Can Lattice QCD account for Charm Flow at PHENIX?

Rajiv V. Gavai[*]
T. I. F. R., Mumbai & Universität Bielefeld

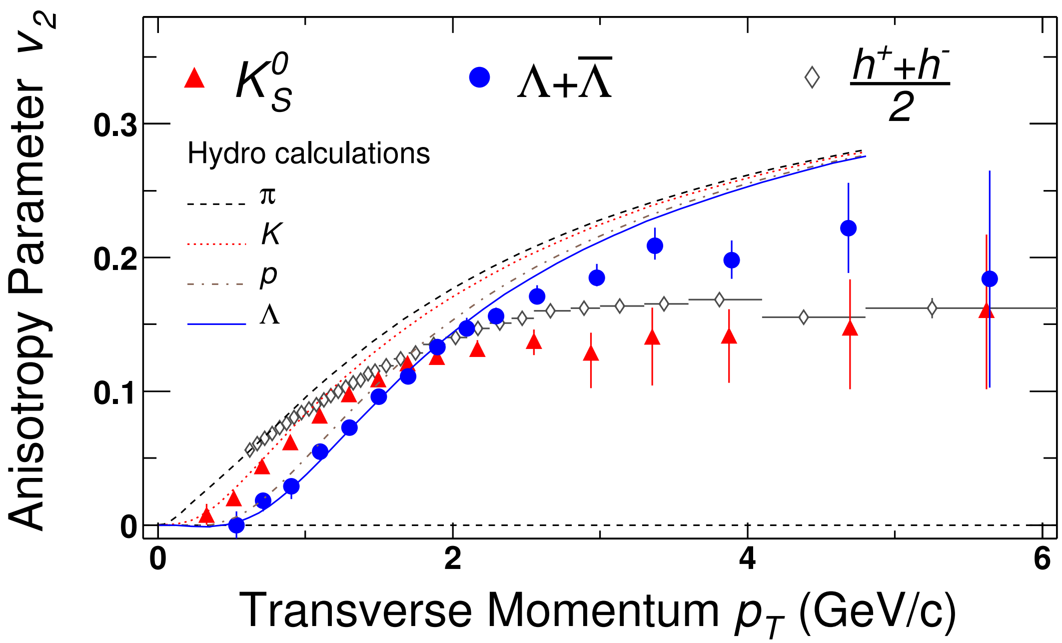
[*] *With Debasish Banerjee, Saumen Datta & Pushan Majumdar, arXiv:1109.5738*

Can Lattice QCD account for Charm Flow at PHENIX?

*Rajiv V. Gavai**
T. I. F. R., Mumbai & Universität Bielefeld

* *With Debasish Banerjee, Saumen Datta & Pushan Majumdar, arXiv:1109.5738*

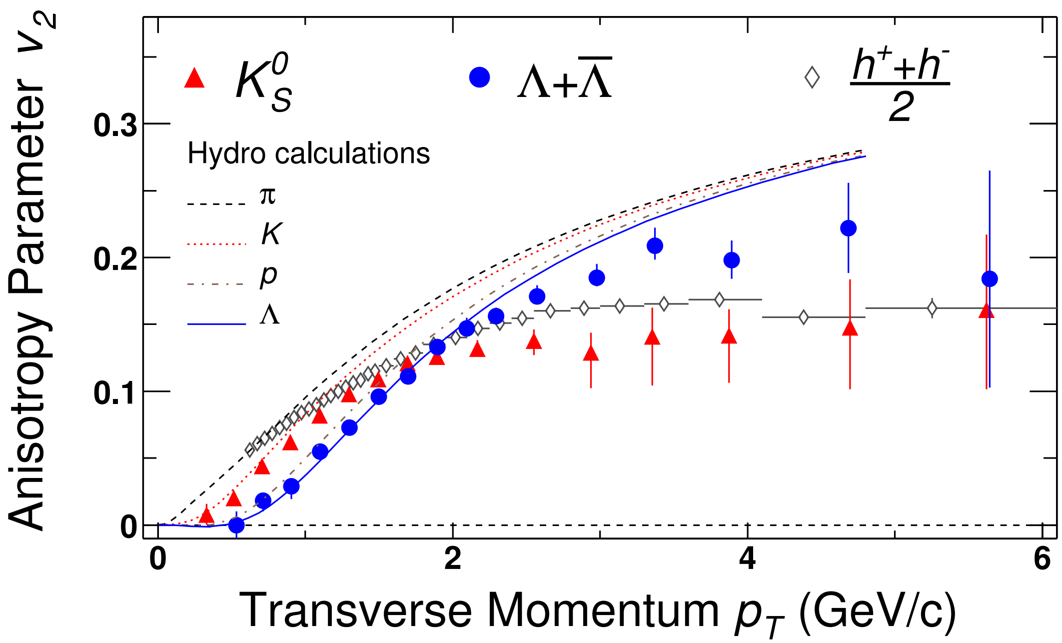
Introduction



(STAR Collaboration, Adams et al., PRL92 (2004) 052302.)

- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.

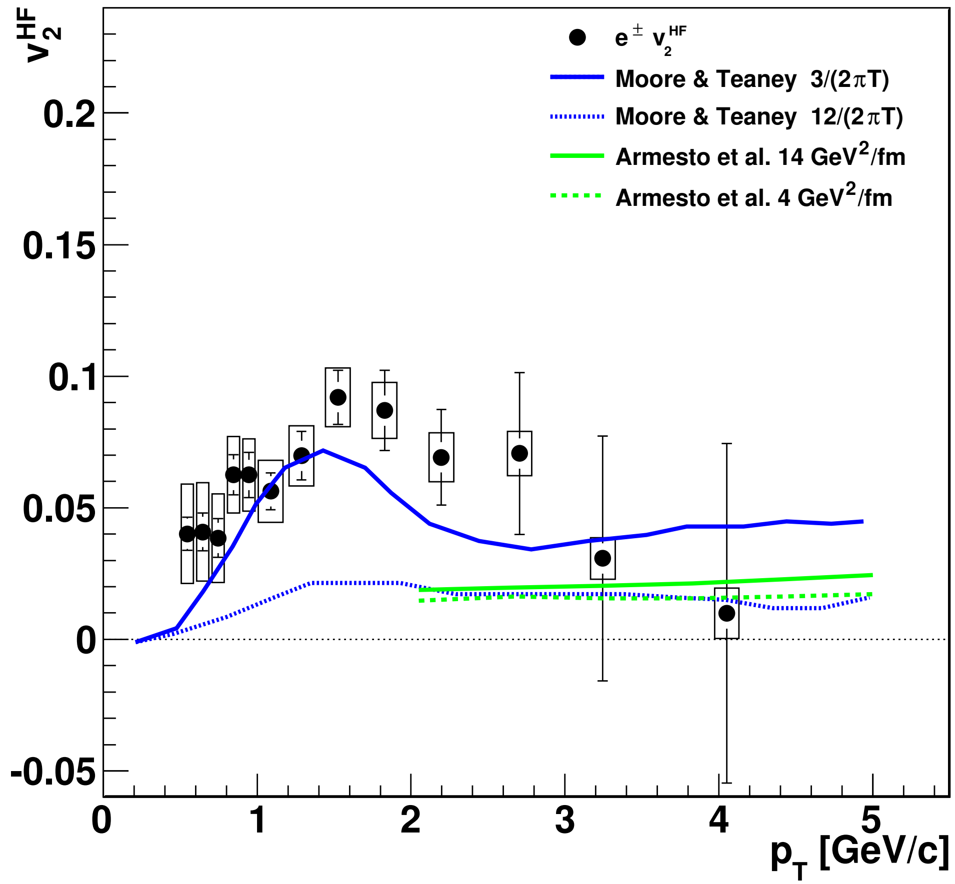
Introduction



(STAR Collaboration, Adams et al., PRL92 (2004) 052302.)

- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.
- Good agreement with ideal hydro: Suggesting early thermalization and perfect fluid and many more interesting things.

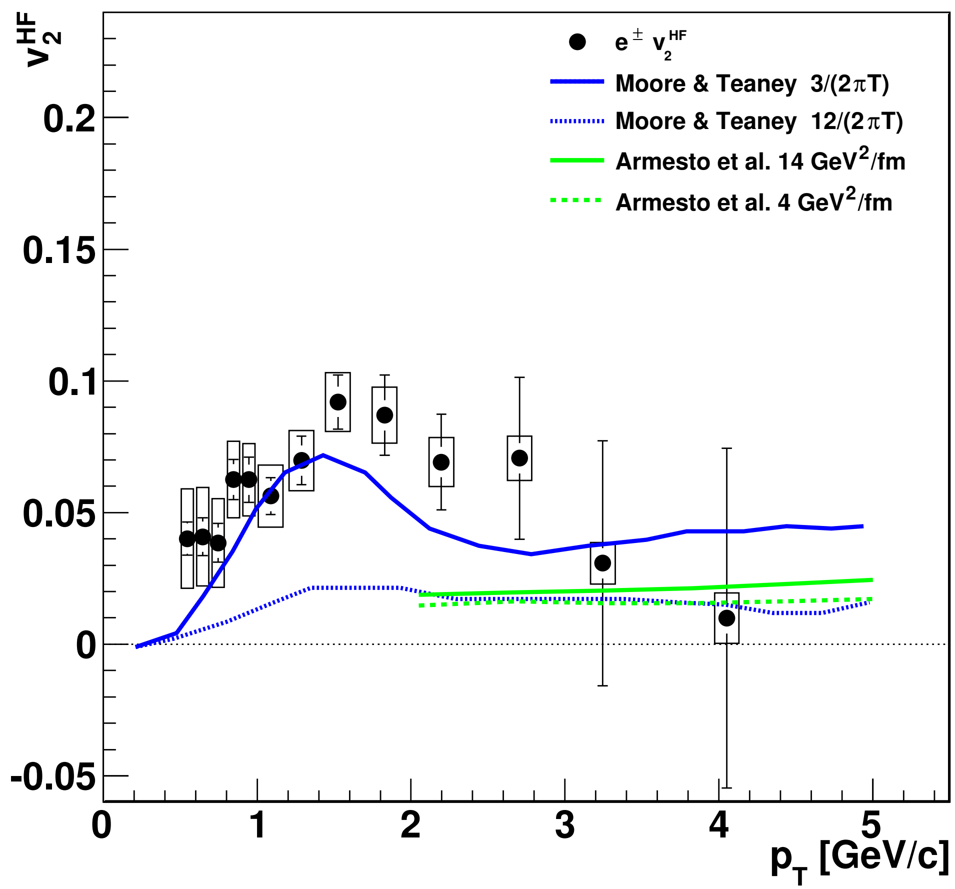
- Naıvely expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.



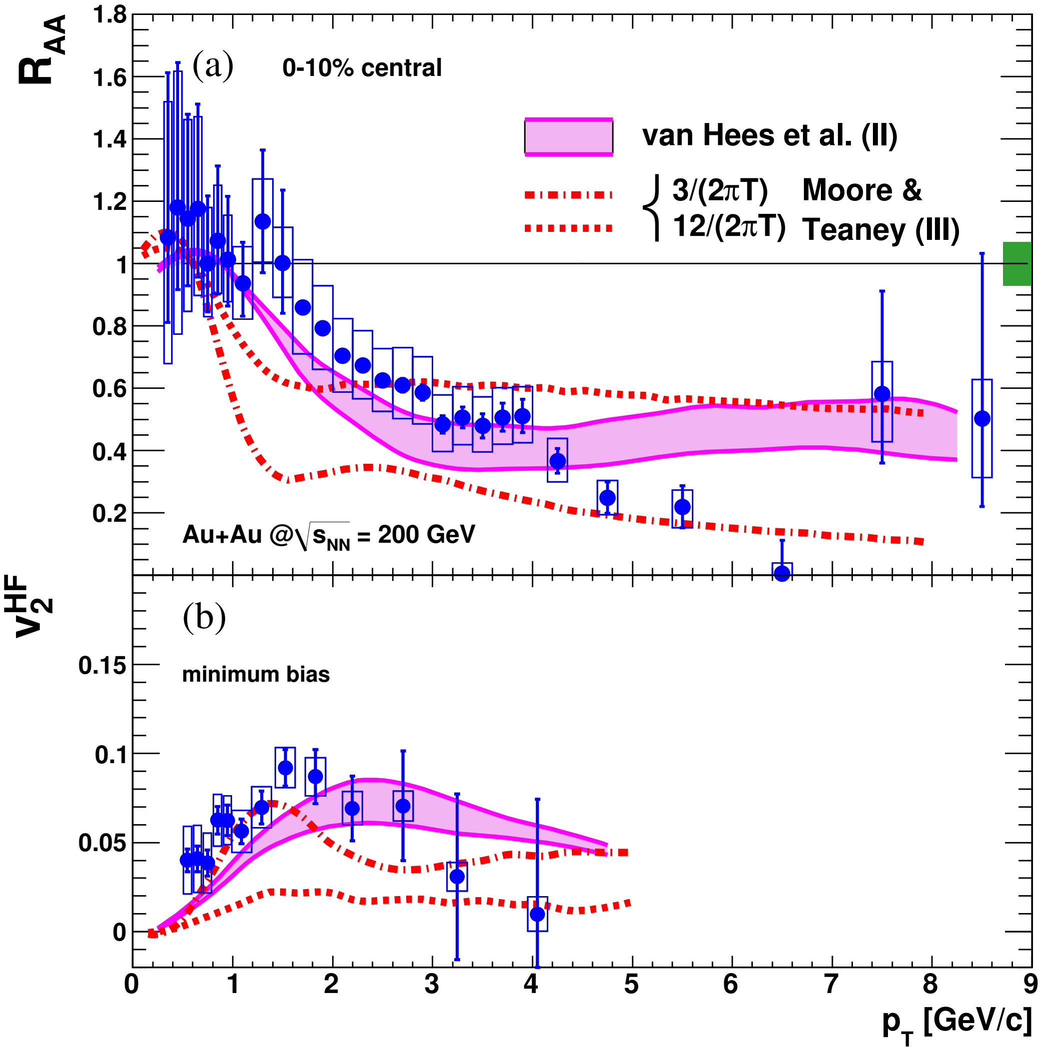
(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

- Naıvely expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.

- In models (Moore-Teaney, PRC 71, 2005), heavy quark diffusion coefficients governs its elliptic flow **and** suppression.



(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)



(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.

- **Smaller** $D \simeq 3/2\pi T$ seems required by data.

- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.

- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.

- **Smaller** $D \simeq 3/2\pi T$ seems required by data.

- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.

- Other models, e.g. van Hees-Greco-Rapp, seem to suggest the same: Heavy Quark Diffusion coefficient is much smaller than perturbative estimates.

- Is it non-perturbative ?

- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.
- **Smaller** $D \simeq 3/2\pi T$ seems required by data.
- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.
- Other models, e.g. van Hees-Greco-Rapp, seem to suggest the same: Heavy Quark Diffusion coefficient is much smaller than perturbative estimates.
- Is it non-perturbative ? Strong coupling models — AdS/CFT based — do lead to values in the desired range under "suitable" assumptions.
- Can Lattice QCD shed some light on the Charm Flow ?

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.
- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.
- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

$$\frac{dp_i}{dt} = -\eta_D \, p_i + \xi_i(t) \qquad \langle \xi_i(t) \xi_j(t') = \kappa \delta_{ij} \delta(t - t') \tag{1}$$

- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time.

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.

- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

$$\frac{dp_i}{dt} = -\eta_D \, p_i + \xi_i(t) \qquad \langle \xi_i(t) \xi_j(t') = \kappa \delta_{ij} \delta(t - t') \tag{1}$$

- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time.

- Diffusion constant D can be found to be $2T^2/\kappa$ with $\eta_D = \kappa/2MT$.

- Moore-Teaney also showed that an initial ($T_0 = 300$ MeV) power-law (LO pQCD) transverse momentum distribution of heavy Q finally ($T_f = 165$ MeV) approximates a thermal one in an ideal Bjorken expansion of the plasma **provided** $D \leq 3/2\pi T$.

- Their comparison of a more realistic hydro-simulation, I showed earlier, also supports such a conclusion.

- Moore-Teaney also showed that an initial ($T_0 = 300$ MeV) power-law (LO pQCD) transverse momentum distribution of heavy Q finally ($T_f = 165$ MeV) approximates a thermal one in an ideal Bjorken expansion of the plasma **provided** $D \leq 3/2\pi T$.

- Their comparison of a more realistic hydro-simulation, I showed earlier, also supports such a conclusion.

- Casalderrey-Solana & Teaney (PRD 74 (2006) 085012) suggested to obtain κ from a correlator of the force exerted on a heavy Q by the medium.

- Moore-Teaney also showed that an initial ($T_0 = 300$ MeV) power-law (LO pQCD) transverse momentum distribution of heavy Q finally ($T_f = 165$ MeV) approximates a thermal one in an ideal Bjorken expansion of the plasma **provided** $D \leq 3/2\pi T$.

- Their comparison of a more realistic hydro-simulation, I showed earlier, also supports such a conclusion.

- Casalderrey-Solana & Teaney (PRD 74 (2006) 085012) suggested to obtain κ from a correlator of the force exerted on a heavy Q by the medium.

- Using Heavy Q Effective Theory, Caron-Huot, Laine & Moore (JHEP 0904, 053) provided a suitable definition for κ for a lattice evaluation:

$$G_E^{\rm Lat}(\tau) \;=\; -\frac{1}{3L} \sum_{i=1}^{3} \left\langle {\rm Re\ tr}\ \left[U(\beta,\tau)\ E_i(\tau,\vec{0})\ U(\tau,0)\ E_i(0,\vec{0}) \right] \right\rangle .$$

- Then momentum diffusion coefficient $\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$, where ρ is the spectral function obtained from G above.

- Then momentum diffusion coefficient $\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$, where ρ is the spectral function obtained from G above.

- They also suggested a suitable discrete version for Lattice QCD :
$E_i(\vec{x}, \tau) = U_i(\vec{x}, \tau) \, U_4(\vec{x} + \hat{i}, \tau) - U_4(\vec{x}, \tau) \, U_i(\vec{x} + \hat{4})$.

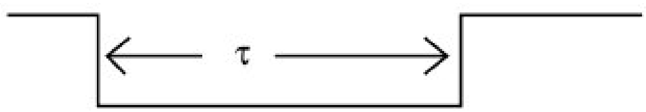
- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.
$G^i_{E,\mathrm{num}}(\tau) = C^i(\tau + 1) + C^i(\tau - 1) - 2C^i(\tau)$
$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t+\tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4)$.

- Then momentum diffusion coefficient $\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$, where ρ is the spectral function obtained from G above.

- They also suggested a suitable discrete version for Lattice QCD :
$E_i(\vec{x}, \tau) = U_i(\vec{x}, \tau) \, U_4(\vec{x} + \hat{i}, \tau) - U_4(\vec{x}, \tau) \, U_i(\vec{x} + \hat{4})$.

- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.
$G^i_{E,\mathrm{num}}(\tau) = C^i(\tau + 1) + C^i(\tau - 1) - 2C^i(\tau)$

$$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t+\tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4).$$



Graphical Representation of $C(\tau)$.

Our Lattice Results

- It is well-known that the Polyakov loop becomes exponentially small with N_τ. The extraction of κ, on the other hand, needs large N_τ.

Our Lattice Results

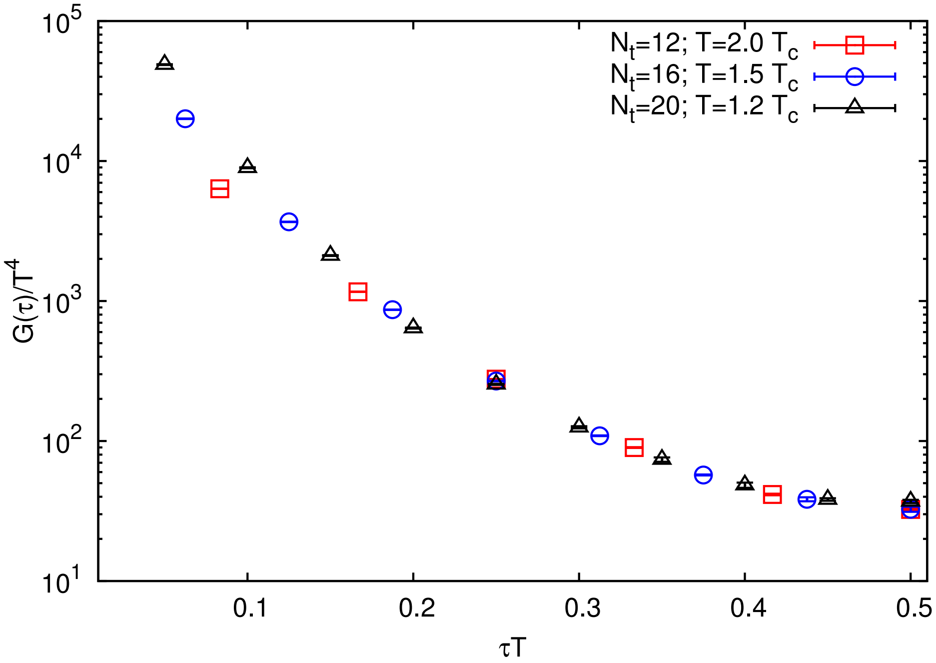
- It is well-known that the Polyakov loop becomes exponentially small with N_τ. The extraction of κ, on the other hand, needs large N_τ.

- We attempted N_τ = 12, 16, 20 and 24. Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.

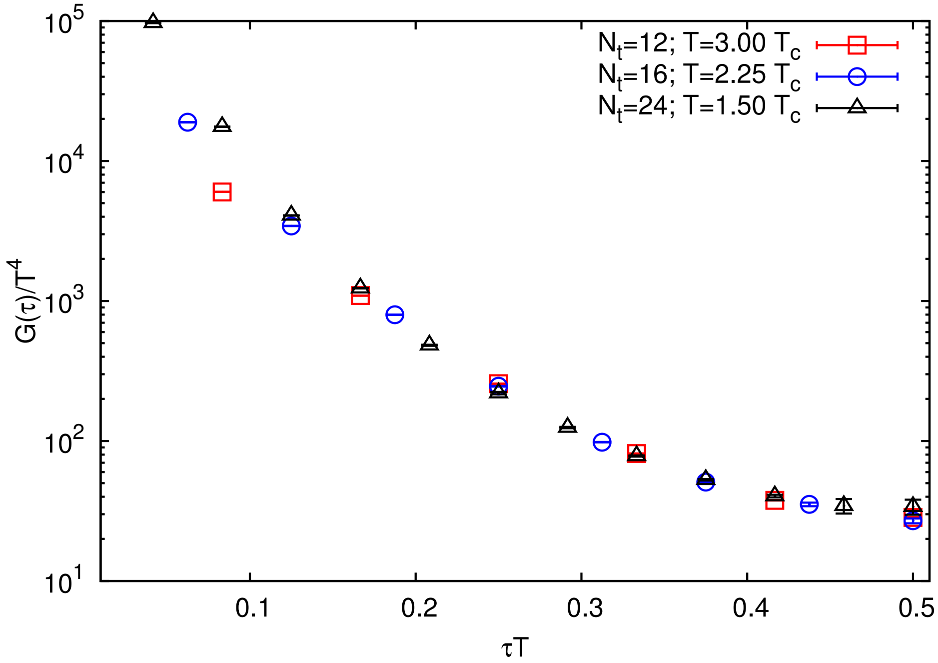
- For same size error on $G(10)$, it was found ~ 2500 times more efficient: Very crucial in getting κ.

Our Lattice Results

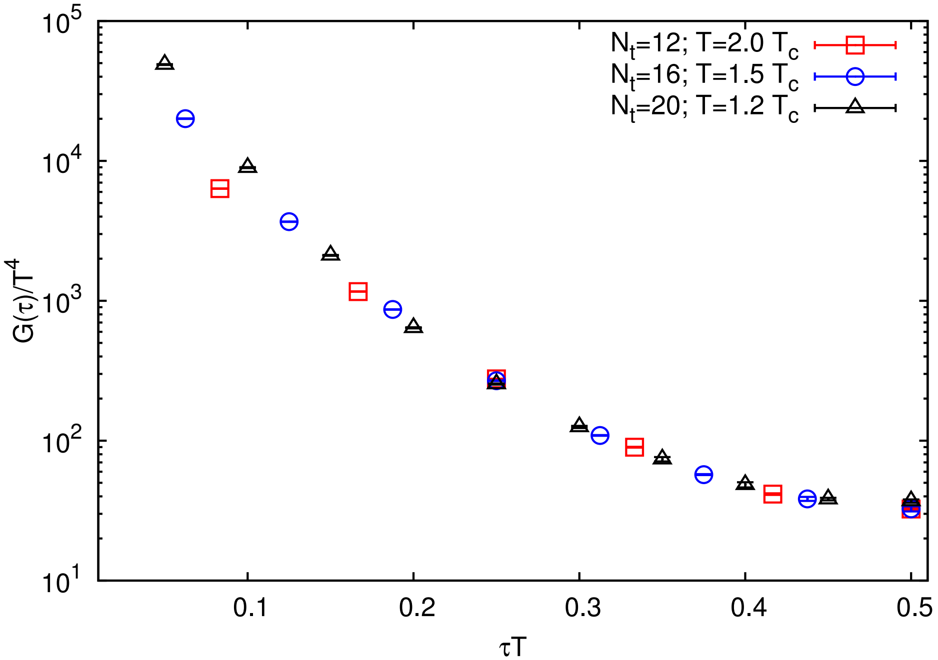
- It is well-known that the Polyakov loop becomes exponentially small with N_τ. The extraction of κ, on the other hand, needs large N_τ.
- We attempted $N_\tau =$ 12, 16, 20 and 24. Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.
- For same size error on $G(10)$, it was found ~ 2500 times more efficient: Very crucial in getting κ.
- Spatial volumes are such that $N_s \geq 2N_\tau$.
- Couplings were chosen suitably to make simulations at $T/T_c =$ 1.04, 1.09, 1.24, 1.5 and 1.96 for the two largest N_τ.
- Typical Statistics : Few hundred Independent Configurations

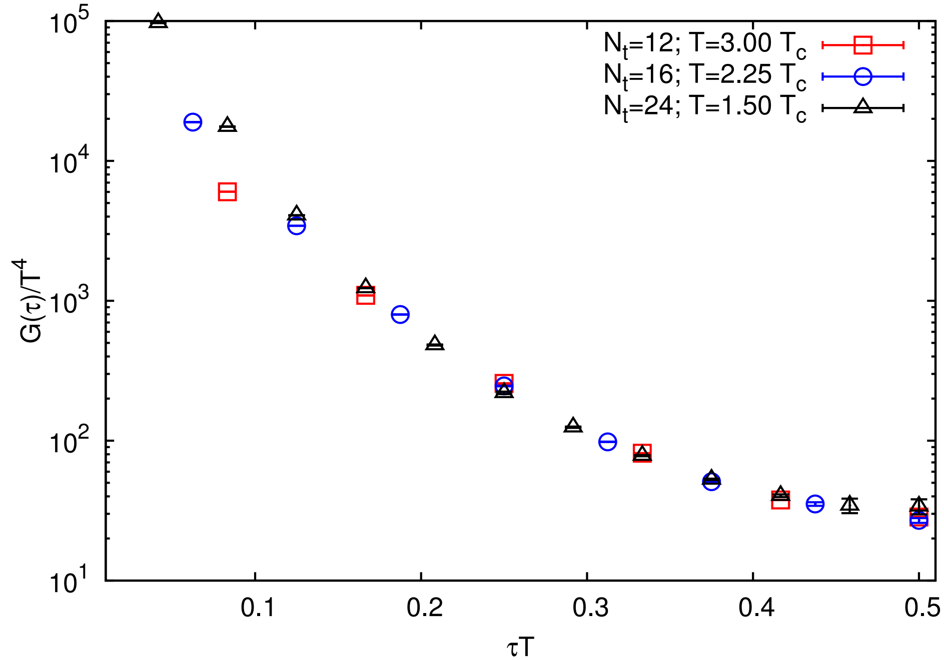


N_t=12; T=2.0 T_c
N_t=16; T=1.5 T_c
N_t=20; T=1.2 T_c
G(τ)/T^4
τT

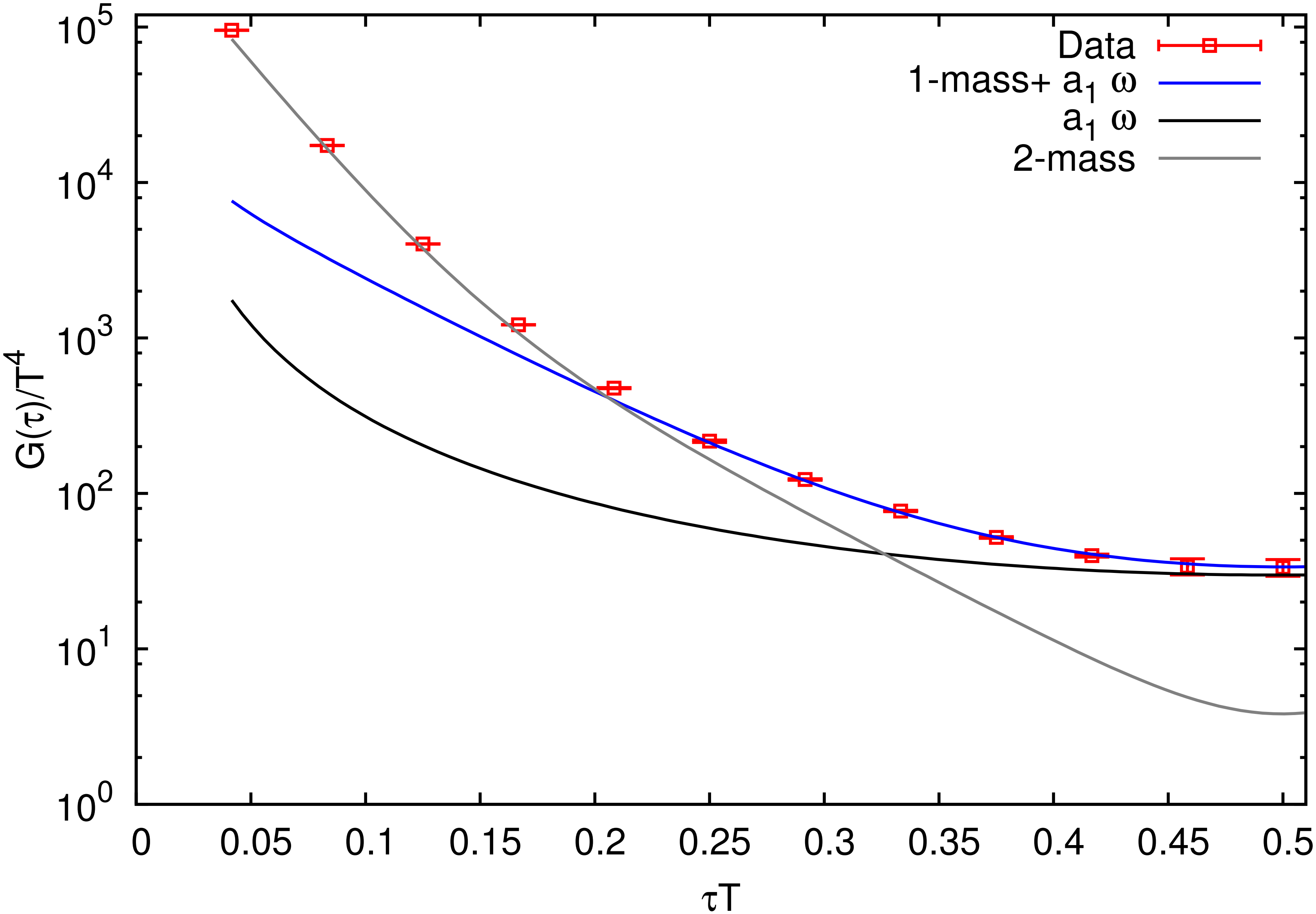


N_t=12; T=3.00 T_c
N_t=16; T=2.25 T_c
N_t=24; T=1.50 T_c
G(τ)/T^4
τT





- Large τ region shows scaling.
- Low τ region, on the other hand, has only lattice artifacts.



Data
1-mass+ a_1 ω
a_1 ω
2-mass
$G(\tau)/T^4$
τT
10^5
10^4
10^3
10^2
10^1
10^0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Extracting D

- Getting to the spectral function ρ, an ill-posed problem, has attracted a lot of attention. Many methods can be tried.

- We use an $ansatz$ for it, obtain G from it, and then fit in the large τ range $[N_\tau/4, N_\tau/2]$

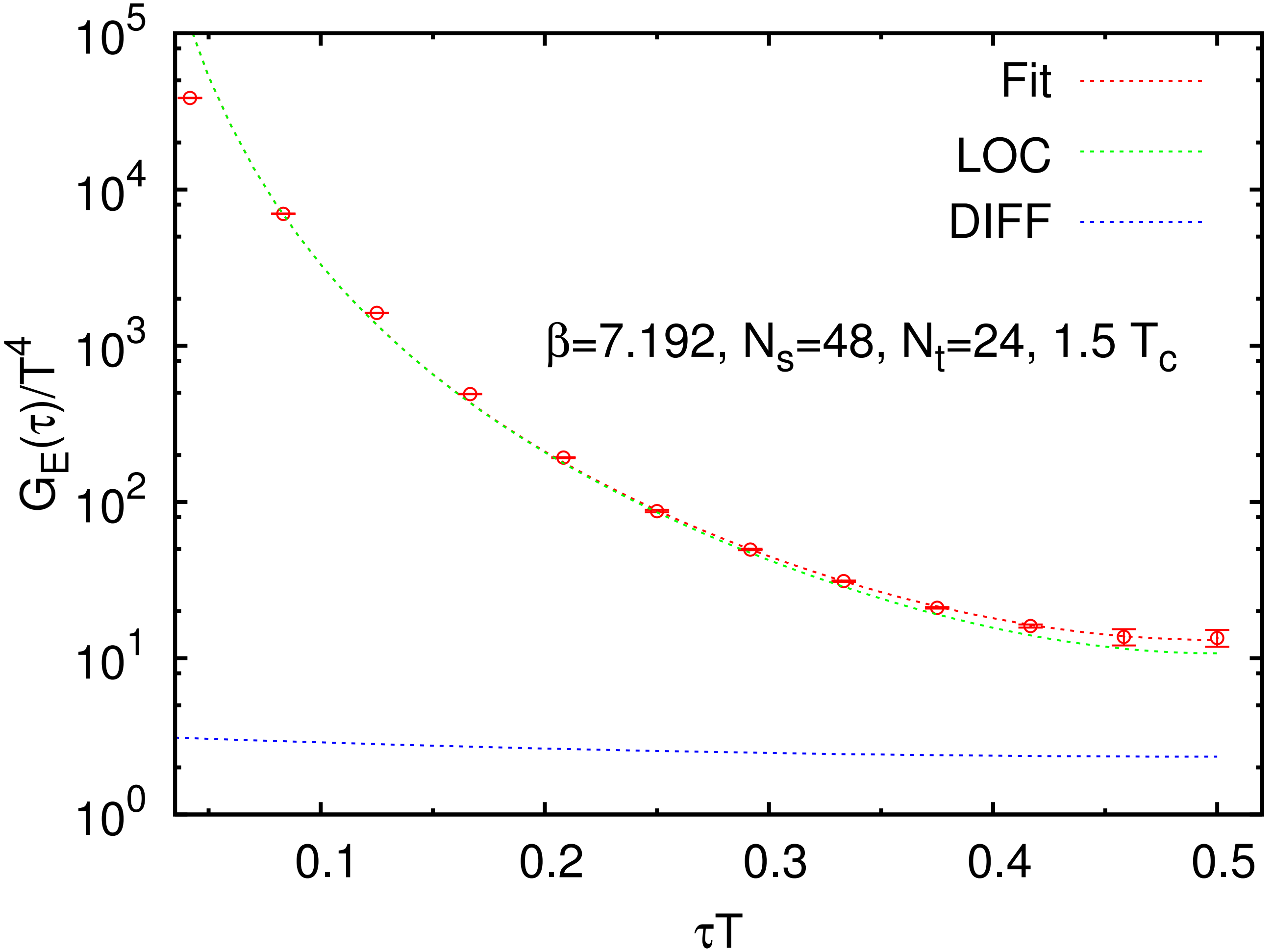
Extracting D

- Getting to the spectral function ρ, an ill-posed problem, has attracted a lot of attention. Many methods can be tried.

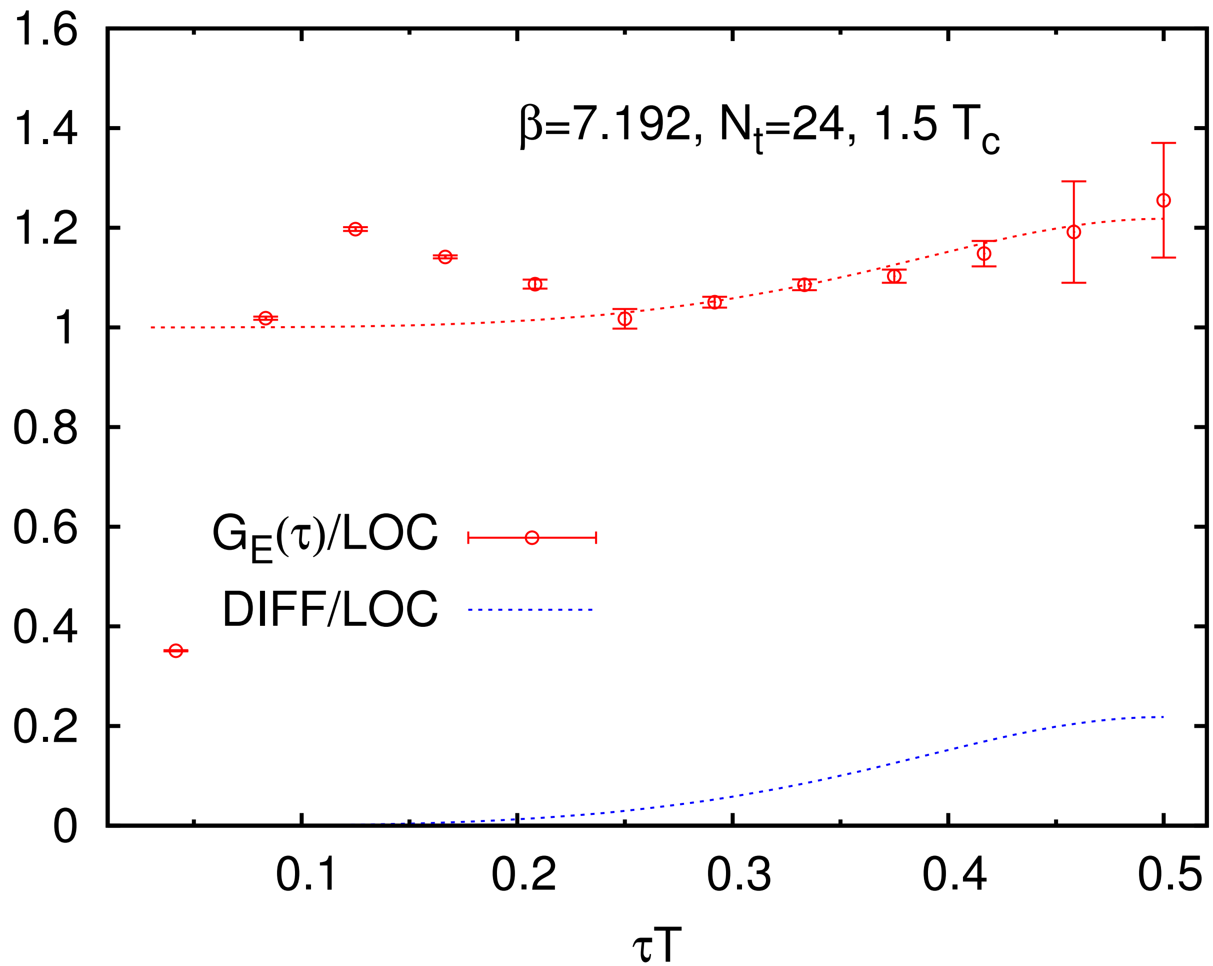
- We use an *ansatz* for it, obtain G from it, and then fit in the large τ range $[N_\tau/4,\ N_\tau/2]$

- $\rho(\omega) \ = \ a\omega\,\Theta(\omega - \Lambda) \ + \ b\omega^3$
  First term is the due to the expected DIFFusion constant, and the second is motivated by leading perturbation theory (LOC)

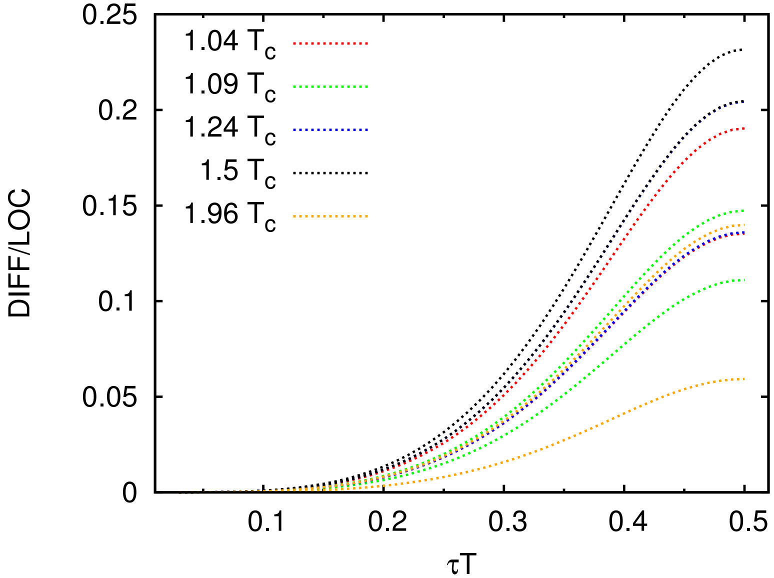
- $\Lambda = 3T$ used; varied from 2 to ∞ for systematic error.

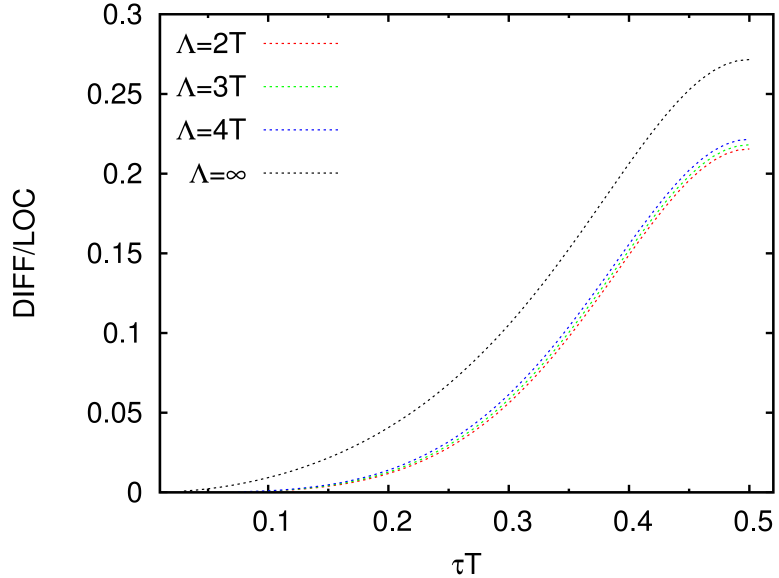


♠ Contribution of the two terms shown as DIFF and LOC.

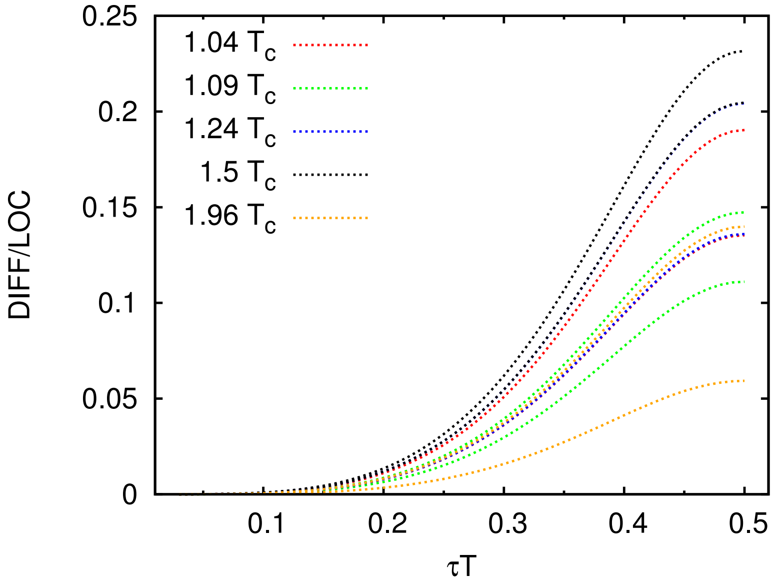


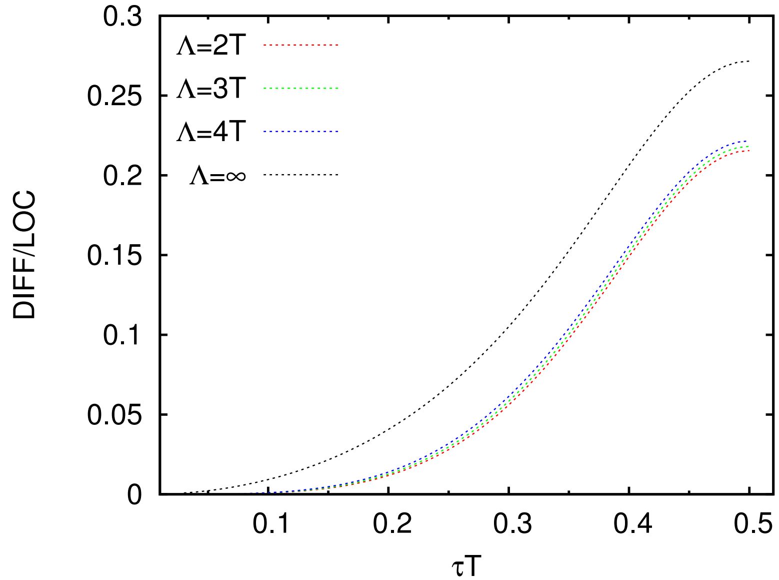
♠ Comparing the DIFF fit with the data after eliminating the LOC.



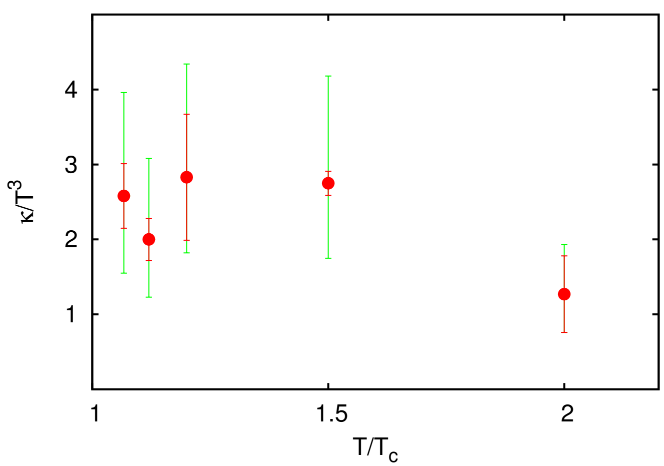


♠ Variation with the temperature and the cut-off Λ.

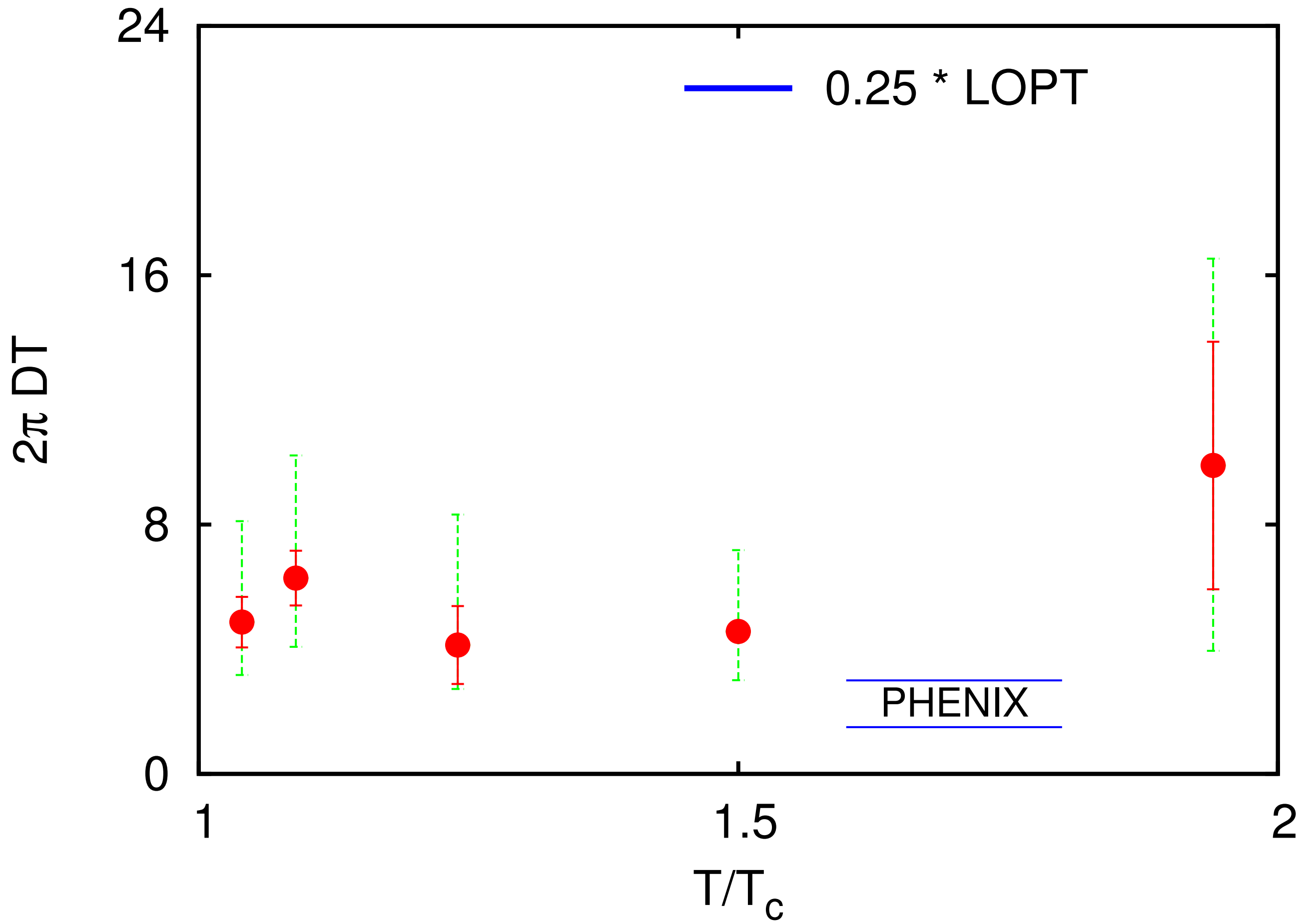




♠ Variation with the temperature and the cut-off Λ.

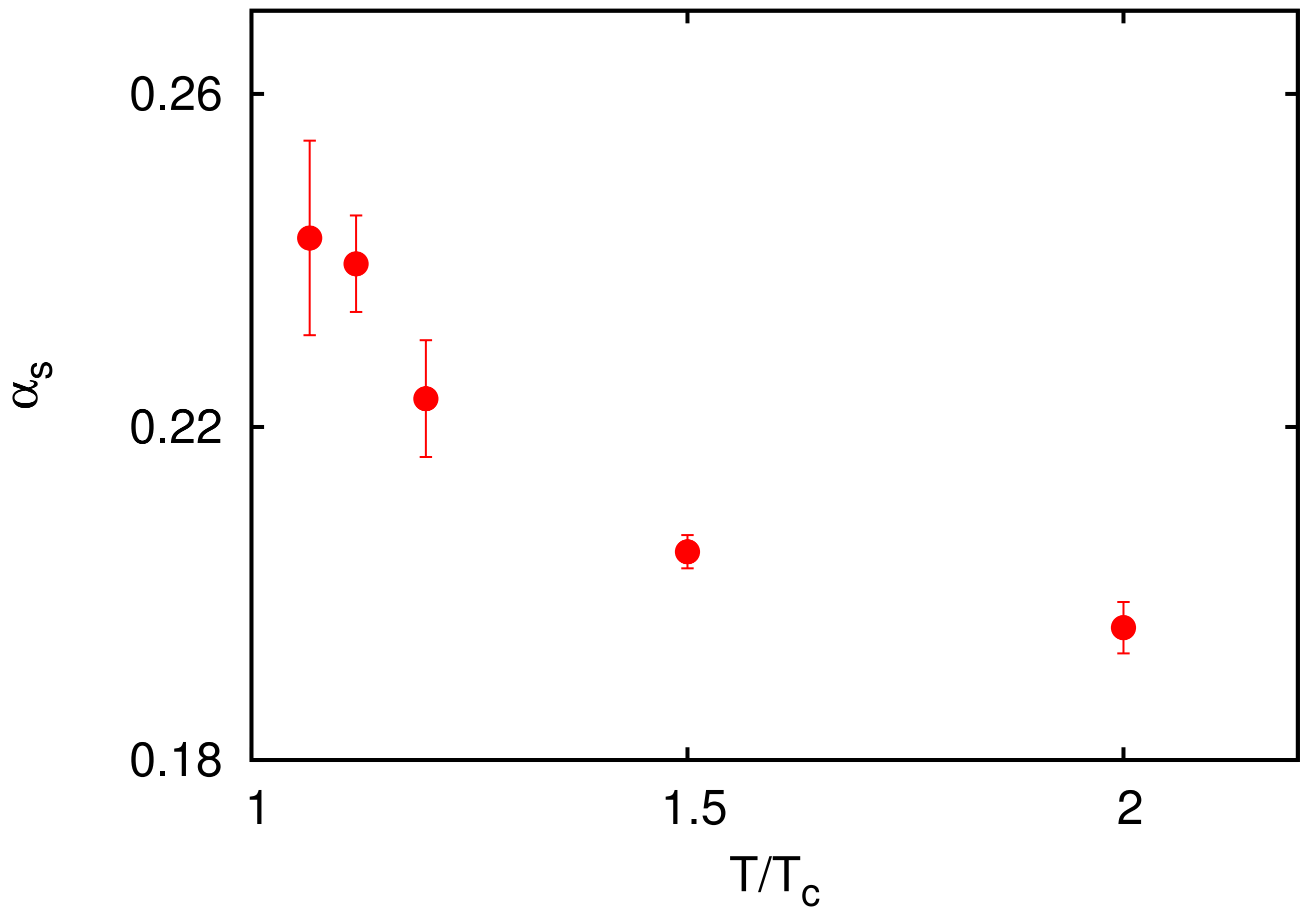


♠ Multiplying by T, obtain a quantity used by Moore-Teaney and PHENIX.



♡ In agreement with preliminary Bielefeld estimates (Ding et al. 1107.0311; Francis et al. 1109.3941).

♠ The ω^3 term comes with g^2. Use as a scheme to define α_s non-perturbatively.



♡ In agreement with other similar estimates (Ding et al. PRD 83 (2011) 034504).

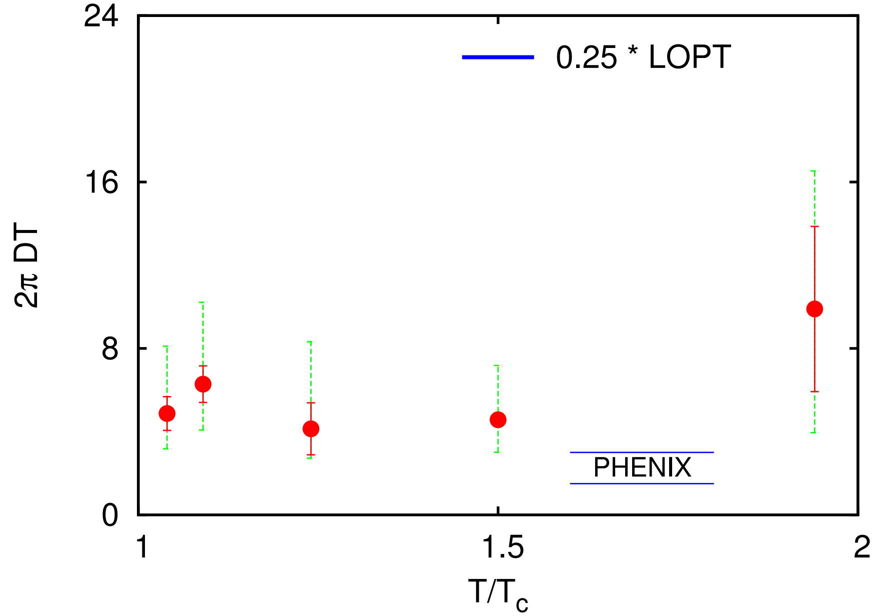
Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.

- Our results for DT are almost constant in the range studied.

Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.
- Our results for DT are almost constant in the range studied.
- The value itself is tantalisingly close to what PHENIX data needs in the Moore-Teaney model.

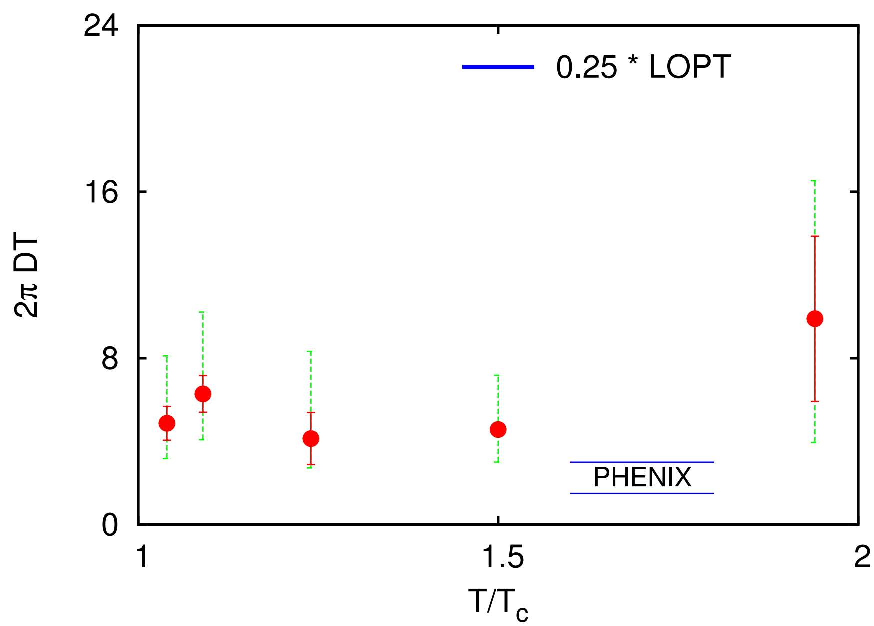


Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.

- Our results for DT are almost constant in the range studied.

- The value itself is tantalisingly close to what PHENIX data needs in the Moore-Teaney model.



It would be interesting to see if DT vs. T/T_c exhibits similar flavour independence as the pressure.