

QCD Critical Point : Synergy of Lattice & Experiments

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Introduction

Lattice QCD Results

Searching Experimentally

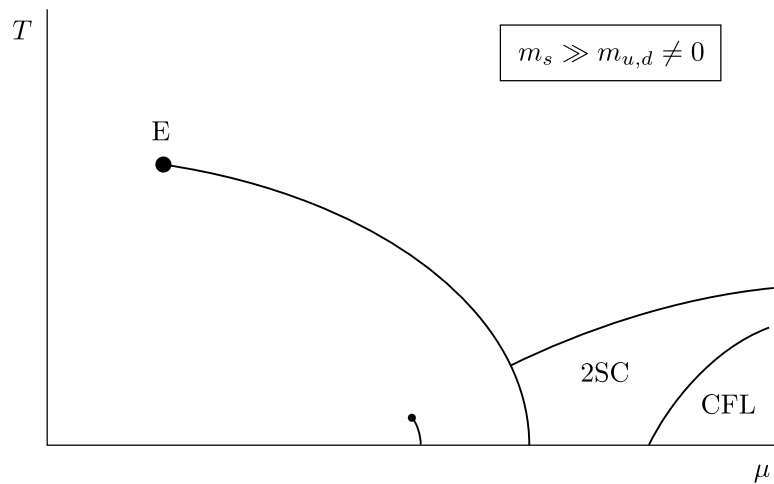
Summary

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- ♠ Many models & Approaches for QCD Phase Diagram
- ♠ QCD Critical Point in $T-\mu_B$ plane.

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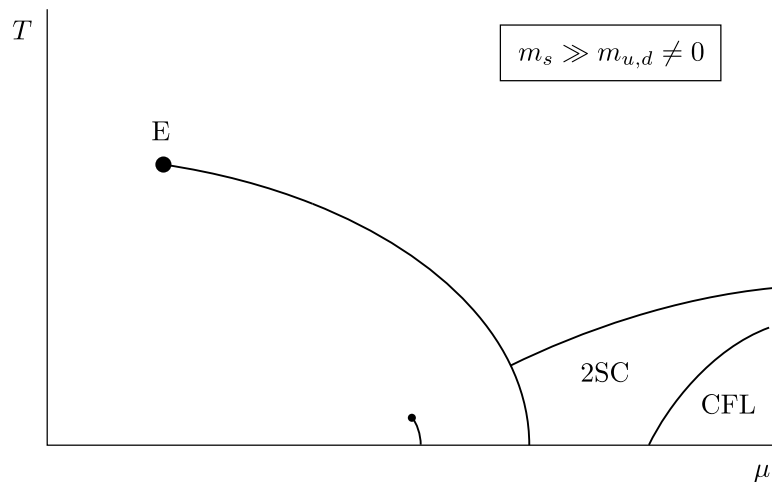


From Rajagopal-Wilczek Review

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Phase Diagram

♠ QCD Critical Point in T - μ_B plane.



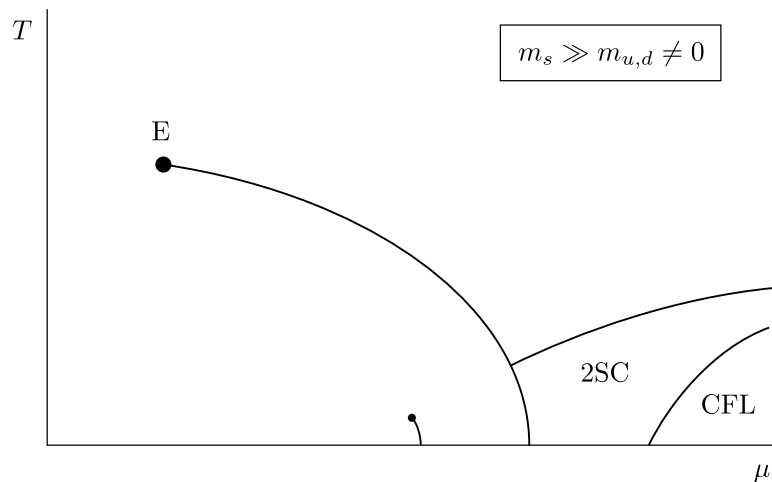
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- Search for its location using *ab initio* methods
- Search for it in the experiments RHIC, FAIR,...
- What hints can Lattice QCD investigations provide ?

The $\mu \neq 0$ problem : Quark Type

- Mostly staggered quarks used in these simulations. Broken flavour and spin symmetry on lattice. Moreover, NO flavour singlet $U_A(1)$ symmetry or anomaly. Critical point needs $N_f = 2$ and anomaly to persist by T_c .

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- Domain Wall or Overlap Fermions better, although computationally expensive.
- Introduction of μ a la Bloch & Wettig (PRL 2006 & PRD2007).
- Unfortunately BW-prescription breaks chiral symmetry ! (Banerjee, Gai & Sharma PRD 2008; PoS (Lattice 2008); PRD 2009) Furthermore, anomaly for it depends on μ unlike in continuum QCD (Gai & Sharma PRD 2010).
- Desperately needed : Formalism with Continuum-like (flavour & spin) symmetries for quarks at nonzero μ and T .

The $\mu \neq 0$ problem : The Measure

$\det M$ is a complex number for any $\mu \neq 0$: The Phase/sign problem
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Lattice Approaches in the past decade —

- Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
- Imaginary Chemical Potential (Ph. de Forcrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).
- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, PoS LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

How Do We Do This Expansion?

Canonical definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \quad \text{and} \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \dots$)

Denoting higher order susceptibilities by χ_{n_u, n_d} , the pressure P has the expansion in μ :

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu, T)}{T^4} - \frac{P(0, T)}{T^4} = \sum_{n_u, n_d} \chi_{n_u, n_d} \frac{1}{n_u!} \left(\frac{\mu_u}{T} \right)^{n_u} \frac{1}{n_d!} \left(\frac{\mu_d}{T} \right)^{n_d} \quad (1)$$

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- We (Gavai-Gupta '05, '09) construct the series for baryonic susceptibility from this expansion. Its radius of convergence gives the nearest critical point.
- Successive estimates for the radius of convergence obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^n}\right)^{1/n}$. We use both these definitions.
- All coefficients of the series must be POSITIVE for the critical point to be at real μ , and thus physical.
- We use up to 8th order. Need 20 inversions of $(D + m)$ on ~ 500 vectors for a single measurement.
- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2010) which save up to 60 % computer time.

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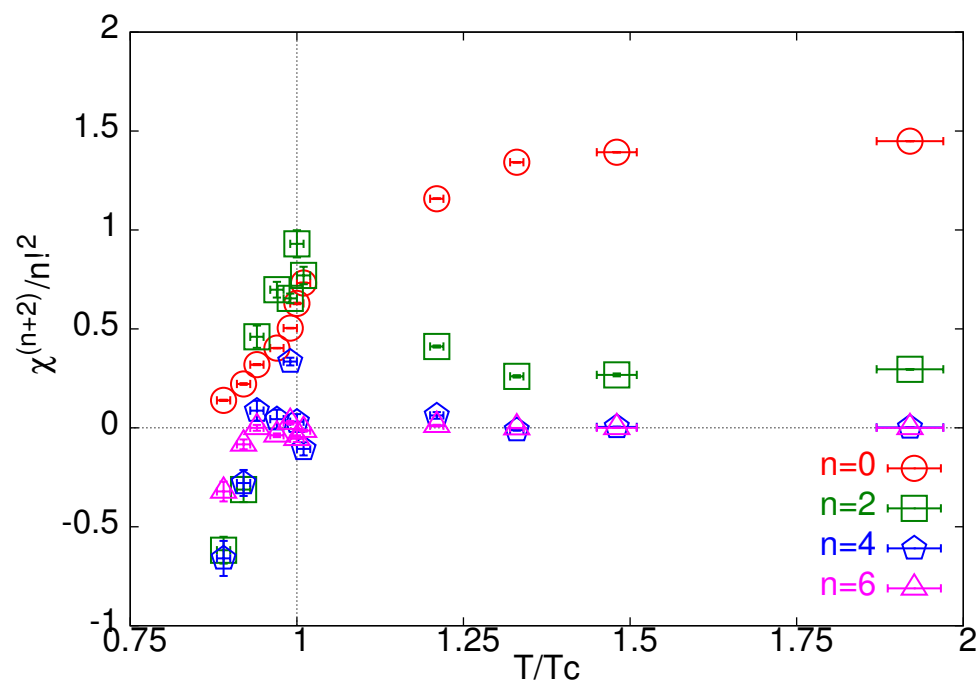
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Lattice QCD Results

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- $m_\pi = 230 \text{ MeV}$ (Gavai-Gupta, PRD 2005, 2009).

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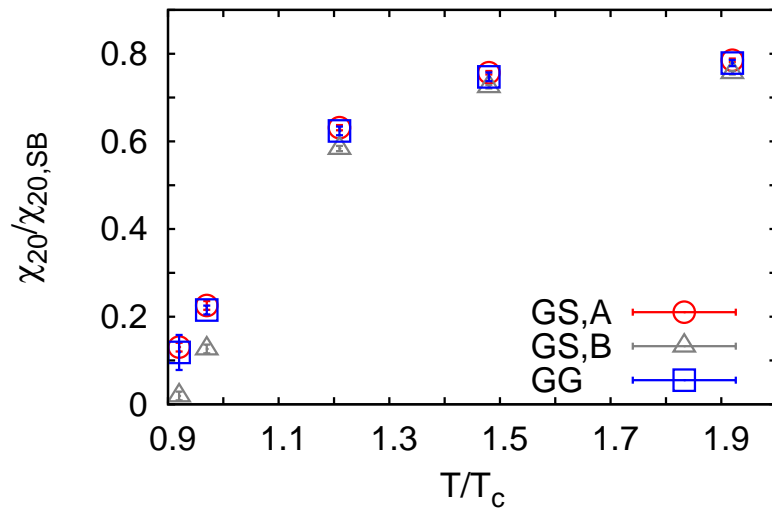
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Window of positivity just below T_c

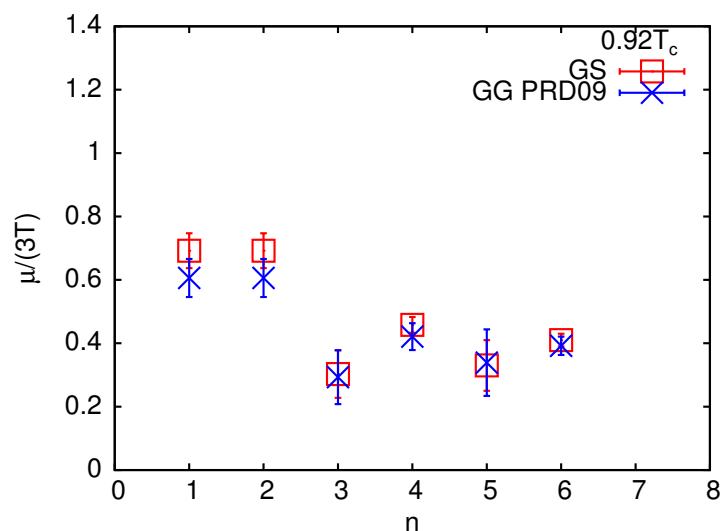
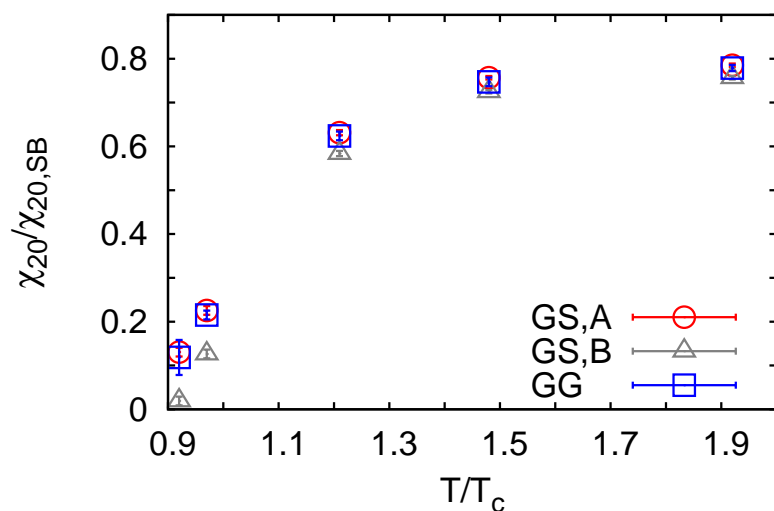
Preliminary Results with μN -idea

♠ Using our proposed μN term (Gavai-Sharma PRD 2010) to evaluate the baryon susceptibility at $\mu = 0$,



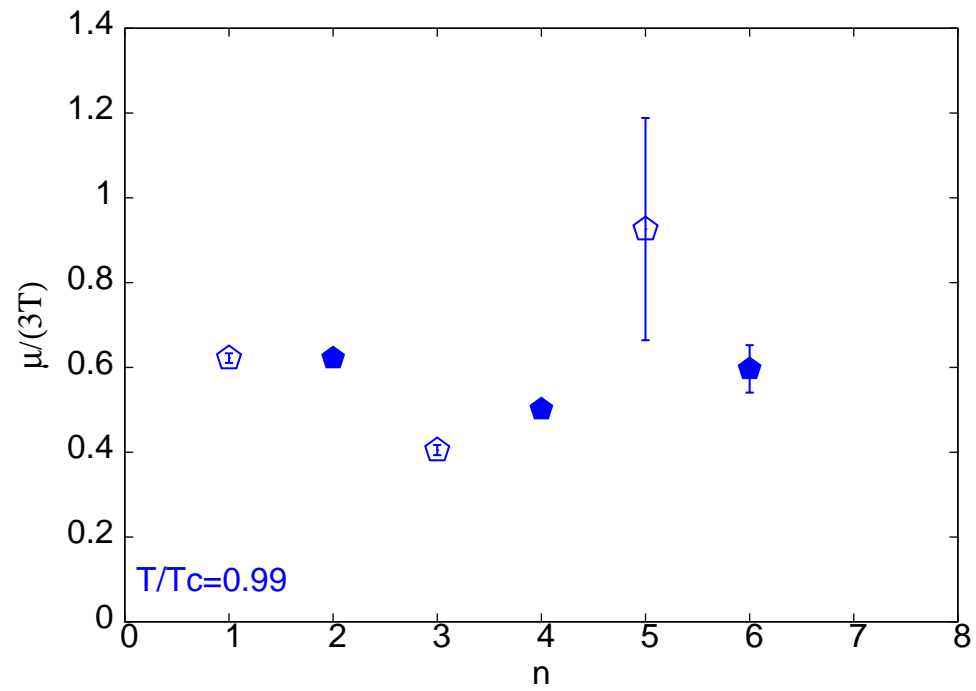
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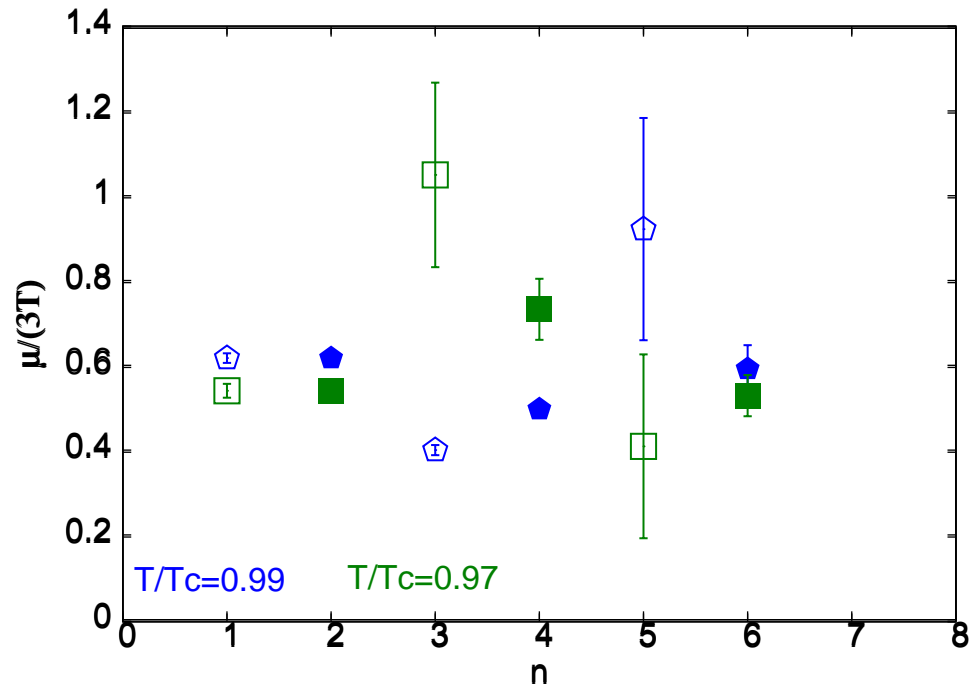
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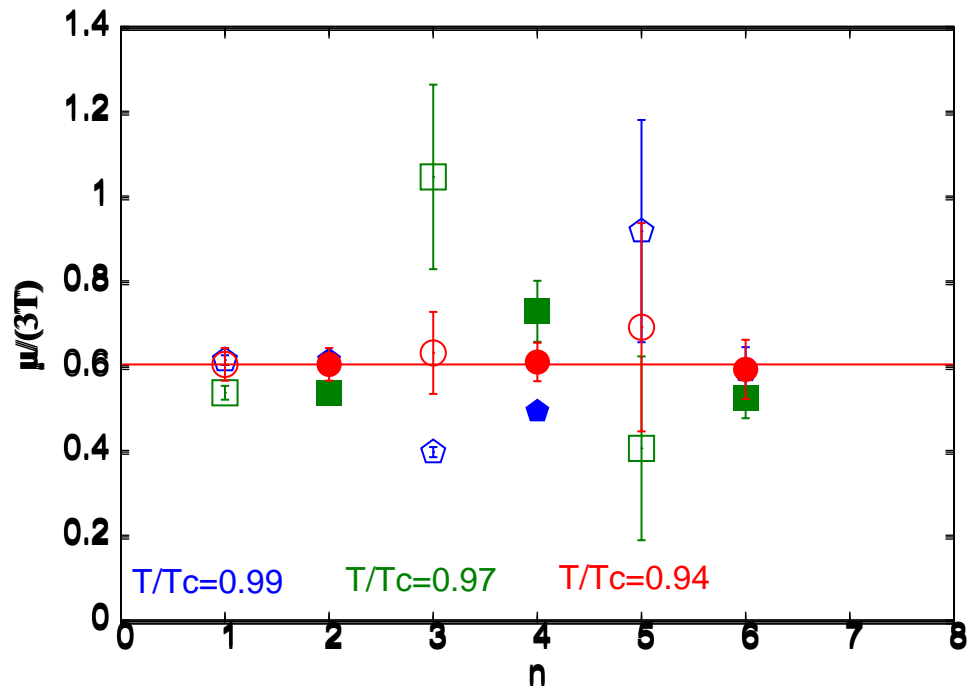


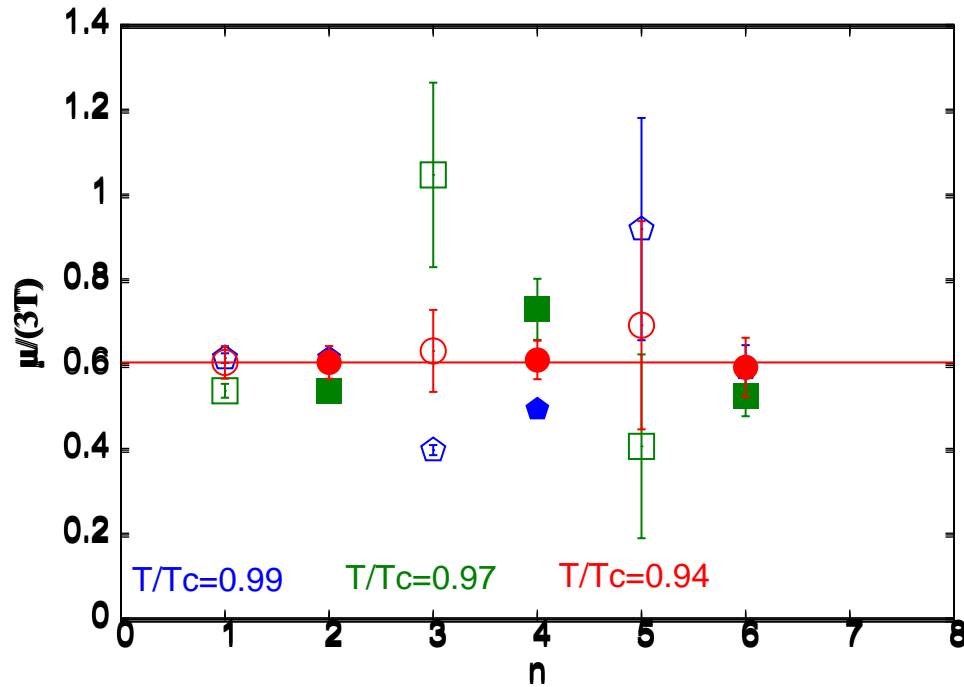
♡ ALL NLS Coefficients do have the same sign for the new method.

♠ The estimates for radius of convergence are comparable as well.





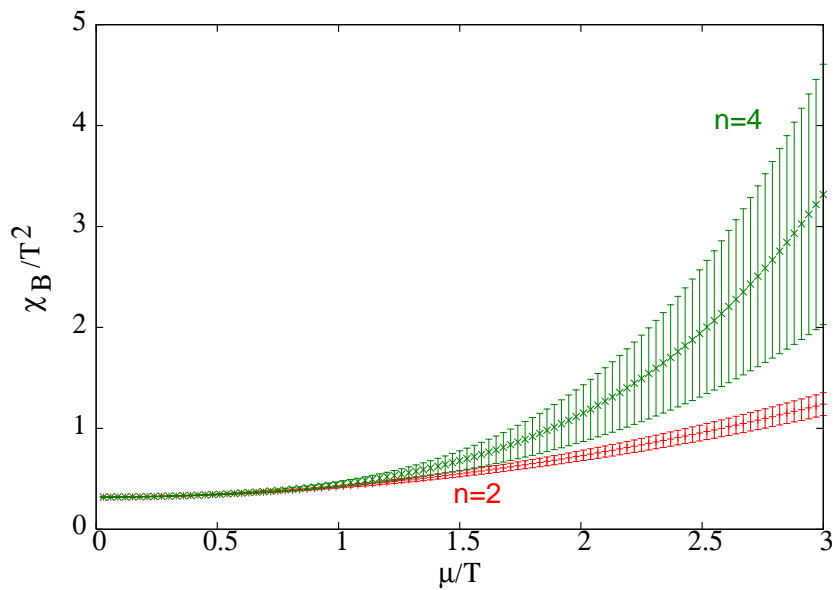




- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: \downarrow to 1.1(1)
- Critical point at $\mu_B/T \sim 1 - 2$.

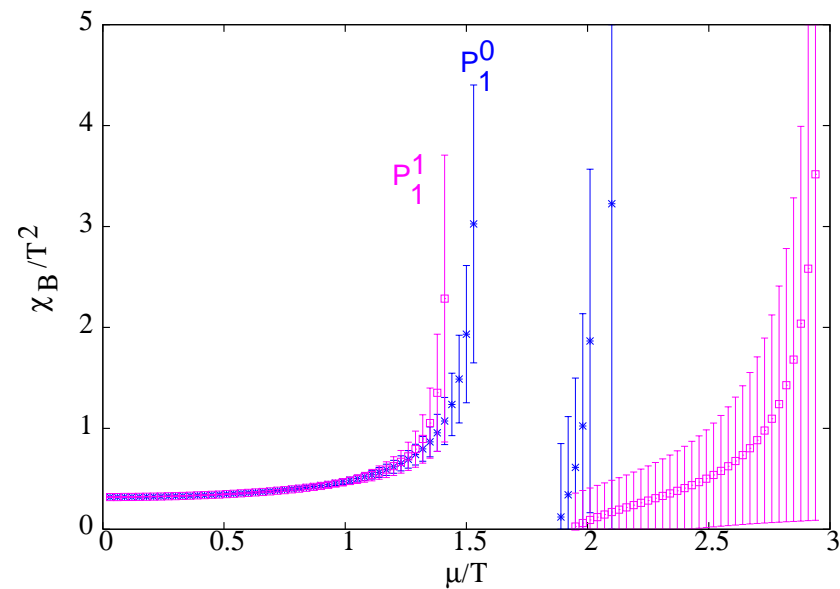
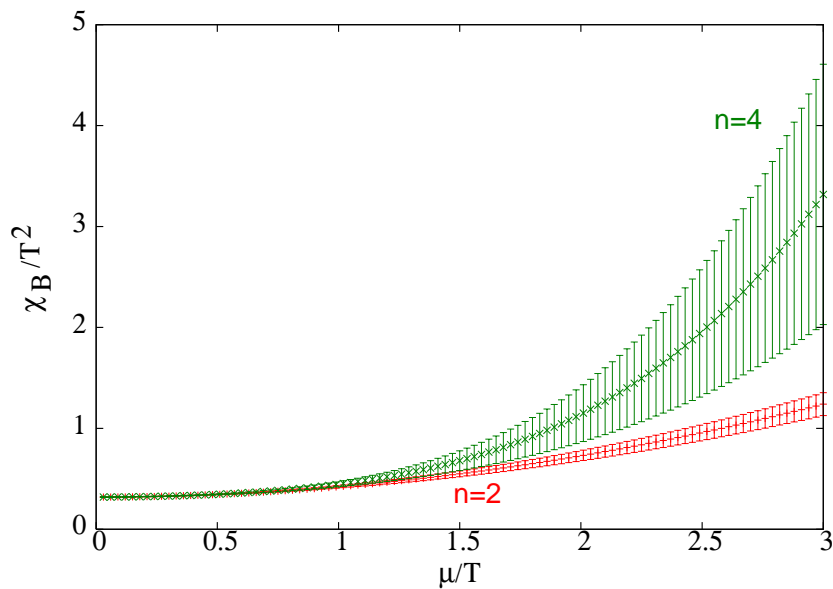
Cross Check on μ^E/T^E

♠ Use the series directly to construct χ_B for nonzero $\mu \rightarrow$ smooth curves with no signs of criticality.



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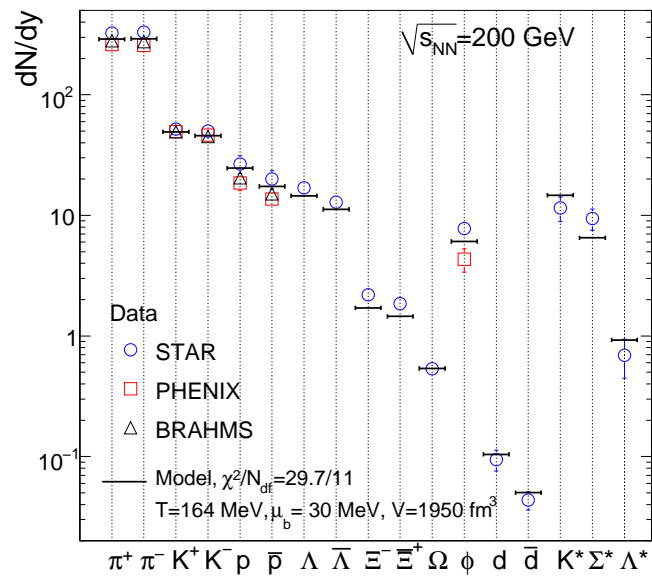


♠ Use Padé approximants for the series to estimate the radius of convergence.

♡ Consistent Window with our other estimates.

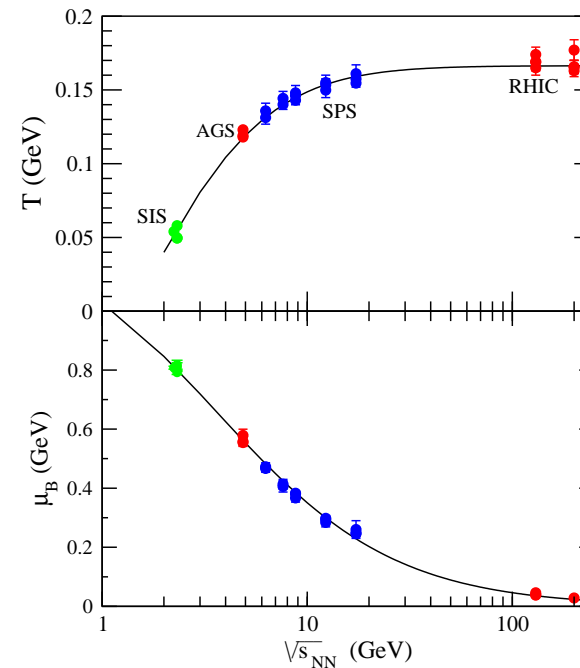
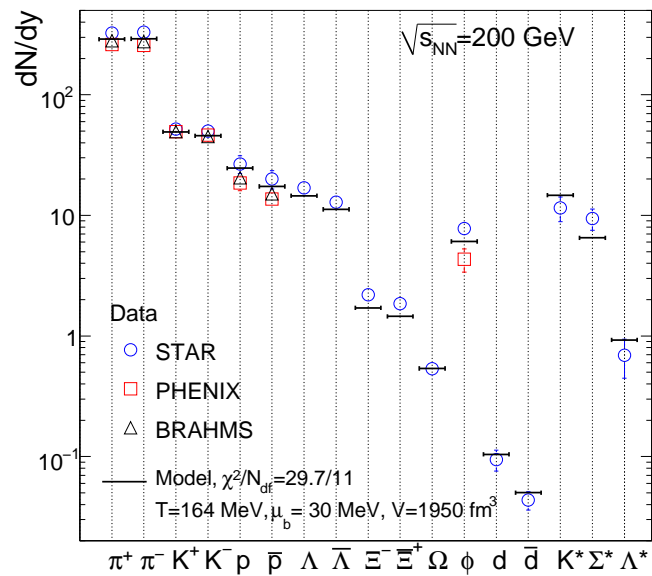
Lattice predictions along the freezeout curve

- Hadron yields well described using Thermodynamical Models, leading to a freezeout curve in the $T-\mu_B$ plane. (Andronic, Braun-Munzinger & Stachel, PLB 2009 ; Oeschler, Cleymans, Redlich & Wheaton, 2009)

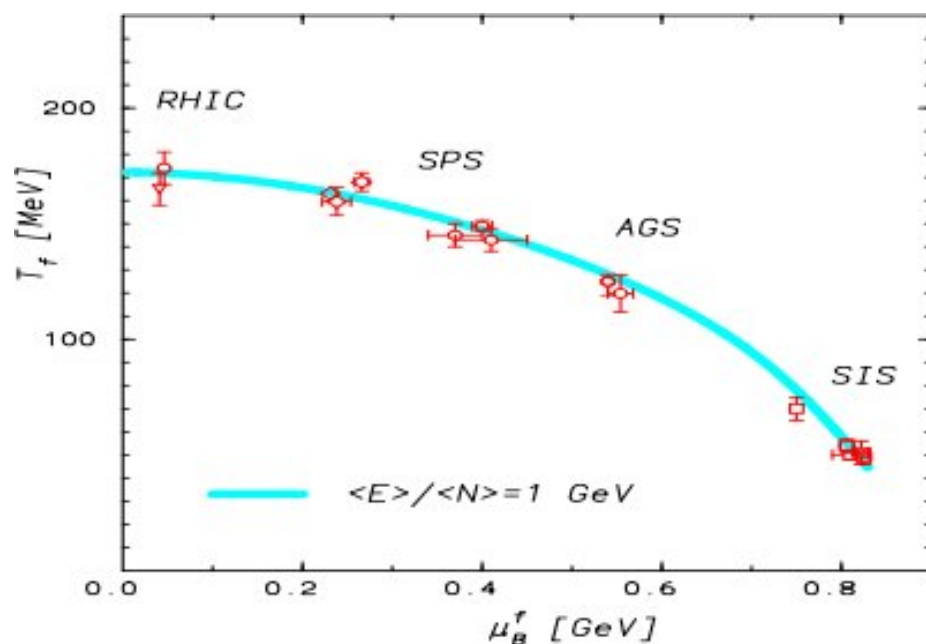


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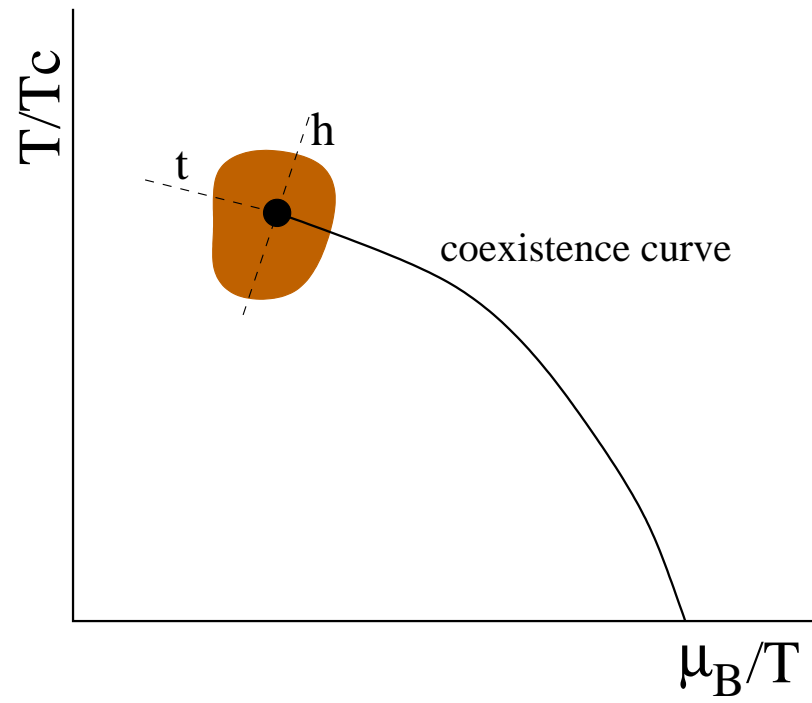


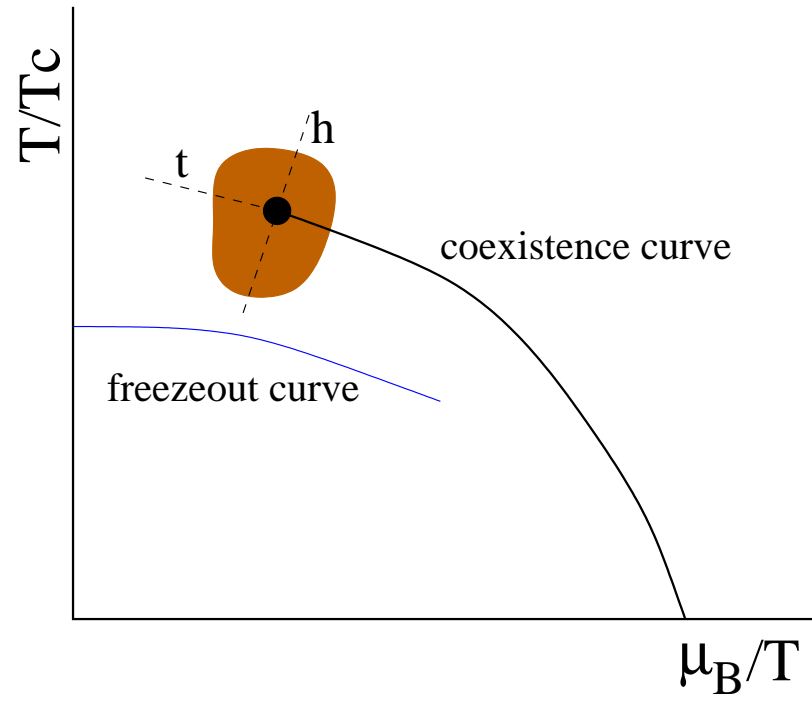
- Plotting these results in the T - μ_B plane, one has the freezeout curve, which was shown to correspond the $\langle E \rangle / \langle N \rangle \simeq 1$. (Cleymans and Redlich, PRL 1998)

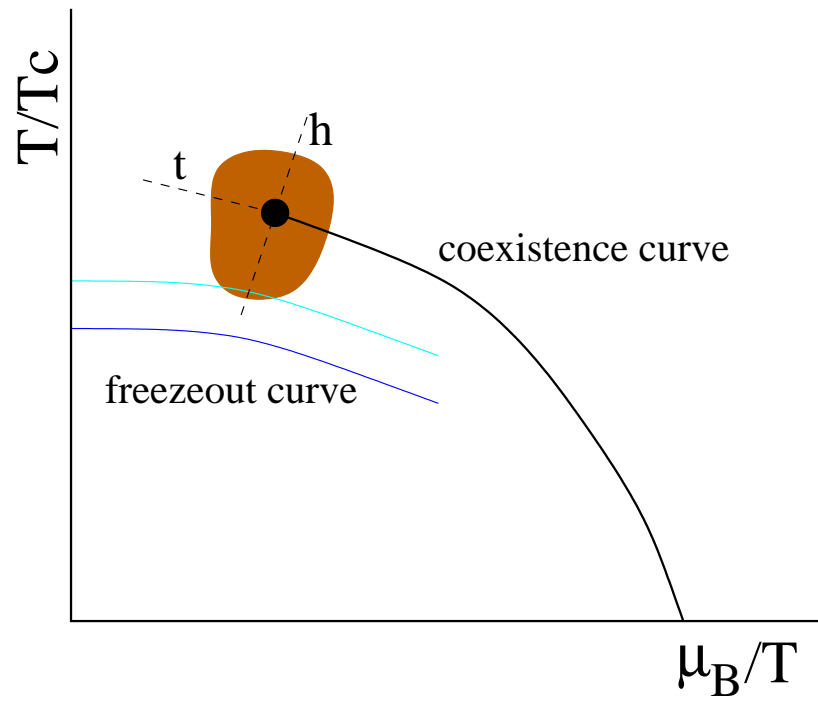


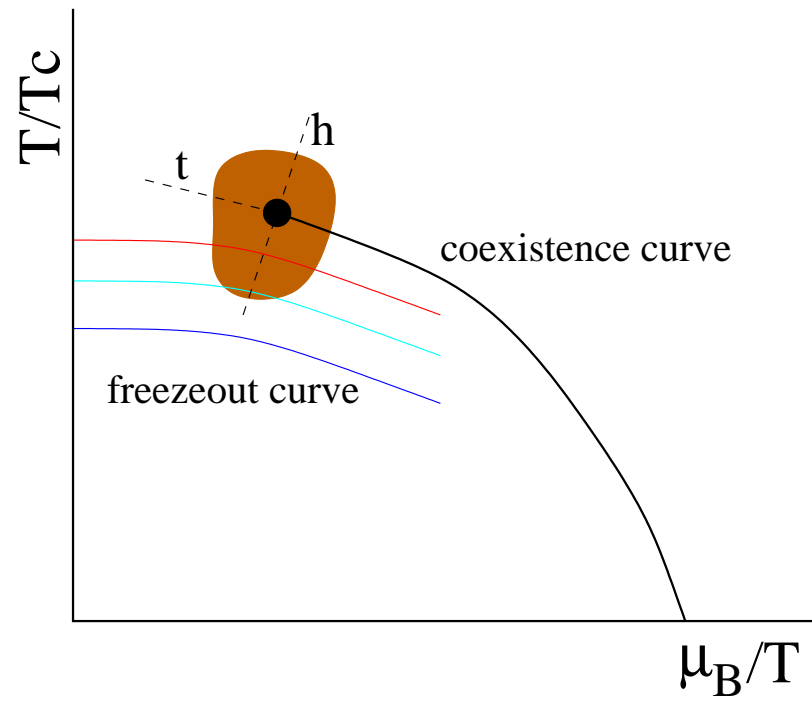
(From Braun-Munzinger, Redlich and Stachel nucl-th/0304013)

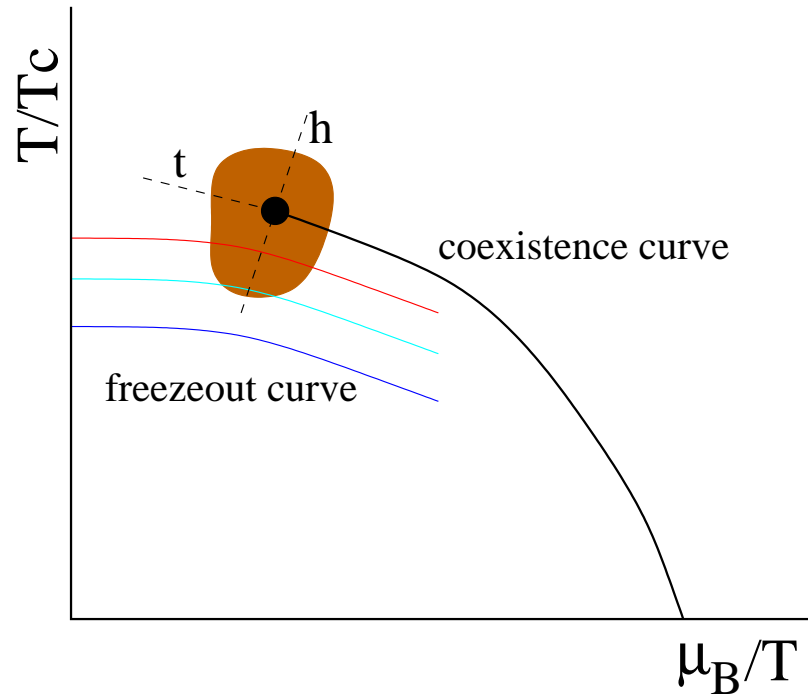
- Our Key Proposal : Use this freezeout curve to relate (T, μ_B) to \sqrt{s} and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)



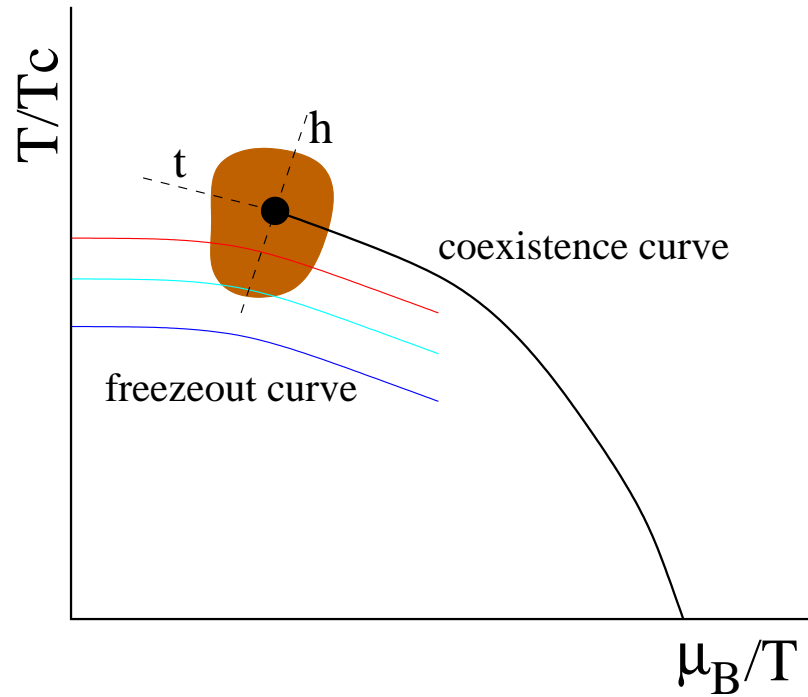








- Define $m_1 = \frac{T\chi^{(3)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)}$, $m_3 = \frac{T\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)}$, and $m_2 = m_1 m_3$ (Gupta, arXiv : 0909.4630) and use the Padè method to construct them.



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- Near the critical point, $\chi_B \sim |\mu - \mu_E|^\delta$. Thus the ratios, m_i , should diverge in the critical region as well.

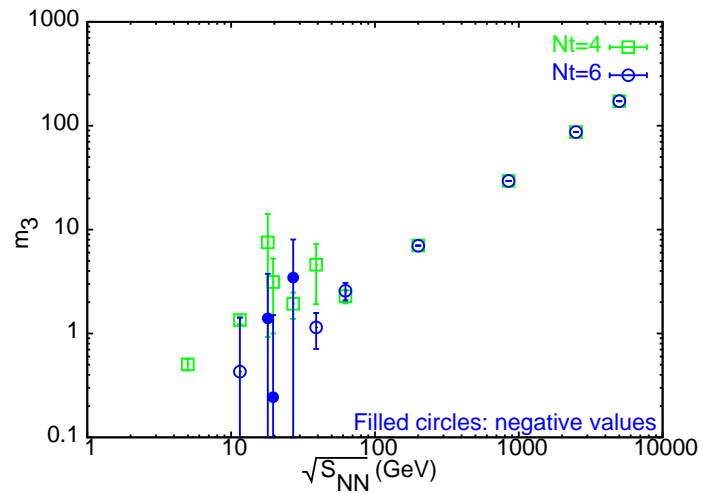
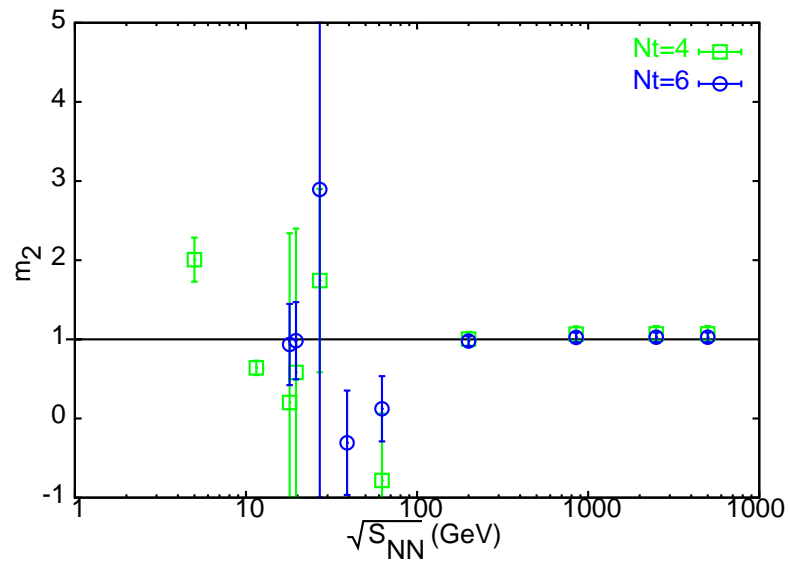
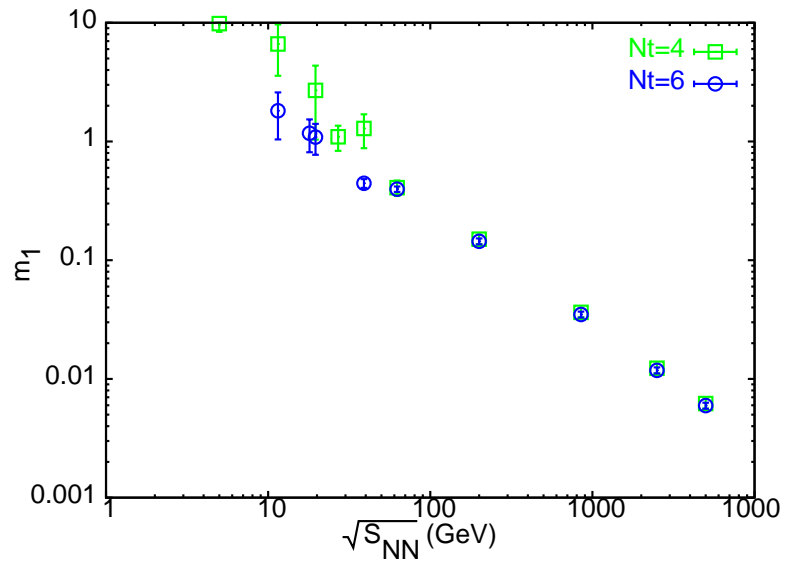
- m_i are dimensionless, and are computed as functions of T/T_c . \implies expect small lattice spacing corrections.
- Spatial Volume cancels out in these ratios \implies Suitable for experiments who can use their favourite proxy for it.

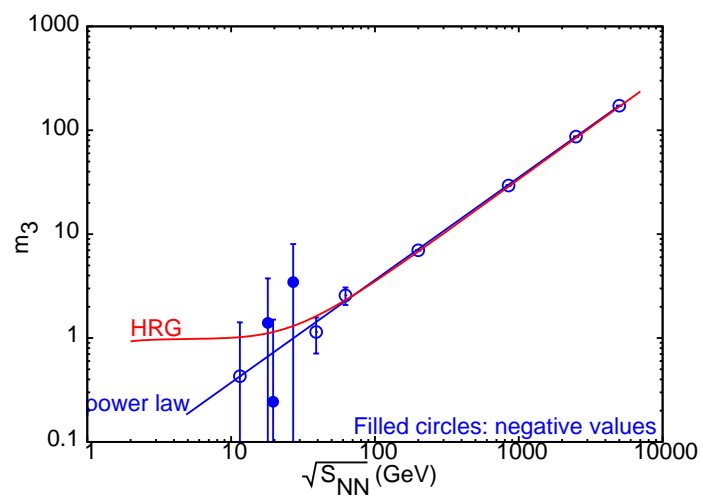
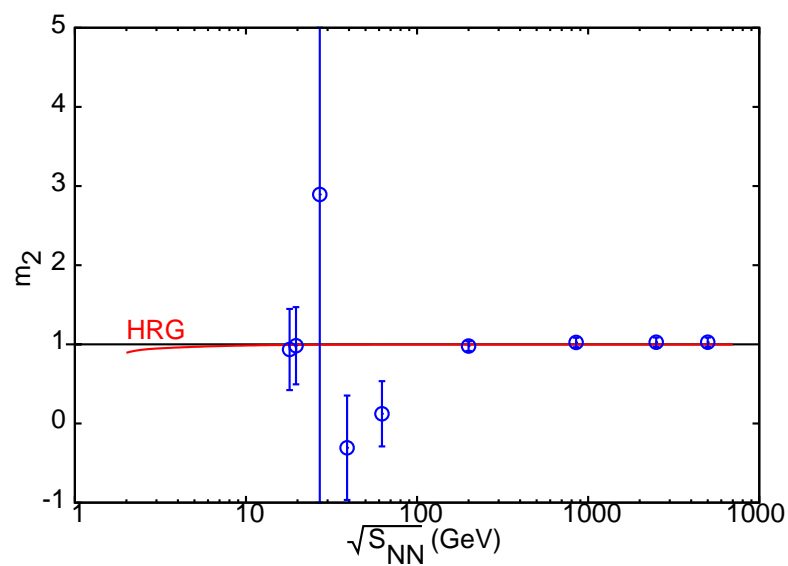
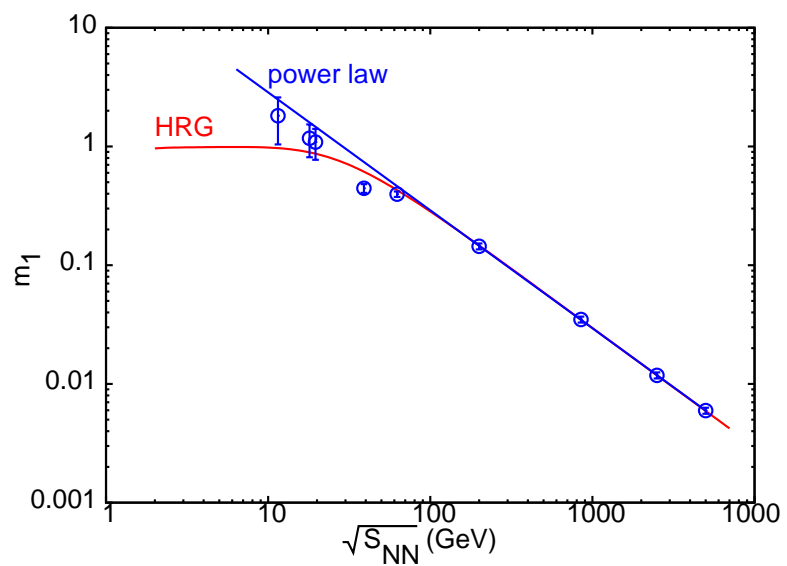
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- Spatial Volume cancels out in these ratios \implies Suitable for experiments who can use their favourite proxy for it.
- Defining $z = \mu_B/T$, and denoting by r_{ij} the estimate for radius of convergence using χ_i, χ_j , one has

$$m_1 = \frac{2z}{r_{24}^2} \left[1 + \left(\frac{2r_{24}^2}{r_{46}^2} - 1 \right) z^2 + \left(\frac{3r_{24}^2}{r_{46}^2 r_{68}^2} - \frac{3r_{24}^2}{r_{46}^2} + 1 \right) z^4 + \mathcal{O}(z^6) \right].$$

- Similar series expressions for m_2 and m_3 . Resum these by Padè ansatz :

$$m_1 = zP_1^1(z^2; a, b), \quad m_3 = \frac{1}{z}P_1^1(z^2; a', b')$$



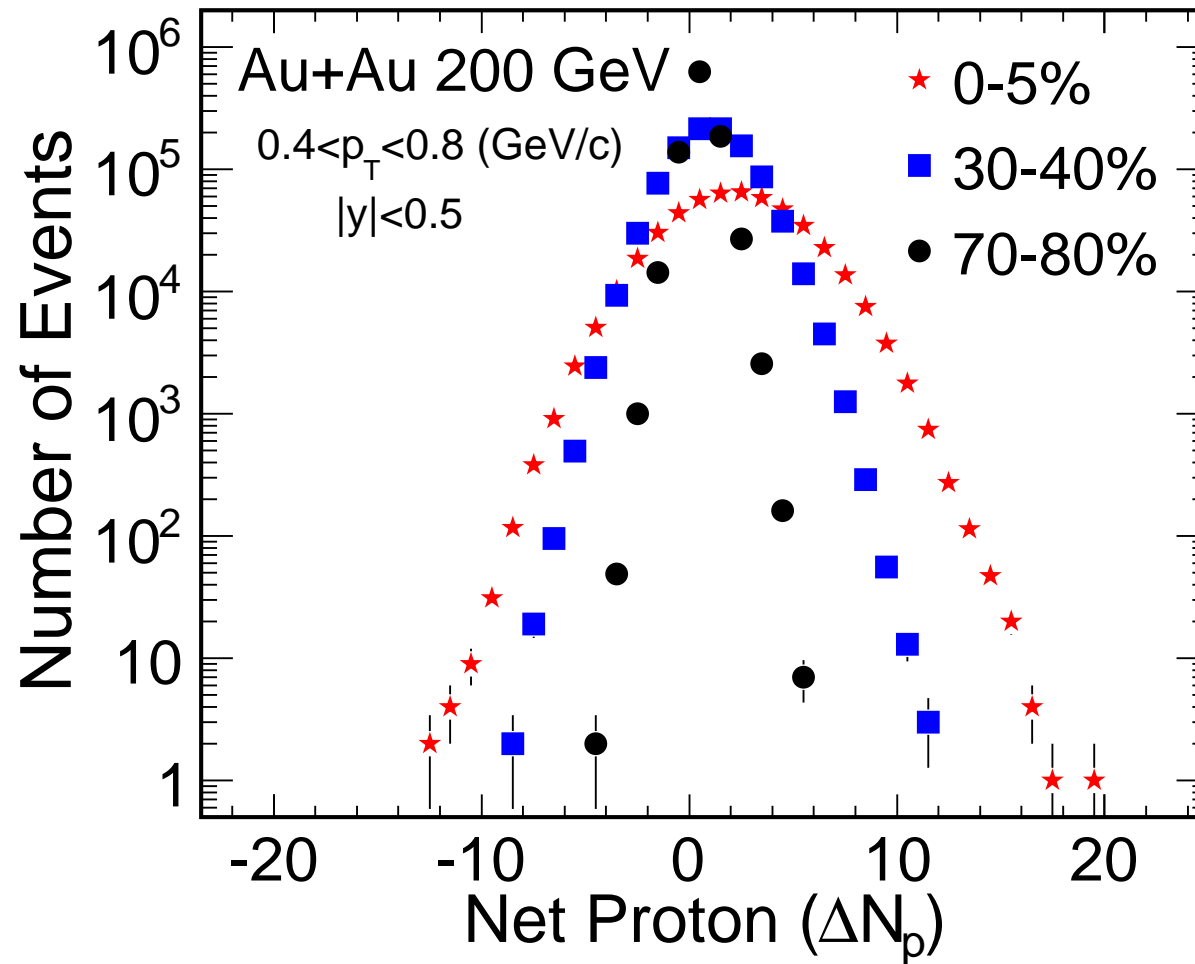


♠ Used $T_c(\mu = 0) = 175$ MeV.

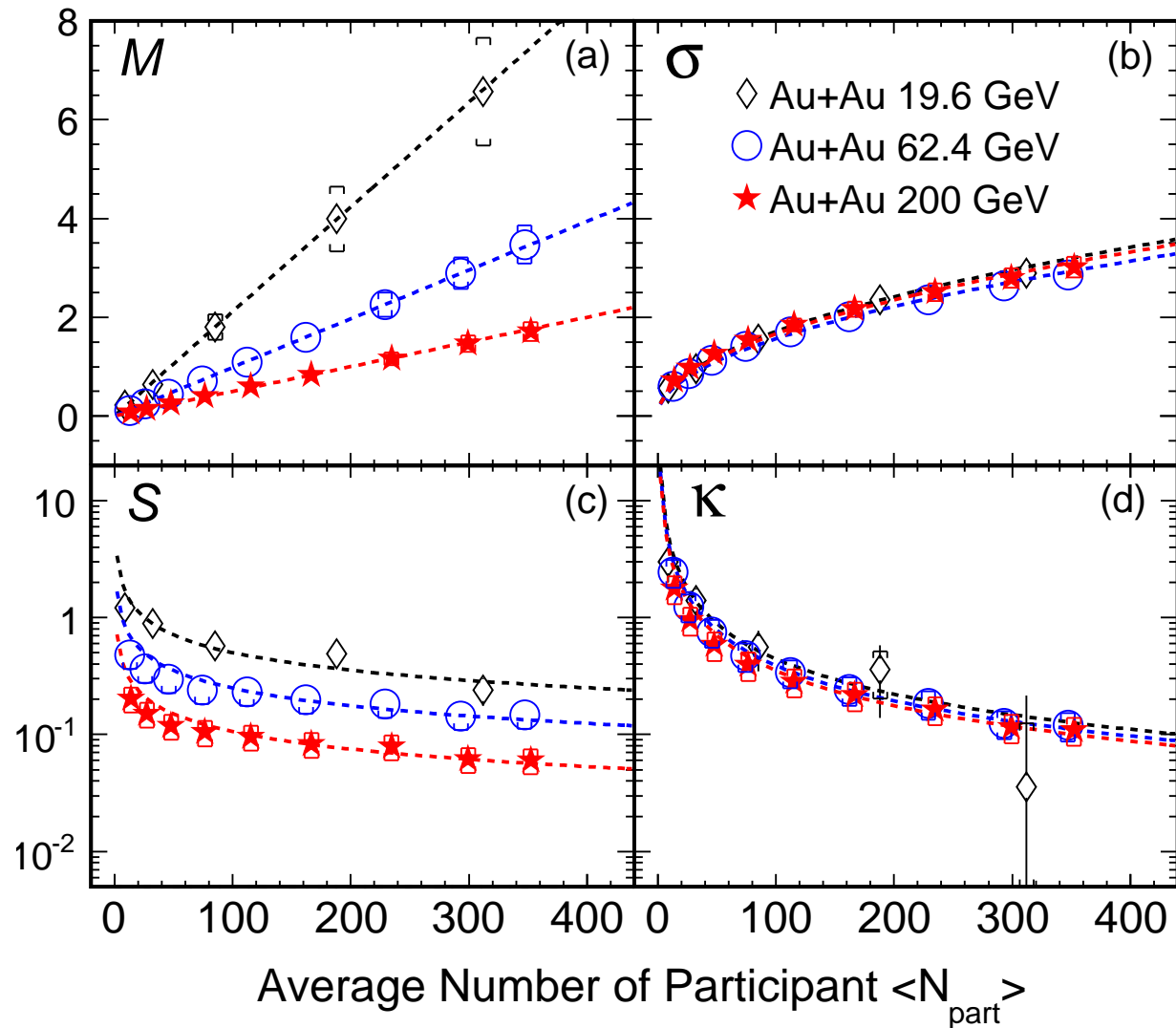
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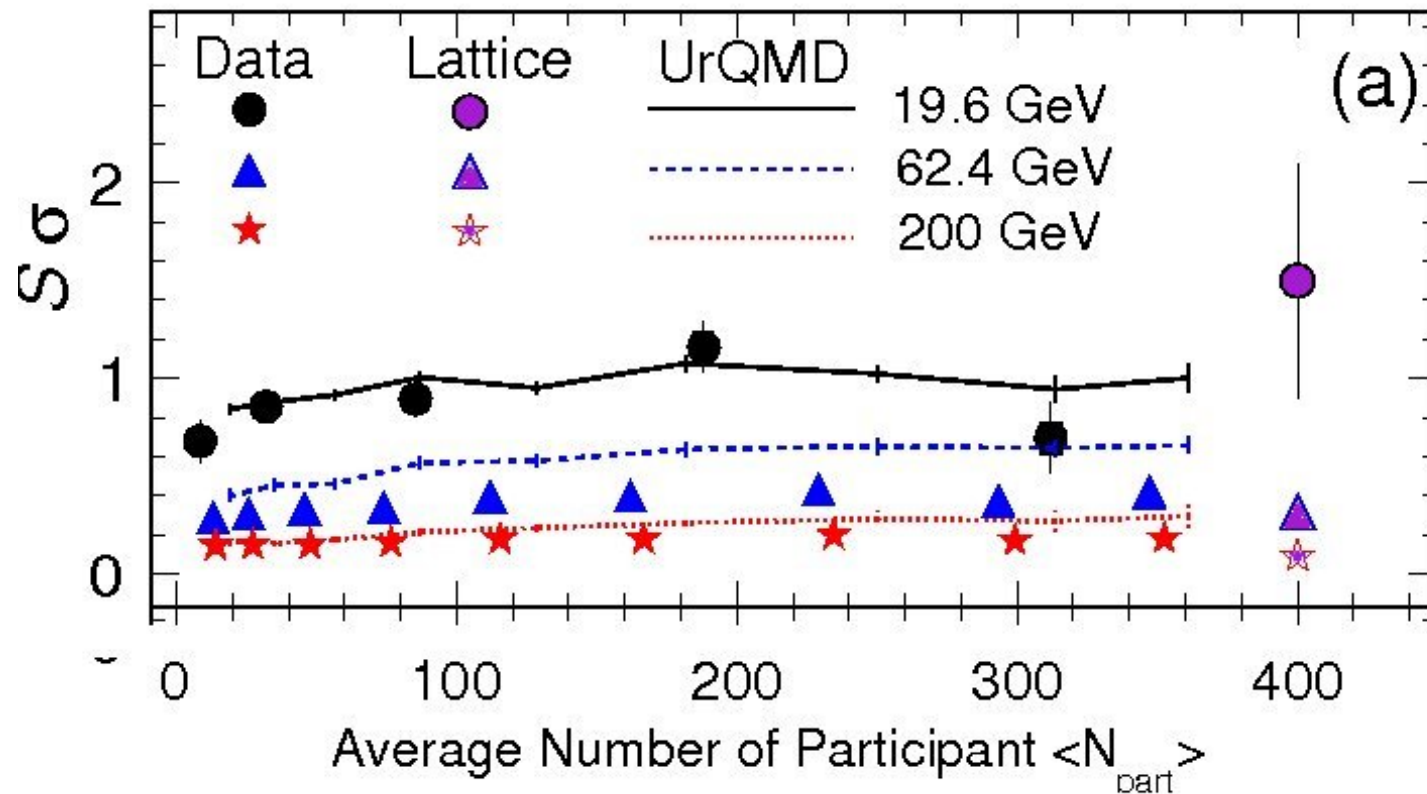
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- Proton number fluctuations (Hatta-Stephenov, PRL 2003): Diverging ξ at critical point is linked to σ mode which cannot mix with any isospin modes $\Rightarrow \chi_I$ to be regular.
- Leads to a ratio $\chi_Q:\chi_I:\chi_B = 1:0:4$
- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be proton number fluctuations only.



Aggarwal et al., STAR Collaboration, arXiv : 1004.4959

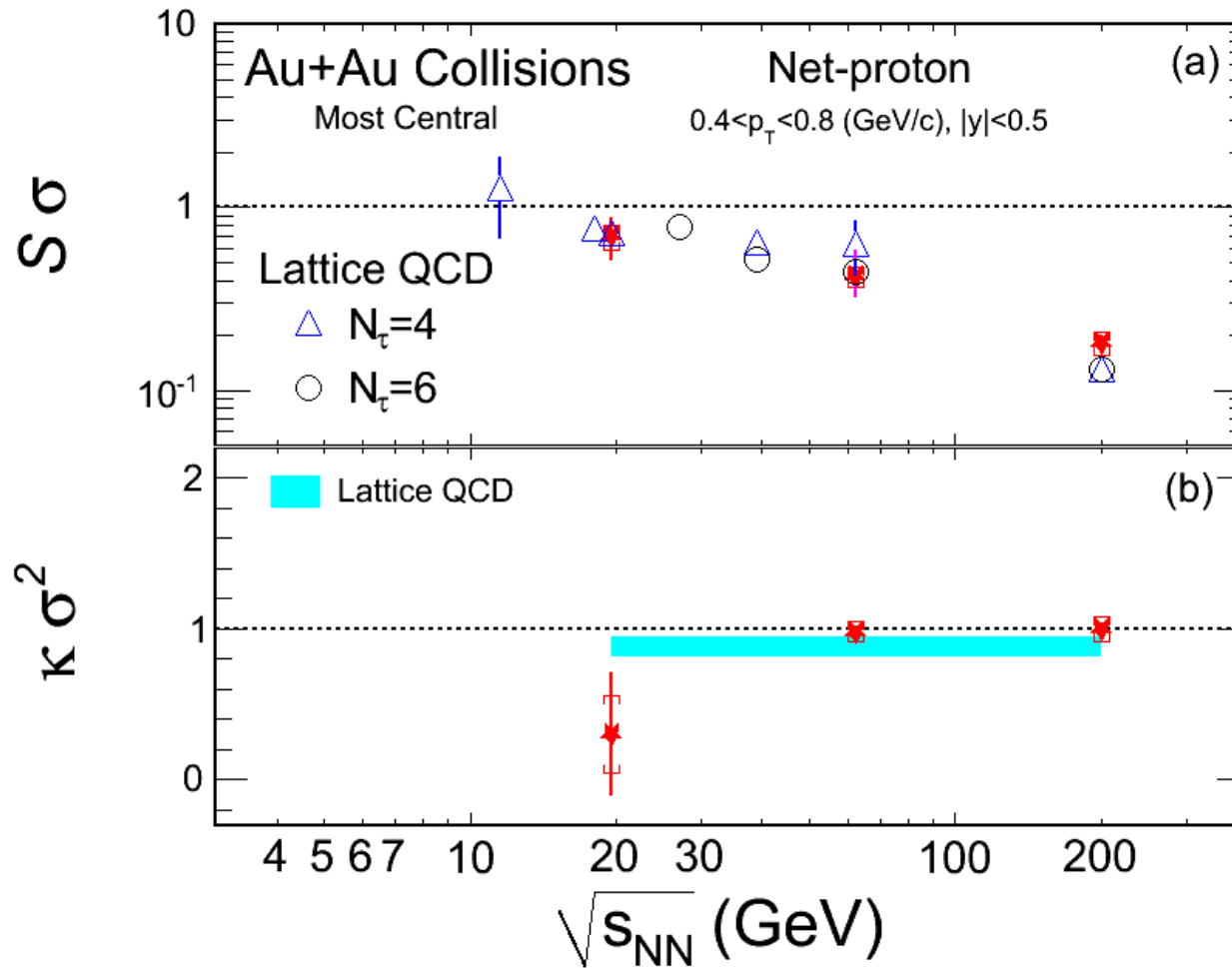


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- Reasonable agreement with our lattice results. Where is the critical point ?



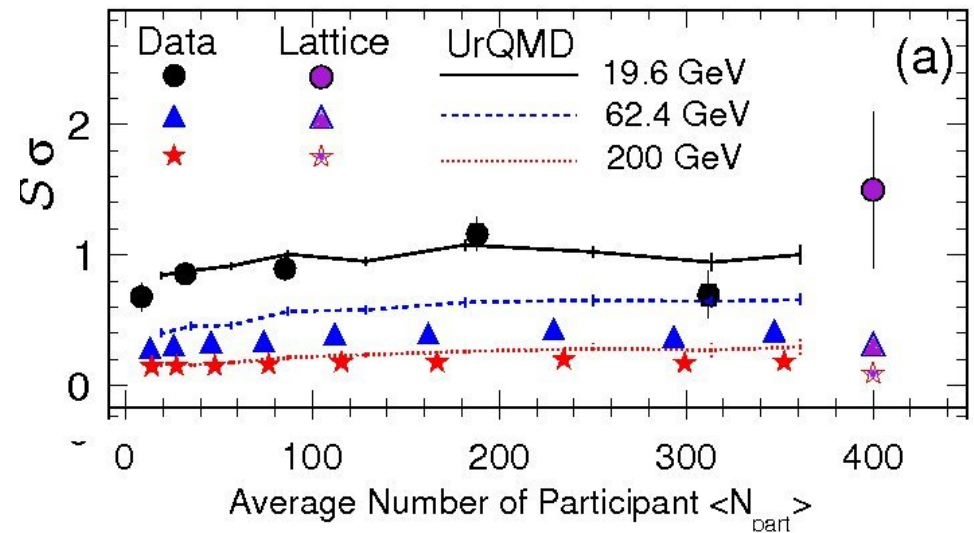
Private communication from STAR Collaboration

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, \rightsquigarrow similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

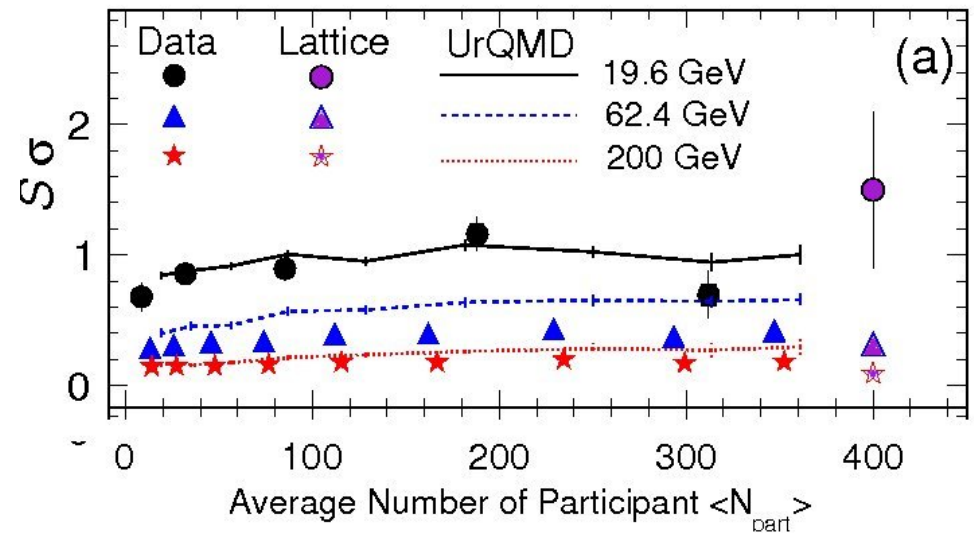
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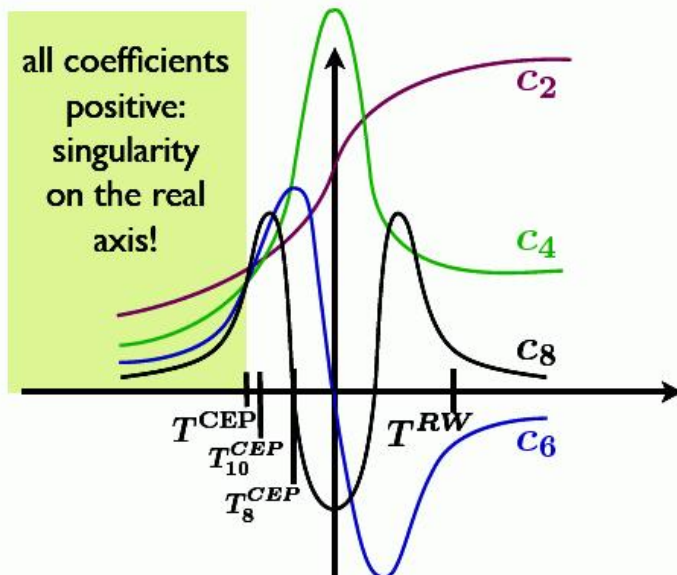


So far no signs of a critical point in the experimental results at CERN.
Will RHIC energy scan deliver it for us ? and/or Will it be FAIR ?



method for locating of the CEP:

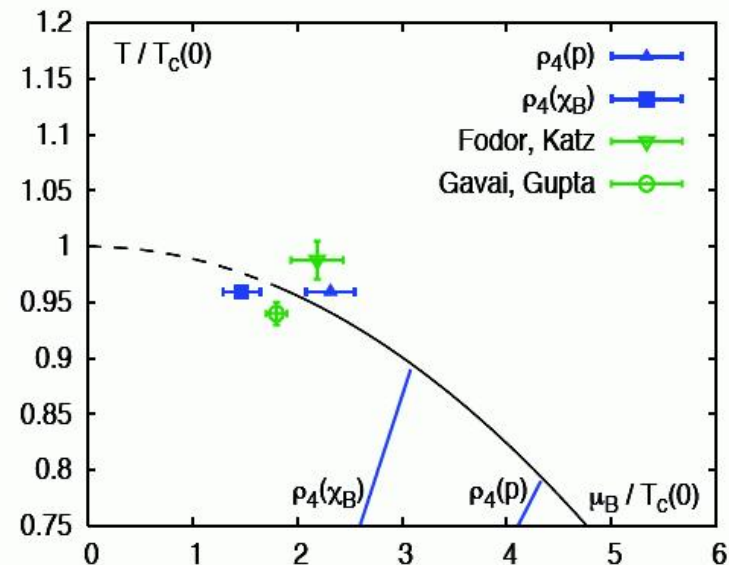
- determine largest temperature where all coefficients are positive $\rightarrow T^{CEP}$
- determine the radius of convergence at this temperature $\rightarrow \mu^{CEP}$



first non-trivial estimate of T^{CEP} by c_8
 second non-trivial estimate of T^{CEP} by c_{10}

$$p = c_0 + c_2 (\mu_B/T)^2 + c_4 (\mu_B/T)^4 + \dots$$

$$\chi_B = 2c_2 + 12c_4 (\mu_B/T)^2 + 30c_6 (\mu_B/T)^4 + \dots$$



$$\rho_n(p) = \sqrt{c_n/c_{n+2}}$$

$$\rho = \lim_{n \rightarrow \infty} \rho_n$$

(Ch. Schmidt FAIR Lattice QCD Days, Nov 23-24, 2009.)

Why Taylor series expansion?

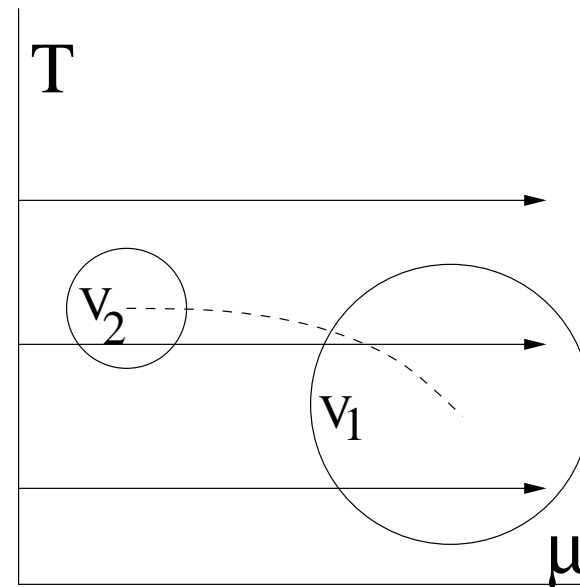
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We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.