Towards QCD Thermodynamics using Exact Chiral Symmetry on Lattice

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GW relation and $\mu \neq 0$

Our Results

Summary

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- Obtained from the long-distance behaviour of the correlator $\langle C_{AB}(z)\rangle = \langle \bar{A}(z)\bar{B}(0)\rangle - \langle \bar{A}(0)\rangle\langle \bar{B}(0)\rangle \sim \exp(-\mu(T)z)$, as $z \to \infty$. Here $\bar{A}(z) = \sum_{x,y,t} A(x,y,z,t)/N_s^2 N_t$ is a local meson or baryon operator.

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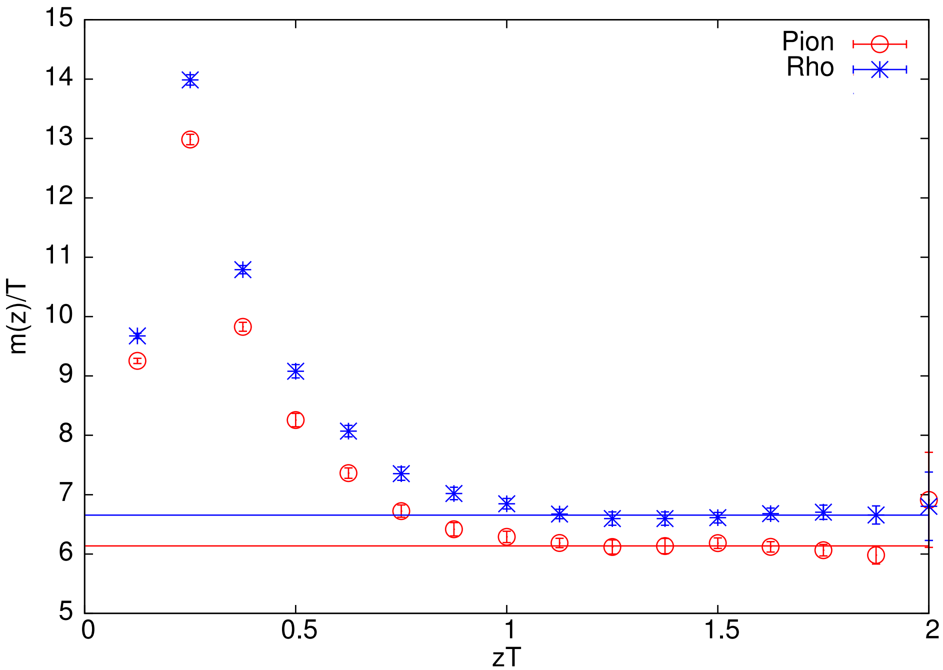
- Overlap fermions appear to do better.

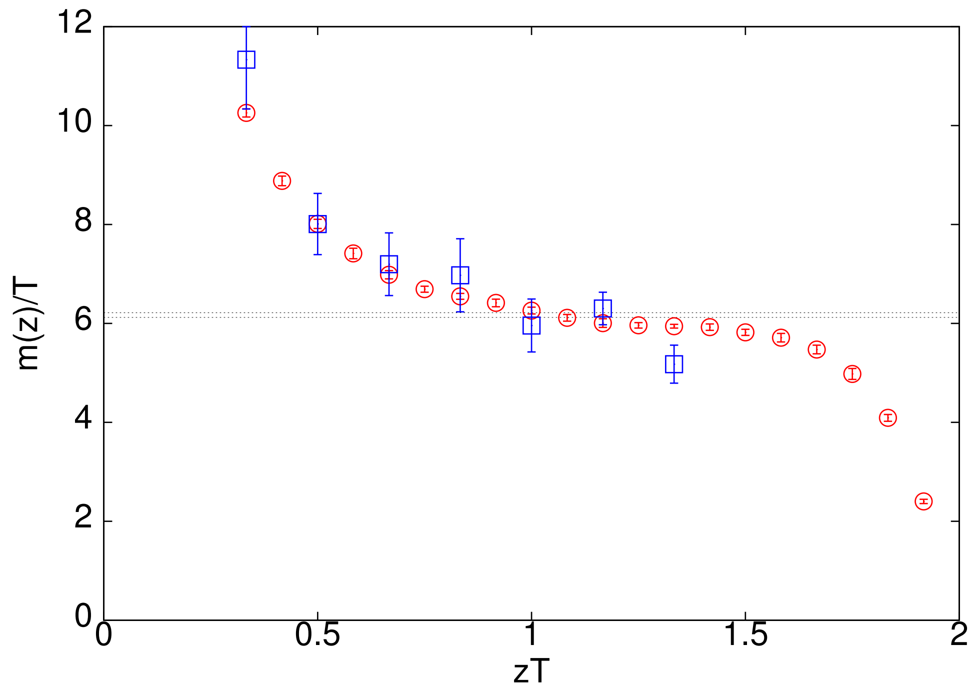
Overlap Compared with Staggered Fermions

♣ Local masses [$\sim \ln(C(r)/C(r+1)$] show nice plateau behaviour for pi & rho for Overlap (left) unlike staggered (right) fermions.

Gavai, Gupta, Lacaze PRD 2008

Gavai, Gupta PRD 2002





Screening Masses Compared

♣ The pionic screening length shows significant a^2 corrections for staggered (left) unlike Overlap (right) fermions.

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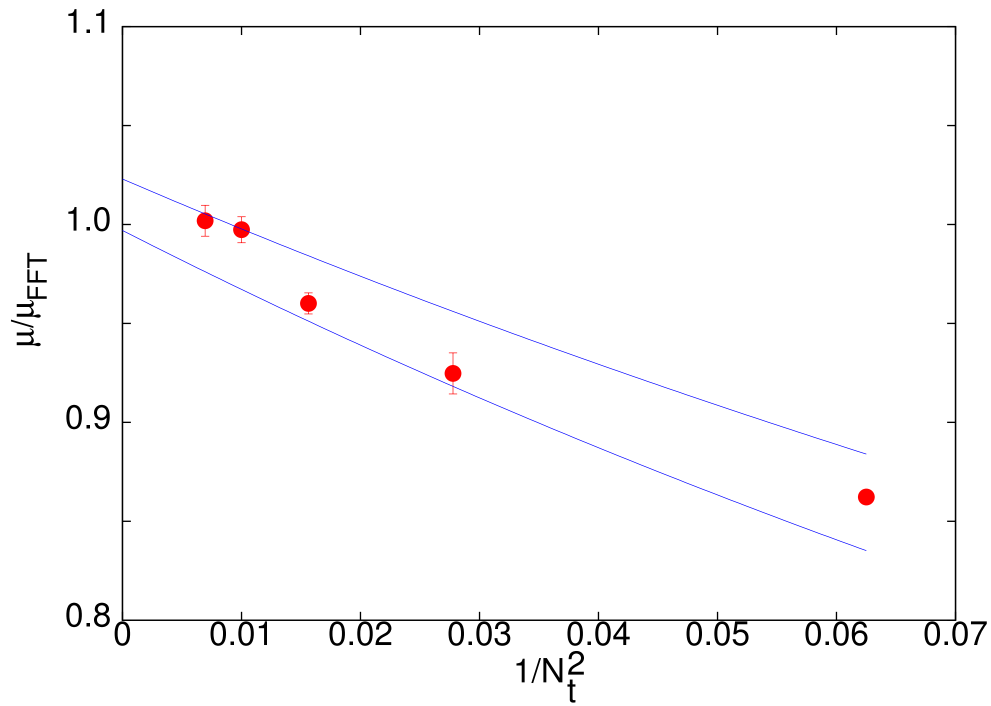

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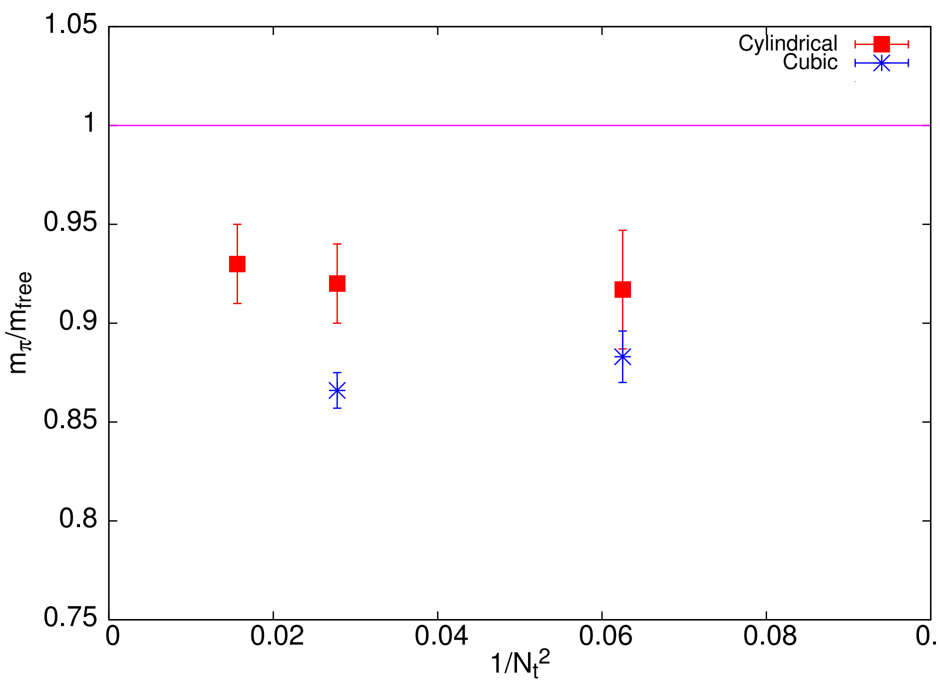

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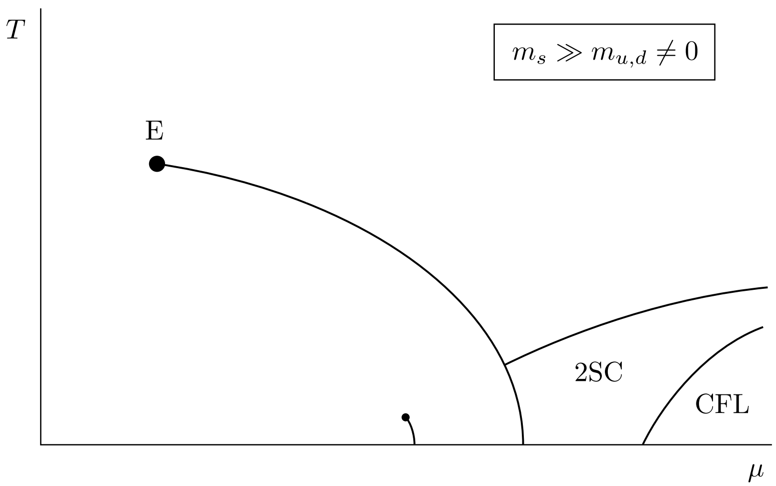
Expected QCD Phase Diagram



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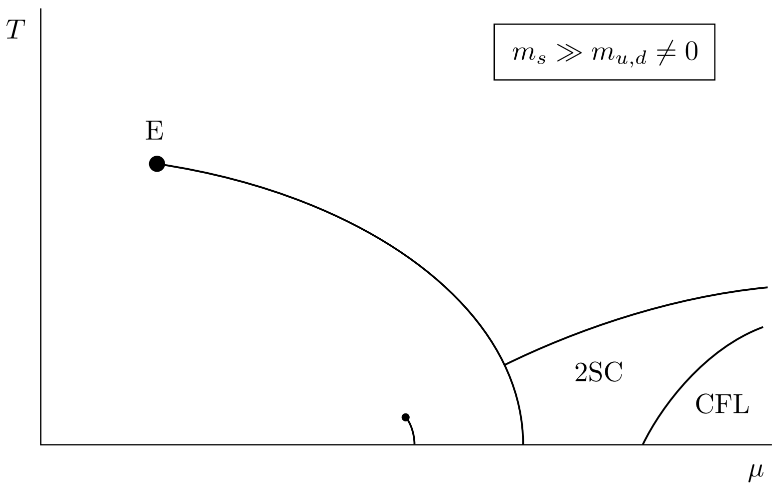

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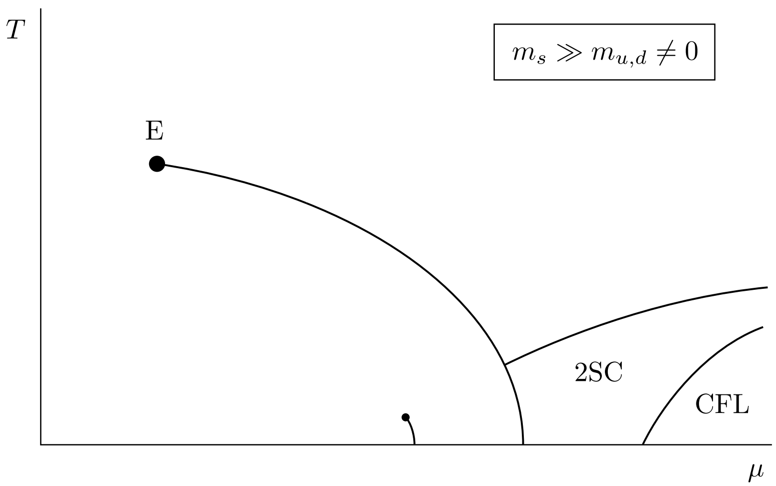

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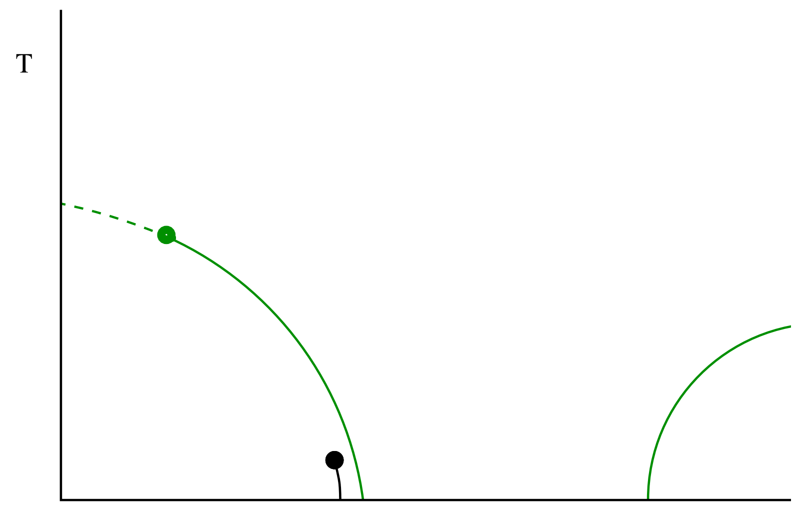

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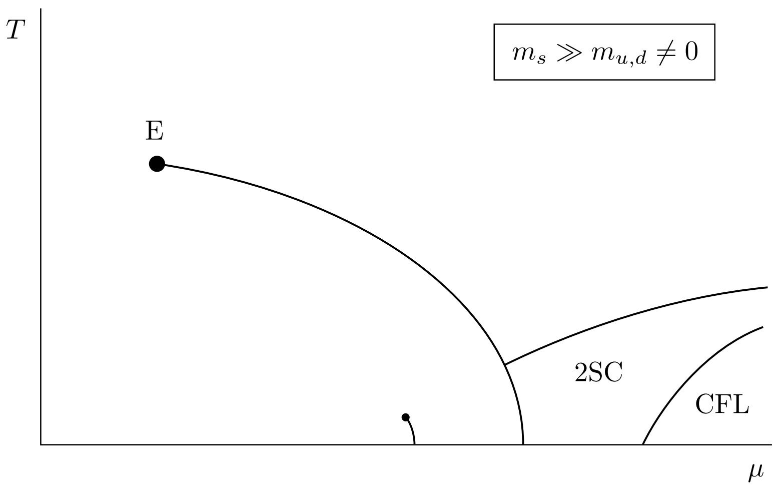

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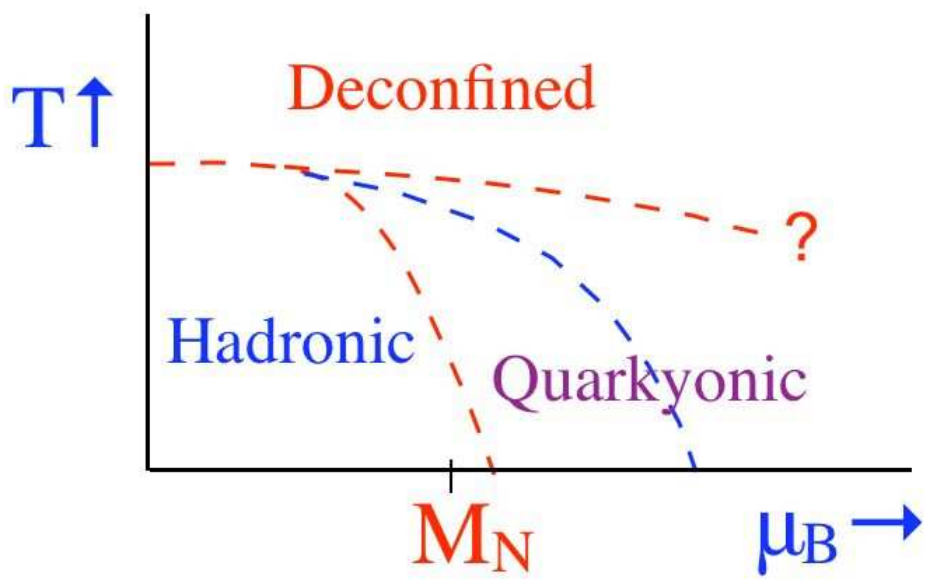

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♠ Exact chiral invariance for a lattice fermion operator D is assured if it satisfies the Ginsparg-Wilson relation : $\{\gamma_5, D\} = aD\gamma_5 D$.

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$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D + D\gamma_5 - \frac{a}{2}D\gamma_5 D - \frac{a}{2}D\gamma_5 D\Big]_{xy} \psi_y = 0 \qquad (1)$$

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♠ Overlap fermions, and Domain Wall fermions in the limit of large fifth dimension satisfy this relation.

Overlap-Dirac Operator

♠ Neuberger (PLB 1998) proposed the overlap-Dirac operator :

$$aD = 1 + A(A^\dagger A)^{-1/2} = 1 + \gamma_5 \,\mathrm{sign}(\gamma_5 A) \qquad \text{with} \quad A = aD_w, \tag{2}$$

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♠ Here D_w is the Wilson-Dirac Operator given by,

$$aD_w = \frac{1}{2}\{\gamma_\mu(\partial^*_\mu + \partial_\mu) - a\partial^*_\mu \partial_\mu\} + M, \tag{3}$$

with $-2 < M < 0$ and ∂_μ and ∂^*_μ as forward and backward gauge-invariant difference operators. An extra a/a_4 factor for $\mu = 4$ at $T \neq 0$.

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♠ quark with a mass : $D(ma) = ma + (1 - ma/2)D$

Domain Wall Fermions

♠ Proposed by Kaplan (PLB 1992), a convenient form for Domain Wall fermion action (Shamir, NPB, 1993) is:

$$S_F = \sum_{s,s'=1}^{N_5} \sum_{x,x'} \bar{\psi}(x,s) D_{dw}(x,s;x',s') \psi(x',s') \ , \tag{4}$$

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$$D_{dw}(x,s;x',s') = [a_5 D_w + 1]\delta_{s,s'} - [P_- \delta_{s,s'-1} + P_+ \delta_{s,s+1'}] \ , \quad (5)$$

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with boundary conditions $P_+ \psi(x,0) = -am\ P_+ \psi(x,N_5)$ and $P_- \psi(x,N_5+1) = -am\ P_- \psi(x,1)$.

♠ Only light modes attached to the wall(s) are physical. Divide out heavy modes by having the $D_{dw}(am)/D_{dw}(am=1)$ as the effective Domain Wall operator in $\mathcal{Z}$.

♡ As outlined in Edwards & Heller (PRD 63, 2001), one can integrate out the fermionic fields in the fifth direction to rewrite the above ratio as

$$[(1 + am) - (1 - am)\gamma_5 \tanh(\frac{N_5}{2} \ln |T|)] \ , \tag{6}$$

with $T = (1 + a_5\gamma_5 D_w P_+)^{-1}(1 - a_5\gamma_5 D_w P_-)$.

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♡ Taking the limit $N_5 \to \infty$ for $a_5 = 1$, one obtains sign function of log $|T|$, proving that the DWF satisfy the Ginsparg-Wilson relation in this limit.

♡ Taking the limit $a_5 \to 0$ such that $L_5 = a_5 N_5 =$ constant, one can show $N_5 \ln T \to L_5 \gamma_5 D_{dw}$. Further, for $L_5 \to \infty$, DWF reduce to the overlap fermions.

♡ We use this form in our numerical work.

Introducing Chemical Potential

- Ideally, one should construct the conserved charge as a first step.
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- Note $\gamma_5 D_w(a\mu)$ is no longer Hermitian, requiring an extension of the sign function. B & W proposal : For complex $\lambda = (x + iy)$, sign(λ) = sign (x).
- Gattringer-Liptak, PRD 2007, showed for $M = 1$ numerically that no μ^2 divergences exist for the free case (U =1).

- We show this to be true analytically and for all M as well. Furthermore, this holds for all functions such that $K(a\mu) \cdot L(a\mu) = 1$ for Overlap (Banerjee, Gavai, Sharma, PRD 2008) and Domain Wall Fermions (Gavai, Sharma 2008).

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- We claim that chiral invariance is lost for nonzero μ. Note that

$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \left[\gamma_5 D(a\mu) + D(a\mu)\gamma_5 - \frac{a}{2} D(0)\gamma_5 D(a\mu) - \frac{a}{2} D(a\mu)\gamma_5 D(0) \right]_{xy} \psi_y \ , \tag{7}$$

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which is not sufficient to make $\delta S = 0$. True for both Overlap and Domain Wall fermions and any K,L.

Consequences

- Exact Chiral Symmetry on lattice lost for any $\mu \neq 0$: Real or Imaginary! Note $D_w(a\mu)$ is Hermitian for the latter case.

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- Only smooth chiral condensates : No (clear) chiral transition for any (large) μ possible. How small a, or large N_T may suffice ?

- All coefficients of a Taylor expansion in μ do have the chiral invariance but the series will be smooth and should always converge.

What if ...

♠ the chiral transformations were $\delta\psi = \alpha\gamma_5(1 - \frac{a}{2}D(a\mu))\psi$ and $\delta\bar{\psi} = \alpha\bar{\psi}(1 - \frac{a}{2}D(a\mu))\gamma_5$?

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- Moreover, symmetry groups *different* at each μ. Recall we wish to investigate $\langle\bar{\psi}\psi\rangle(a\mu)$ to explore if chiral symmetry is restored.

- The symmetry group remains *same* at each T with $\mu = 0$ $\Longrightarrow \langle\bar{\psi}\psi\rangle(am = 0, T)$ is an order parameter for the chiral transition.

Our Results

- We investigated thermodynamics of free overlap and domain wall fermions with an aim to examine the continuum limit analytically and numerically.

- Analytically, we prove the absence of μ^2-divergences for general K and L. Our numerical results were for tuning the irrelevant parameter M to obtain small deviations from continuum limit on coarse lattices.

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- Energy density and pressure can be obtained from $\ln Z = \ln\ \det\ D_{ov}$ by taking T and V, or equivalently a_4 and a, partial derivatives.

- Dirac operator is diagonal in momentum space. Use its eigenvalues to compute $\mathcal{Z}$:
$\lambda_\pm = 1 - [sgn\left(\sqrt{h^2+h_5^2}\right) h_5 \pm i\sqrt{h^2}]/\sqrt{h^2+h_5^2}$, with
$h_i = -\sin ap_i$, i =1, 2 and 3, $h_4 = -a\ \sin(a_4 p_4)/a_4$ and
$h_5 = M - \sum_{i=1}^{3}[1-cos(ap_i)] - a[1-cos(a_4 p_4)]/a_4$.

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$$\epsilon a^4 = \frac{2}{N^3 N_T} \sum_{p_i,n} F(\omega_n) = \frac{2}{N^3 N_T} \sum_{p_i,n} \left[(g + \cos\omega_n) + \sqrt{d + 2g\cos\omega_n} \right] \times \left[\frac{(1-\cos\omega_n)}{d + 2g\cos\omega_n} + \frac{\sin^2\omega_n (g+\cos\omega_n)}{(d+2g\cos\omega_n)(f+\sin^2\omega_n)} \right] \quad (9)$$

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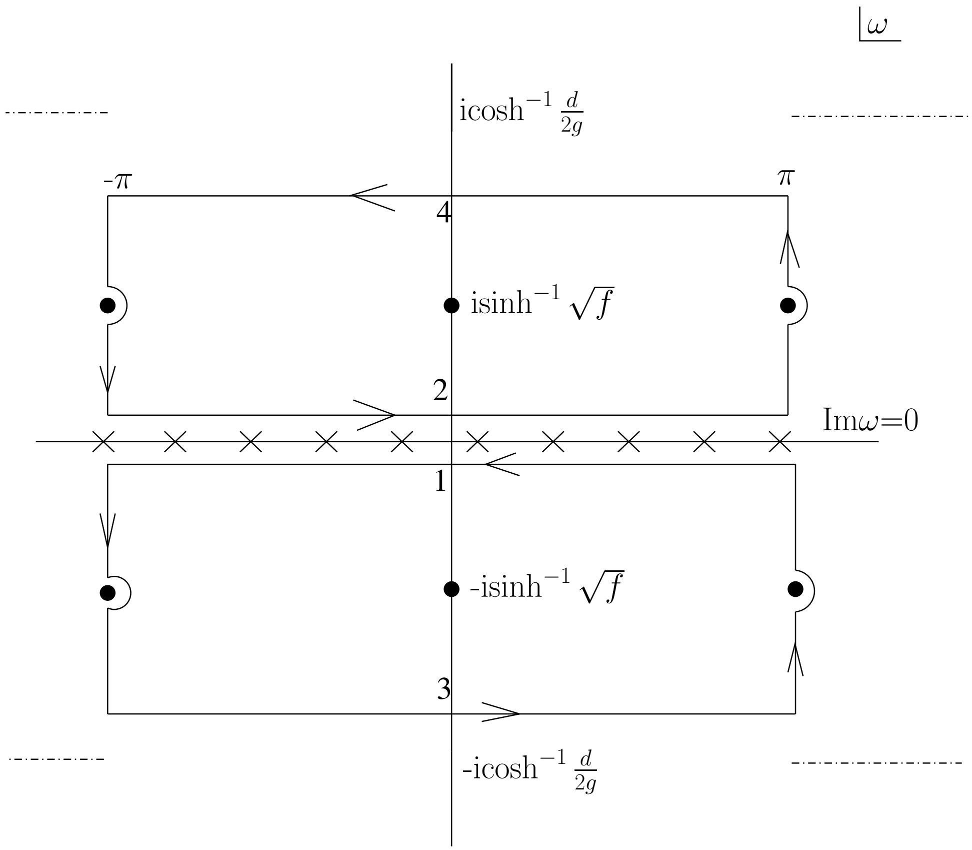
- Can be evaluated using the standard contour technique or numerically.

Analytic Evaluation : $\mu = 0$.



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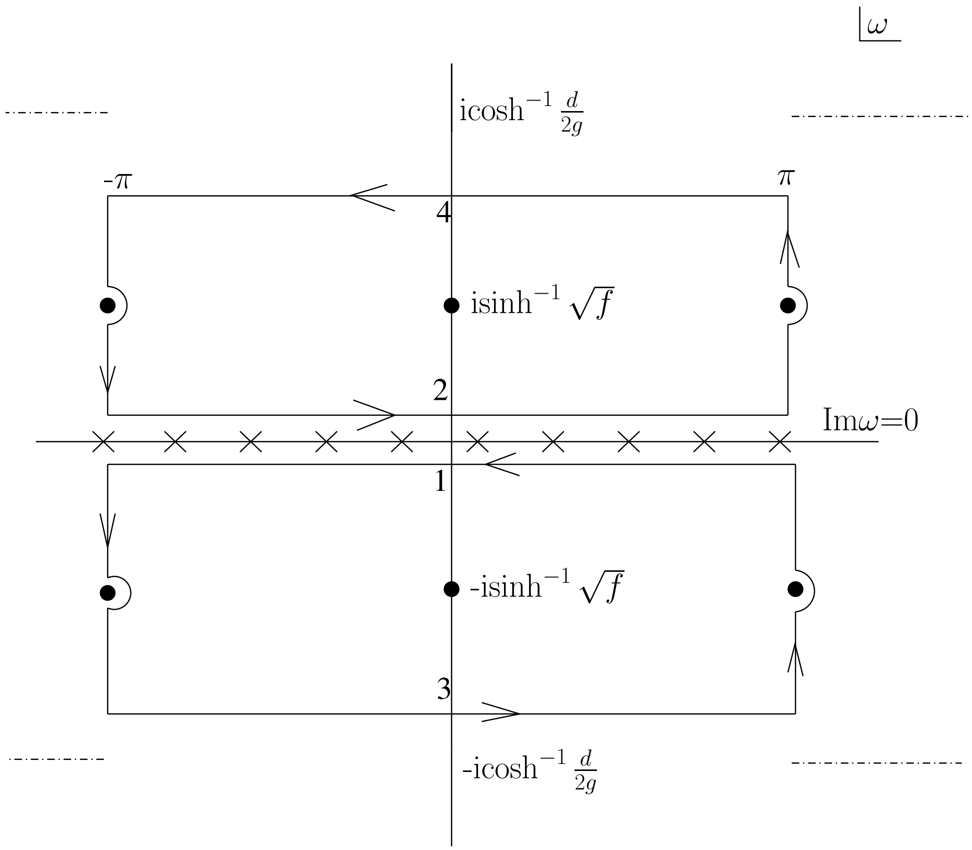

- Poles at $\omega = \pm i\ \sinh^{-1}\sqrt{f}$ and Poles (branch points) at $\pm i\ \cosh^{-1}\frac{d}{2g}$.

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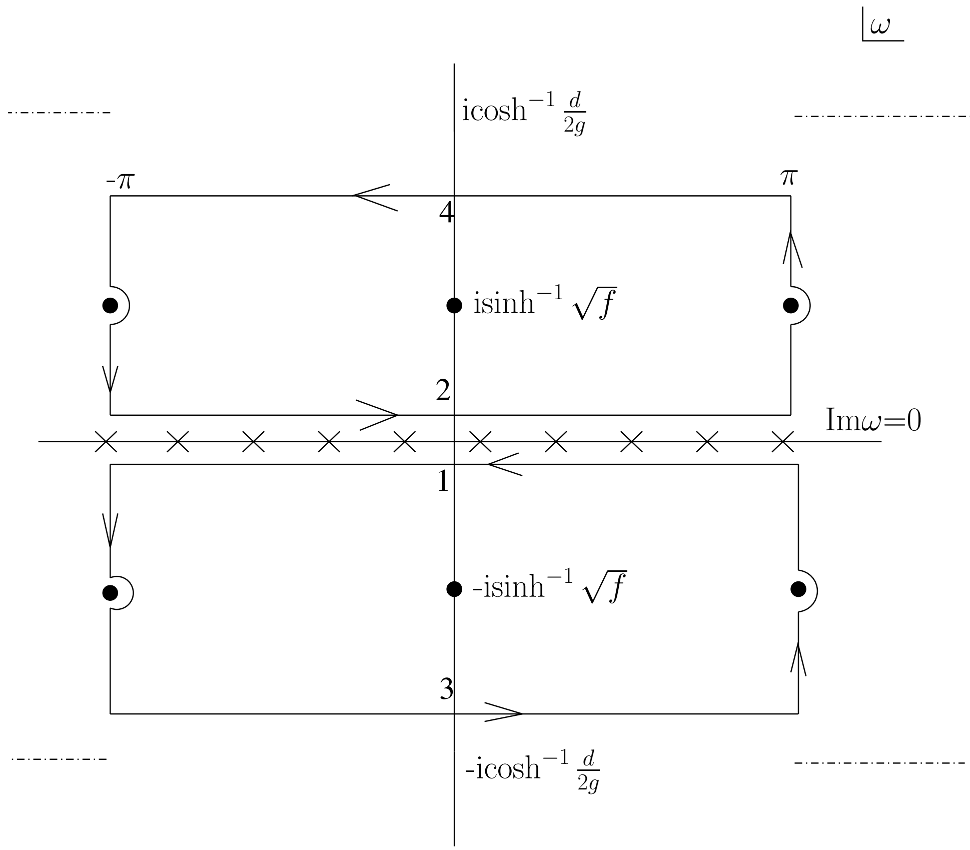

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- Evaluating integrals, $\epsilon a^4 = 4N^{-3}\sum_{p_j}\left[\sqrt{f/1+f}\right]$ $[\exp(N_T \sinh^{-1}\sqrt{f})+1]^{-1}$ $+\epsilon_3 + \epsilon_4$, where $f = \sum_i \sin^2(ap_i)$.

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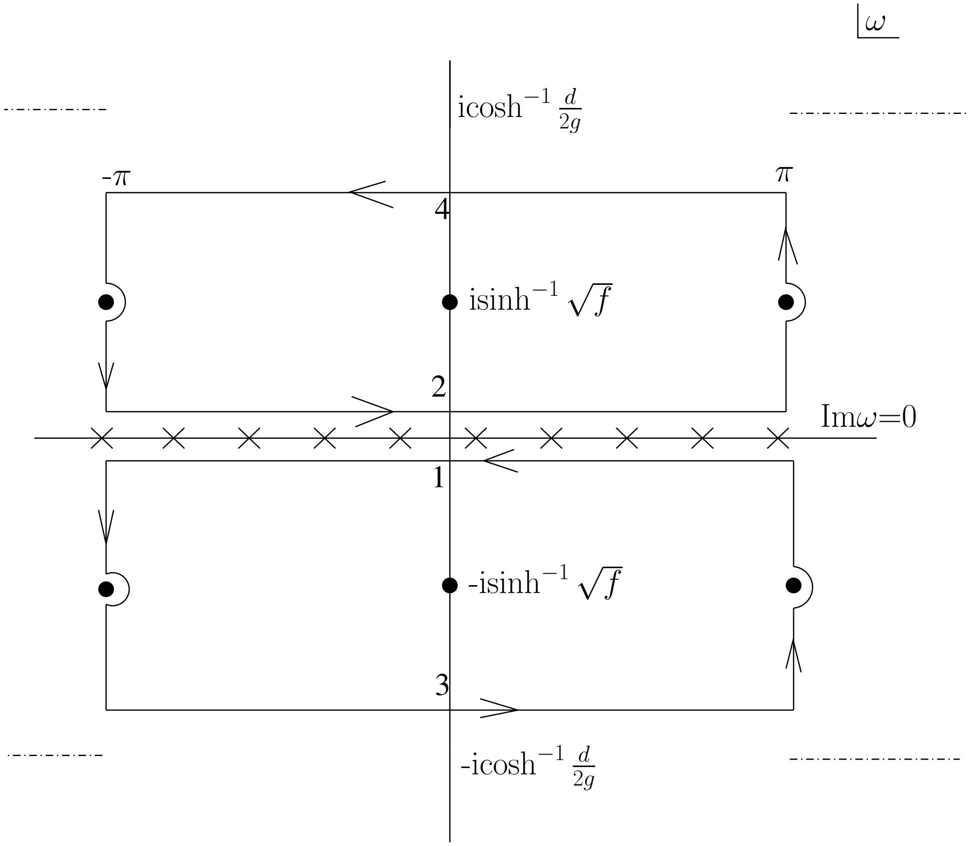

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- Evaluating integrals, $\epsilon a^4 = 4N^{-3}\sum_{p_j}\left[\sqrt{f/1+f}\right]$ $[\exp(N_T\sinh^{-1}\sqrt{f})+1]^{-1}$ $+\epsilon_3 + \epsilon_4$, where $f = \sum_i \sin^2(ap_i)$.
- Can be seen to go to ϵ_{SB} as $a \to 0$ for all M.

More Details : $T = 0$, $\mu \neq 0$

- Defining $K(\mu) + L(\mu) = 2R\cosh\theta$ and $K(\mu) - L(\mu) = 2R\sinh\theta$, the same treatment as above goes through by substituting $\sin\omega_n \to R\sin(\omega_n - i\theta)$ and $\cos\omega_n \to R\cos(\omega_n - i\theta)$.

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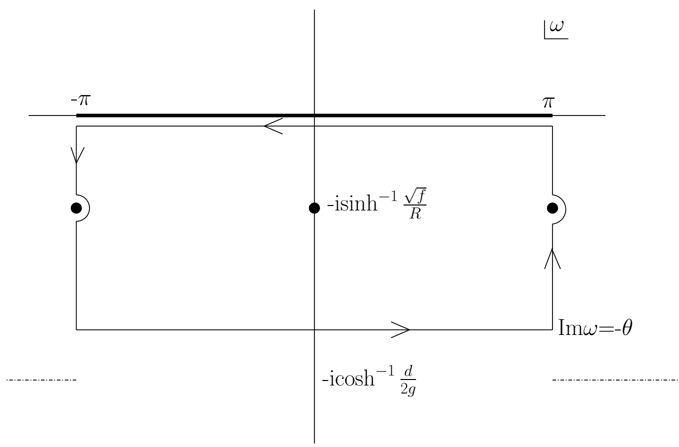
- Energy density is also functionally the same with $F(1, \omega_n) \to F(R, \omega_n - i\theta)$.

- Additional observable, number density : Has the same pole structure so similar computation.



Divergence Cancellation at $T = 0$, $\mu \neq 0$

- Doing the contour integral, the energy density turns out to be :

$$\epsilon a^4 = (\pi N^3)^{-1} \sum_{p_j} \Big[2\pi \mathrm{Res}\ F(R,\omega)\Theta\left(K(a\mu) - L(a\mu) - 2\sqrt{f}\right) + \int_{-\pi}^{\pi} F(R,\omega) d\omega - \int_{-\pi}^{\pi} F(1,\omega) d\omega \Big].$$

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- Generalization to $T \neq 0$ and $\mu \neq 0$ case straightforward. One merely needs two different contours depending on pole locations and value of θ.

Numerical Evaluation

♣ Zero temperature contribution : as $N_T \to \infty$, ω sum becomes integral which we estimated numerically.
♣ Continuum limit by holding $\zeta = N/N_T = LT$ fixed and increasing N_T.

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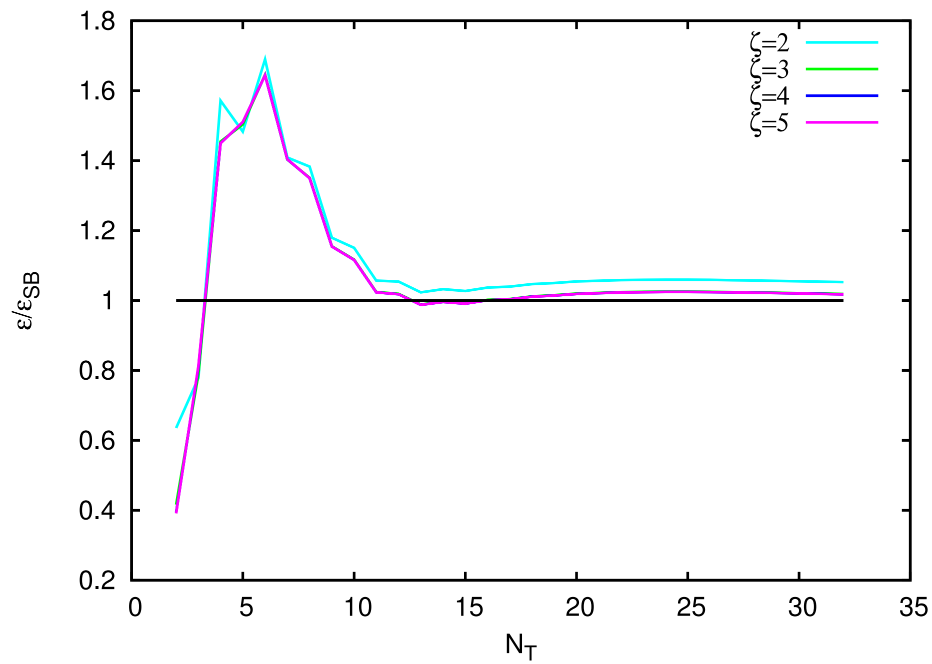

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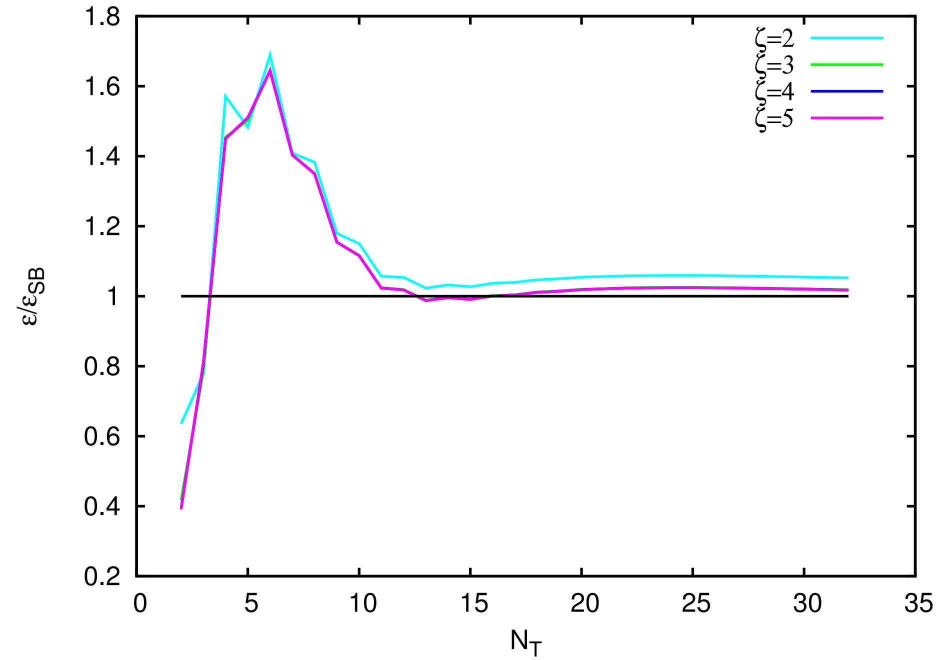
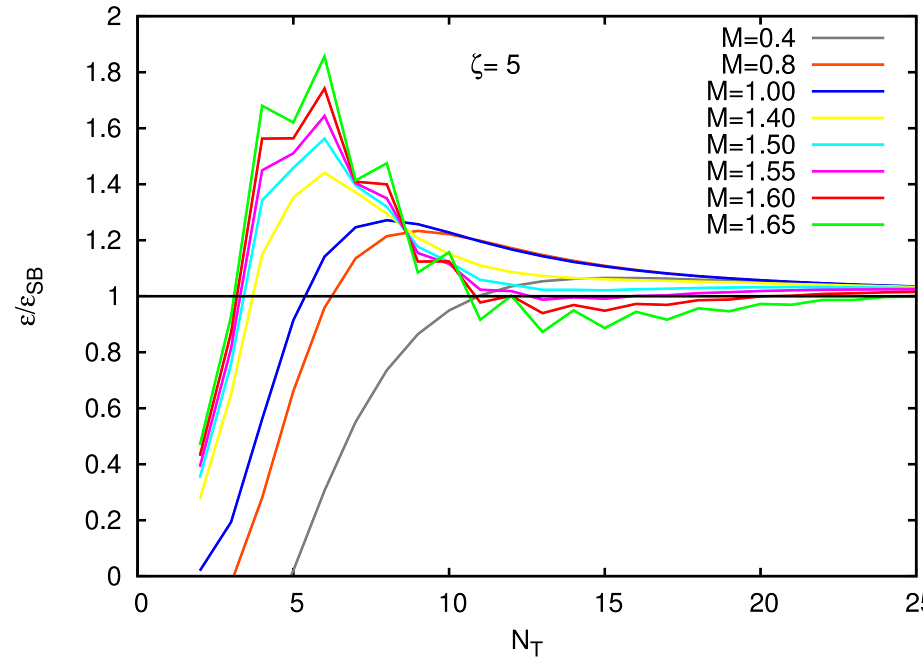
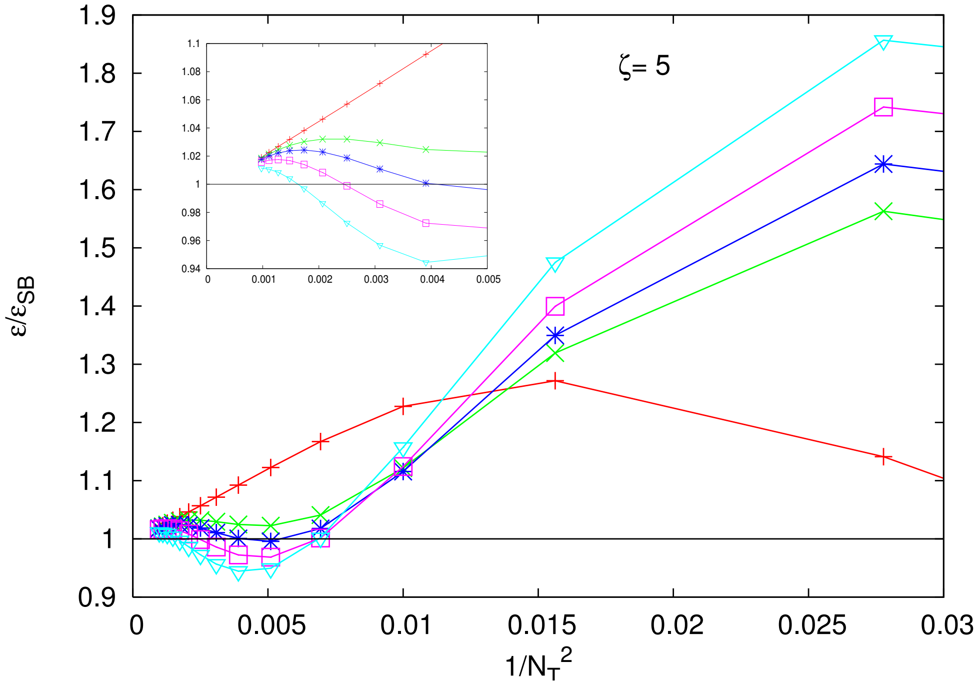




Approach to SB-Limit



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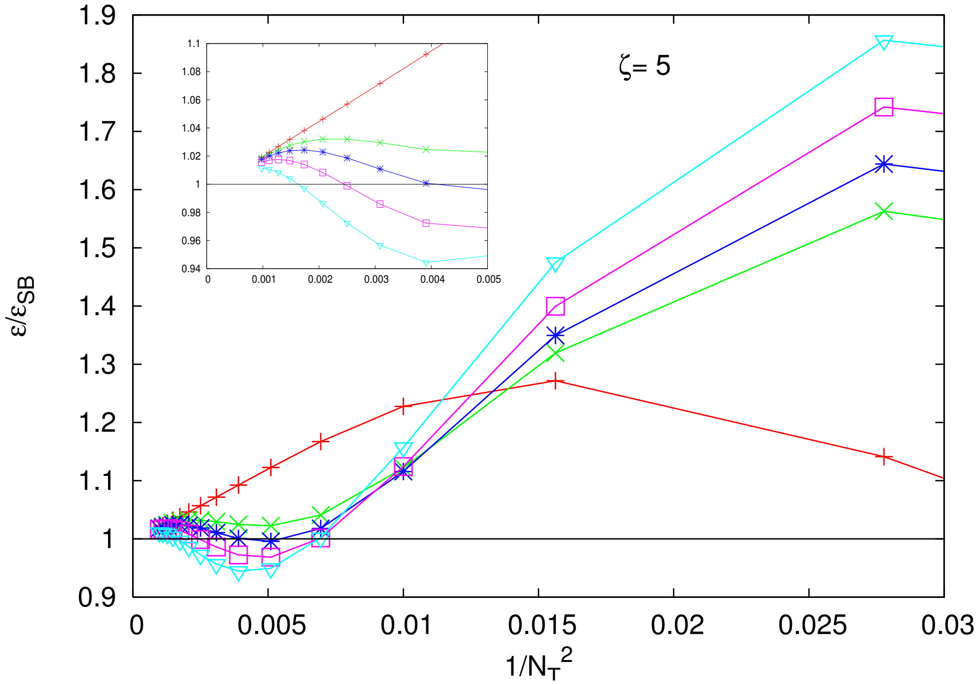
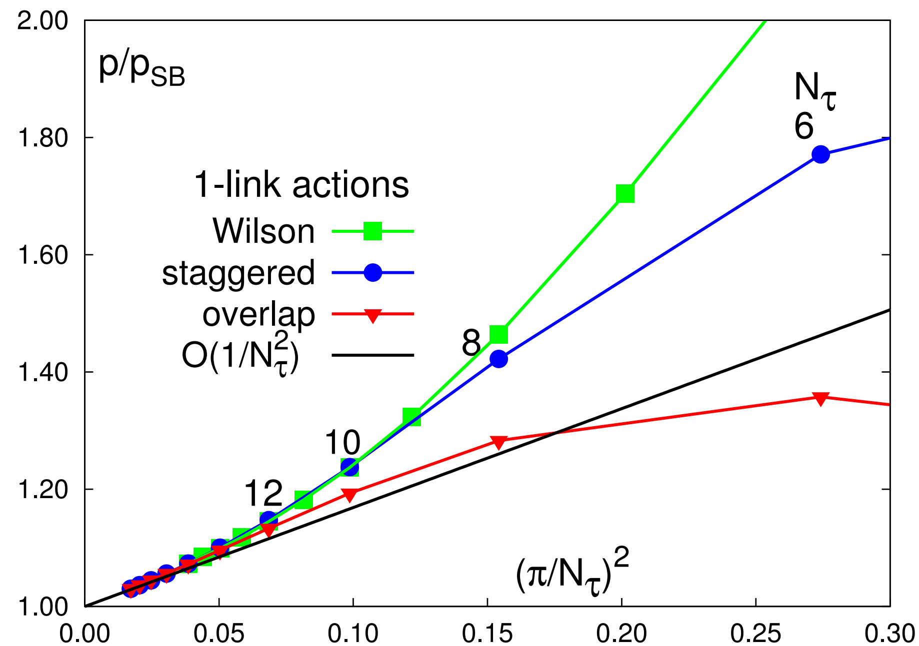


Banerjee, Gavai & Sharma , PRD78, 2008



Hegde, Karsch, Laermann & and Shcheredin, arXiv:0801.4883

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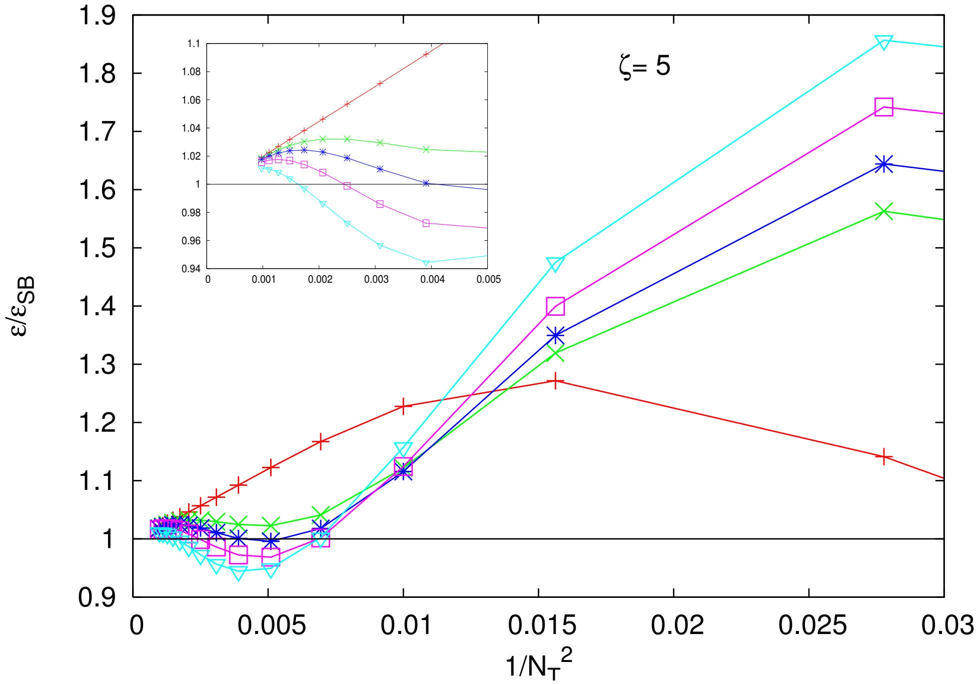

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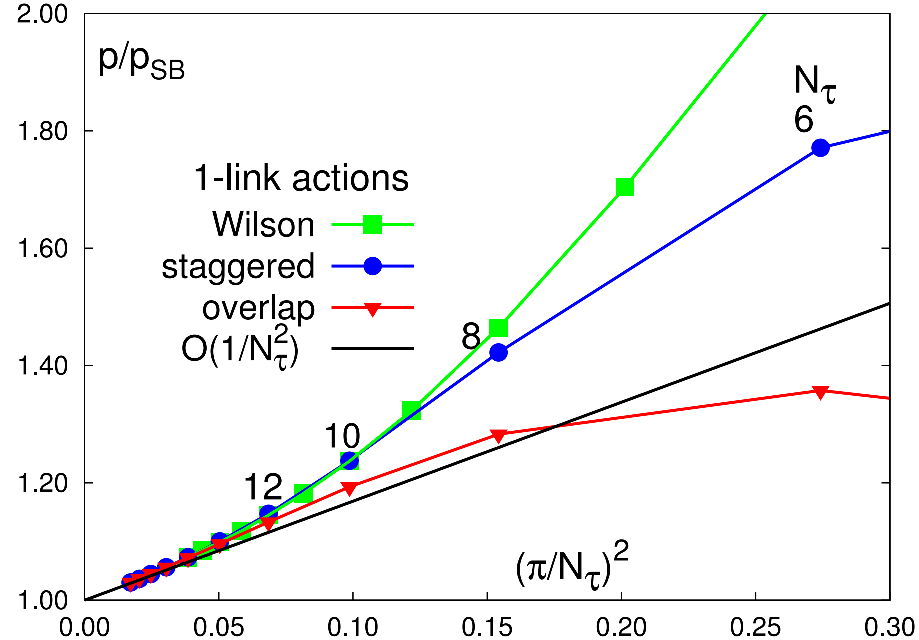

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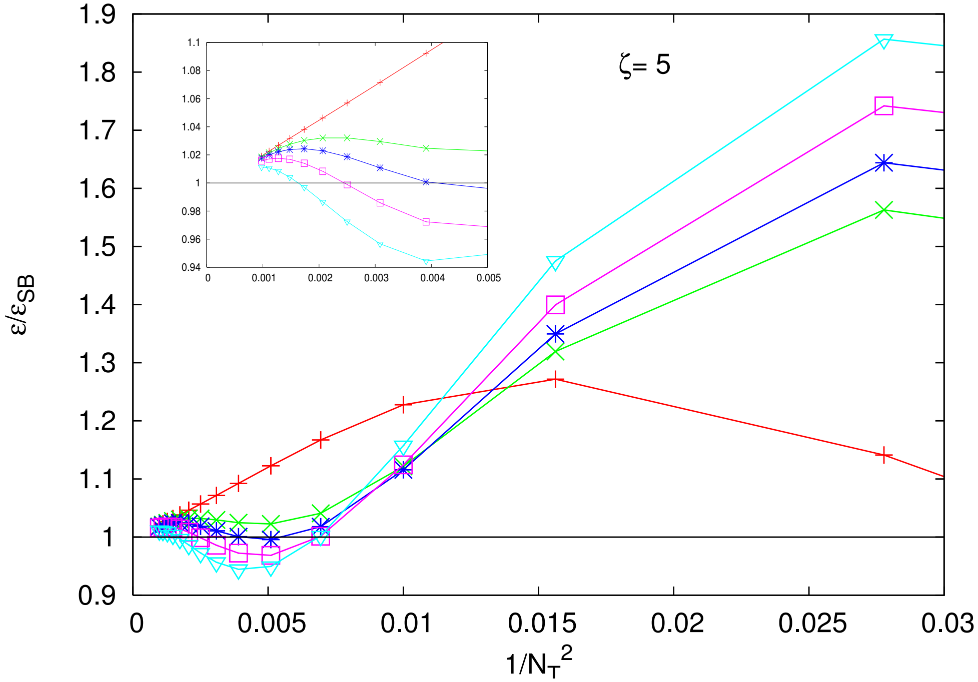

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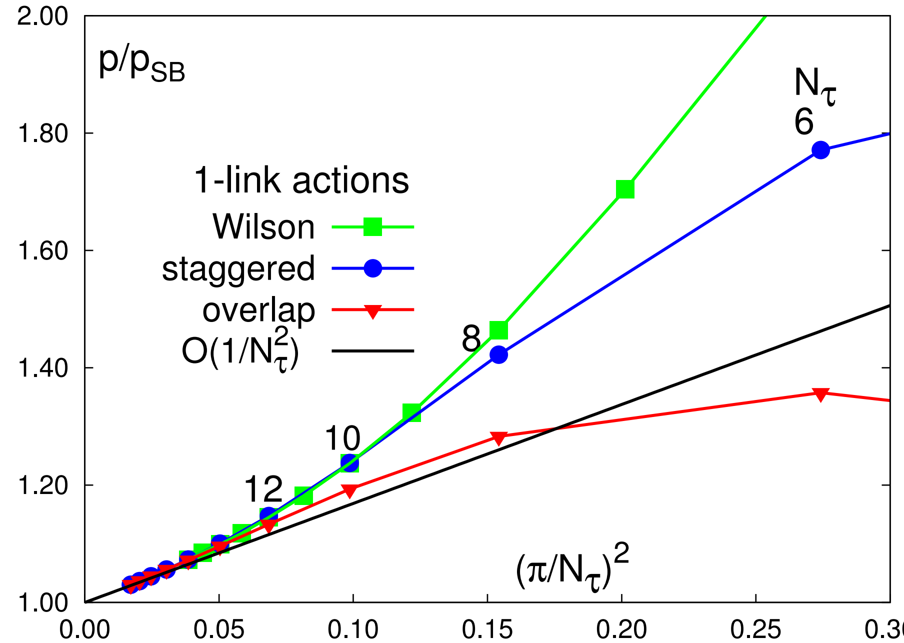

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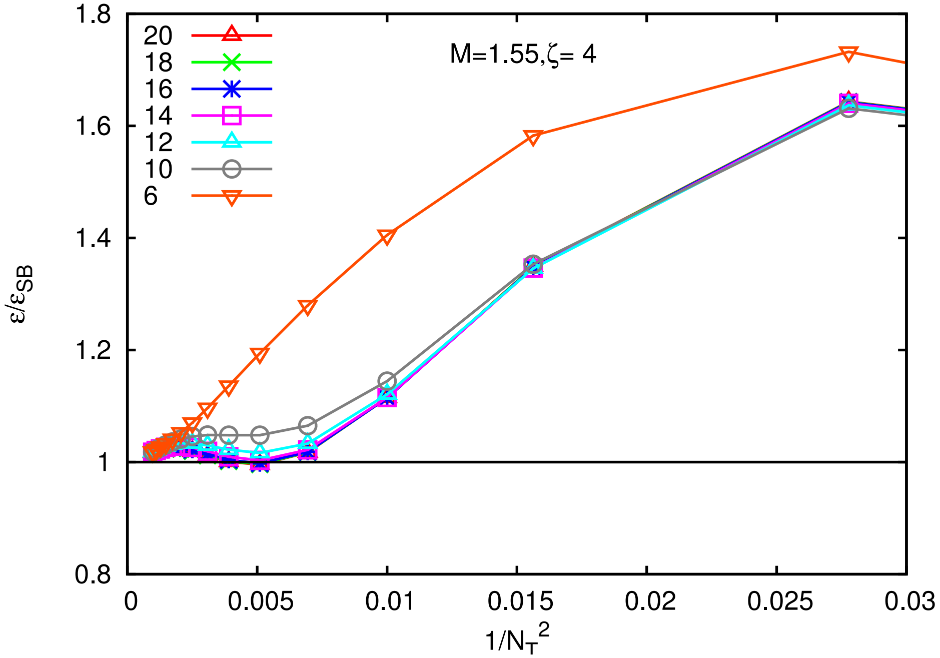
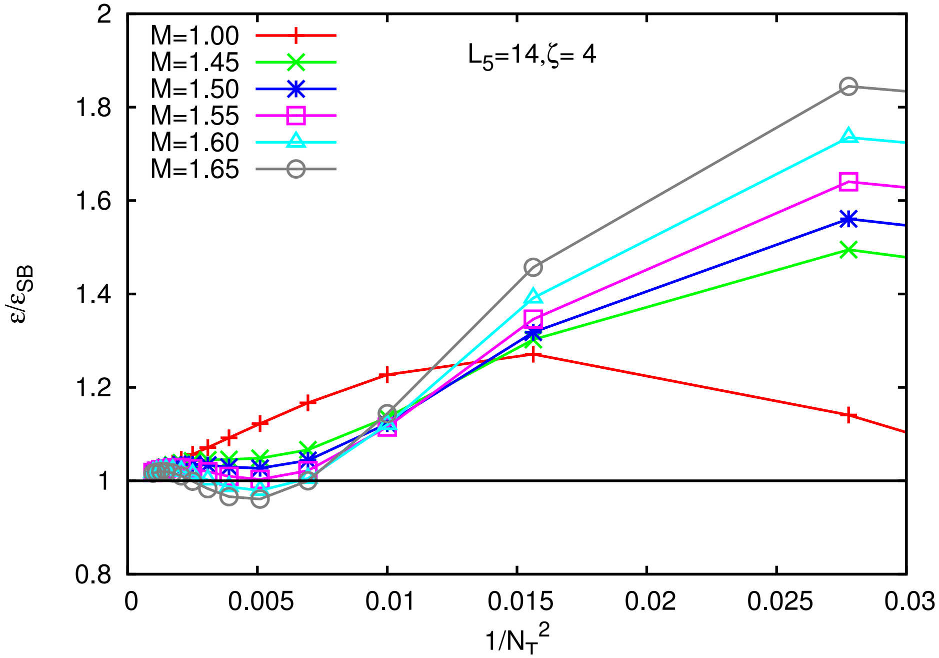
Domain Wall Fermions ($a_5 \to 0$)





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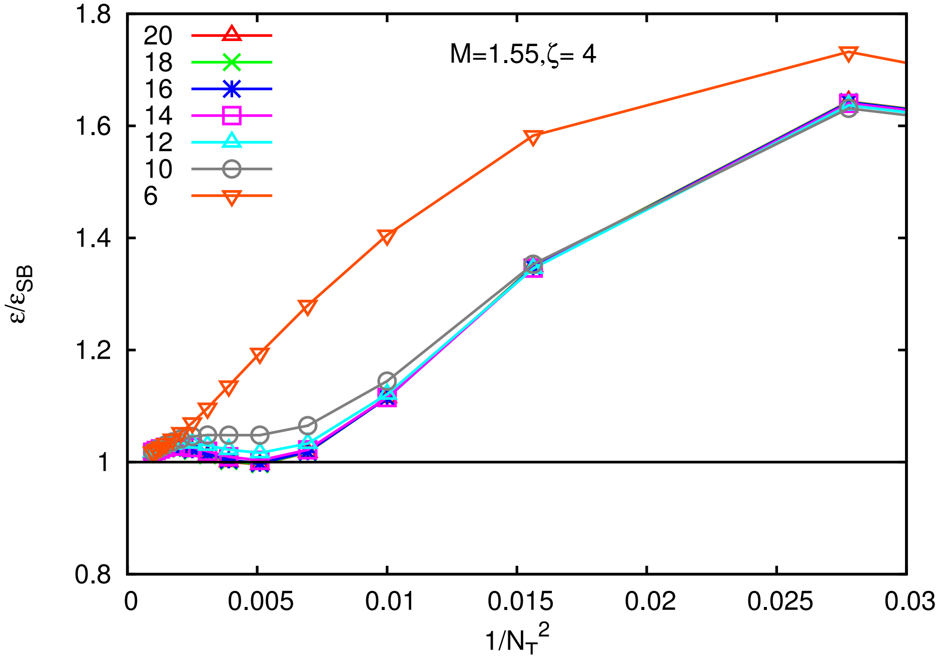

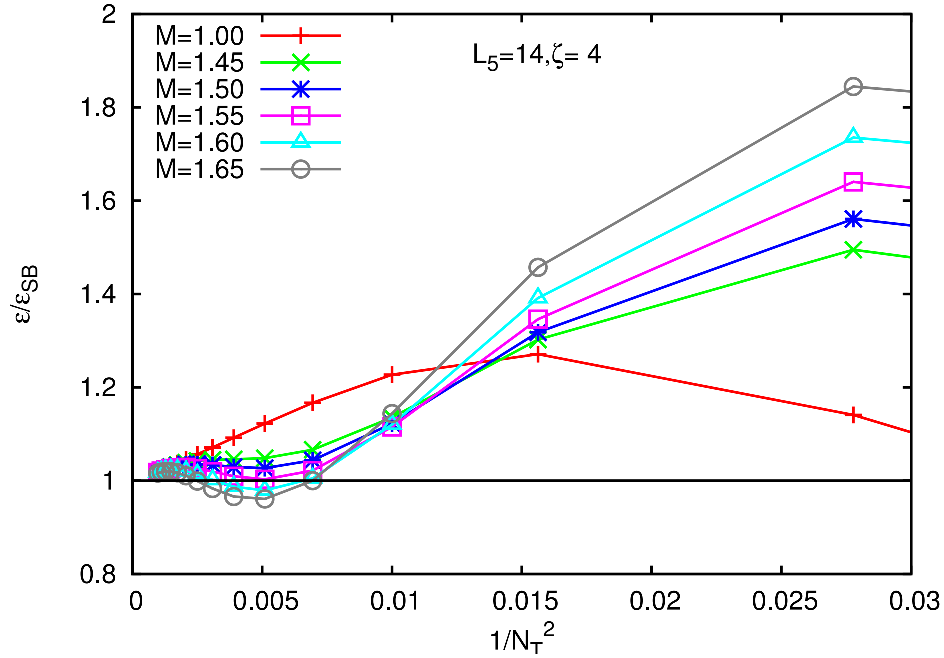

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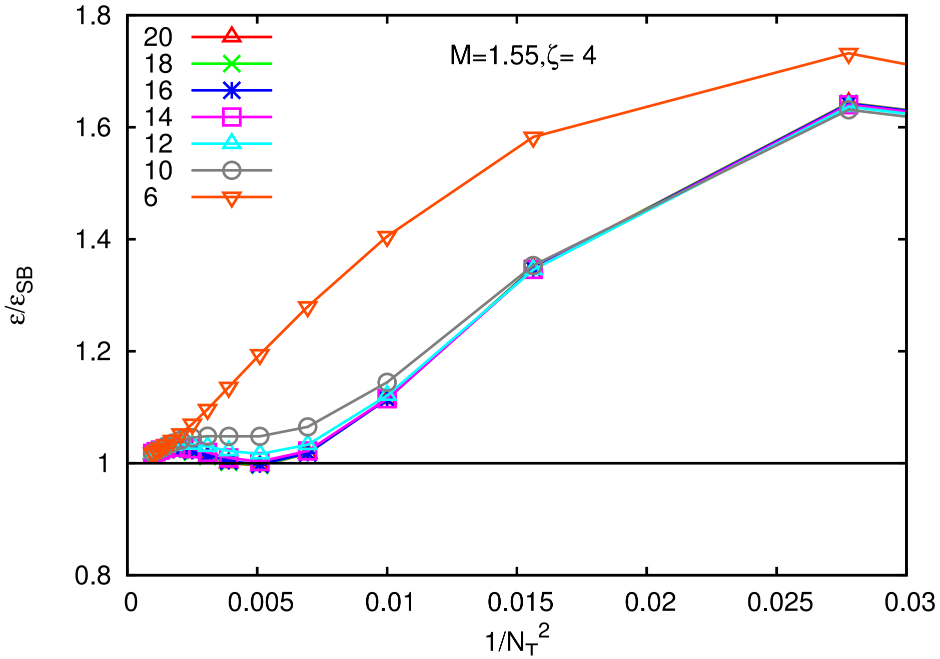

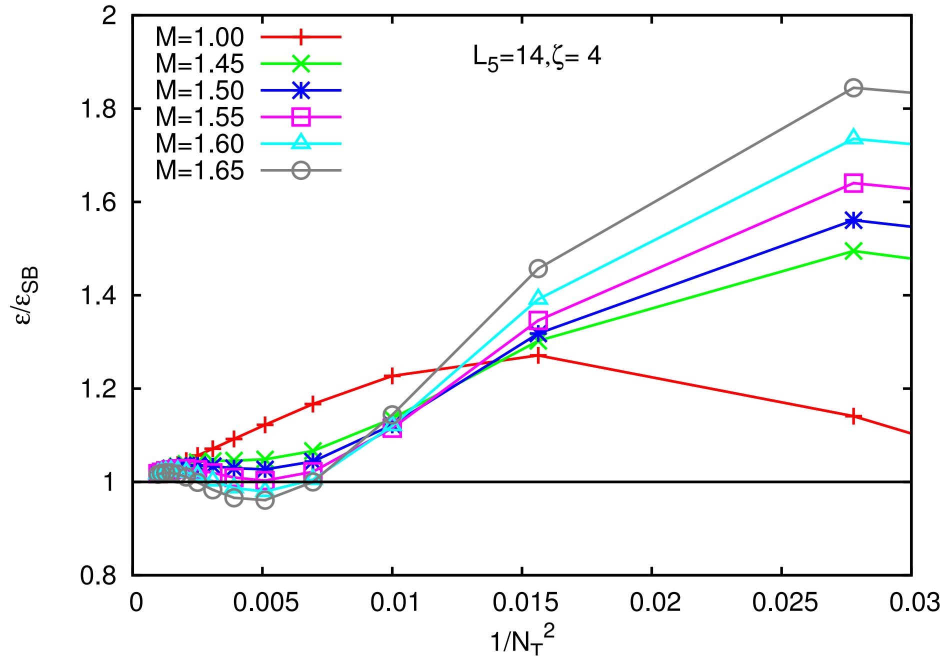

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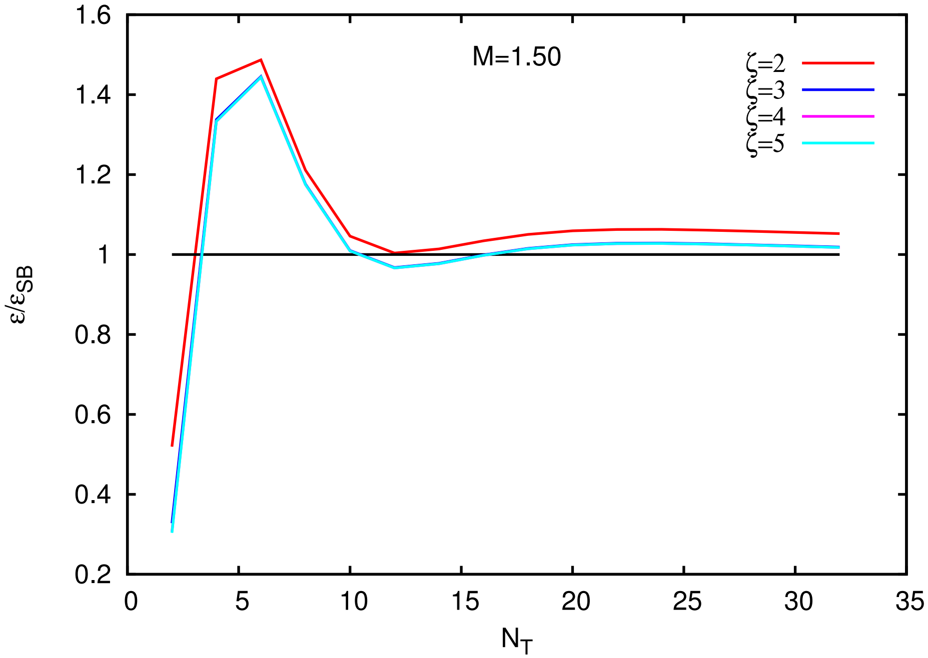

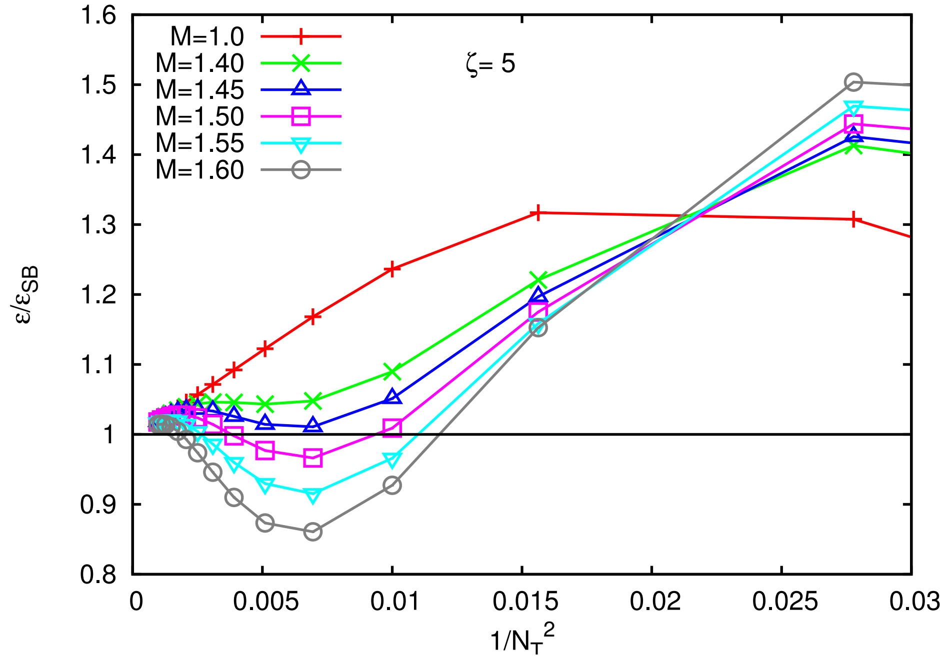

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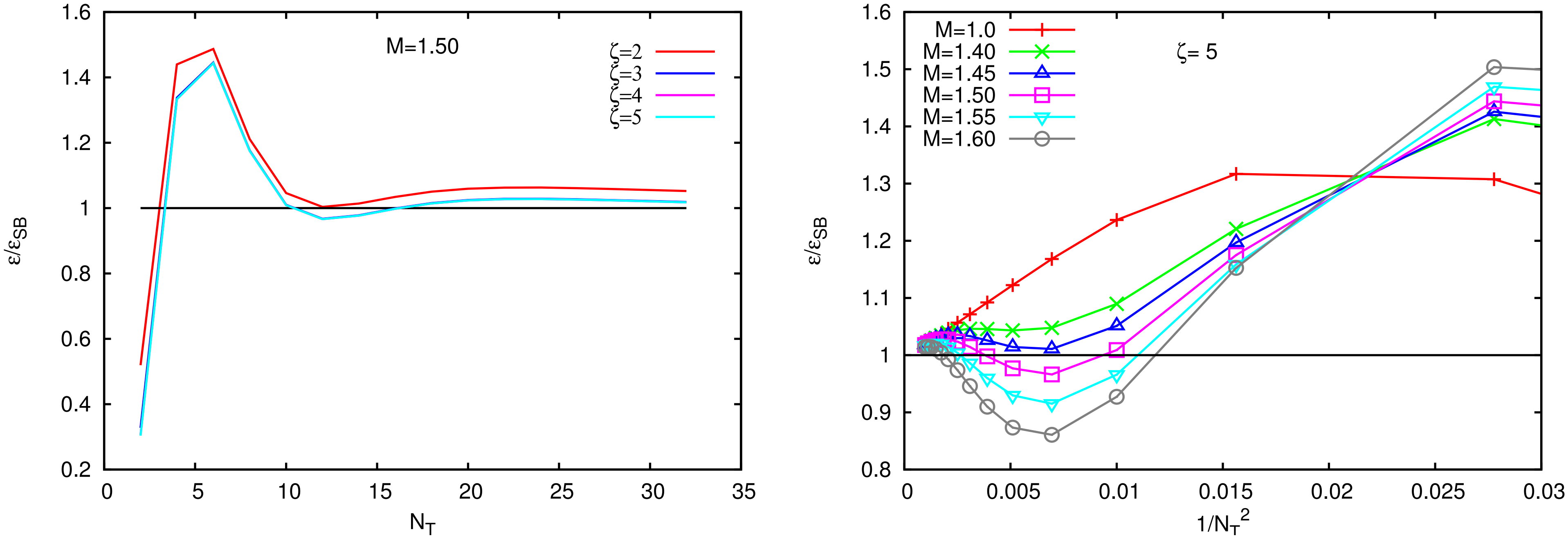

Rajiv V. Gavai and Sayantan Sharma, in preparation.

♢ $\zeta \geq 4$ seems to be large enough to get thermodynamic limit.
♢ Optimal range now seems to be $1.40 \leq M \leq 1.50$; $M = 1.9$ used by Chen et al. (PRD 2001) in their study of order parameters of FTQCD.

Numerical Evaluation

$\diamondsuit$ Two Observables : $\Delta\epsilon(\mu, T) = \epsilon(\mu, T) - \epsilon(0, T)$ and Susceptibility, $\sim \partial^2 \ln \mathcal{Z}/\partial\mu^2$.

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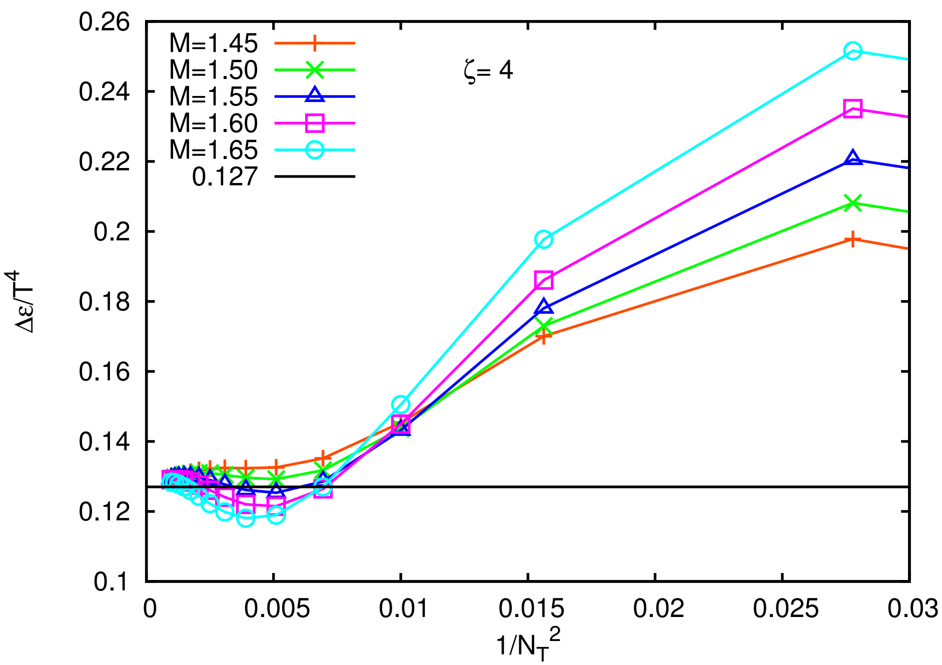
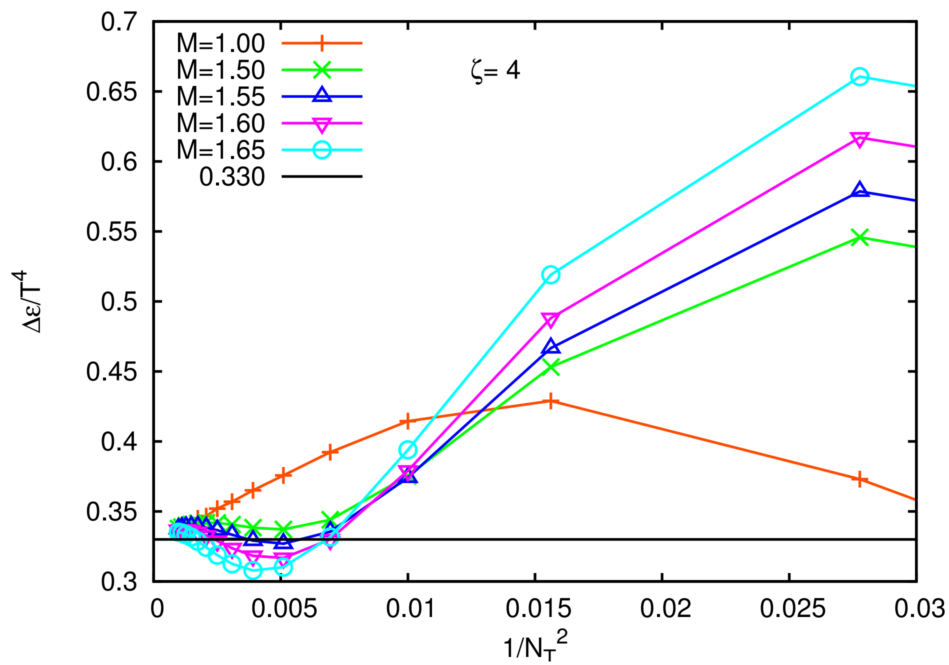
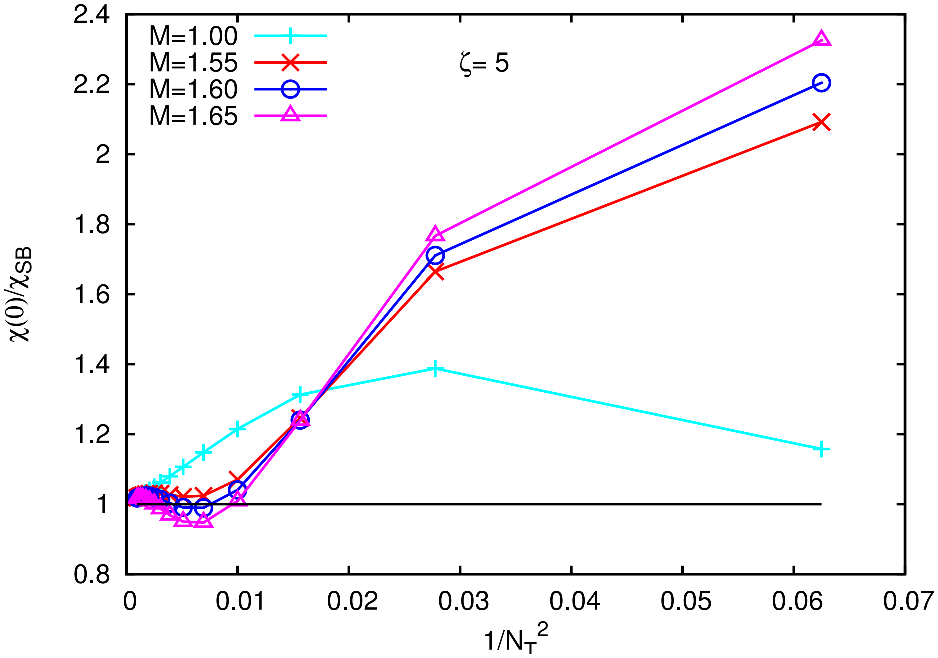
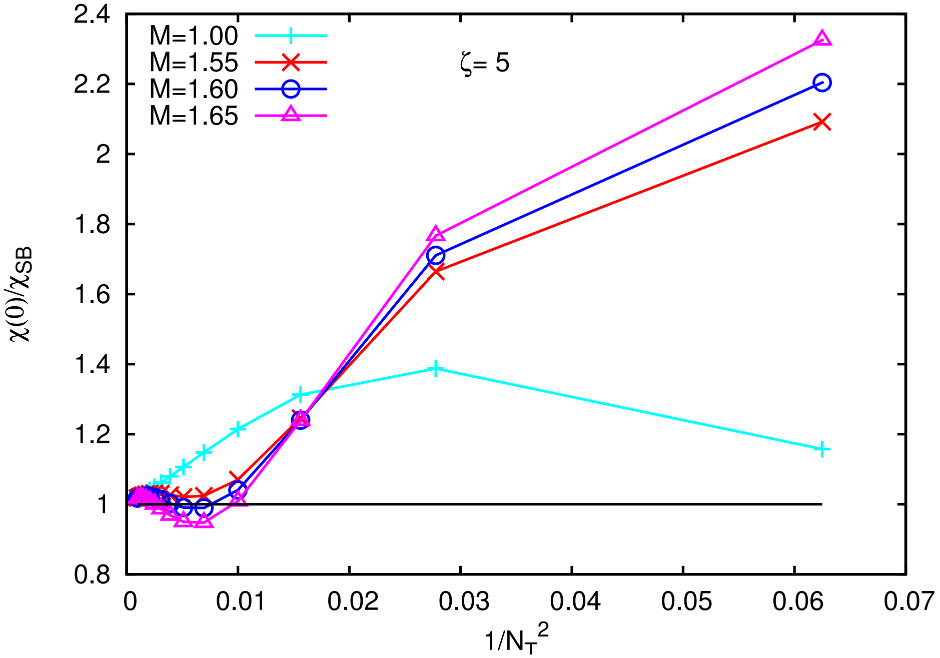
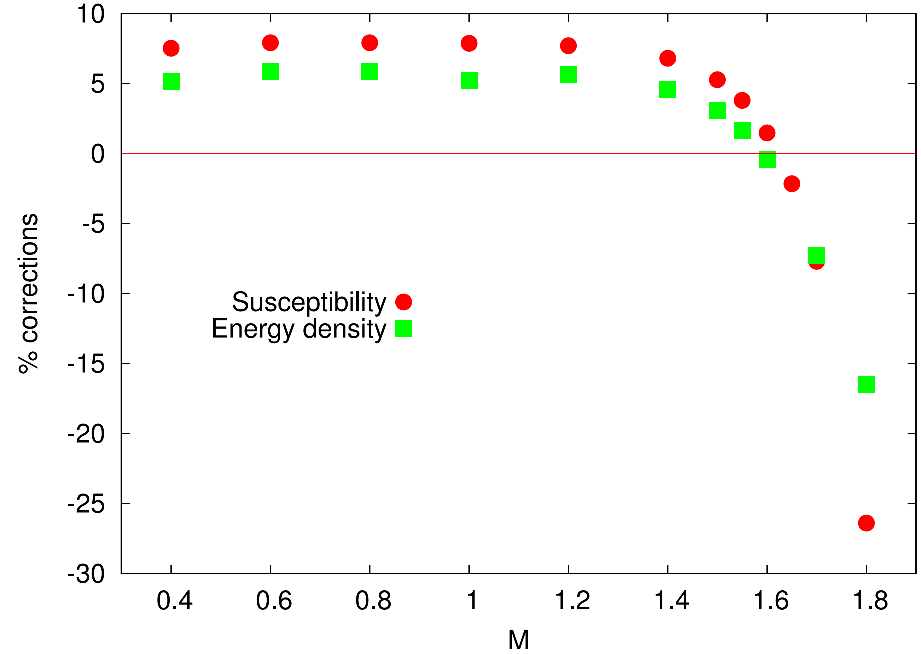




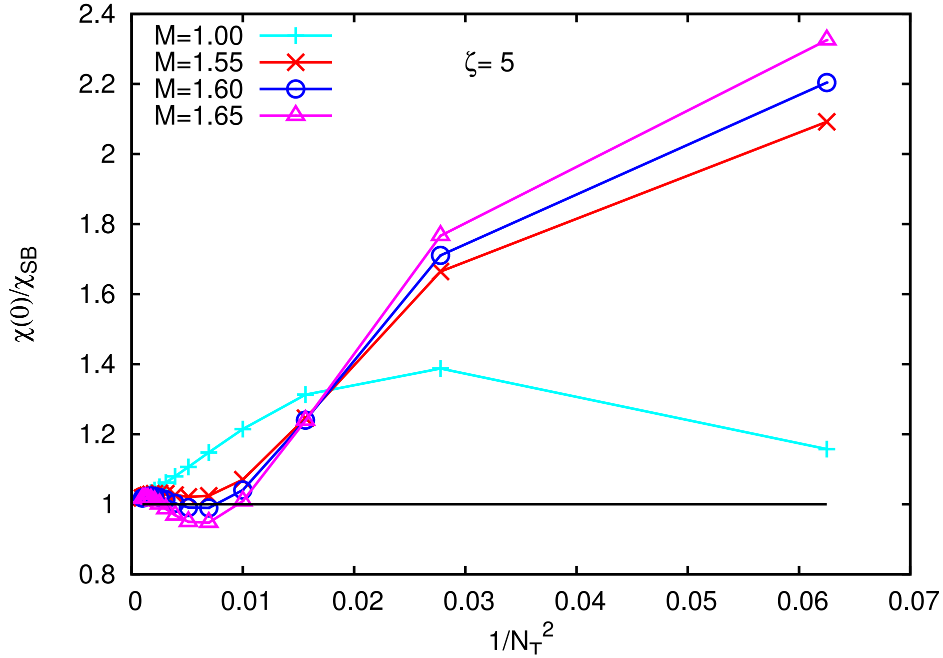


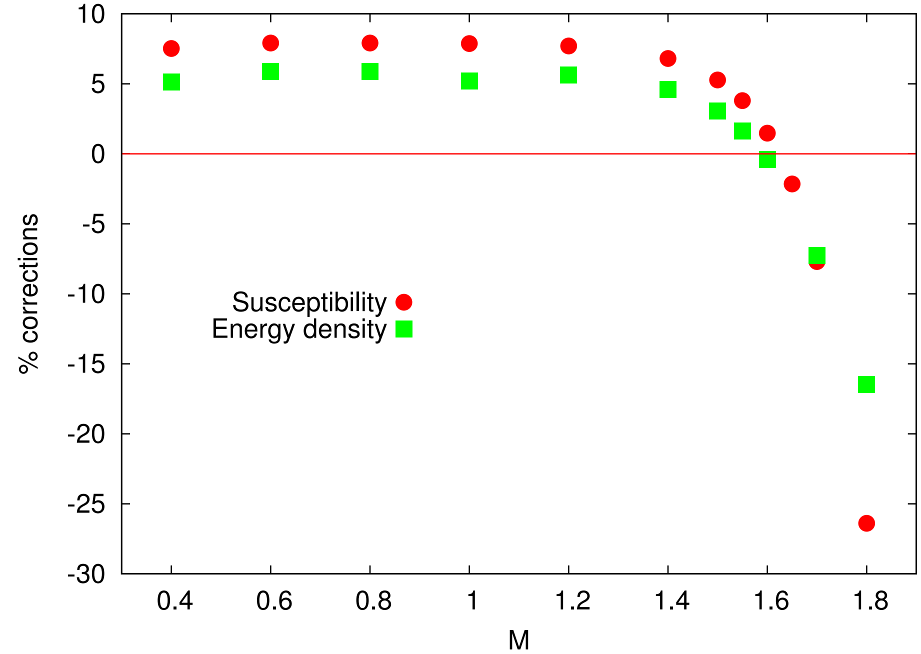
M=1.00
M=1.55
M=1.60
M=1.65
ζ= 5
χ(0)/χSB
1/NT2
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07





♡ Susceptibility too behaves the same way as the energy density.





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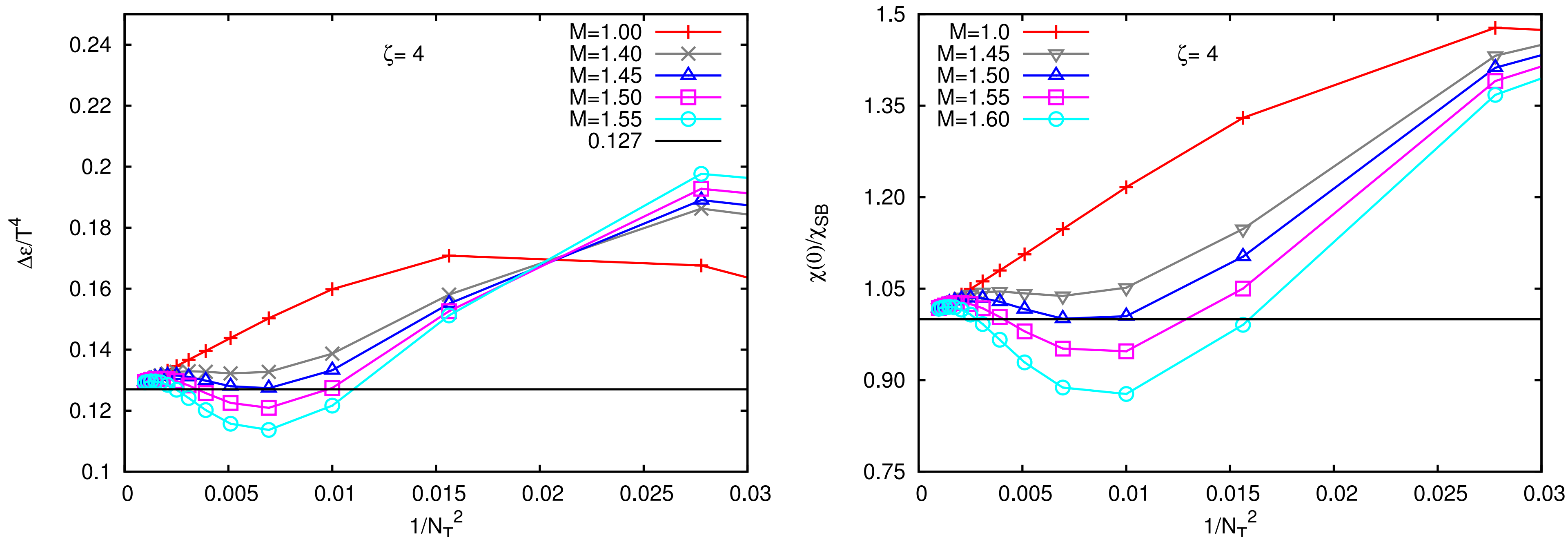

Domain Wall Fermions ($a_5 = 1$)

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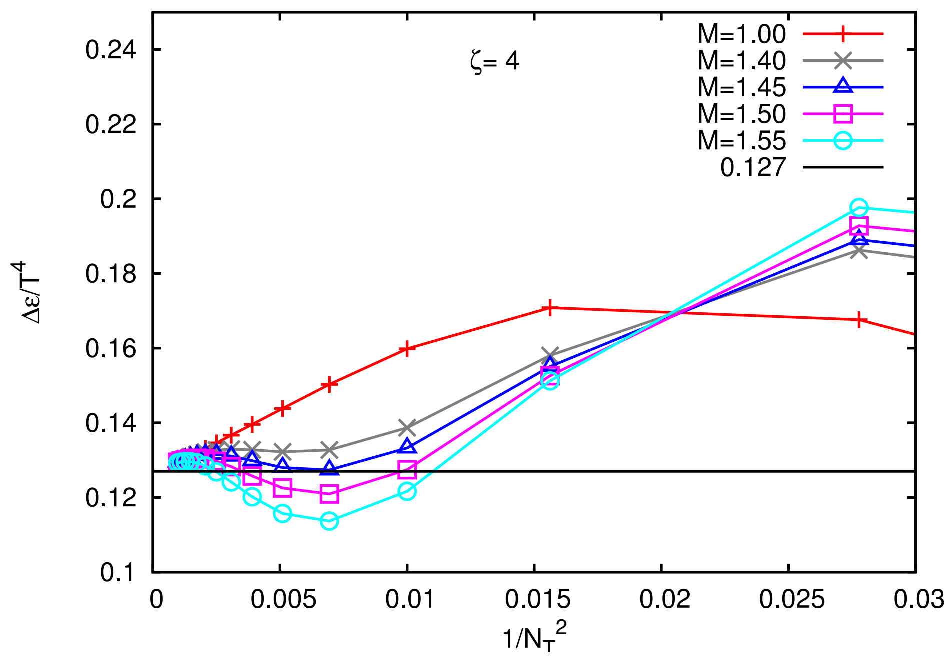

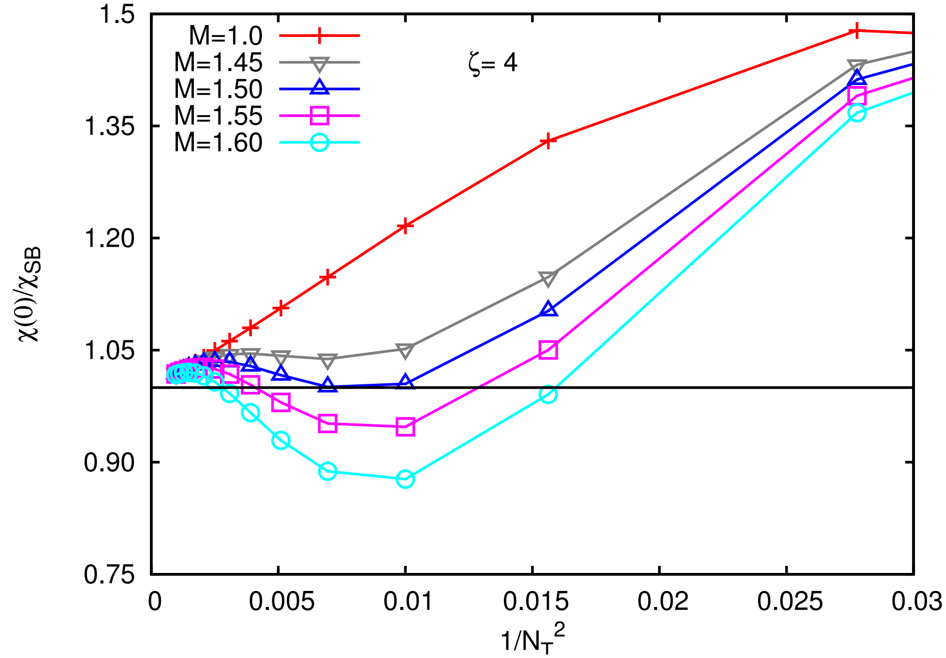

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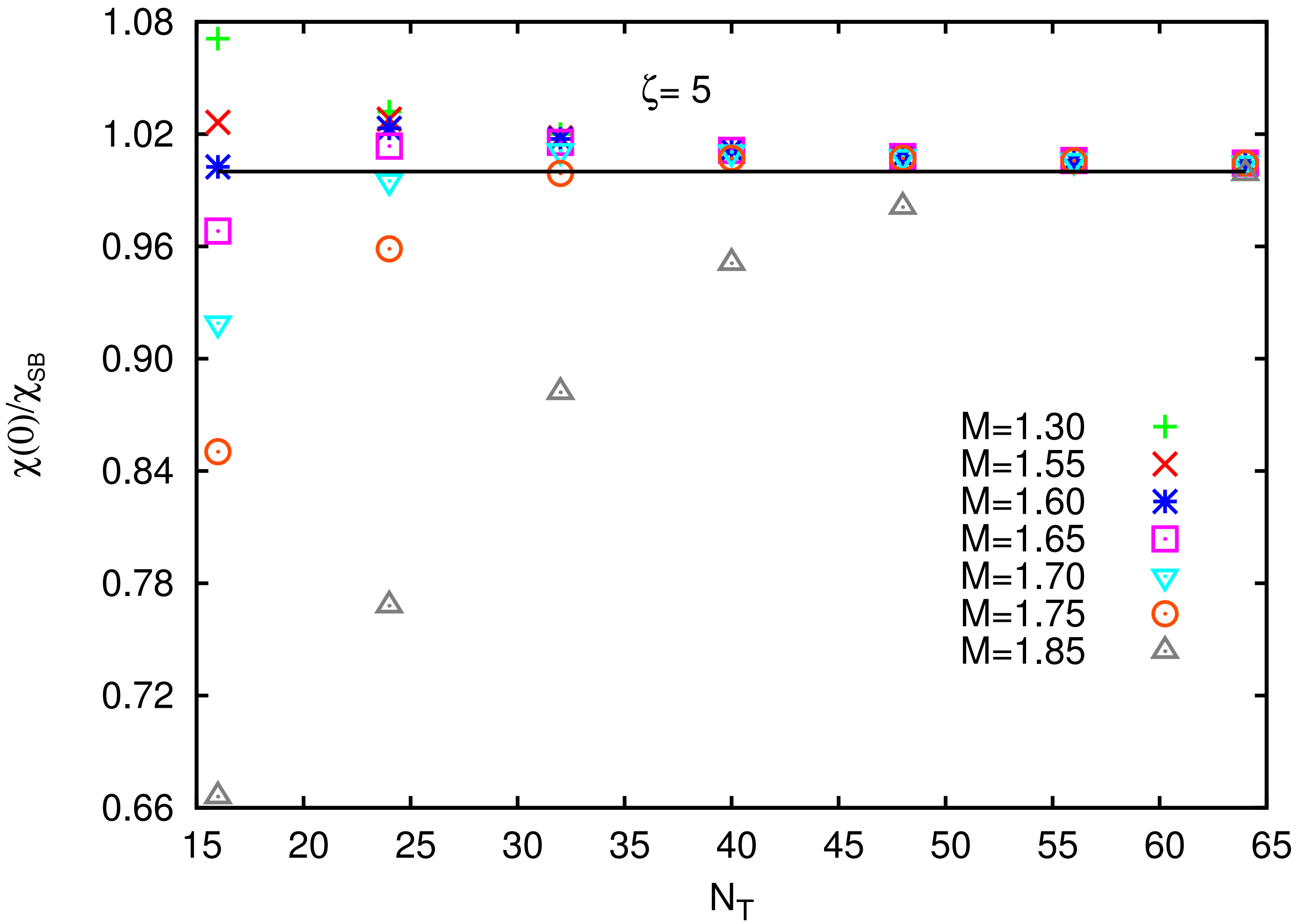

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M=1.75
M=1.85
1.08
1.02
0.96
0.90
0.84
0.78
0.72
0.66
15
20
25
30
35
40
45
50
55
60
65

Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

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Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

- However, any μ^2-divergence in the continuum limit is avoided for it and an associated general class of functions $K(\mu)$ and $L(\mu)$ with $K(\mu) \cdot L(\mu) = 1$.

- For the choice of $1.5 \leq M \leq 1.6$ ($1.4 \leq M \leq 1.5$), both the energy density and the quark number susceptibility at $\mu = 0$ exhibited the smallest deviations from the ideal gas limit for $N_T \geq 12$ for Overlap (Domain Wall) Fermions.