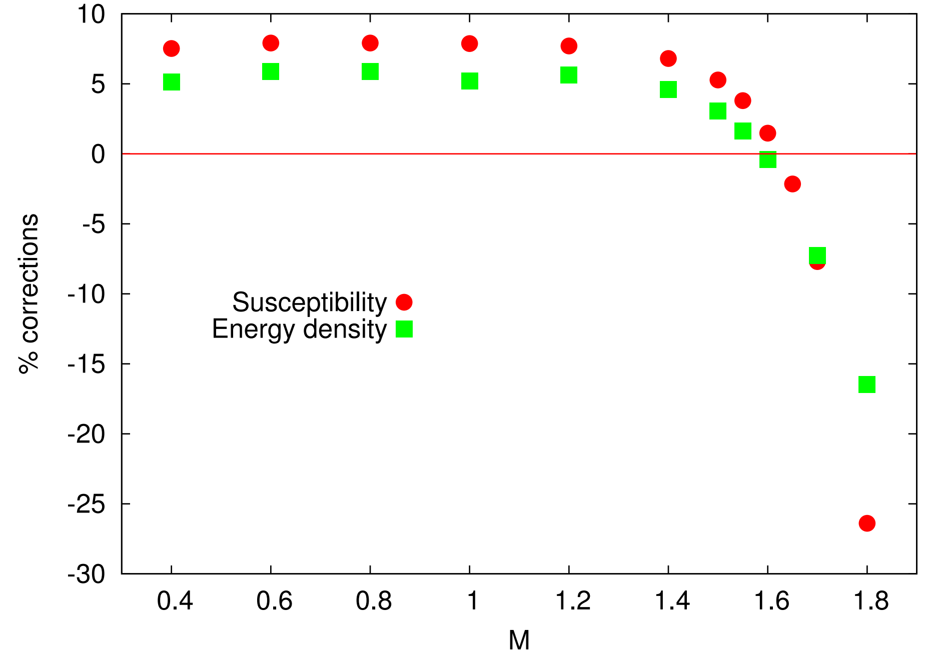




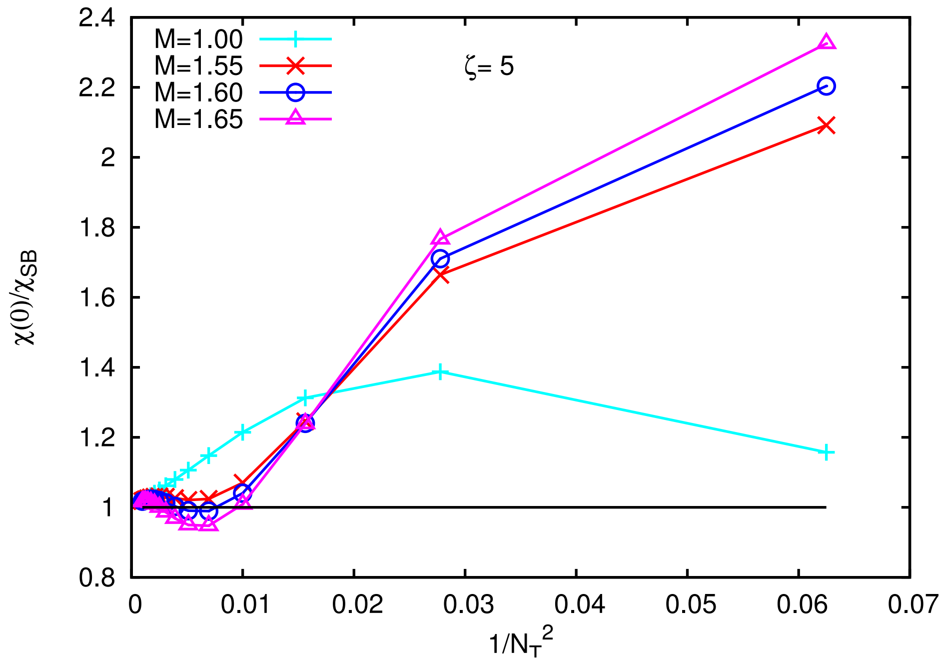


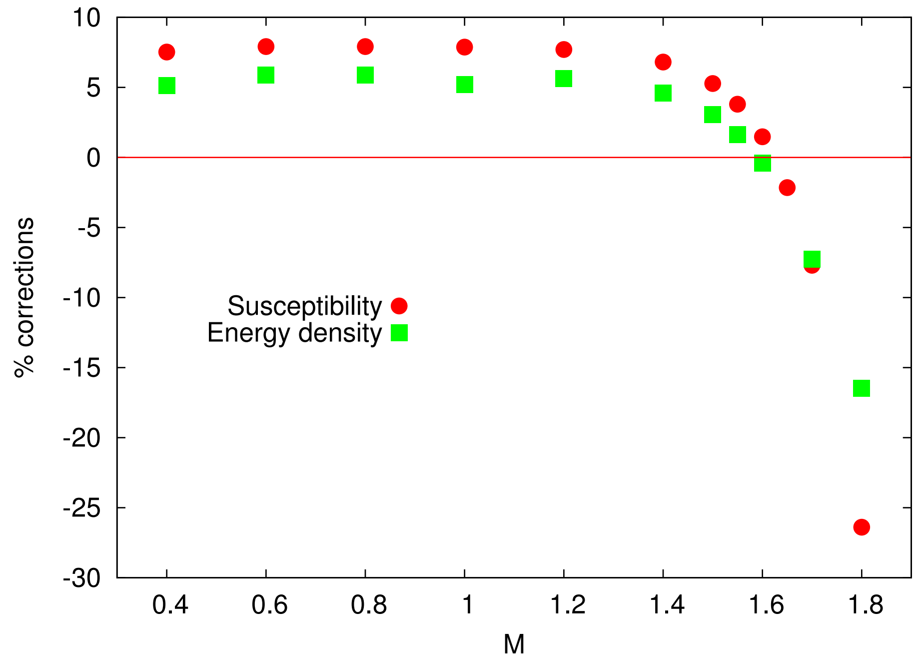
M=1.00
M=1.55
M=1.60
M=1.65
ζ= 5
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
χ(0)/χSB
1/NT2





♡ Susceptibility too behaves the same way as the energy density.





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Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.
- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

Summary

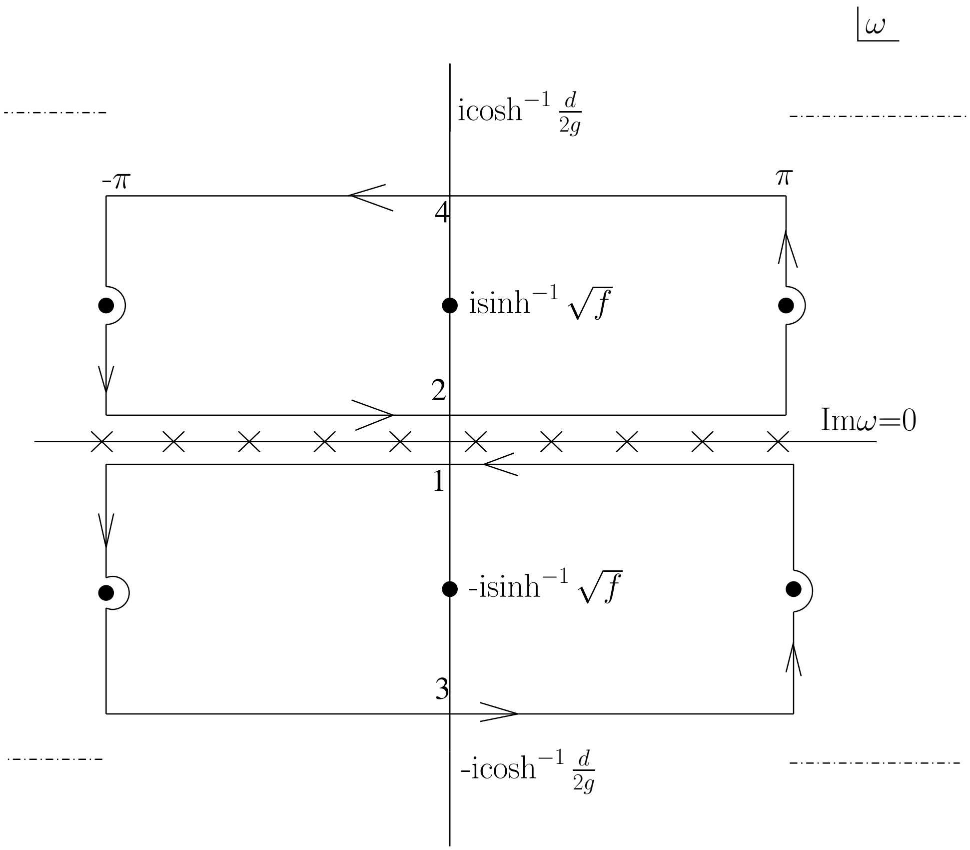
- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

- However, any μ^2-divergence in the continuum limit is avoided for it and an associated general class of functions $K(\mu)$ and $L(\mu)$ with $K(\mu) \cdot L(\mu) = 1$.

- For the choice of $1.5 \leq M \leq 1.6$, both the energy density and the quark number susceptibility computed for $\mu = 0$ exhibited the smallest deviations from the ideal gas limit for $N_T \geq 12$.

Analytic Evaluation



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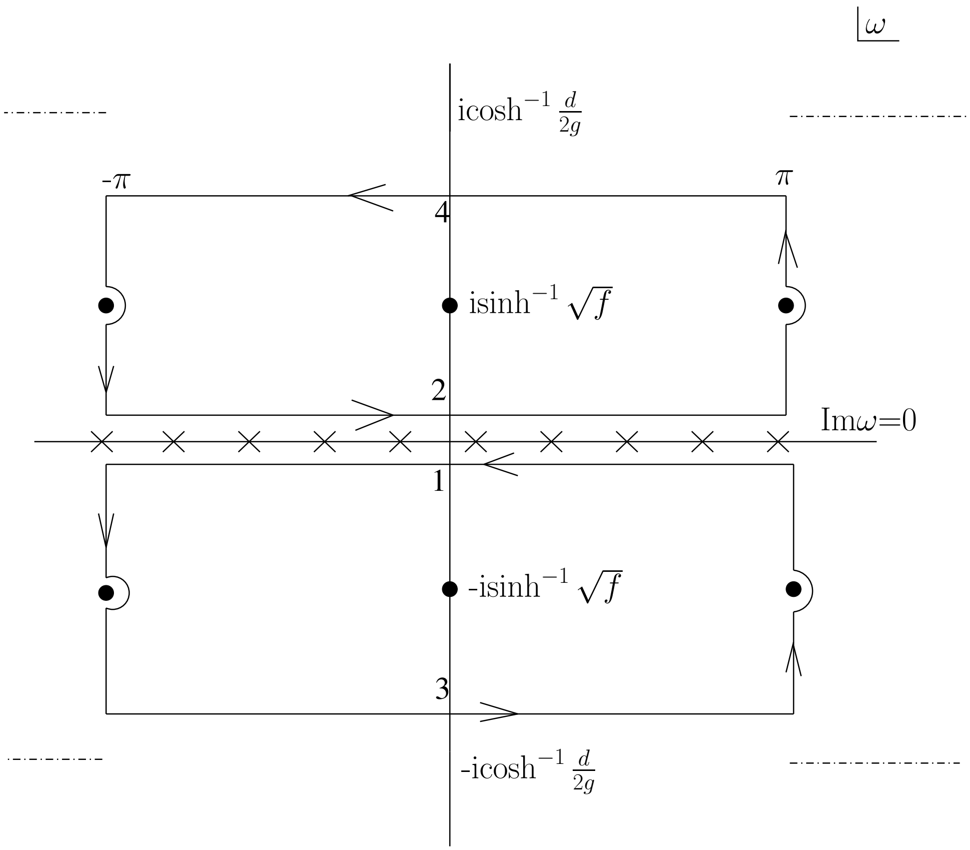
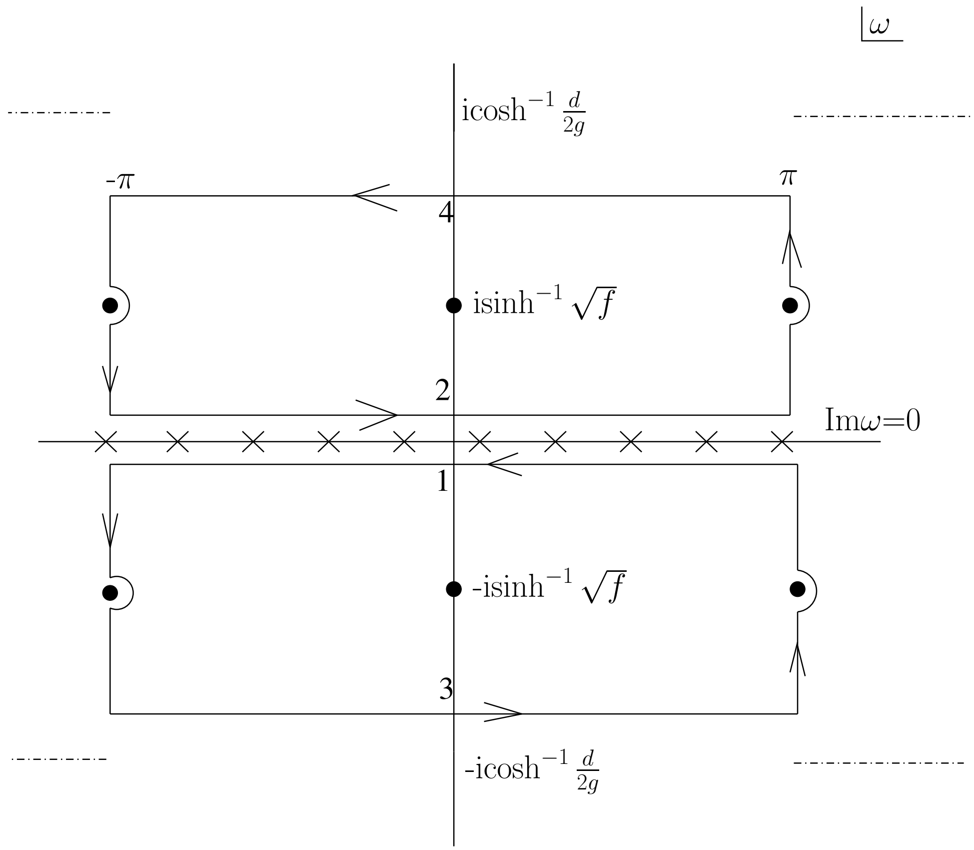


- Poles at $\omega = \pm i\ \sinh^{-1}\sqrt{f}$ and Poles (branch points) at $\pm i\ \cosh^{-1}\frac{d}{2g}$.

Analytic Evaluation



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- Evaluating integrals, $\epsilon a^4 = 4N^{-3}\sum_{p_j}\left[\sqrt{f/1+f}\right]$ $[\exp(N_T \sinh^{-1}\sqrt{f}) + 1]^{-1}$ $+\epsilon_3 + \epsilon_4$, where $f = \sum_i \sin^2(ap_i)$.

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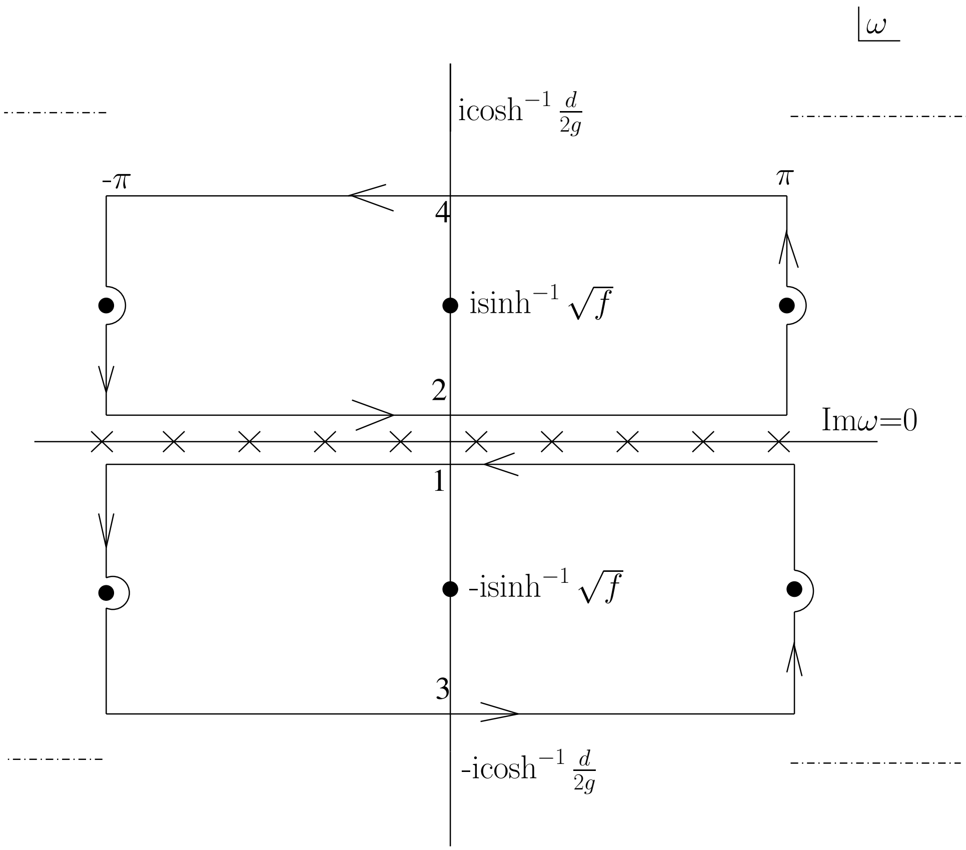

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- Can be seen to go to ϵ_{SB} as $a \to 0$ for all M.

More Details : $T = 0$, $\mu \neq 0$

- Defining $K(\mu) + L(\mu) = 2R \cosh\theta$ and $K(\mu) - L(\mu) = 2R \sinh\theta$, the same treatment as above goes through by substituting $\sin\omega_n \to R\sin(\omega_n - i\theta)$ and $\cos\omega_n \to R\cos(\omega_n - i\theta)$.

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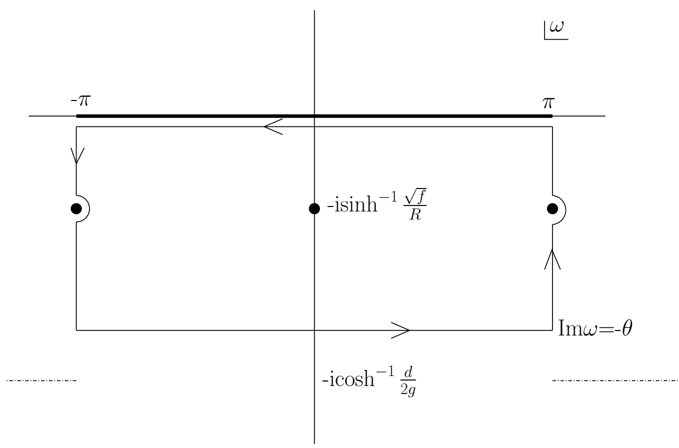
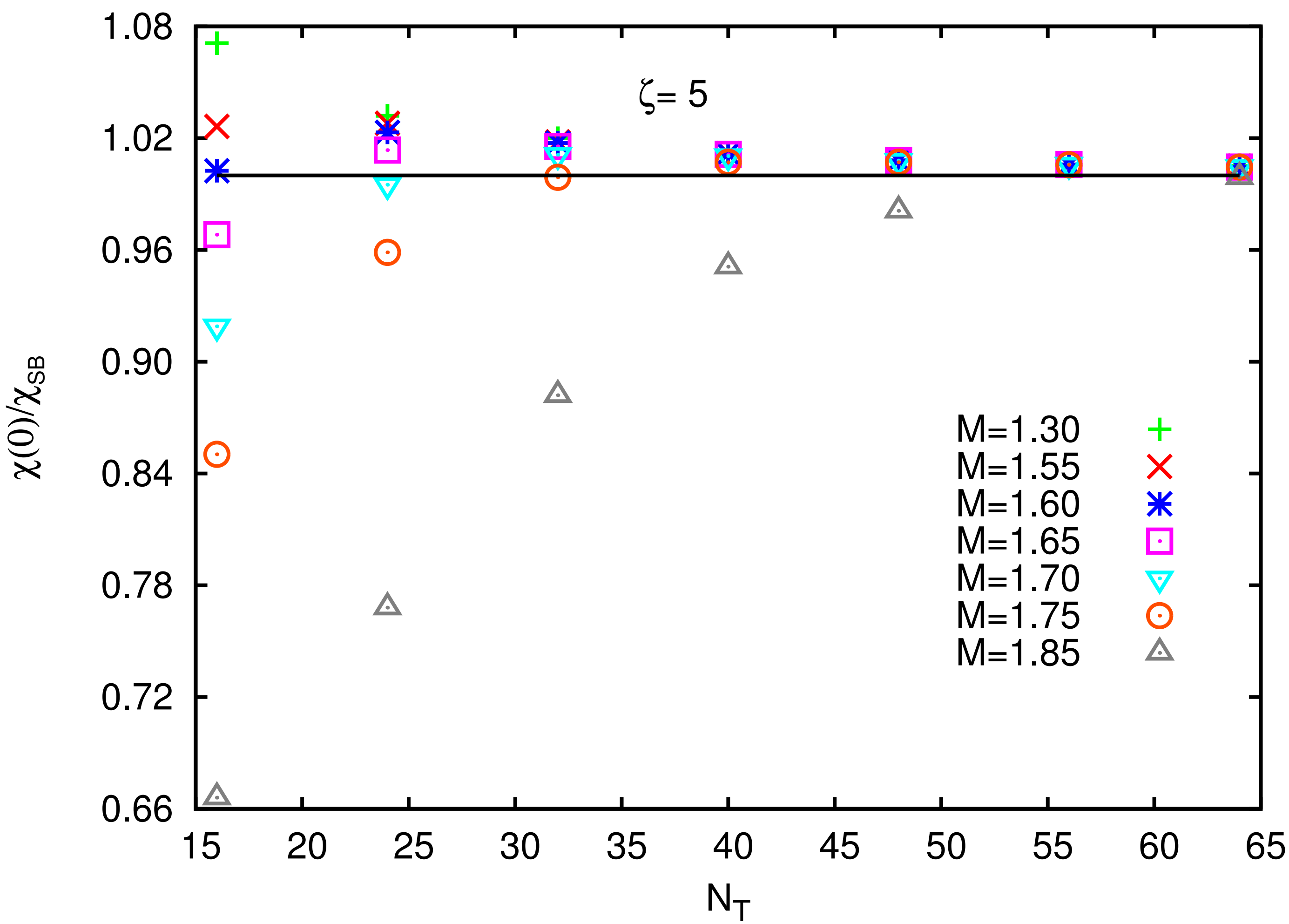
- Energy density is also functionally the same with $F(\omega_n) \to F(R, \omega_n - i\theta)$.

- Additional observable, number density : Has the same pole structure so similar computation.





ζ= 5
1.08
1.02
0.96
0.90
0.84
0.78
0.72
0.66
15
20
25
30
35
40
45
50
55
60
65
N_T
χ(0)/χ_SB
M=1.30
M=1.55
M=1.60
M=1.65
M=1.70
M=1.75
M=1.85