

# Computing Challenges in Lattice QCD

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Introduction : Why & How

Computing Challenges

Current Scenario

Summary

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- QCD — A (Gauge) Theory of interactions of quarks-gluons.
- Similar to structure in theory of electrons & photons (QED).

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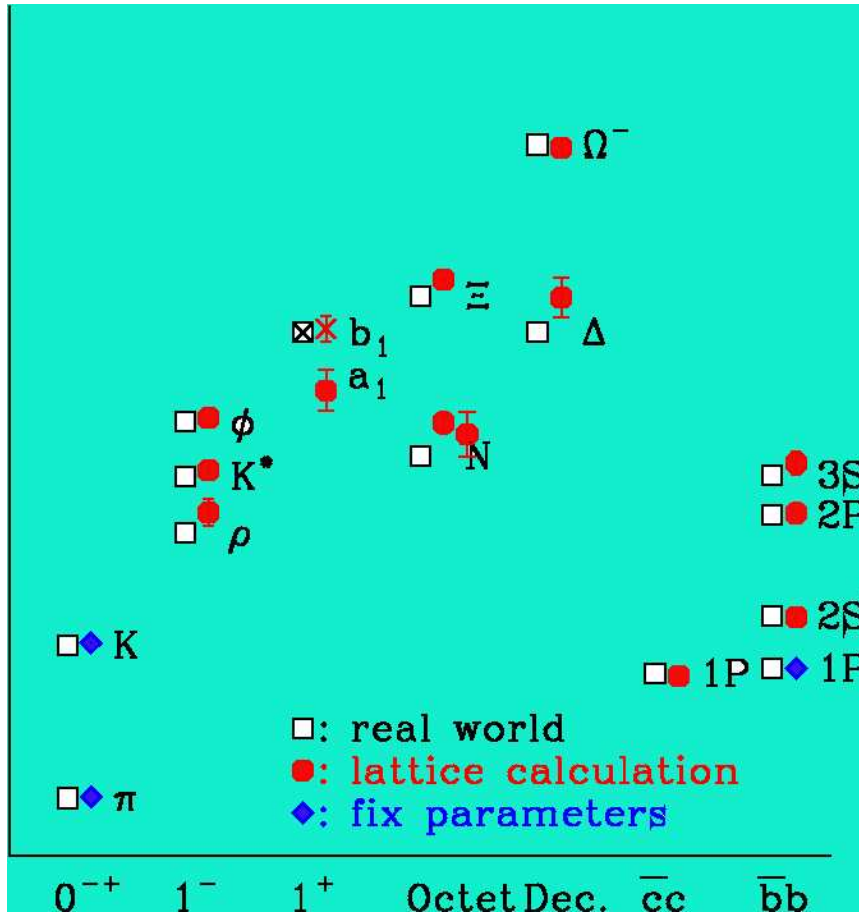
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- Similar to structure in theory of electrons & photons (QED).
- *Unlike QED*, the coupling is usually very large and its eight “photons” interact amongst themselves.
- Very high interaction (binding) energies. E.g.,  $M_{Proton} \gg (2m_u + m_d)$ , by a factor of 100 → Understanding it is knowing where the visible mass of Universe comes from.
- Much richer structure and phenomena : Quark Confinement, Dynamical Symmetry Breaking, Quark-Gluon Plasma, Colour Superconductivity..

QCD defined on a space time lattice – Best and Most Reliable way to extract  
**Predictions** for non-perturbative physics.

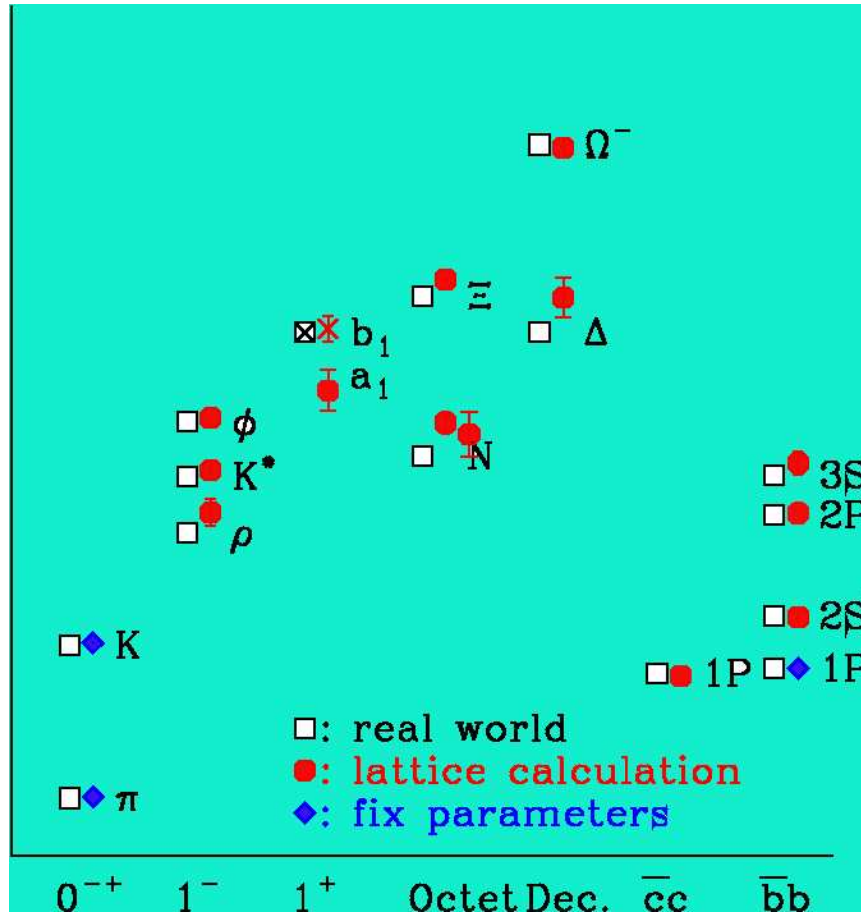
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♡ Strong Coupling Constant  $\alpha_s$  computed from underlying theory.

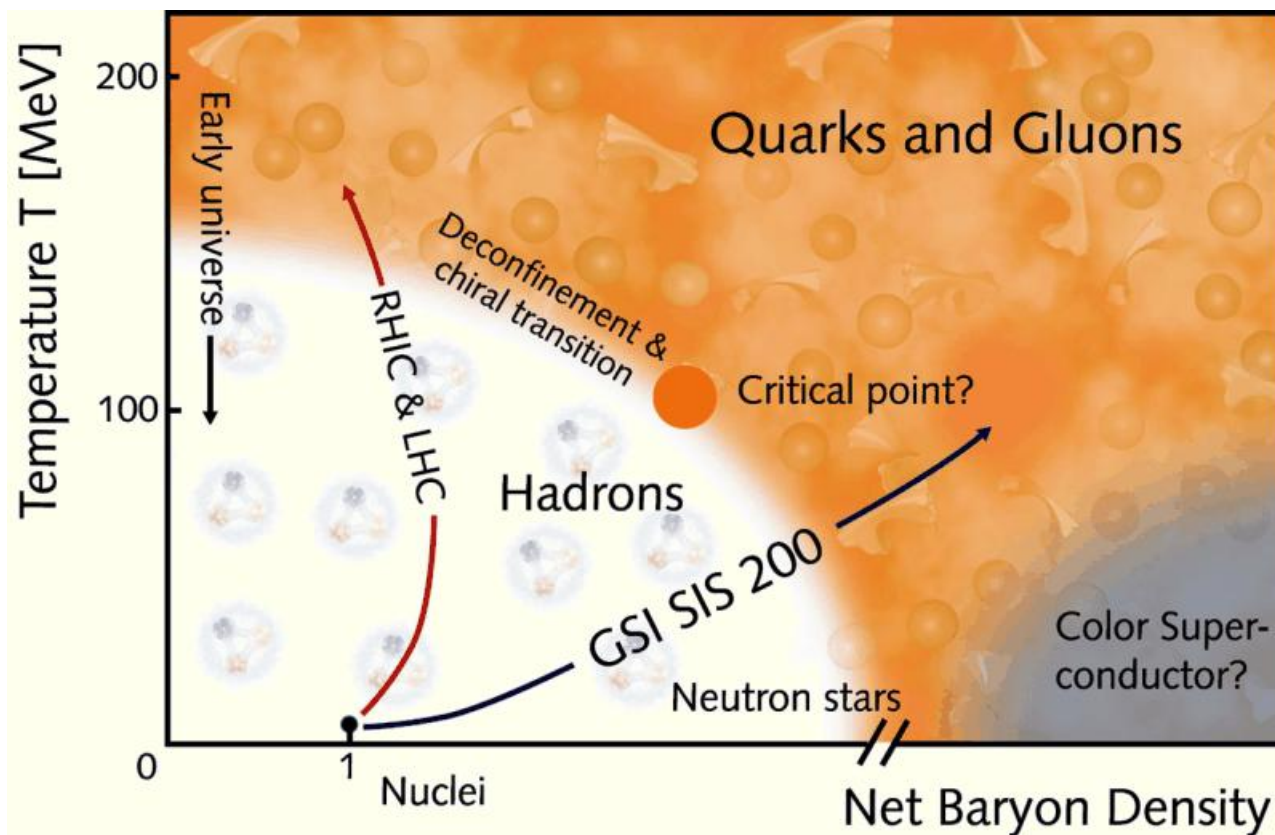
♡ Heavy Meson properties predicted :  $m_{B_c}, f_B, f_{D..}$



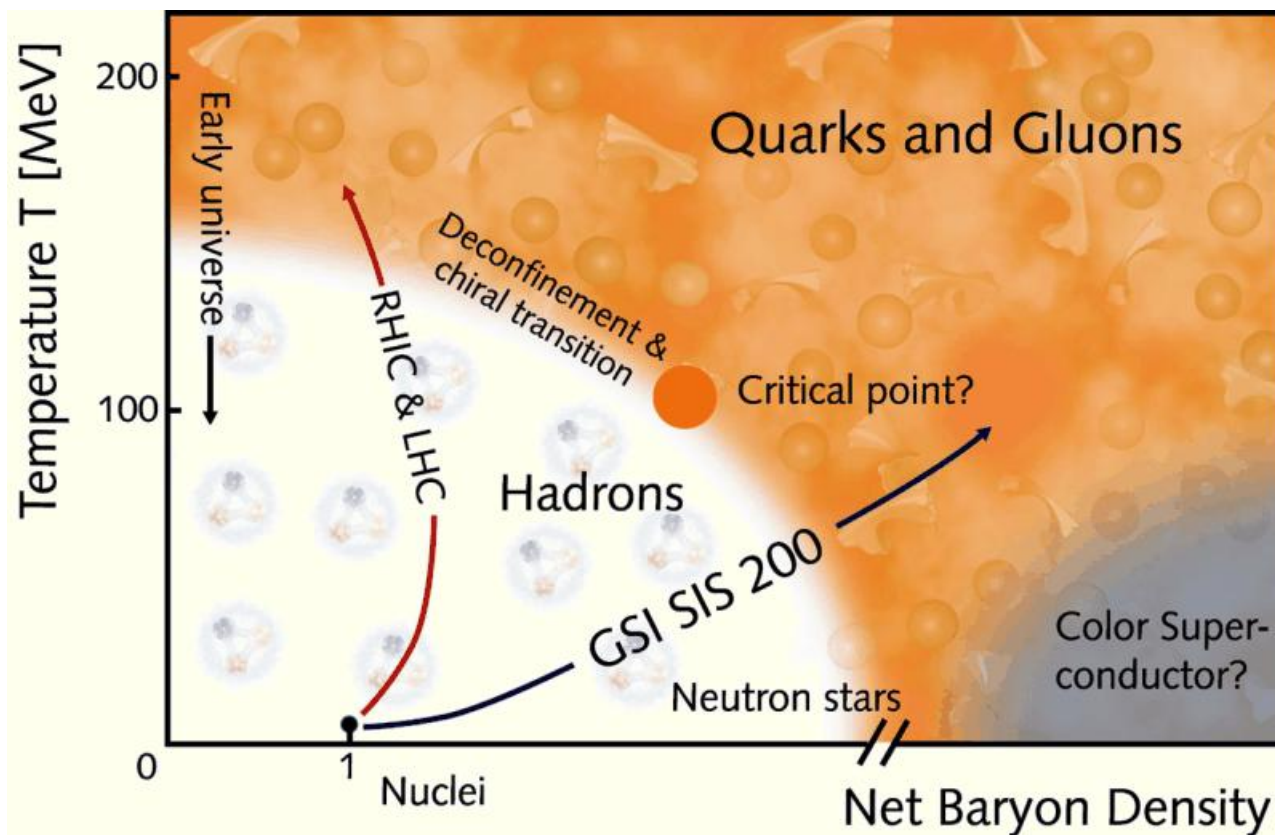
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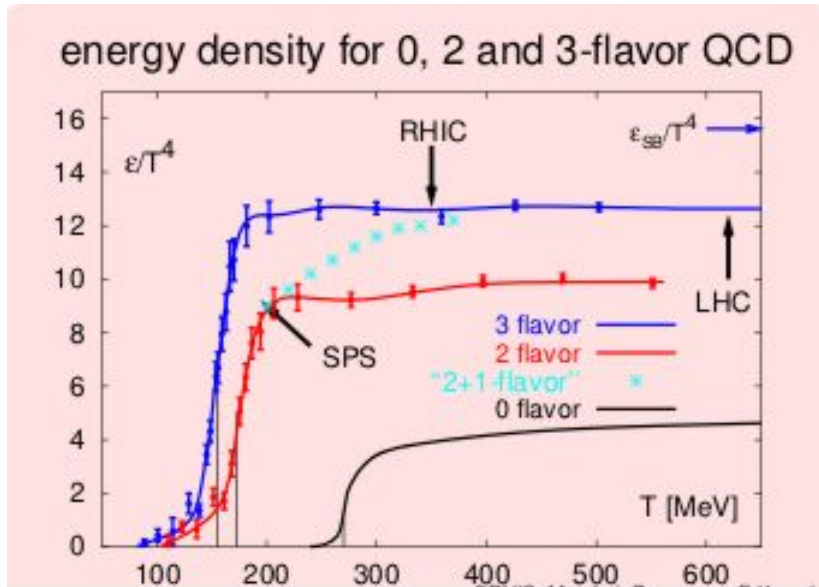
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- New States at High Density & Temperatures, expected on basis of models. (Figure from H. Stöcker)

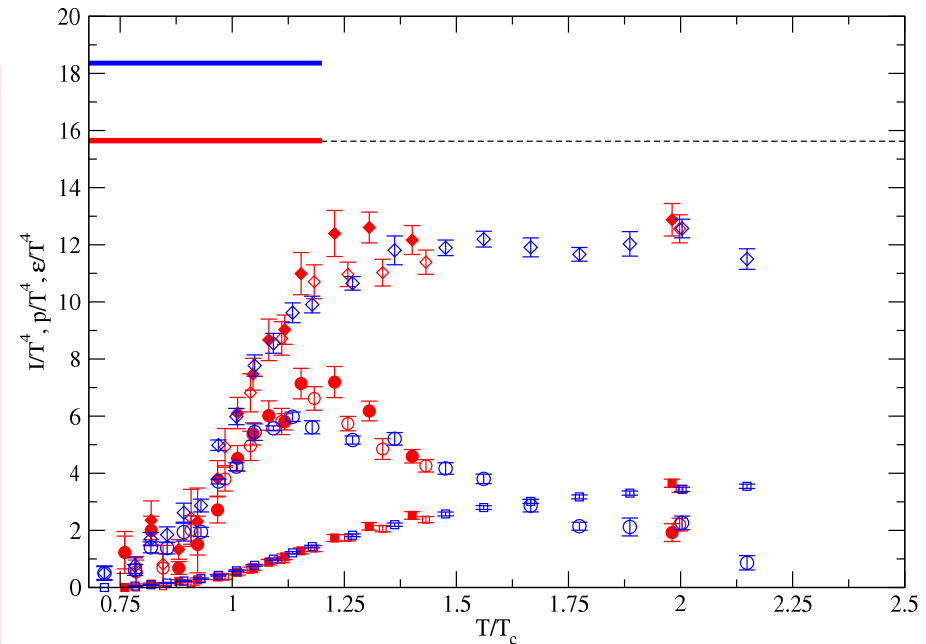
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$N_t = 4$  Lattices

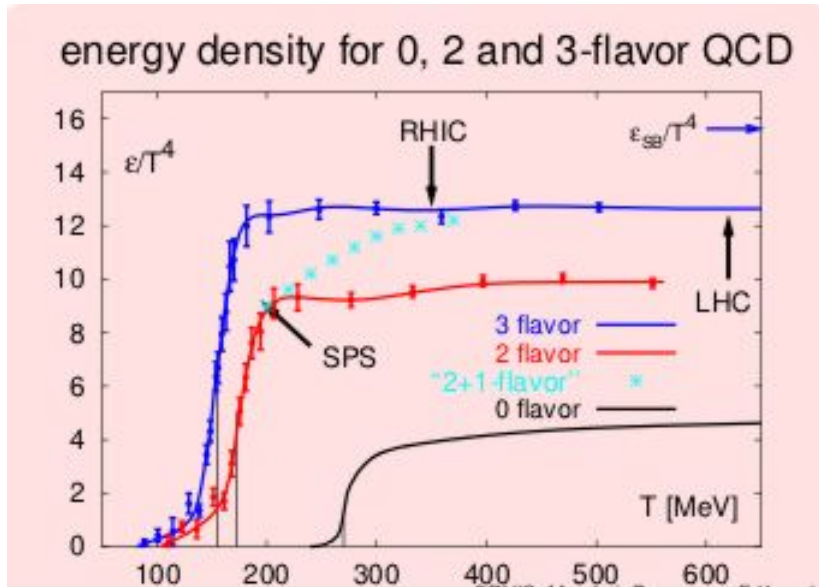
Bielefeld 2001 (Karsch hep-ph/0103314)



$N_t = 6$  Lattices

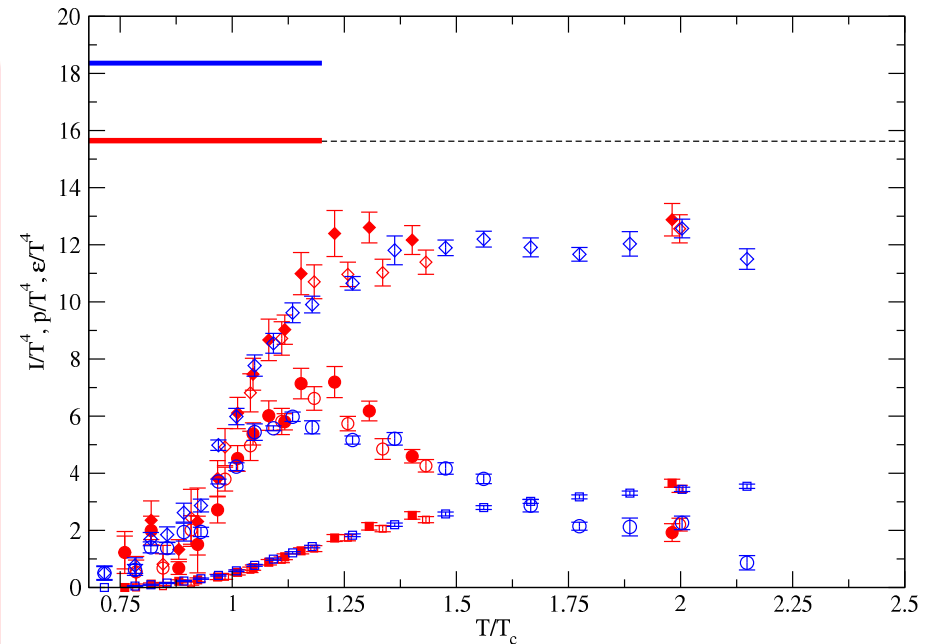
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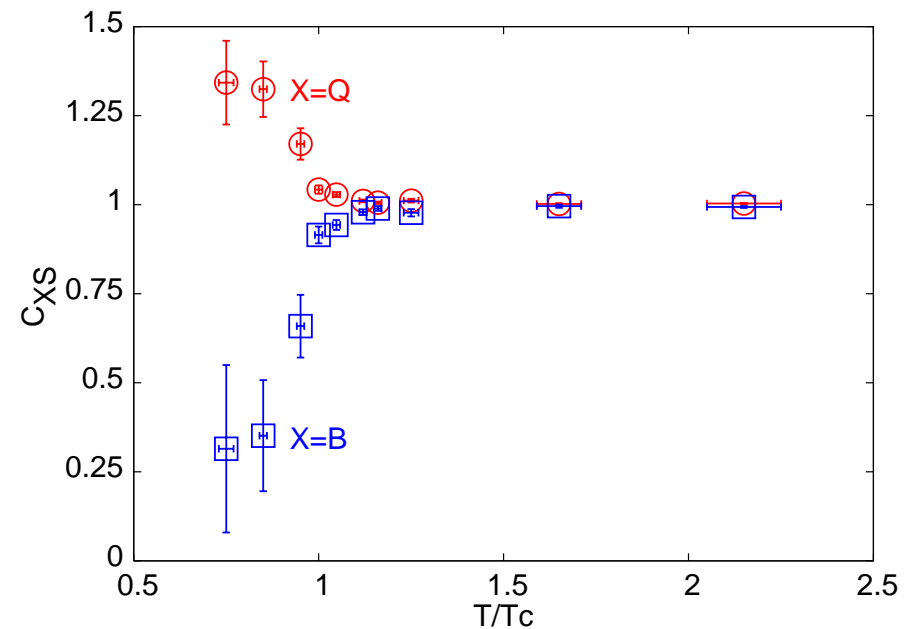
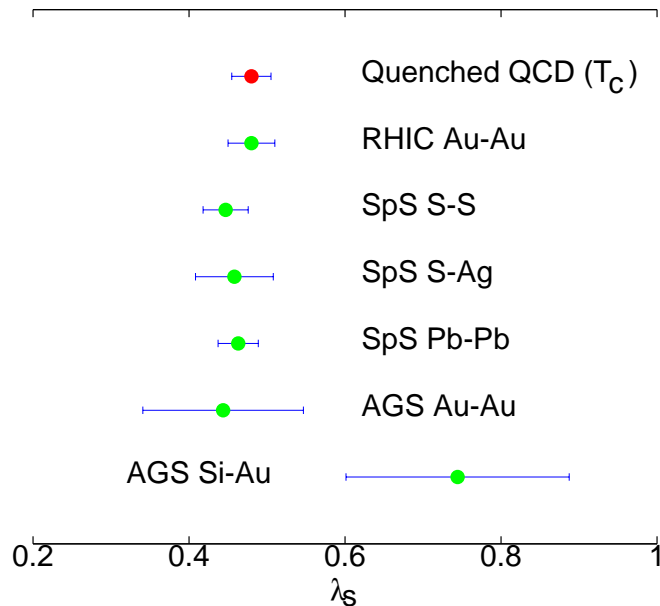
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- Other quantities for Heavy Ion Physics: the Wróblewski Parameter  $\lambda_s$ ,  $J/\psi$ -dissolution, dileptons, speed of sound, transport coefficients... etc.

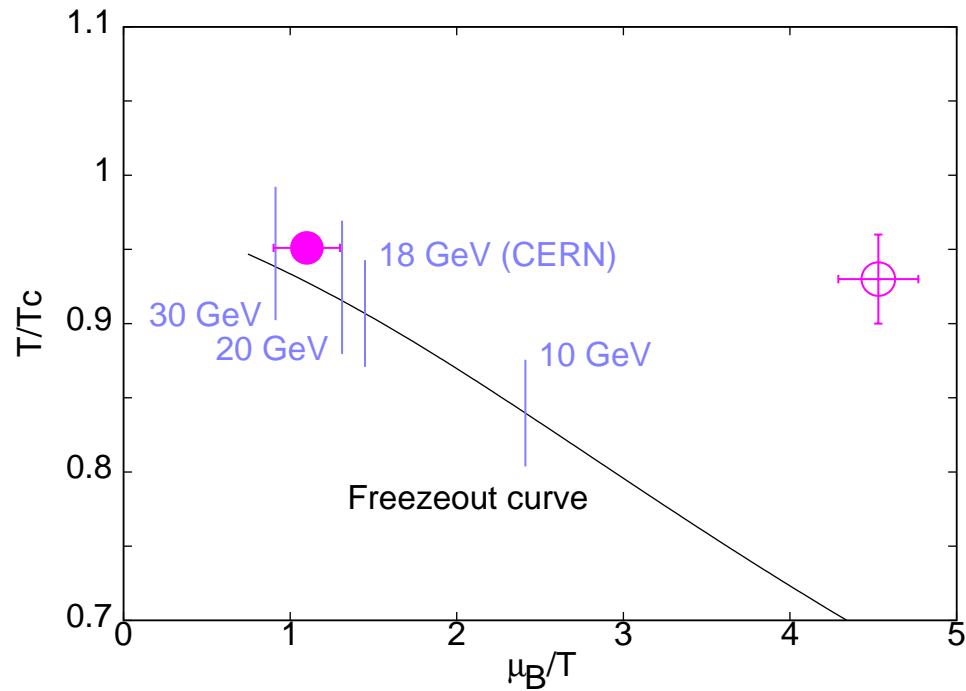
- $\lambda_s$  — Measure of Production of strange quark-antiquark pairs; Expts agree with estimates from the new state Quark-Gluon Plasma.
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R. V. Gavai and Sourendu Gupta, Phys Rev D65, 2002 and hep-lat/0510044.

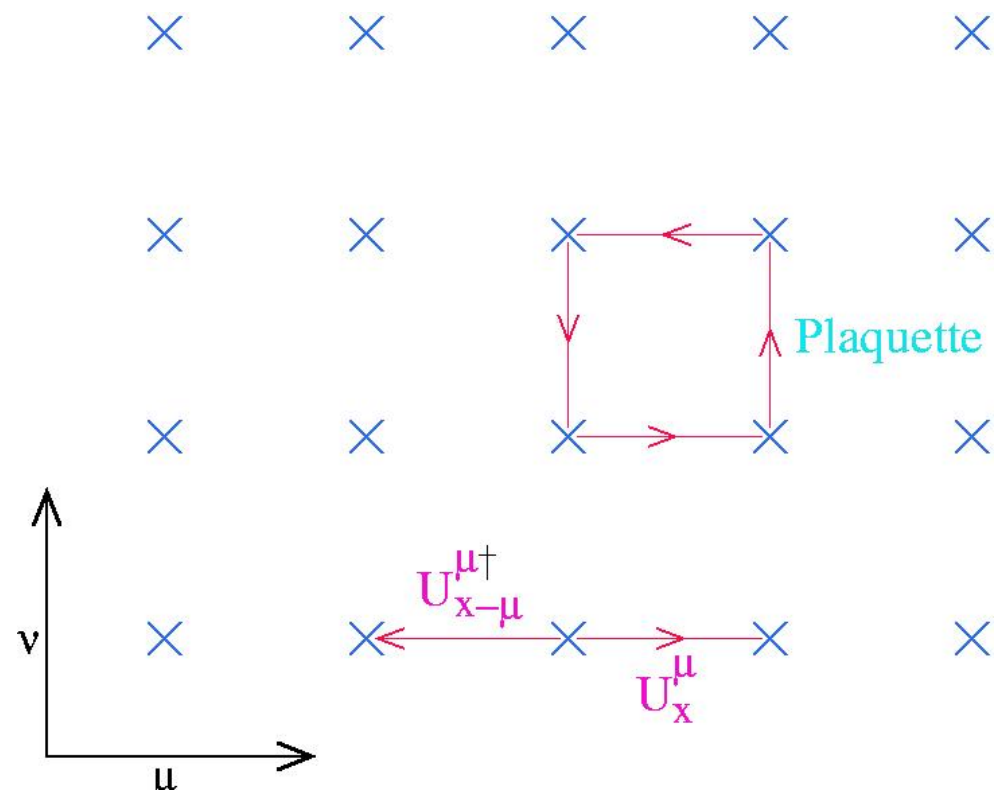


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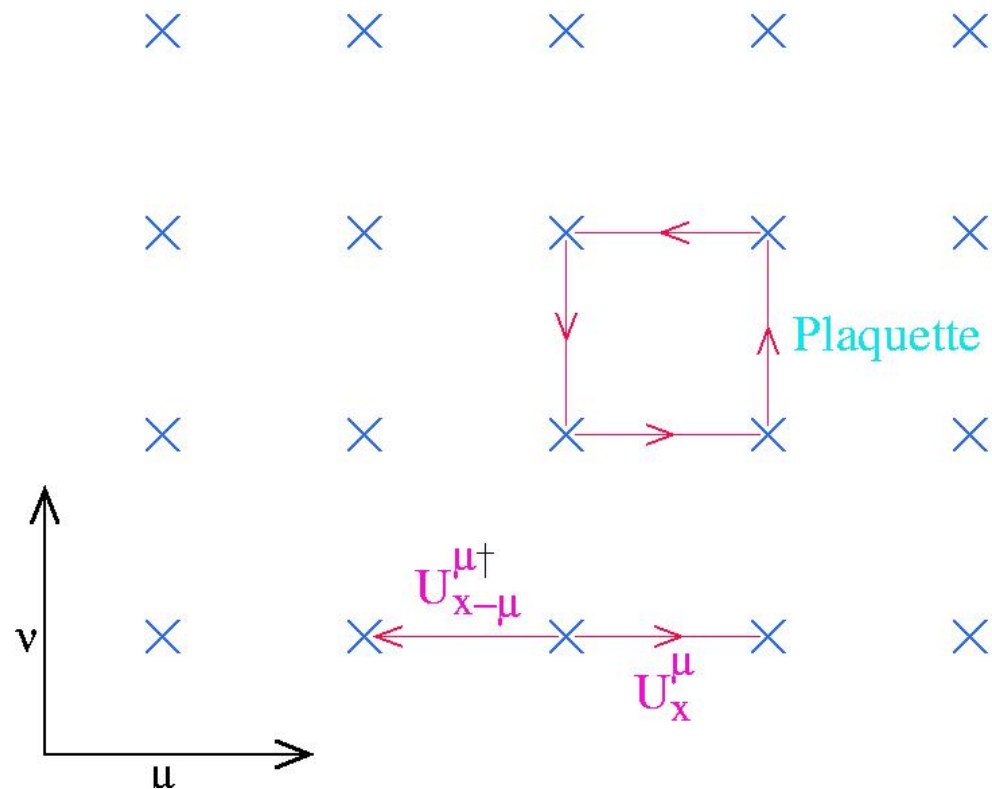
♠ Lattice QCD has yielded information on the critical point of QCD, which may be discovered in energy scans at RHIC (Open circle from Fodor-Katz JHEP 2002).

# Basic Lattice QCD

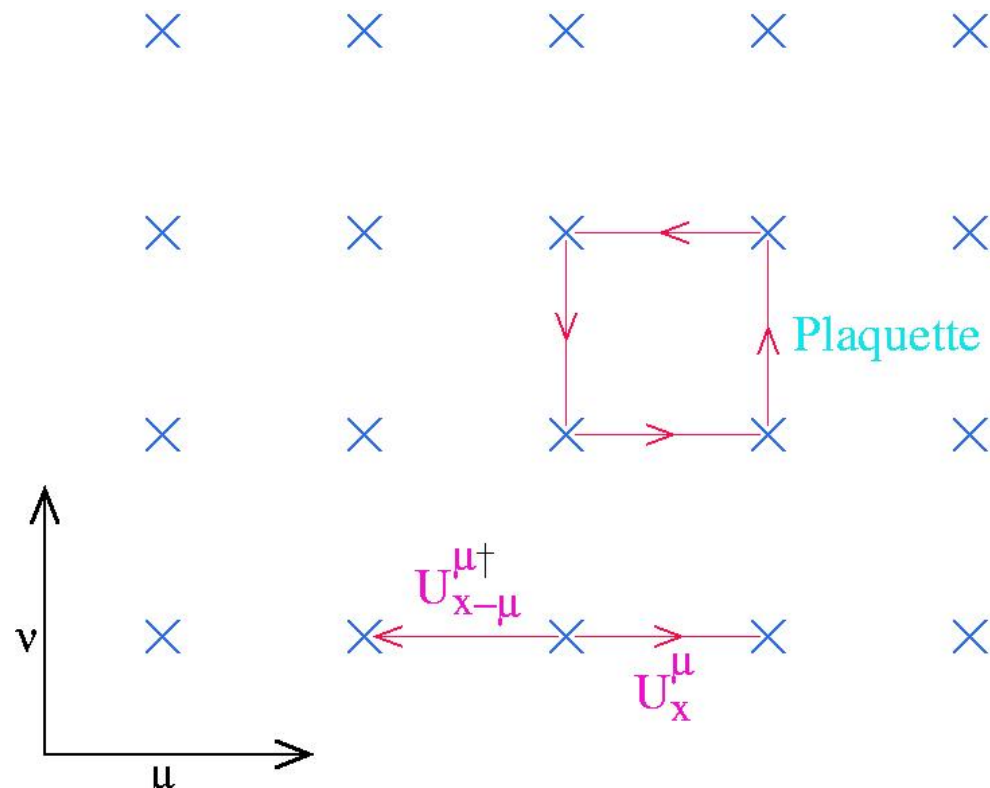
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 $\Rightarrow \psi'(x) = V_x \psi(x)$ ,  
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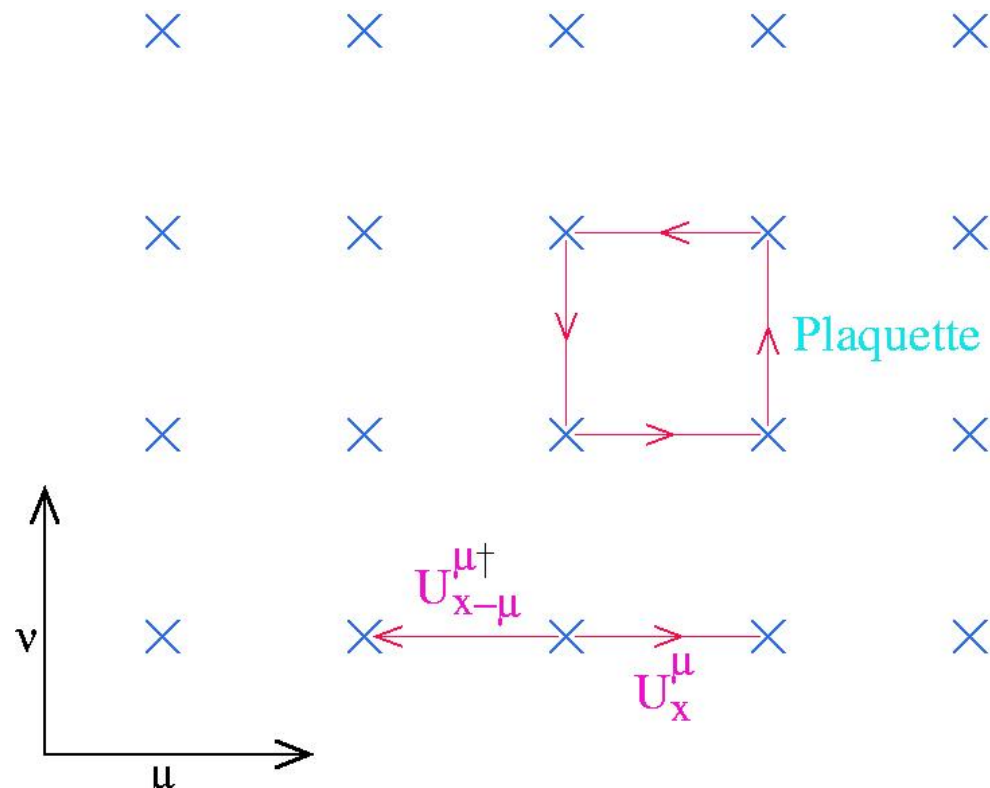
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- Fermion Actions : Staggered, Wilson, Overlap..

Typically, we need to evaluate

$$\langle \Theta(m_v) \rangle = \frac{\int DU \exp(-S_G) \Theta(m_v) \text{Det } M(m_s)}{\int DU \exp(-S_G) \text{Det } M(m_s)}, \quad (1)$$

where  $M$  is the Dirac matrix in  $x$ , colour, spin, flavour space for fermions of mass  $m_s$ ,  $S_G$  is the gluonic action, and the observable  $\Theta$  may contain fermion propagators of mass  $m_v$ .



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Complexity of evaluation of  $\text{Det } M \implies$  approximations : Quenched ( $m_s = \infty$  limit) and Full (low  $m_s = m_u = m_d$ )  $\rightsquigarrow$  Computer time  $\uparrow$  and Precision  $\downarrow$ .

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- Large-scale— such as QCD spectrum or thermodynamics for realistic light quark (physical pion) masses. Need large lattices to have reasonable box size in units of  $m_\pi^{-1}$ , i.e., more computational power.

- Most CPU time in full QCD simulations goes in obtaining the quark propagator  $M^{-1}$  by using Conjugate Gradient, i.e., in solving  $M \cdot X = r$ , for a given source vector  $r$ .
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- Suitable for both parallelization and vectorization. Both aspects have been exploited efficiently in Lattice QCD computations.
- Lattice QCD experts are actively involved in design and development of new parallel technology in hardware and software.

# Current Scenario

- Factors governing choice of machines :
  - Processor – high sustained performance for QCD code, large cache, fast interface to memory/network.
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  - Ease of Programming – Standard Languages (C/C++, Fortran), Efficient compilers and system libraries.
  - Costs – Machine (Hardware/Software) and Operational costs like Power and Cooling.
  - Space Requirement.

- Lattice results have been (will be) obtained with
  - Custom-Design machines, e.g.,
    - \* CP-PACS(Tsukuba)
    - \* QCDSP/QCDOC (Columbia),
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  - PC Clusters, e.g., Wuppertal, JLAB, Fermilab

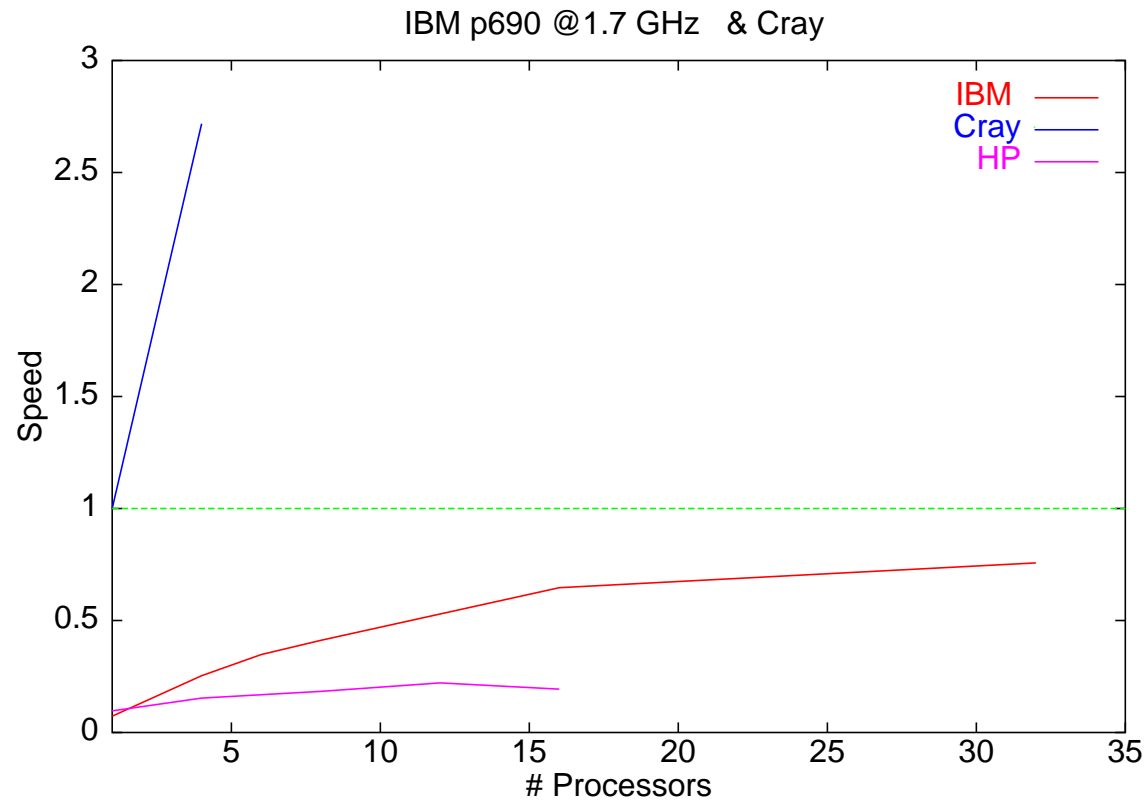
# Our Main Workhorse



CRAY X1 of I L G T I , T I F R, Mumbai

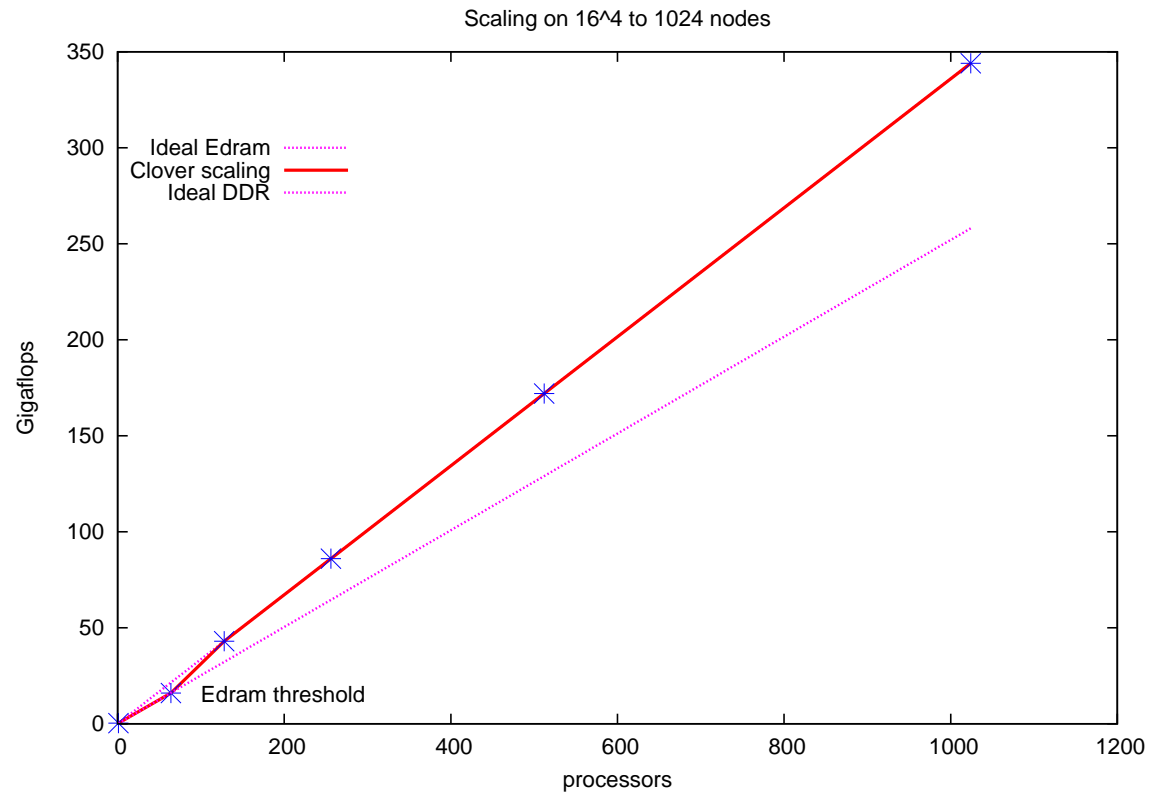
# Strong Scaling : Problem size fixed

## Hybrid Monte Carlo – Full QCD



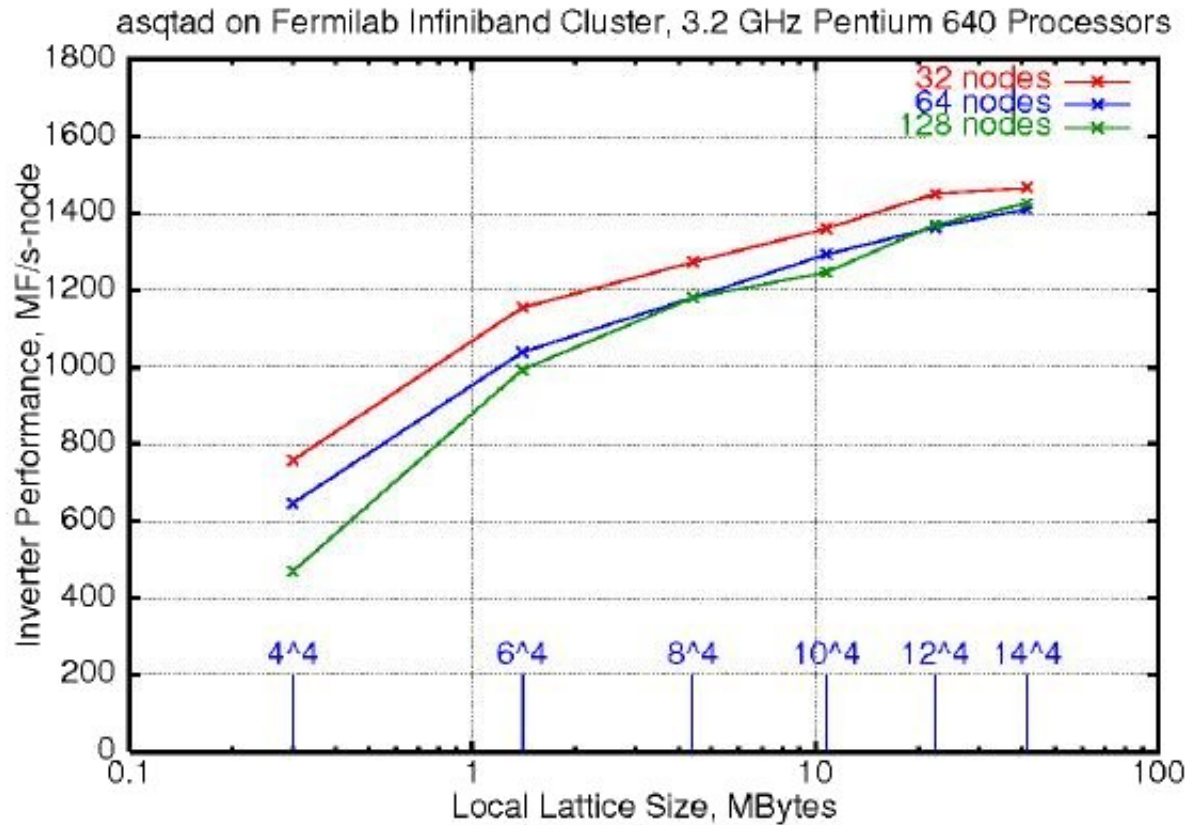


## Strong Scaling : Problem size fixed QCDOC - Clover Conjugate Gradient



P. A. Boyle et al., NP B(PS) 140 (2005) 169.

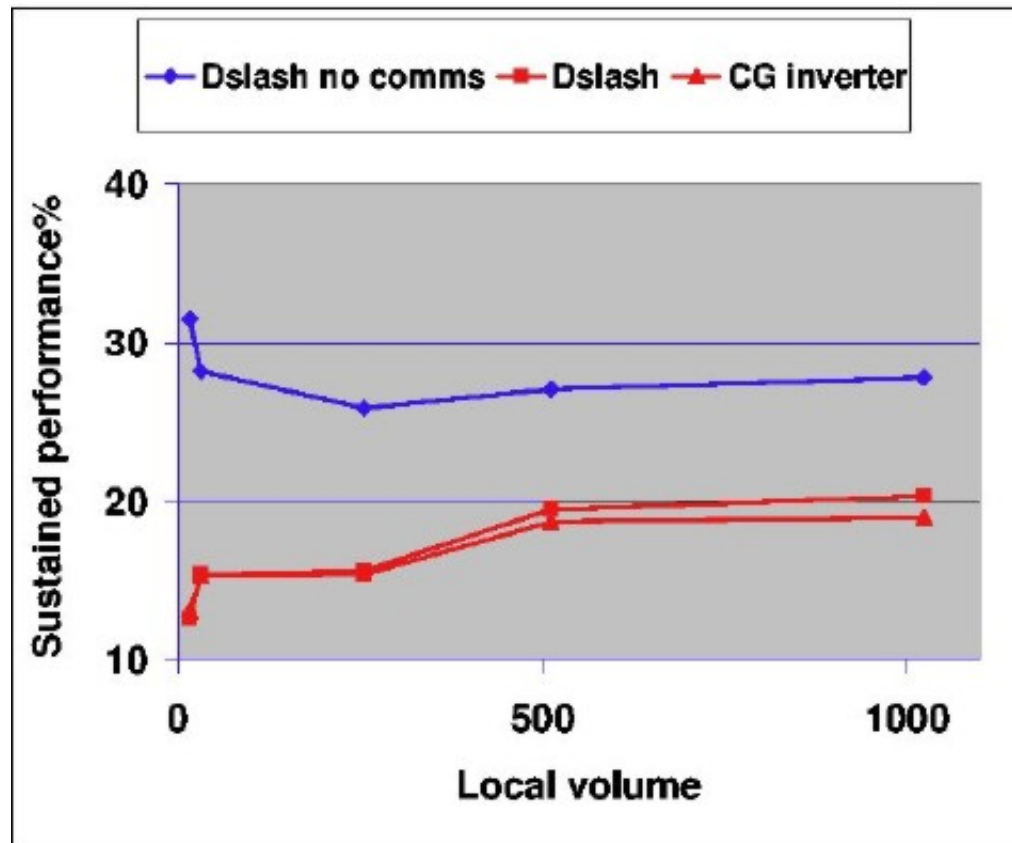
## Weak Scaling : Local volume fixed Staggered Fermion Conjugate Gradient



Weak  
scaling  
curves on  
new cluster

From D. J. Holmgren.

## Scaling : BlueGene/L Wilson Fermion Conjugate Gradient



G. Bhanot, D. Chen, A. Gara and P. Vranas, NP B(PS) 140 (2005) 823.

# Future Prospects

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- So will commercial supercomputers, e.g.,
  - IBM BlueGene/P, successor to BlueGene/L
  - CRAY Black Widow successor to X1, Strider 3
  - Fujitsu has plans for 3 PFlops by 2010
- Custom-Design machines ?

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All three avenues, i.e., custom-designed machines, commercial supercomputers and PC clusters, likely to continue playing important role in future.