QCD Critical Point : Marching towards continuum

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Introduction

Our Lattice Results

Summary

* *Work done with Saumen Datta & Sourendu Gupta*

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- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics.

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- Mostly staggered quarks used in these simulations. Broken flavour and spin symmetry on lattice.

- The expectation value of an observable $\mathcal{O}$ computed by importance sampling :

$$\langle \mathcal{O} \rangle = \frac{\int DU \exp(-S_G) \ \mathcal{O} \prod_f \mathrm{Det} \ M(m_f, \mu_f)}{\mathcal{Z}}.$$

Simulations can be done IF Det $M > 0$. However, det M is a complex number for any $\mu \neq 0$: The Phase/sign problem.

Lattice Approaches

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- Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
- Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
- Taylor Expansion (C. Allton et al., PR D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).
- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

Detail of Expansion

Text-book definitions yield various number densities and susceptibilities :

$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}$ and $\chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$.

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \quad (1)$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$. We use both and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8^{th} order. Bielefeld-RBC so far has up to 6^{th} order.

- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2012 & PRD 2010) which save up to 60 % computer time.

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Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\pi/m_\rho = 0.31 \pm 0.01$; Kept the same as $a \to 0$ (on all N_t).
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
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- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: ↓ to 1.1(1)

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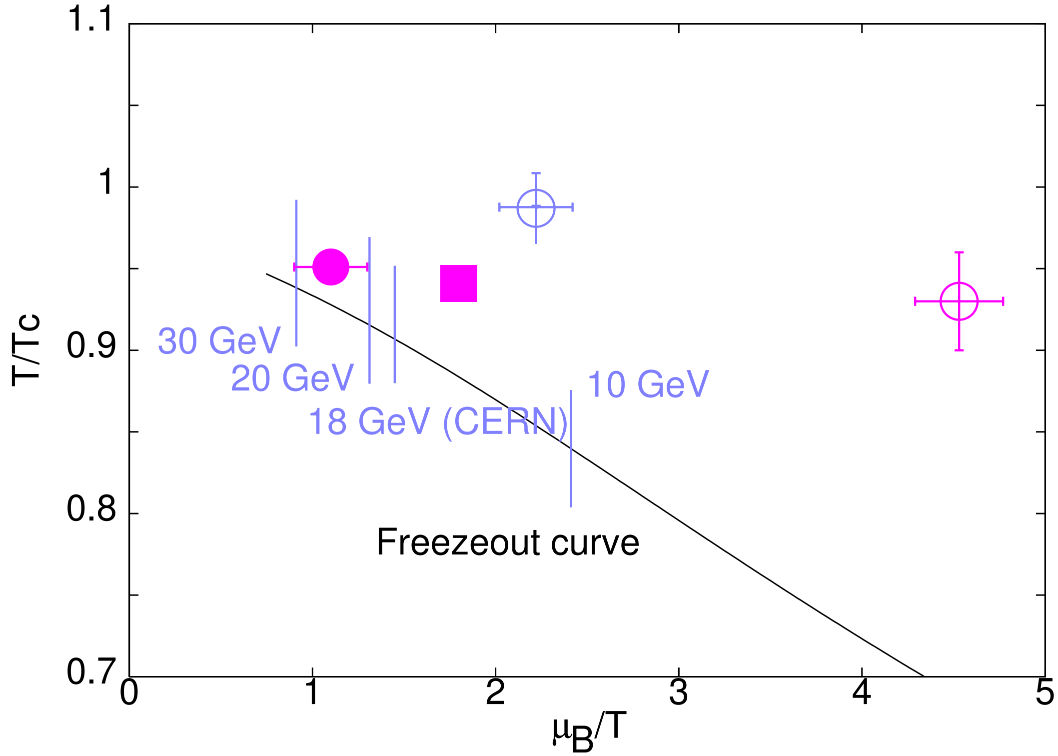
- T_c — defined by the peak of Polyakov loop susceptibility.
- Even finer Lattice : 8 $\times 32^3$ — This Talk
  Aspect ratio, N_s/N_t, maintained four to reduce finite volume effects.

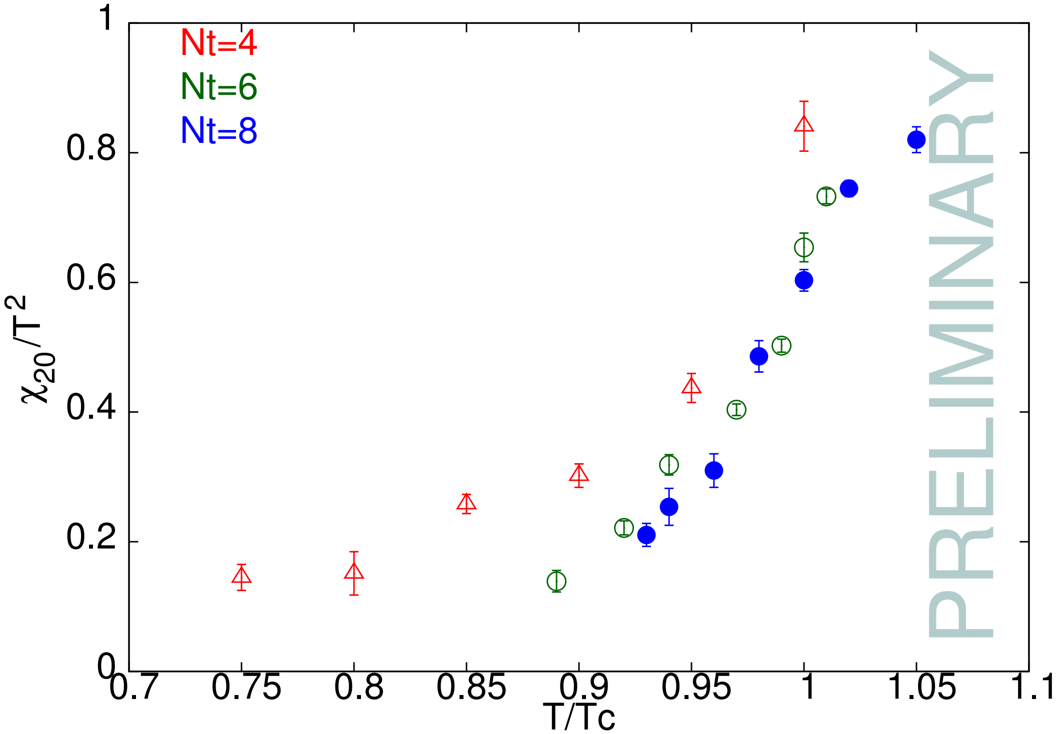
Critical Point : Story thus far



♠ $N_f = 2$ (magenta) and 2+1 (blue) (Fodor-Katz, JHEP '04).

♡ $N_t = 4$ Circles (GG '05 & Fodor-Katz JHEP '02), $N_t = 6$ Box (GG '09).

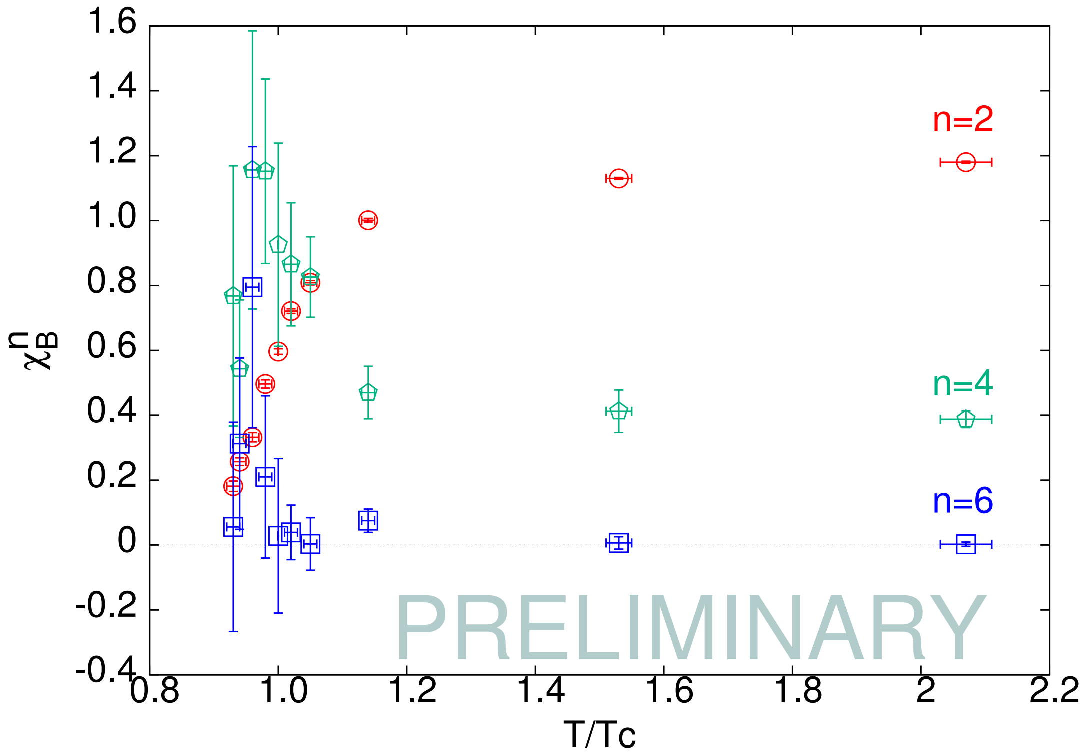
χ_2 for $N_t = 8$, 6, and 4 lattices



♠ $N_t = 8$ and 6 results agree

♡ $\beta_c(N_t = 8)$ agrees with Gottlieb et al. PR D47,1993.

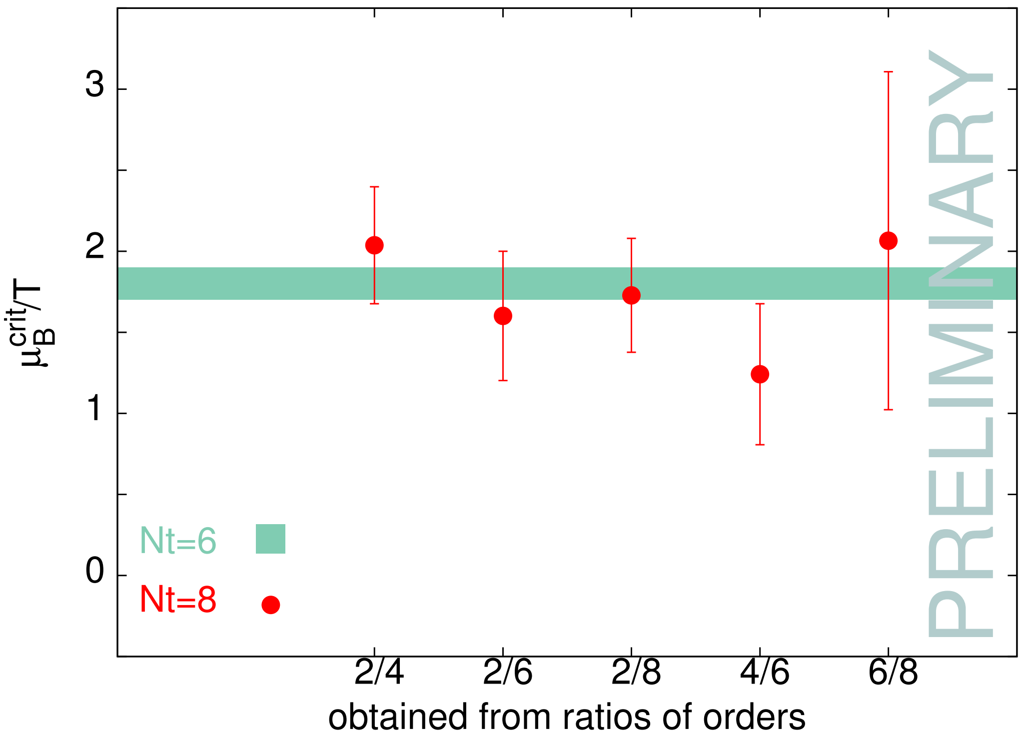
χ_B^n **for** $N_t = 8$ **lattice**



♠ 100 configurations & 1000 vectors at each point employed.

♡ More statistics coming in critical region. Window of positivity in anticipated region.

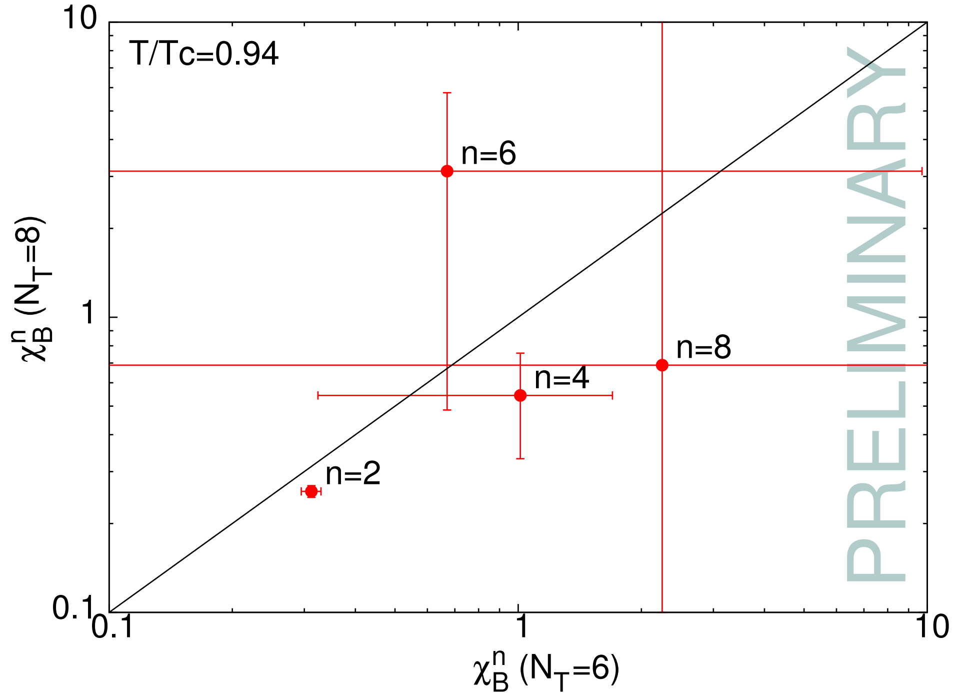
Radius of Convergence result



♠ At our (T_E, μ_E) for $N_t = 6$, the ratios display constancy for $N_t = 8$ as well.

♡ Currently : Similar results at neighbouring $T/T_c \Longrightarrow$ a larger ΔT at same μ_B^E.

Consistence check for critical point



♠ Ideally, all coefficients of the series must be the same at the critical point for both $N_t = 8$ and 6.

♡ Too far from checking this as errors have to be reduced. Encouraging signs none the less.

Summary

- The method we advocated, and employed for $N_t = 4$ and 6, works for $N_t = 8$ as well, yielding similar qualitative picture.

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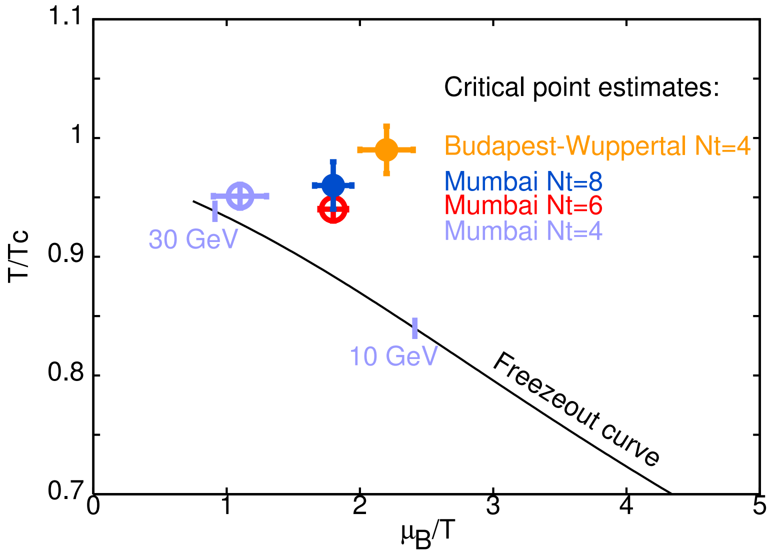

Critical Point location appears the same for $N_t = 8$ and 6 at $\mu_B/T \sim 1.8(1)$. Slight shift in temperature to $\frac{T^E}{T_c} = 0.96 \pm 0.02$; Agrees with $N_t = 6$ within errors.

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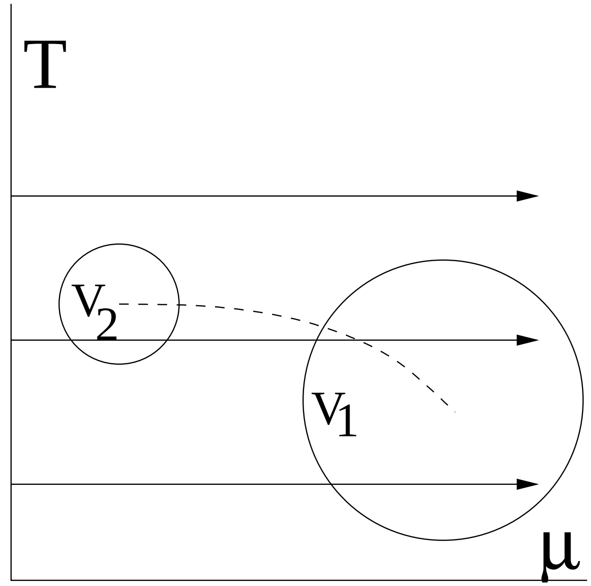


We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

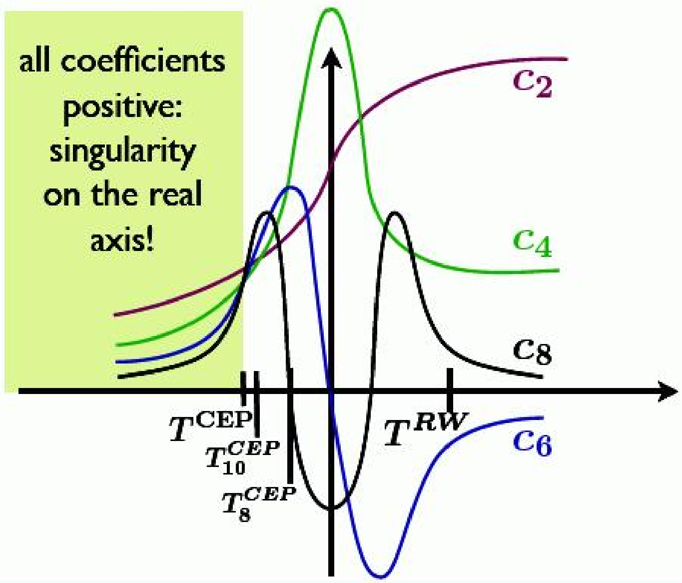
The critical endpoint (II)

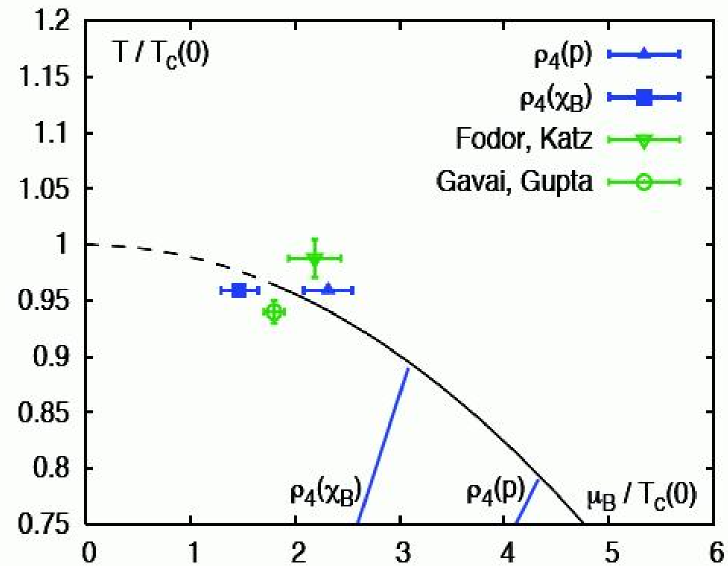
method for locating of the CEP:

- determine largest temperature where all coefficients are positive $\rightarrow T^{\mathrm{CEP}}$
- determine the radius of convergence at this temperature $\rightarrow \mu^{\mathrm{CEP}}$

$$p = c_0 + c_2 (\mu_B/T)^2 + c_4 (\mu_B/T)^4 + \cdots$$

$$\chi_B = 2c_2 + 12c_4 (\mu_B/T)^2 + 30c_6 (\mu_B/T)^4 + \cdots$$





first non-trivial estimate of T^{CEP} by c_8
second non-trivial estimate of T^{CEP} by c_{10}

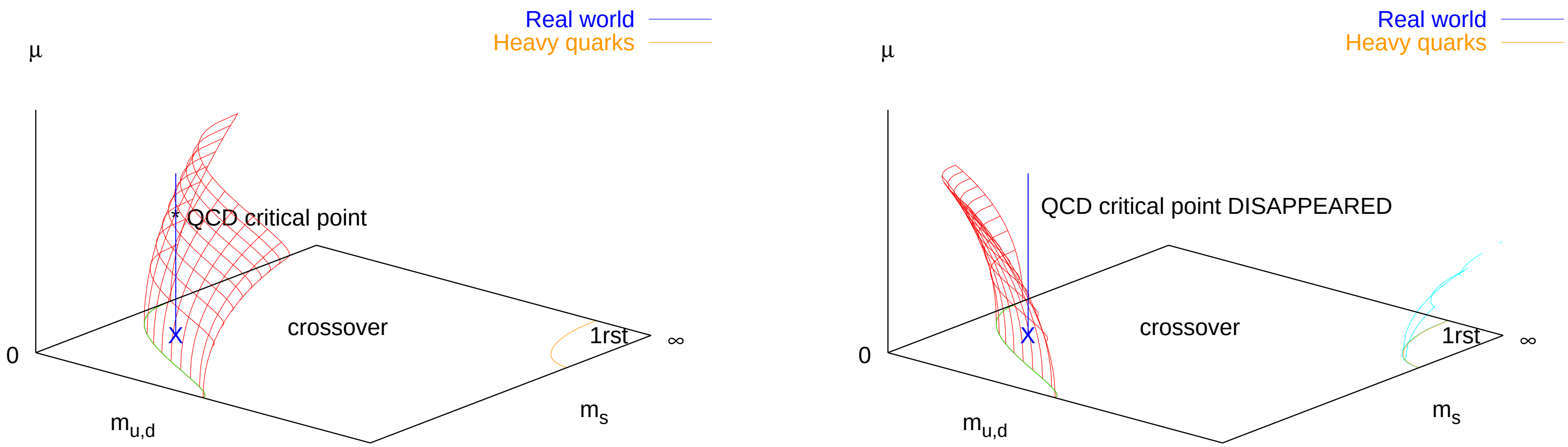
$$\rho_n(p) = \sqrt{c_n / c_{n+2}}$$

$$\rho = \lim_{n\to\infty} \rho_n$$

(Ch. Schmidt FAIR Lattice QCD Days, Nov 23-24, 2009.)

Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811



For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T_c}\right)^2 - 47(20)\left(\frac{\mu}{\pi T_c}\right)^4$, i.e., m_c shrinks with μ.

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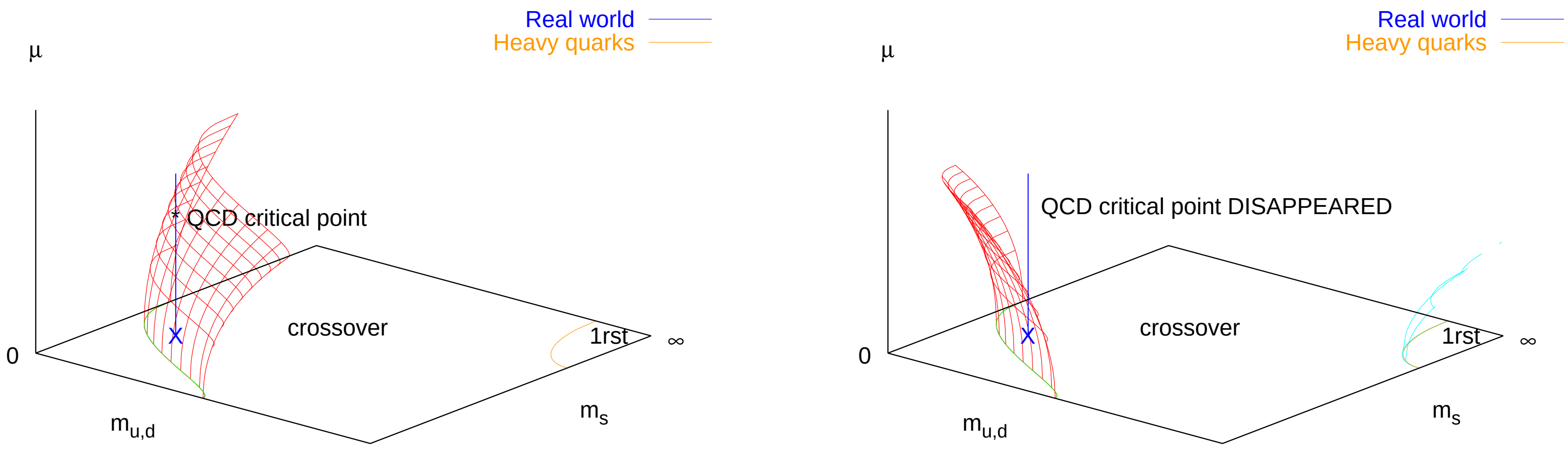

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Problems : i) Positive coefficient for finer lattice (Philipsen, CPOD 2009), ii) Known examples where shapes are different in real/imaginary μ, "The Critical line from imaginary to real baryonic chemical potentials in two-color QCD", P. Cea, L. Cosmai, M. D'Elia, A. Papa, PR D77, 2008