

Renormalized Polyakov loop in the Fixed Scale Approach

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T. I. F. R., Mumbai, India

** arXiv : 1001.4977, submitted to Phys. Lett. B & in preparation.*

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Introduction

Results

Summary

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Introduction

- Polyakov loop $L(\vec{x})$ — Deconfinement Order Parameter (Spontaneous Breaking of $Z(N)$) (McLerran & Svetitsky, PRD 1981)
- One hopes to construct effective theories (Pisarski, PRD 2006) of L for investigations of deconfinement phase transitions and many models employ L .

Introduction

- Polyakov loop $L(\vec{x})$ — Deconfinement Order Parameter (Spontaneous Breaking of $Z(N)$) (McLerran & Svetitsky, PRD 1981)
- One hopes to construct effective theories (Pisarski, PRD 2006) of L for investigations of deconfinement phase transitions and many models employ L .
- On an Euclidean $N_\sigma^3 \times N_\tau$ lattice $L(\vec{x})$ is defined at a site \vec{x} as
$$L(\vec{x}) = \frac{1}{N_c} \text{Tr} \prod_{x_0=1}^{N_\tau} U^4(\vec{x}, x_0).$$
- No SSB on finite lattices/volumes. Usually one defines $\bar{L} = \sum_{\vec{x}} L(\vec{x})/N_\sigma^3$, and employs $\langle |\bar{L}| \rangle$, or its susceptibility, to locate the deconfinement phase transition.
- $\langle |\bar{L}| \rangle \rightarrow 0$ as $1/\text{Volume}$ in the confined phase, and $\langle |\bar{L}| \rangle \neq 0$ in the deconfined phase.

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- Like any Wilson loop, Polyakov loop needs to be renormalized.
- More so, since as an order parameter it seeks to label phases by being zero or nonzero.

Earlier Work

♣ The physical interpretation of L as relate to the free energy of a single static quark offers a clue.

♠ The single quark free energy $F_b(N_\tau, a)$ is obtained from $\ln\langle|\bar{L}|\rangle = -F_b(T)/T = -aN_\tau F_b(N_\tau, a)$.

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- Use of quark-antiquark (Polyakov loop) correlations (Kaczmarek et al. PLB 2002)
- Use of N_τ -grids and fits to L (Dumitru et al. PRD 2004)
- Use of renormalization group iteratively (S. Gupta et al. PRD 2008)

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♡ Let β_c , corresponding to the position of the peak of the $|L|$ -susceptibility for some fixed $N_{\tau,c}$, be the choice of the fixed scale a_c .

♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

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♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

♠ Write the single quark free energy as a sum of a would-be divergent and a regular contribution,

$$F_b(T, a_c) = F(T, a_c) - A(a_c),$$

where A is the would-be divergent free energy in physical units.

♠ Since

$$\frac{T}{T_c} \ln \langle |\bar{L}| \rangle = -\frac{F(T, a_c)}{T_c} + \frac{A(a_c)}{T_c},$$

the free energy at any two different scales, a_{c1} and a_{c2} , differs by the *same* constant at all T .

◇ Use $\langle |L| \rangle$ at just one temperature to eliminate the relative shift \implies All cut-off dependence of the order parameter is gone in the entire T -range.

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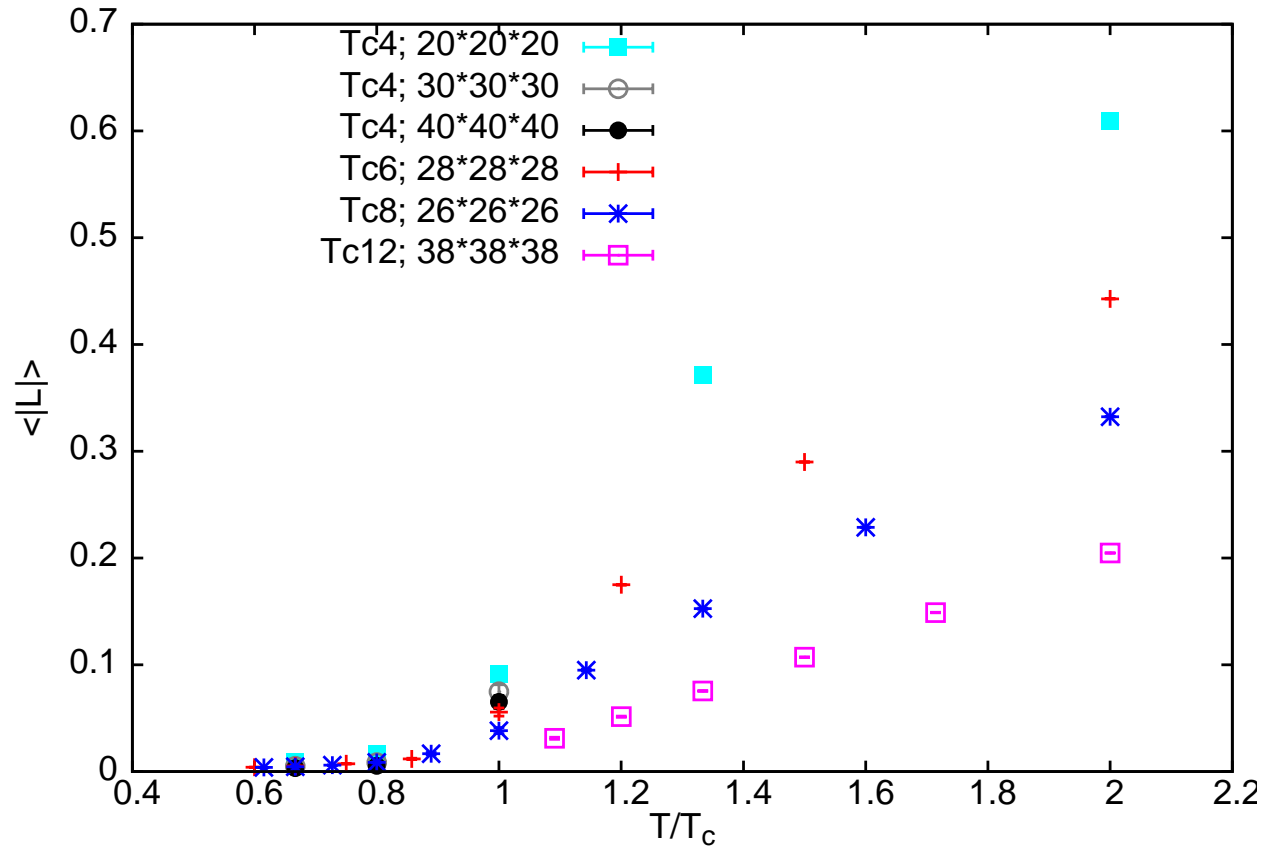
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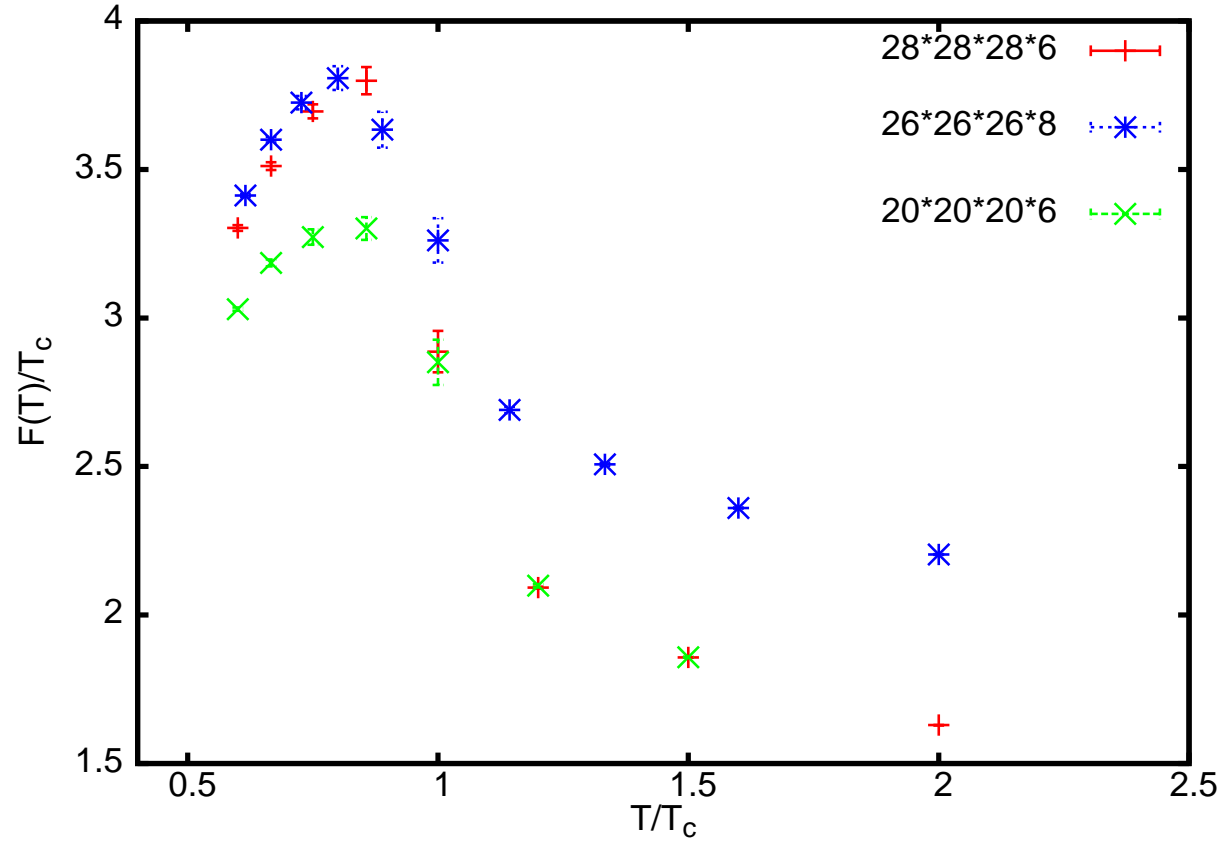
♣ In the following, I consider the simple case of $SU(2)$ to demonstrate how well it works. It should work similarly for any N_c or QCD.

♥ I employ the critical β for $N_\tau = 4, 6, 8$ and 12 from the table of Velytsky, IJMP C19, (2008), 1079, which agree with earlier results where available.

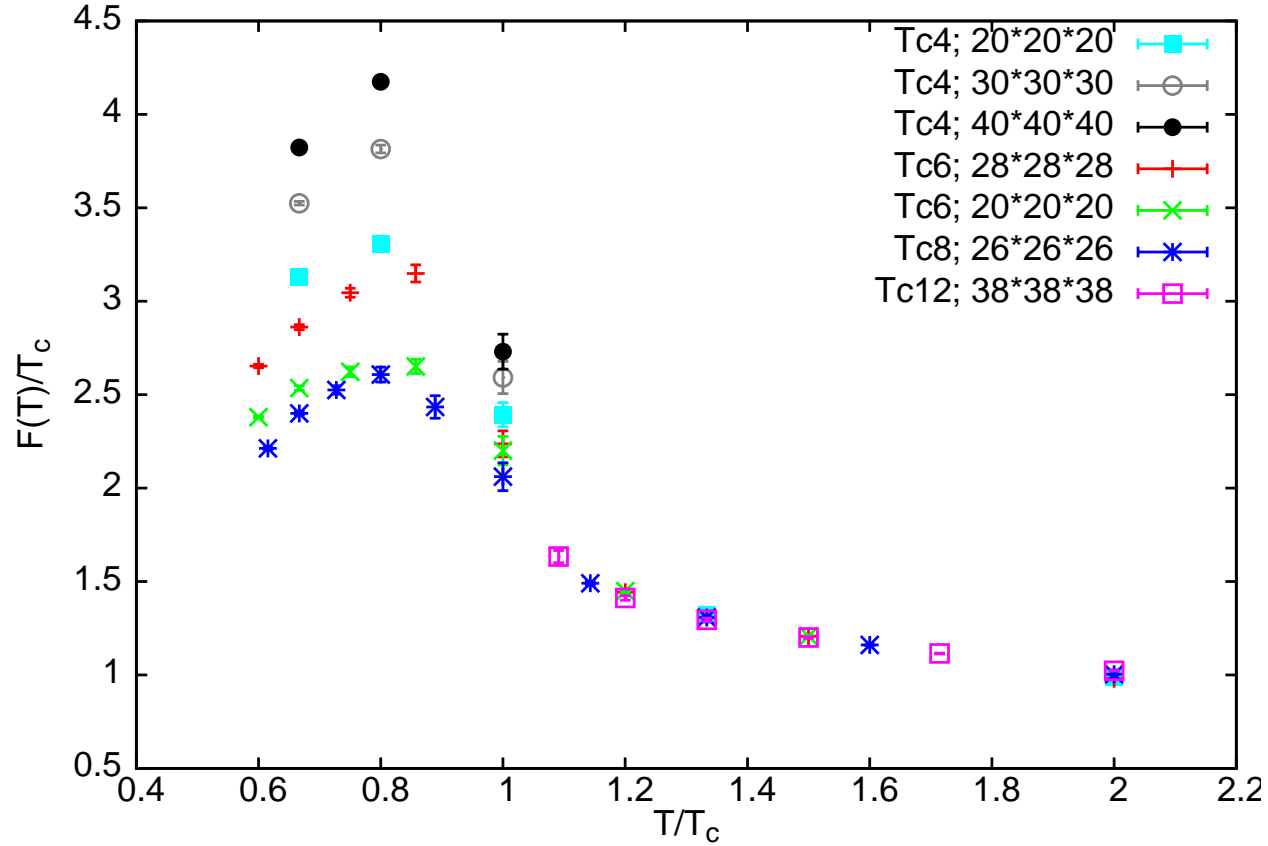
Results for $SU(2)$



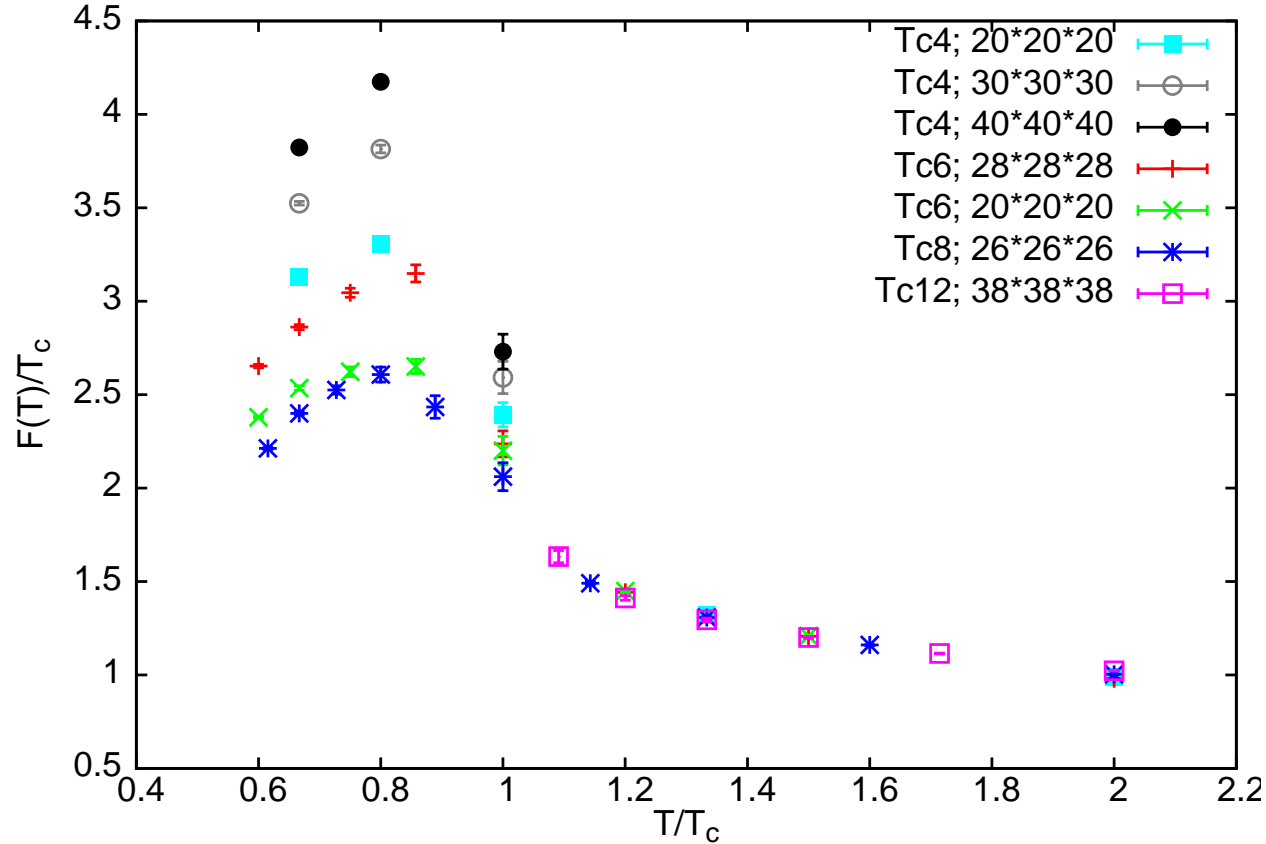
- 4 different scales : Tc4, Tc6, Tc8 and Tc12 with $a \rightarrow 0$ progressively. Increasing Spatial Volume leads to decrease in L for $T < T_c$.



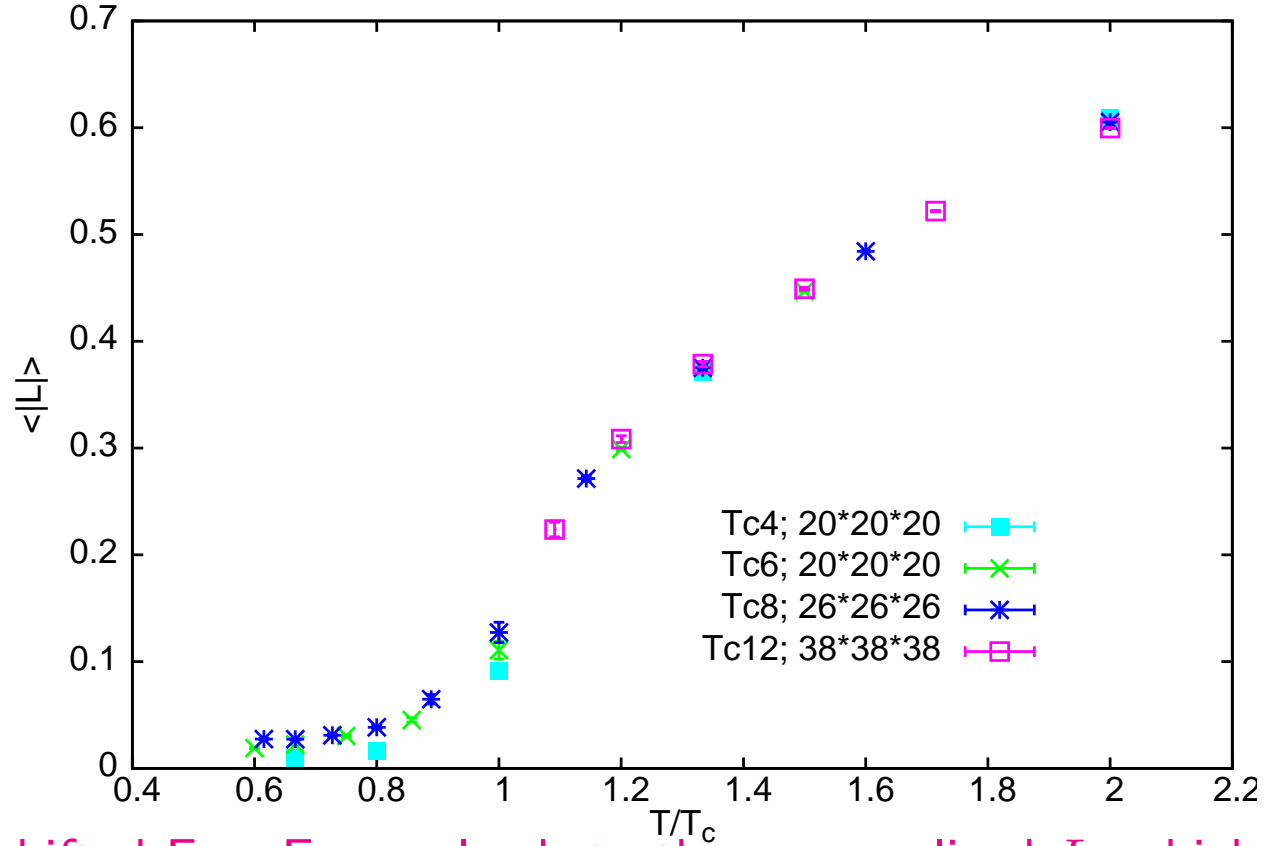
- Illustrate for two scales : Different behaviour in T for the Free Energy. Shift F by $\Delta F(2T_c)$.



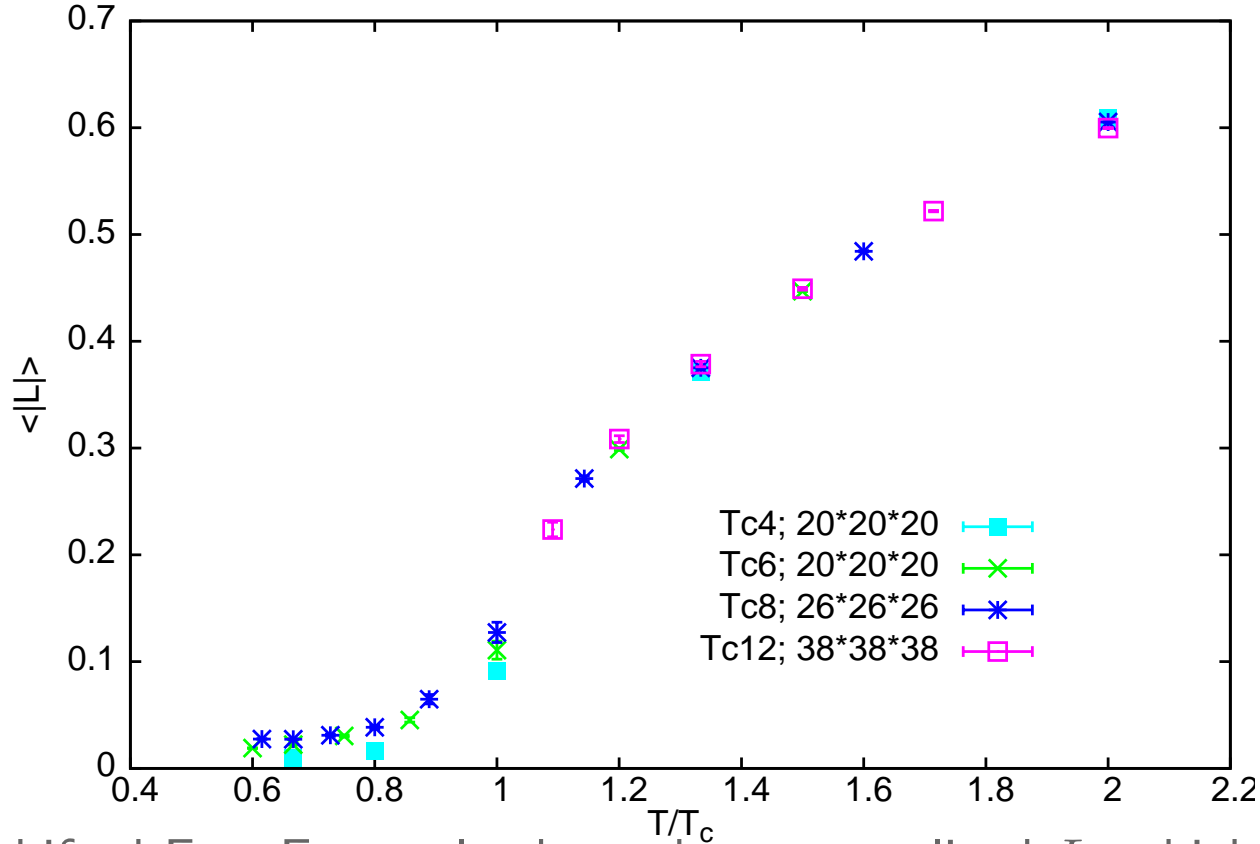
- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.



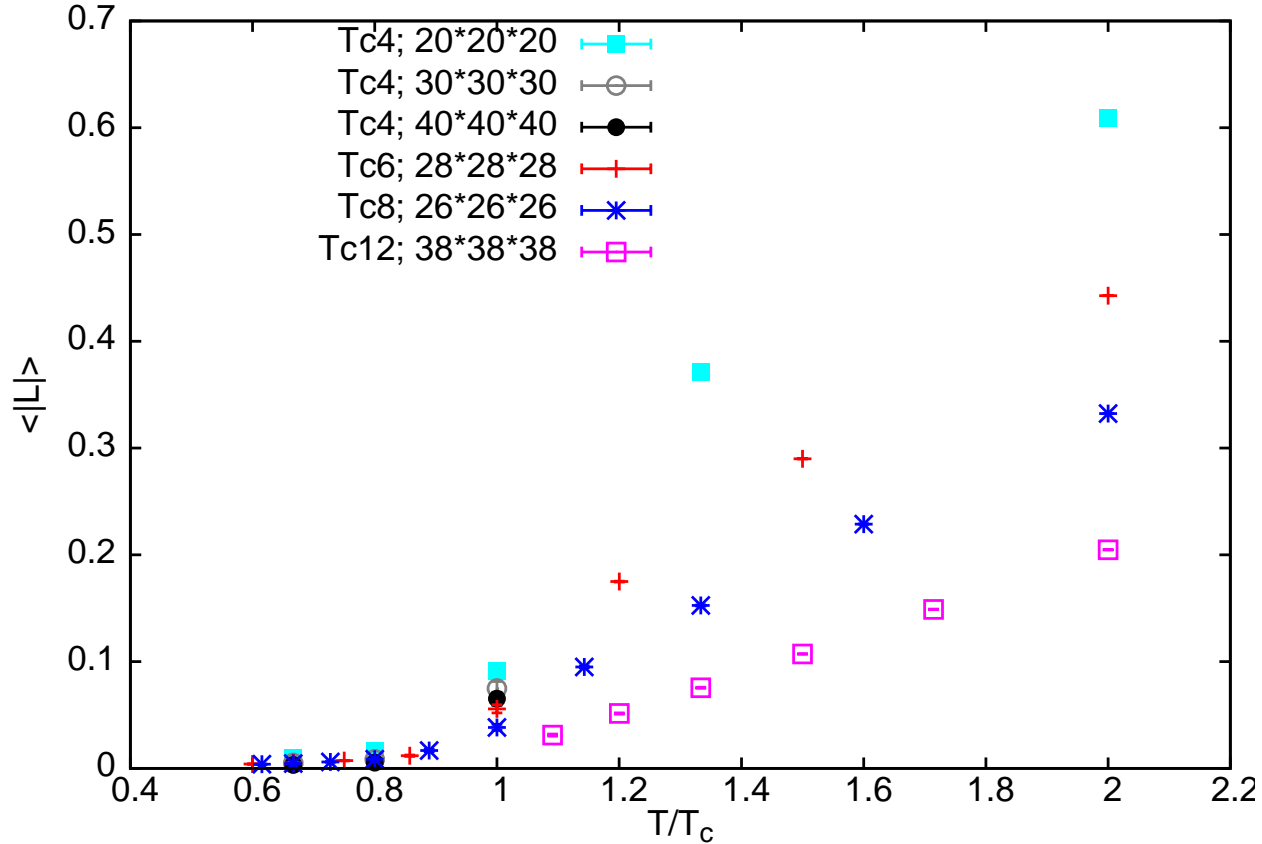
- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.
- For $T \leq T_c$, F increases with the spatial volume but scale-independent.



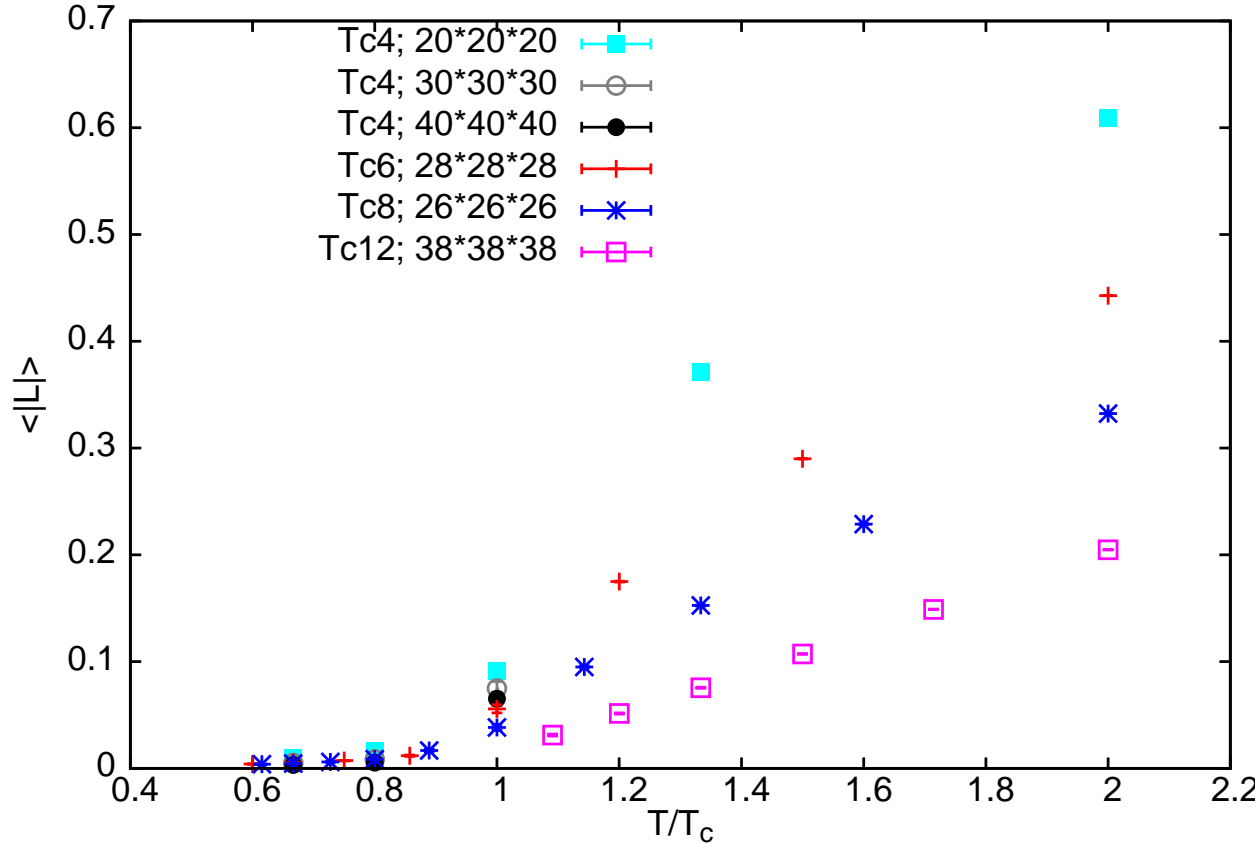
- The shifted Free Energy leads to the renormalized L , which is independent of cut-off for $\beta \geq 2.2991$.



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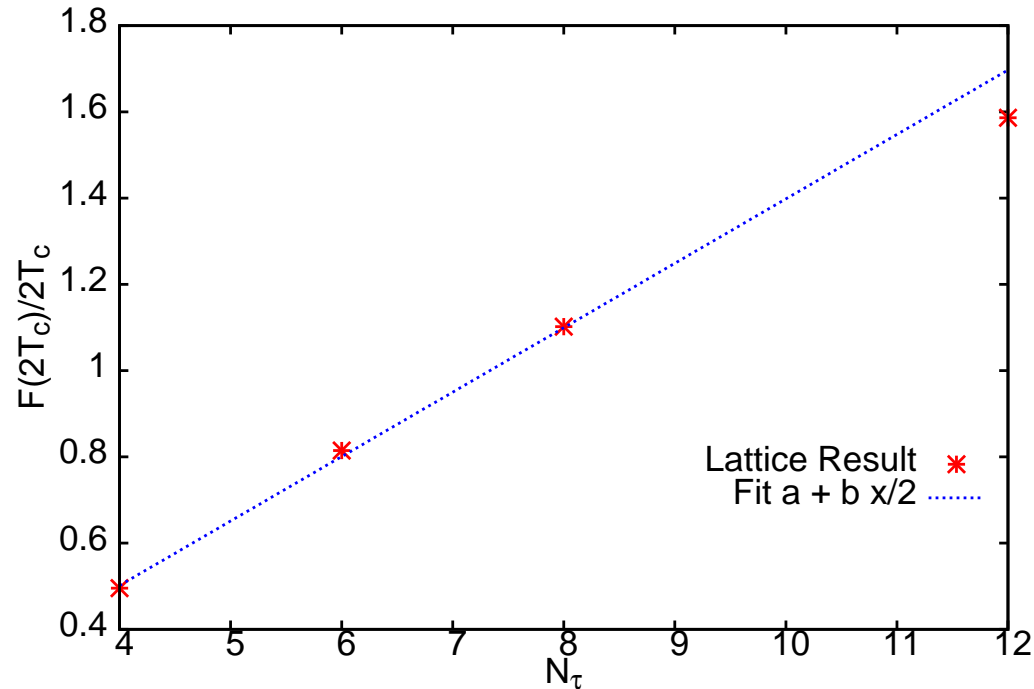
- I chose 3 constants to shift all the data to the Tc4 scale : The Tc6, Tc8, Tc12 results have simply jumped to their appropriate place on the $\langle |L| \rangle$ for it.



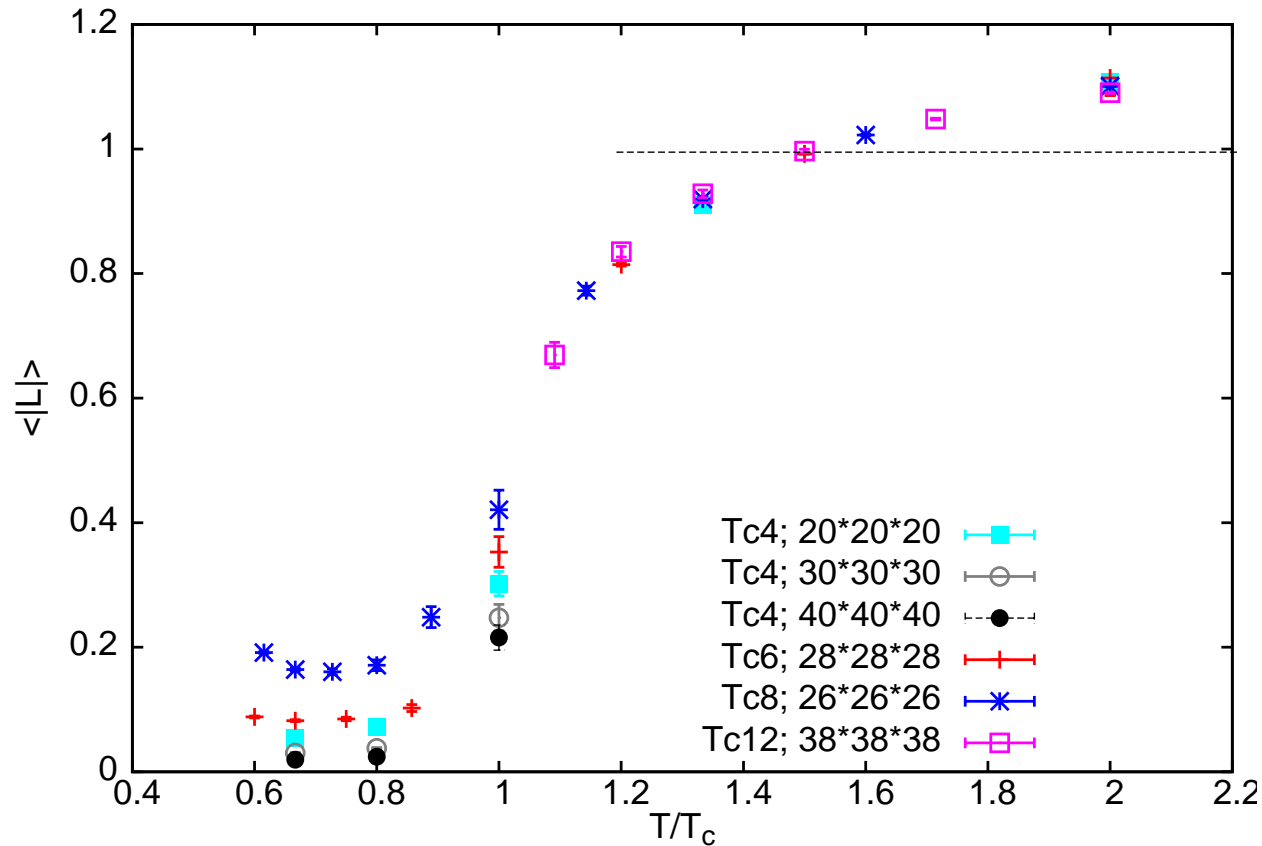
- I chose 3 constants to shift all the data to the Tc4 scale : The Tc6, Tc8, Tc12 results have simply jumped to their appropriate place on the $\langle |L| \rangle$ for it.
- Does the renormalized L then climb to unity slowly?

- High Temperature Perturbation Theory (Gava-Jengo, PLB 1981) tells us that $L \rightarrow 1$ from above at very large T : $L = 1 + C_3 g^3 + \mathcal{O}(g^4)$, where $c_3(N_c) > 0$ is a constant.

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- In stead of shifts at $2T_c$ for varying scales, try a fit $-\ln\langle|\bar{L}_j|\rangle = F(2T_c)/2T_c + B \cdot N_{\tau j}/2$.



♣ Eliminating the B -dependent divergent term for the Tc4-scale in addition to the shifts, one has,



♠ L now does go to unity from above at large T . Large volumes, aspect ratio of ~ 10 , needed for $L \simeq 0$ for low T .

Summary

- I showed that the fixed scale approach leads to a natural definition of a physical, N_τ -independent, order parameter which is defined in both the confined and the deconfined phases.
- It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).

Summary

- I showed that the fixed scale approach leads to a natural definition of a physical, N_τ -independent, order parameter which is defined in both the confined and the deconfined phases.
- It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).
- The definition itself does not depend on any lattice artifacts or the lattice size in the deconfined phase.
- It displays the expected behaviour in both the phases, i.e., volume dependence in the low T -phase and approach to unity from above in high T -phase.