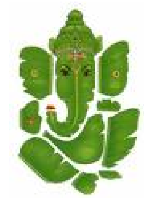
Happy Birthday, Guys!



प्रिय जाँ-पॉल व प्रिय लॅरी

जीवेत् शरदः शतम् !

Dear Jean-Paul, and Dear Larry,
May You Live a Hundred Autumns !

One more step towards the QCD Critical Point

Rajiv V. Gavai
T. I. F. R., Mumbai, India

Introduction : QCD Phase diagram

♠ A fundamental aspect – Critical Point in T-μ_B plane;

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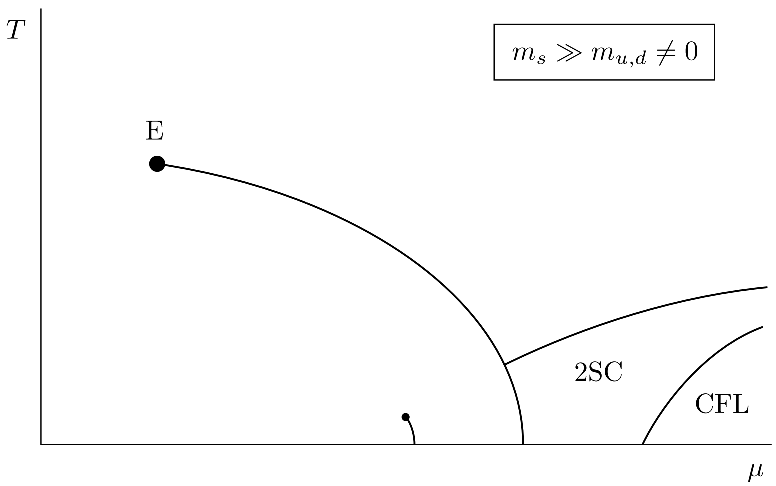

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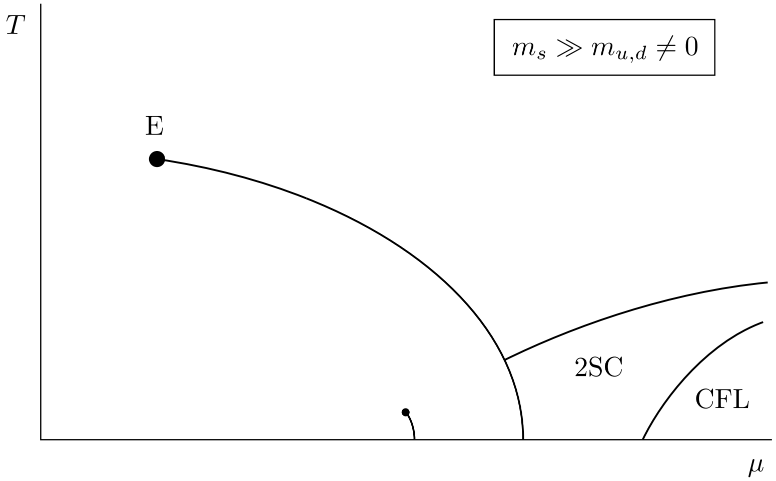

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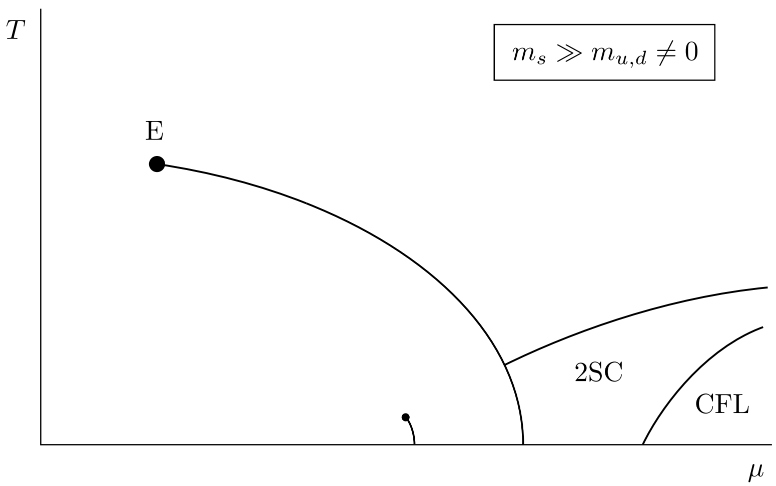

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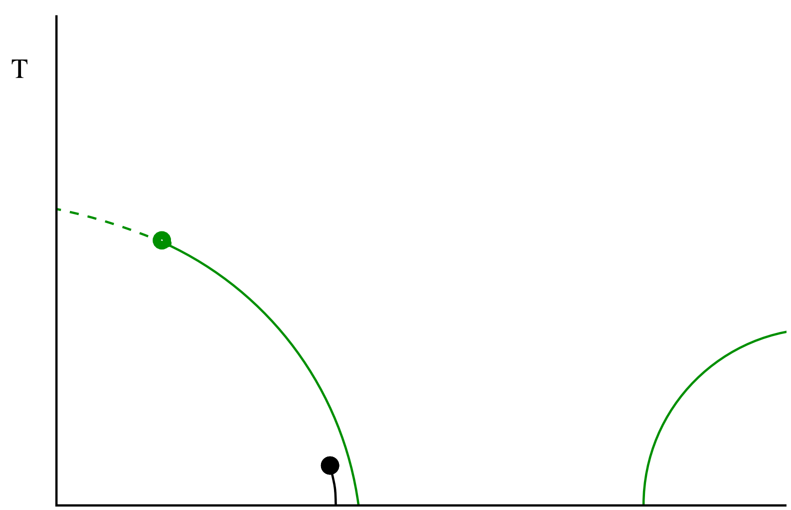

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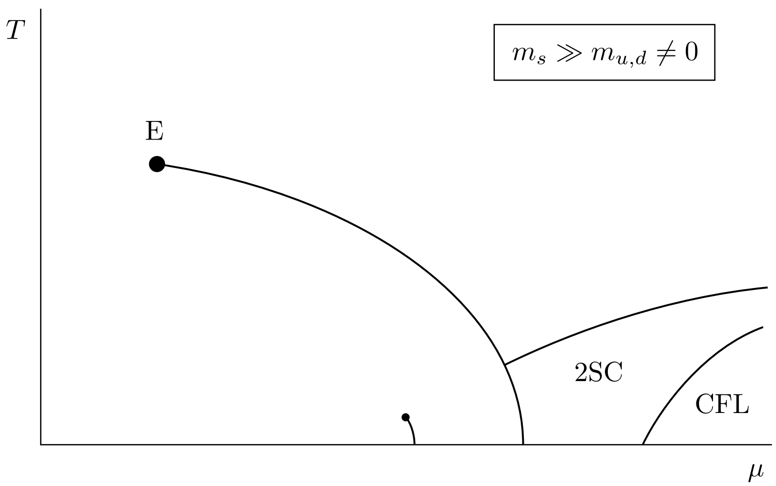

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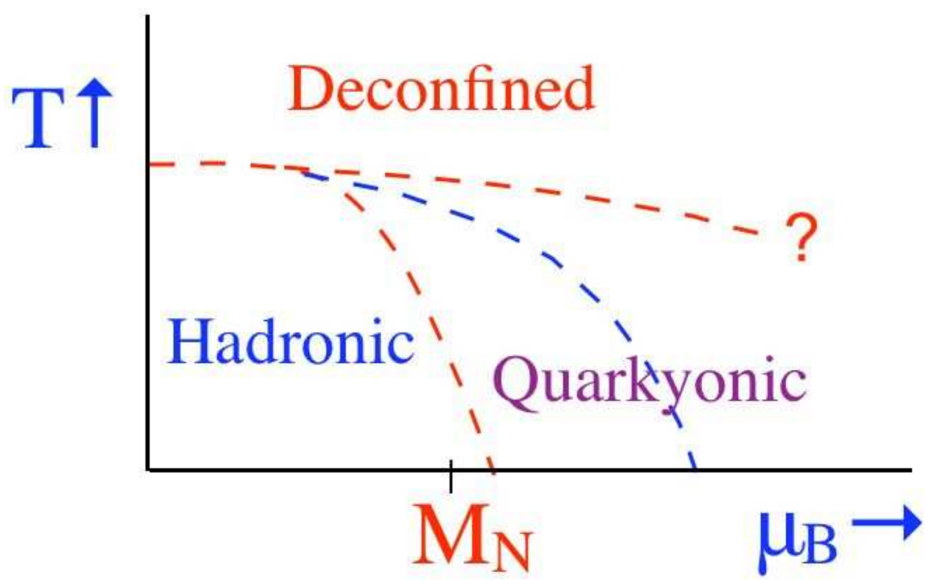
... but could, however, be ... McLerran-Pisarski 2007



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- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics.

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- Mostly staggered quarks used in these simulations.

  - exact chiral symmetry for all lattice spacings.
  - Broken flavour and spin symmetry on lattice
  - $N_f = 4$ or multiples straightforward but need "rooting trick" for other N_f $\Longrightarrow N_f = 2$ simulations may be fine in $a \rightarrow 0$ limit but 3 or 2 +1 maybe problematic (Creutz, arXiv:0901.0150[hep-ph]).

- Domain Wall or Overlap Fermions better, in principle.
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  - *No* order parameter to explore the $T - \mu$ phase diagram.
  - Thorny technical issues : Non-Hermiticity, Valid for a limited range of μa, ...
- Staggered Fermions, howsoever problem-ridden they may be, appear to be our best bet so far.
- Graphene-inspired fermions (Creutz JHEP 2008, Boriçi PRD 2008) could be better ?

The Phase Problem for $\mu \neq 0$

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det} \ M(m_f, \mu_f) \quad ,$$

and the thermal expectation value of an observable $\mathcal{O}$ is

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However, det M is a complex number for any $\mu \neq 0$: The Phase/sign problem.

Lattice Approaches

Several Approaches proposed in the past : None as satisfactory as the usual $T \neq 0$ simulations. Still scope for a good/great idea !

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- Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
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- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

Why Taylor series expansion?

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Our Strategy : i) Study volume dependence at several T to bracket the critical region and then to ii) track its change as a function of volume.

Taylor Expansion

Canonical definitions yield various number densities and susceptibilities :

$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}$ and $\chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$.

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!} \left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!} \left(\frac{\mu_d}{T}\right)^{n_d} \qquad (1)$$

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Chiral-symmetry order parameter, the lattice, and nucleosynthesis

Larry McLerran
Fermi National Laboratory, P. O. Box 500, Batavia, Illinois 60510
(Received 11 September 1987)

I discuss an order parameter for the chiral-symmetry restoration phase transition which may be useful in computations of big-bang nucleosynthesis, a phenomenon which requires a finite baryon-number density. This parameter is, strictly speaking, an order parameter in the large-N limit, and distinguishes between a parity-doubled and a massless-fermion realization of chiral-symmetry restoration. This order parameter may be evaluated at a zero net baryon-number density at finite temperature, and is useful as long as the baryon chemical potential μ is much less than the temperature T.

Recent work on the hadronization phase transition in cosmology has shown that if there is a first-order chiral transition then it may be possible that this transition can affect nucleosynthesis.[1-3] A proper treatment of this problem shows that it may be possible to quantitatively explain the abundances of ^{2}H, ^{3}He, and ^{4}He for a variety of values of Ω, unlike the case for a conventional computation of element abundances. Here Ω is the fraction of matter compared to the amount needed for closure. These

$\mu/T \sim 10^{-9}$. If we define the net baryon-number density to be ρ_B^{CS} in the chiral-symmetric phase, ρ_B^{CB} in the symmetry-broken phase, then the quantity of interest is

$$r = \rho_B^{CS}/\rho_B^{CB} . \qquad (1)$$

Although the numerator and denominator of this expression both depend upon μ, the ratio r is finite in the limit μ approaches zero.

We can understand the physics of the parameter r using

Physical Review D36, 3291 (1987).

20 December 2001

PHYSICS LETTERS B

Physics Letters B 523 (2001) 143–150

www.elsevier.com/locate/npe

Quark number susceptibilities from HTL-resummed thermodynamics

J.-P. Blaizot [a], E. Iancu [a], A. Rebhan [b]

[a] *Service de Physique Théorique, CE Saclay, F-91191 Gif-sur-Yvette, France*

[b] *Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8-10/136, A-1040 Vienna, Austria*

Received 30 October 2001; accepted 31 October 2001

Editor: P.V. Landshoff

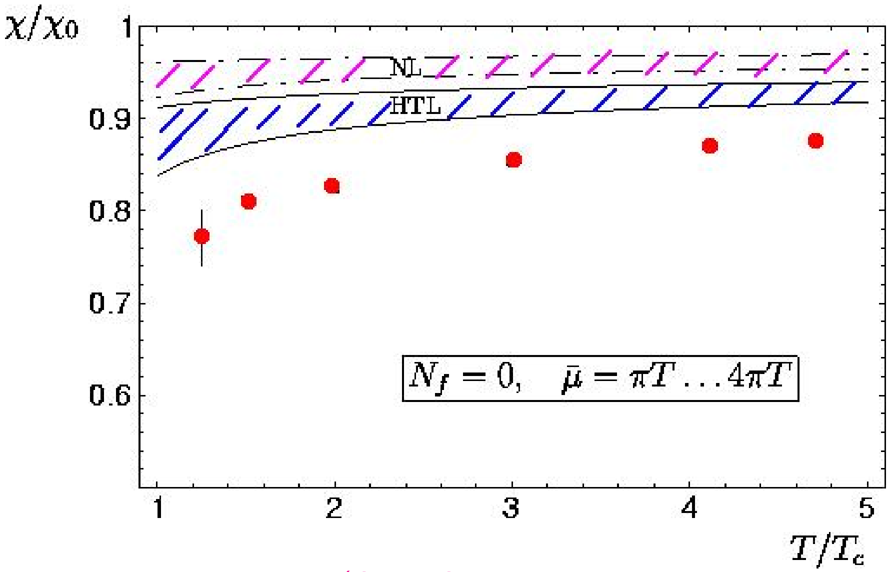
Abstract

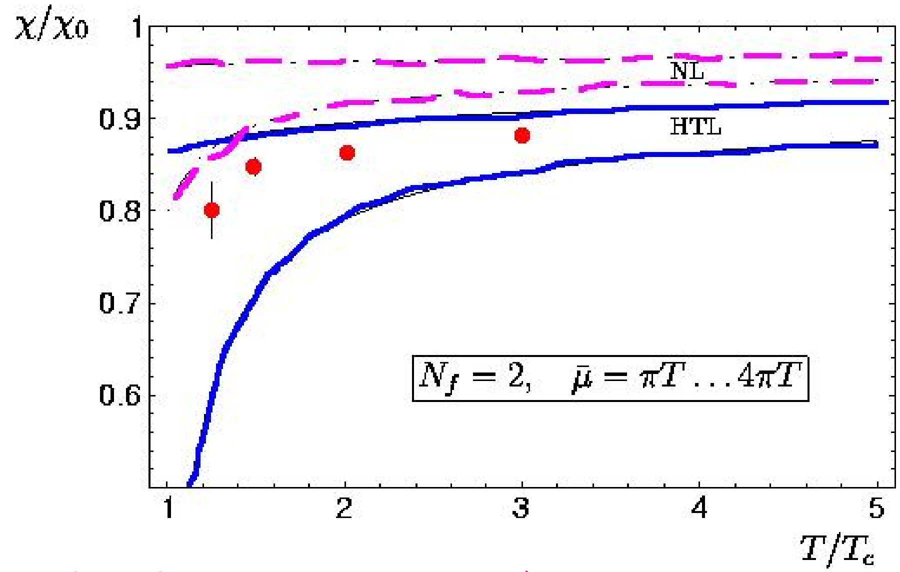
We compute analytically the diagonal quark number susceptibilities for a quark–gluon plasma at finite temperature and zero chemical potential, and compare with recent lattice results. The calculation uses the approximately self-consistent resummation of hard thermal and dense loops that we have developed previously. For temperatures between 1.5 to $5T_c$, our results follow the same trend as the lattice data, but exceed them in magnitude by about 5–10%. We also compute the lowest order contribution, of order $\alpha_s^3 \log(1/\alpha_s)$, to the off-diagonal susceptibility. This contribution, which is not a part of our self-consistent calculation, is numerically small, but not small enough to be compatible with a recent lattice simulation.

Resumed Perturbation Theory

Hard Thermal Loop & Self-consistent resummation give :

(Blaizot, Iancu & Rebhan, PLB '01)





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Our results (•) above were for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Continuum limit from Larger N_t and improved actions agrees better.
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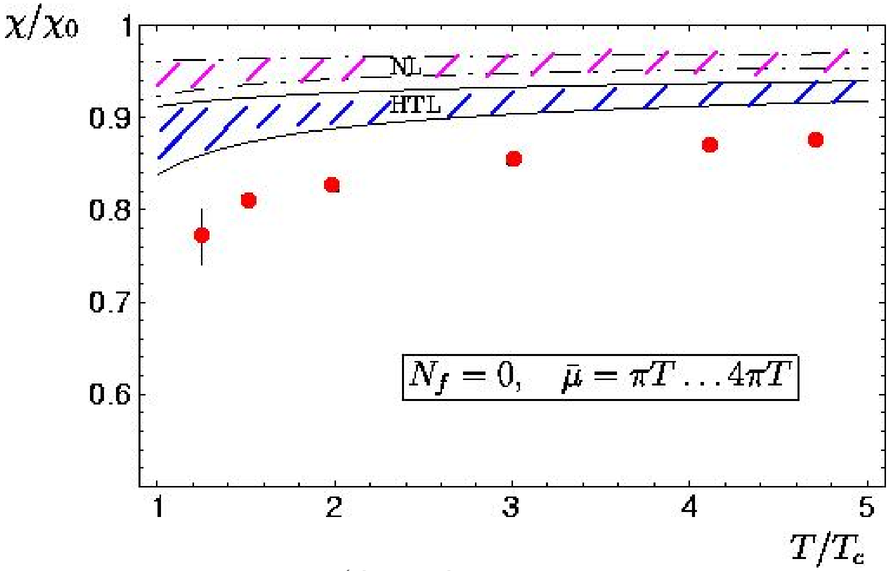

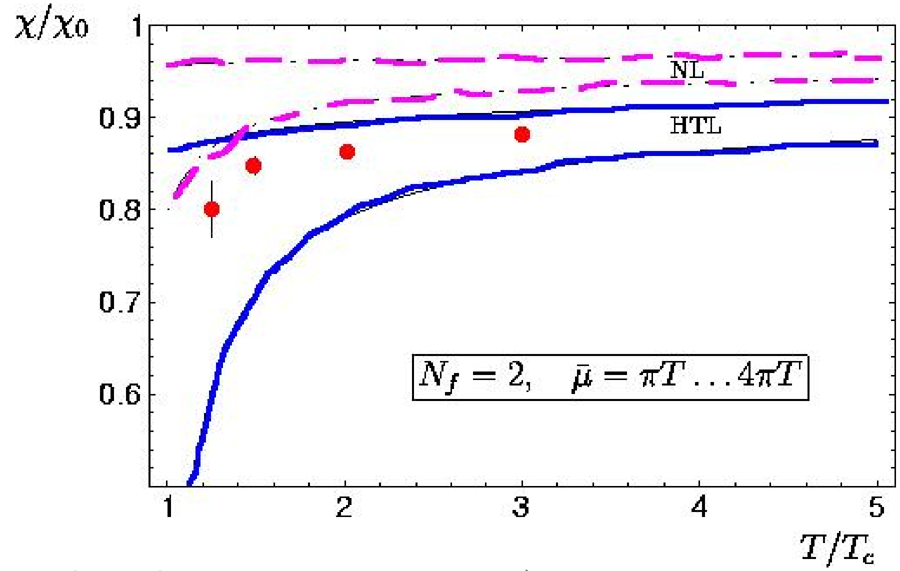

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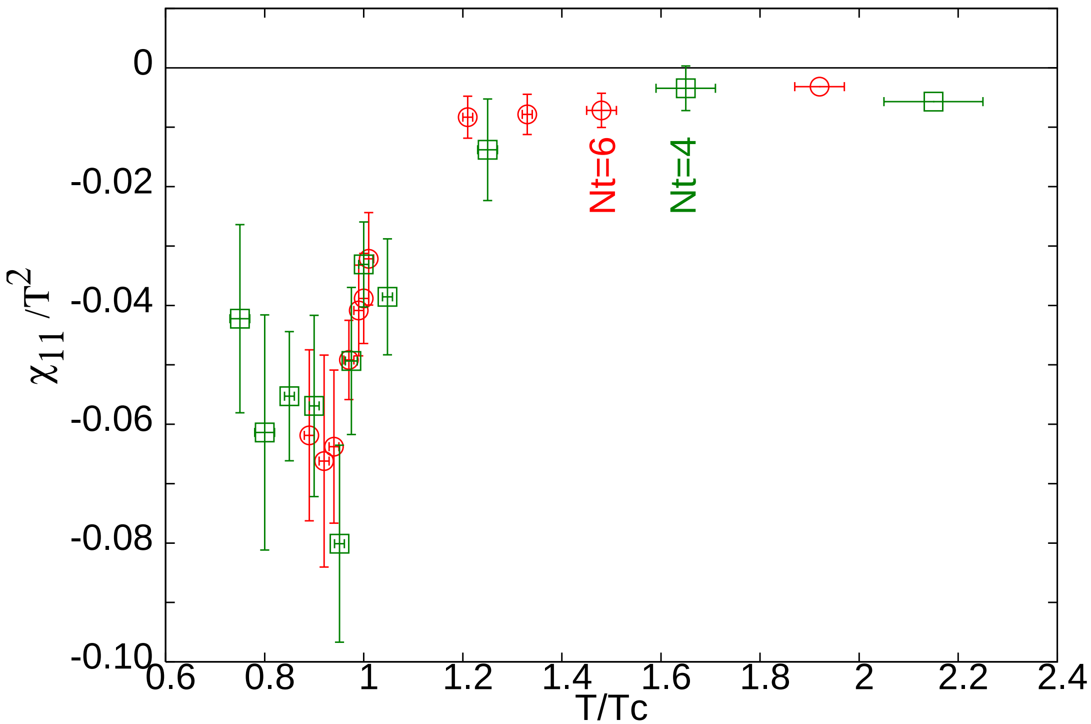
χ_{ud}

Our $N_t = 4$ & 6 agree for $\chi_{ud} \Rightarrow$ Small lattice artifact effects.
Measure of the seriousness of sign problem.
Blaizot-Iancu-Rebhan result : $\chi_{ud} = -\frac{10}{9\pi^3}\alpha_s^3 \ln(1/\alpha_s)$.



Towards the Critical Point

- From the expansion above, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence can be obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}}\right)^{1/n}$. We use both the definitions and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- In the window of positive coefficients, we locate the critical point by looking for the independence of our estimates of the order n and the method.

- We further check for the finite size effects : Estimates of radius of convergence increase with order for small volumes, becoming flat on our largest volume.

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How Do We Do This Expansion?

CRAY X1 of I L G T I , T I F R, Mumbai

Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\rho/T_c = 5.4 \pm 0.2$ and $m_\pi/m_\rho = 0.31 \pm 0.01$ (MILC)
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
- Lattice used : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, arXiv:0806.2233, PRD in press). Needed to determine β_c. Our result ($\beta_c = 5.425(5)$) well bracketed by MILC for $m/T_c = 0.075$ and 0.15.

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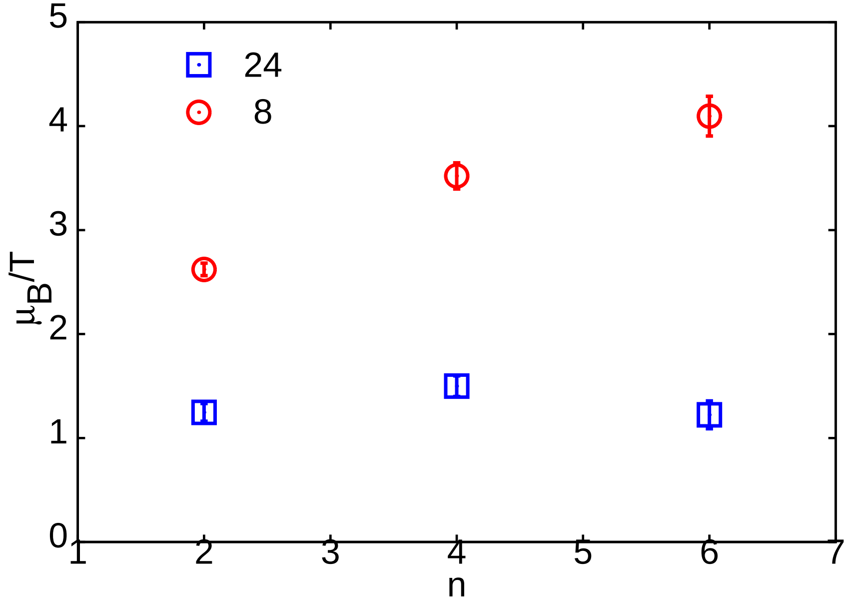
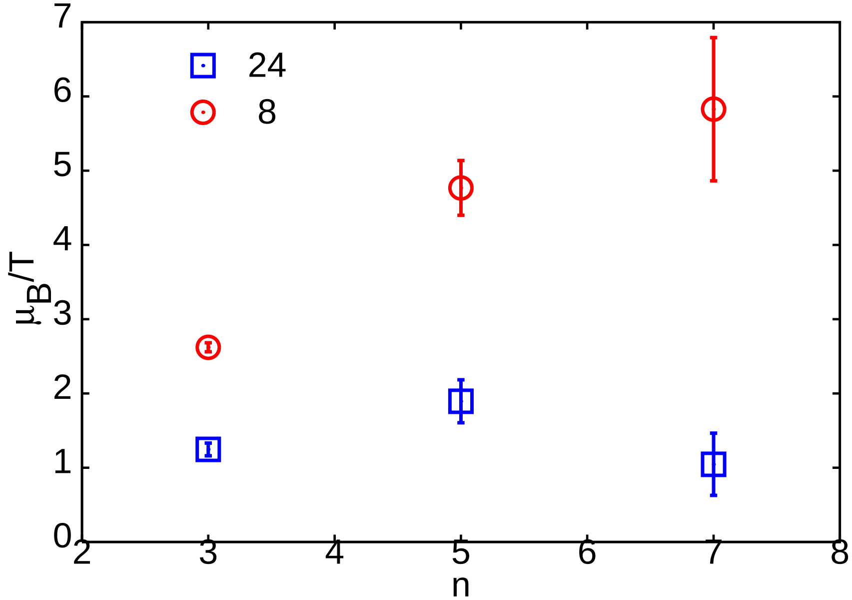
- New Simulations made at $T/T_c =$ 0.89(1), 0.92(1), 0.94(1), 0.97(1), 0.99 (1) 1.00(1), 1.21(1), 1.33(1), 1.48(3) and 1.92(5)
- Typical stat. 50-200 in max autocorrelation units.

$N_t = 4$ (Gavai & Gupta PRD 2005)





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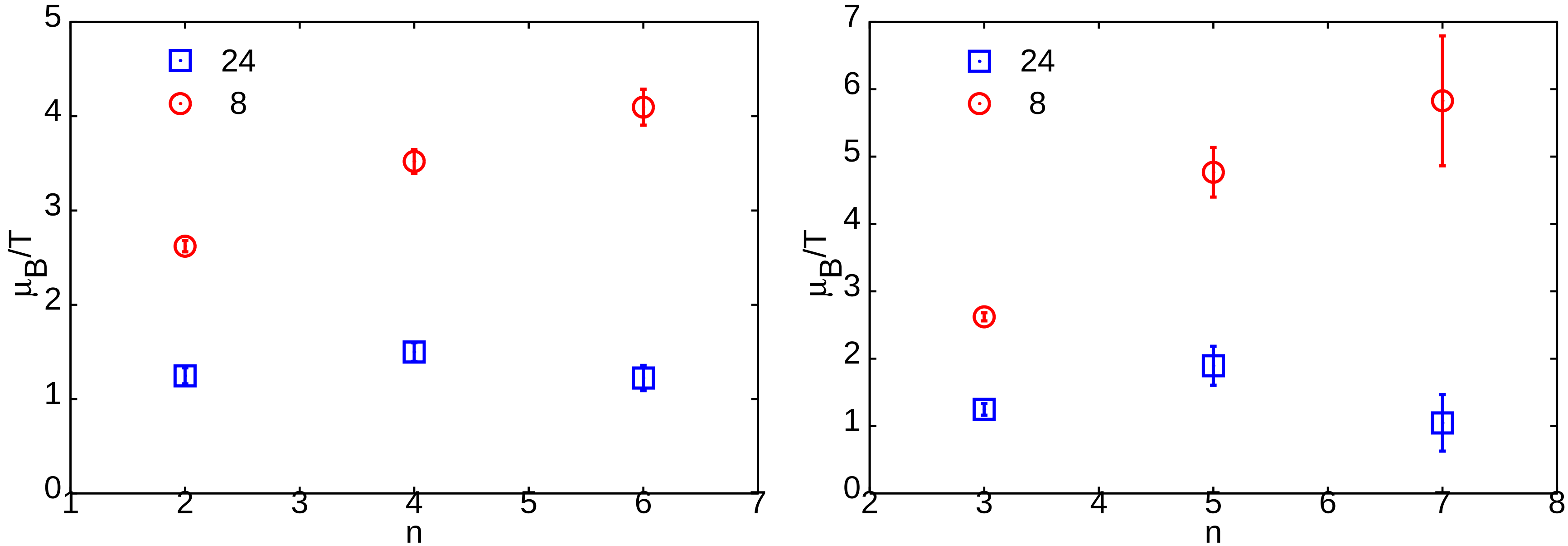

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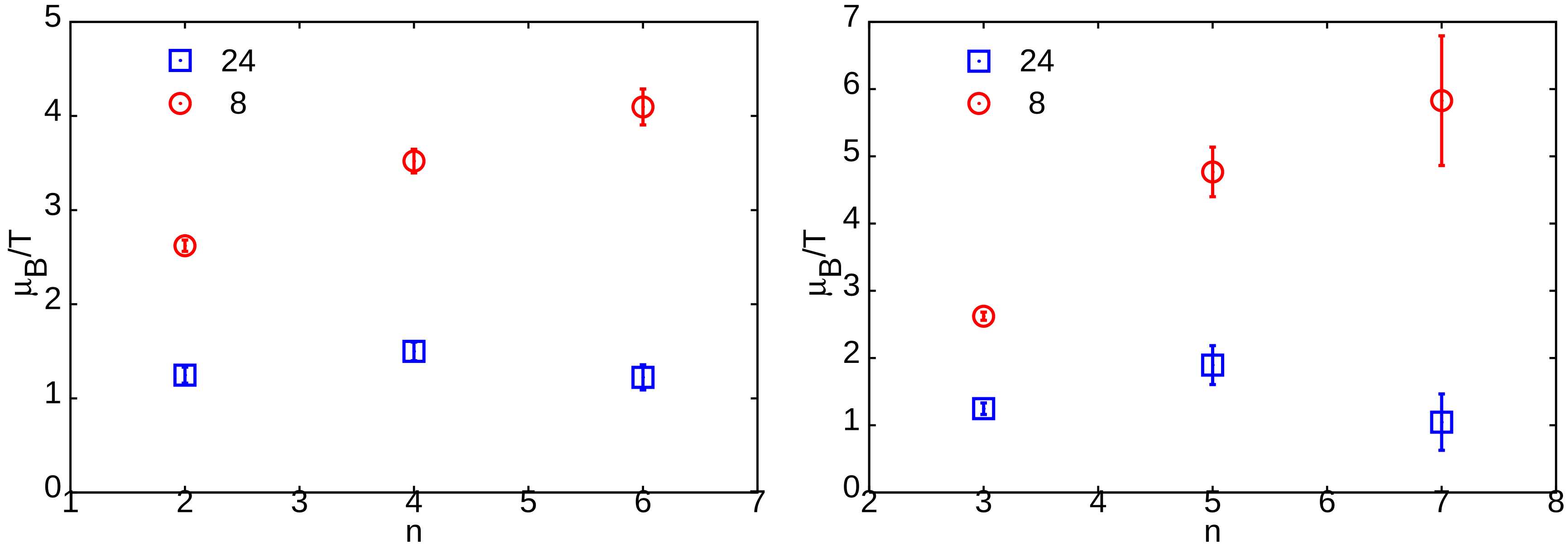

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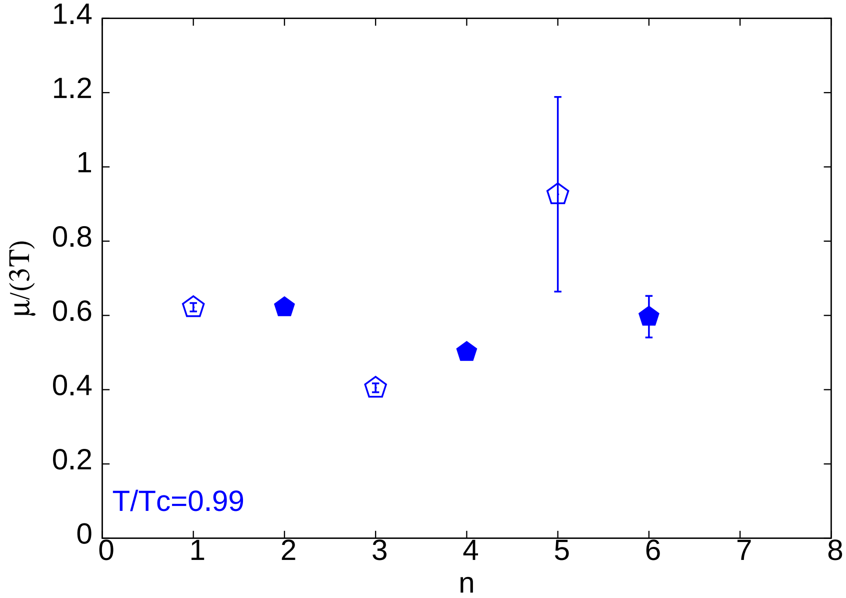
- Our estimate consistent with Fodor & Katz (2002) [$m_\pi/m_\rho = 0.31$ and $N_s m_\pi \sim$ 3-4].
- Strong finite size effects for small N_s. A strong change around $N_s m_\pi \sim 6$.

$N_t = 6$ **(Gavai & Gupta PRD 2008)**



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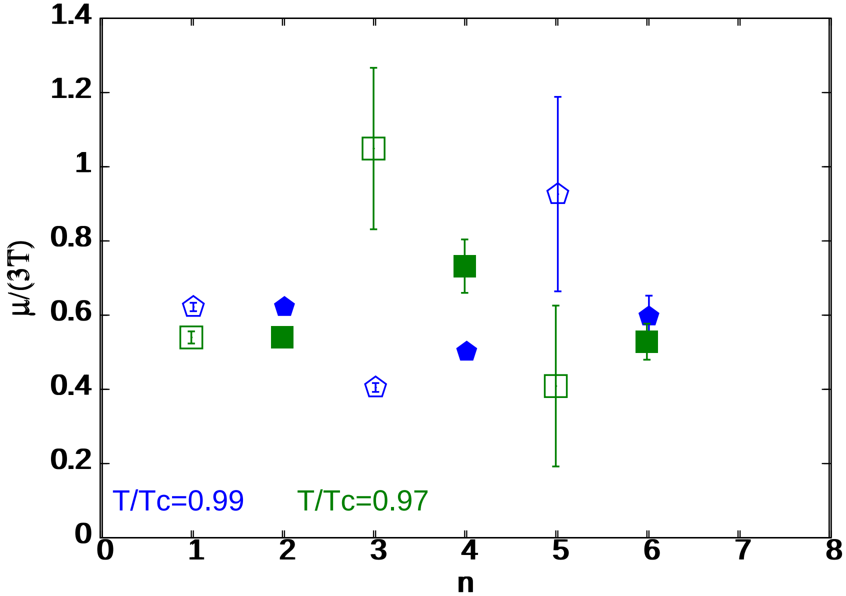

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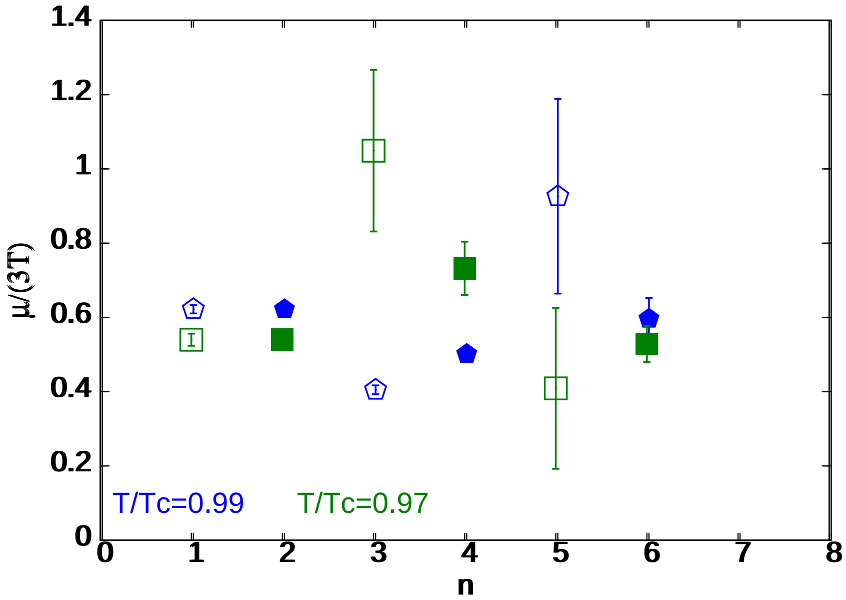

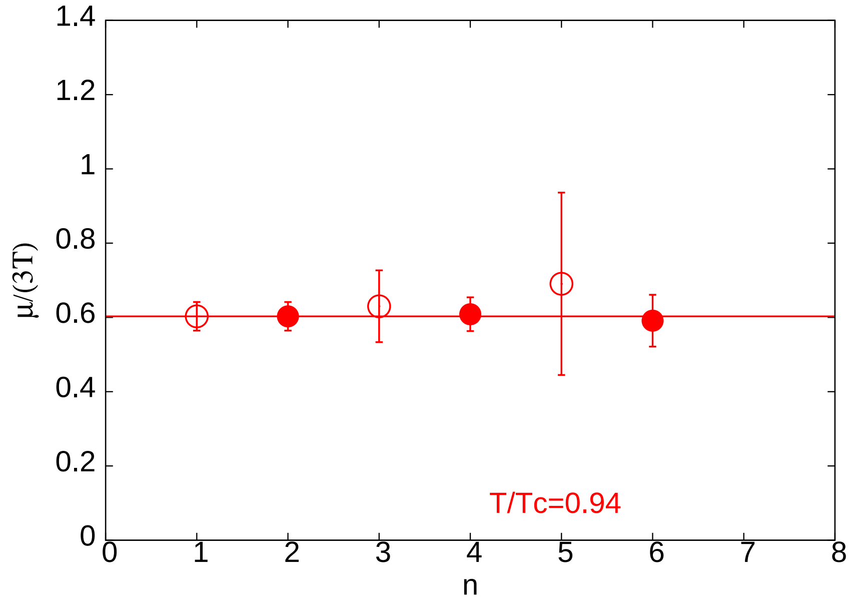

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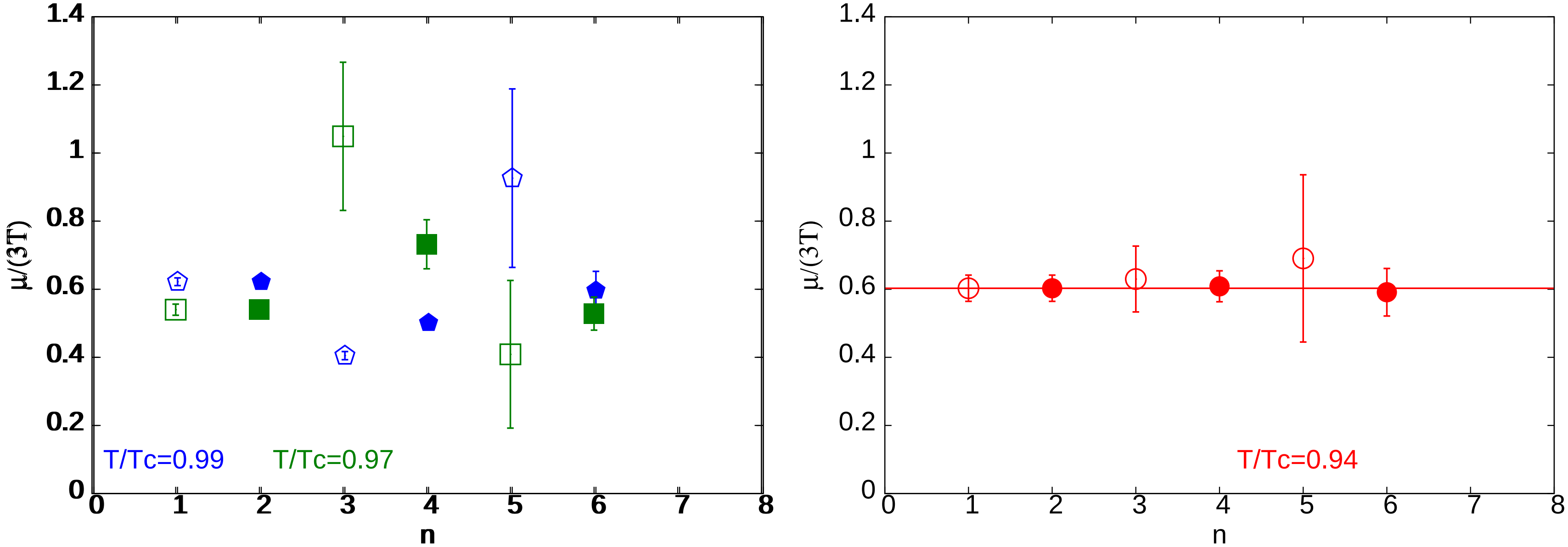
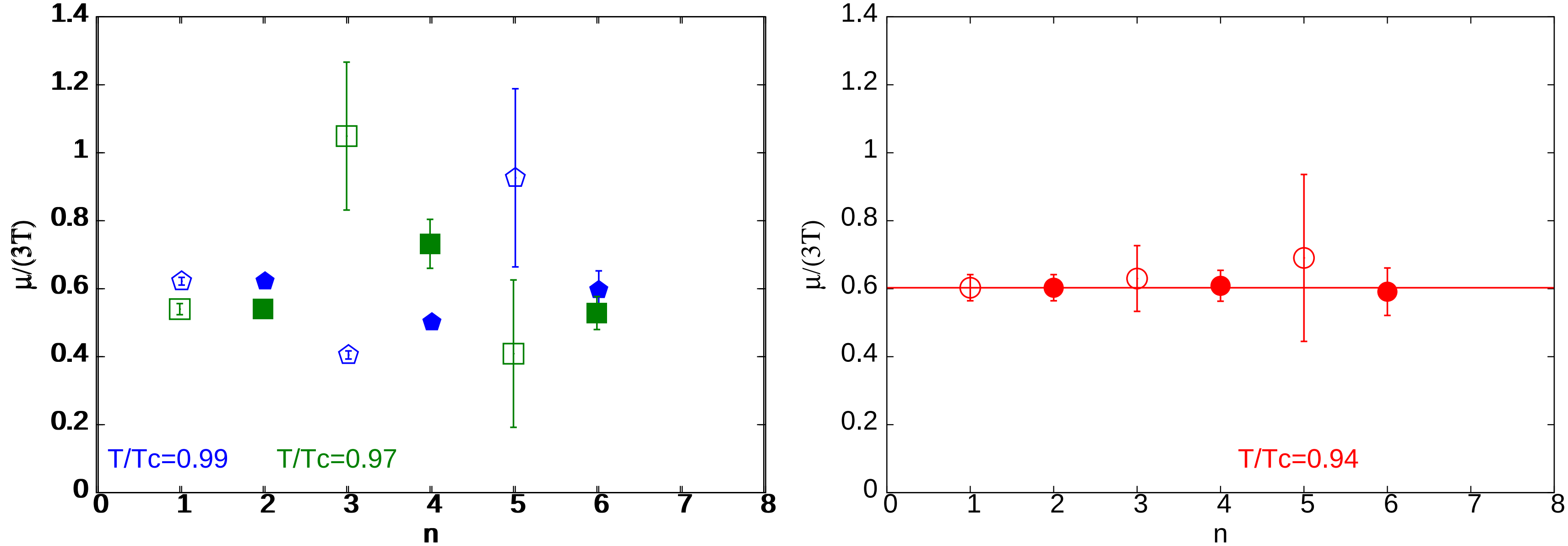


- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier result on the coarser lattice for same volume was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume limit brought it down to 1.1(1). Still to be done for $N_t = 6$.

$N_t = 6$ (Gavai & Gupta PRD 2008)



- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier result on the coarser lattice for same volume was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume limit brought it down to 1.1(1). Still to be done for $N_t = 6$.

- Critical point at $\mu_B/T \sim 1 - 2$.

Cross Check on μ^E/T^E

♠ Use Padé approximants for the series to estimate the radius of convergence.

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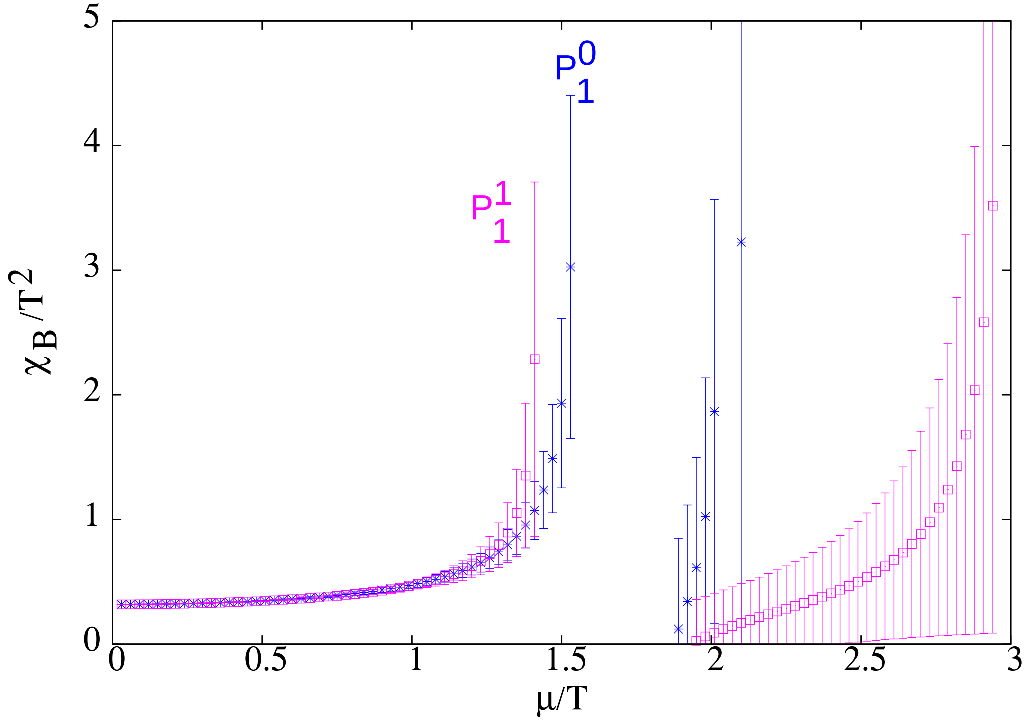

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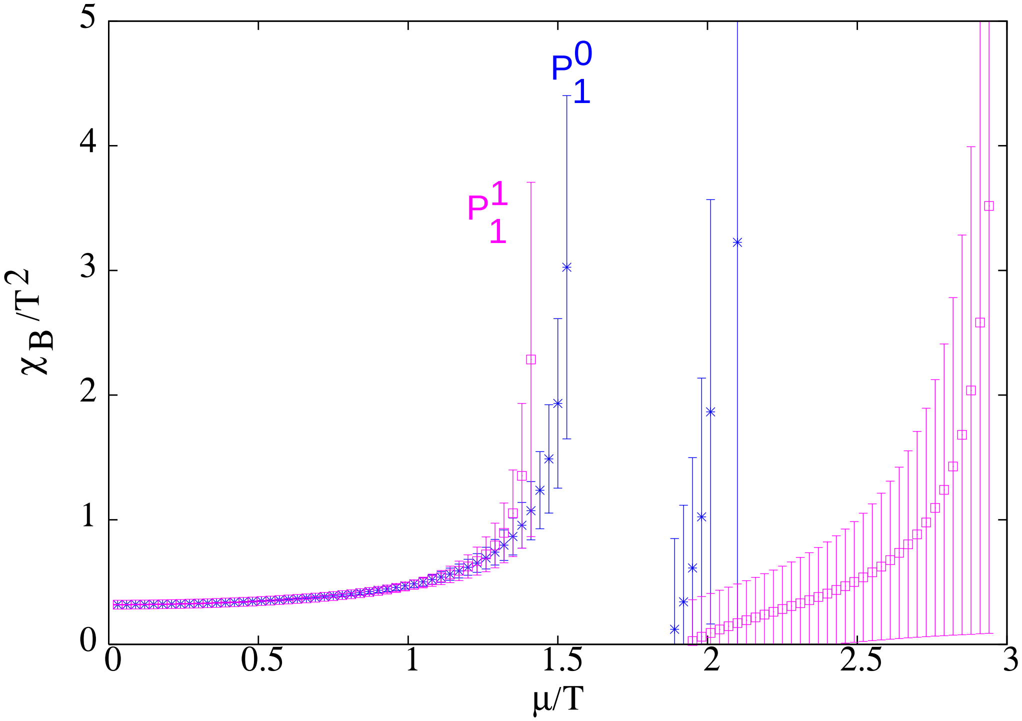
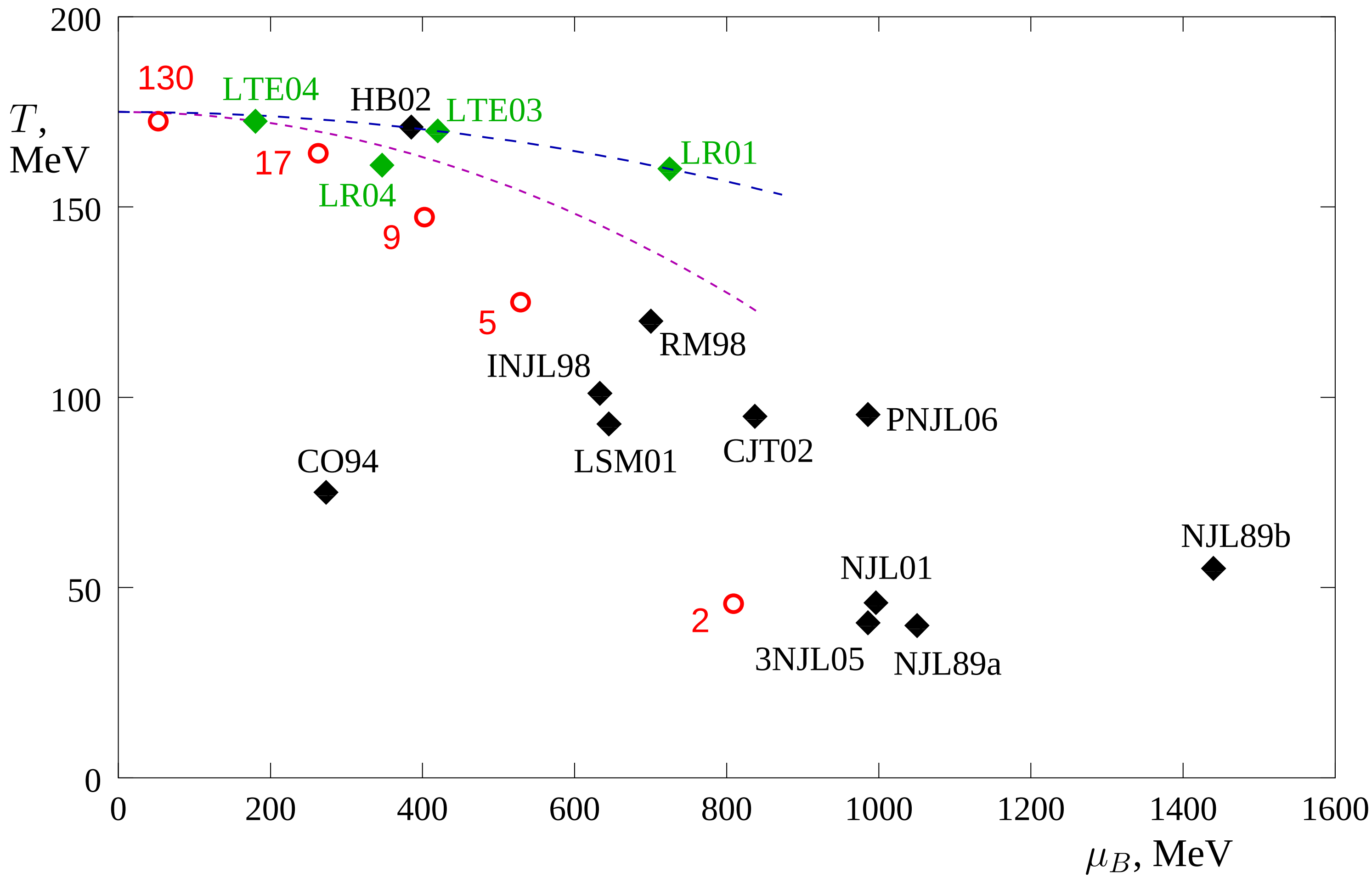


♡ Consistent Window with our other estimates.

Comparison with Other Results

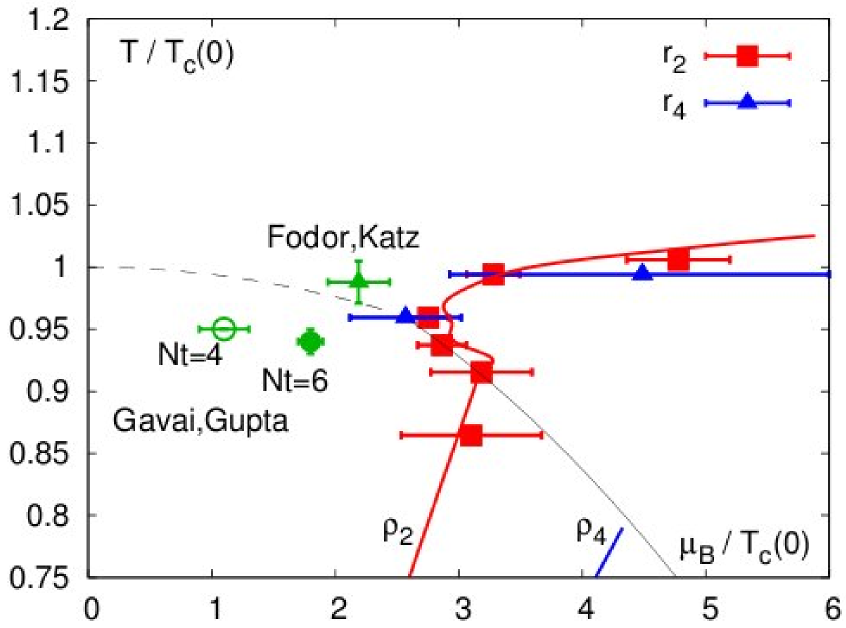


From M. Stephanov, Lattice 2007 Plenary.

Estimating $T_c(\mu_c)$ and μ_c/T

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)
- estimator for μ_c: $$\left(\frac{\mu_c(T)}{T_c(0)}\right)_n \equiv \rho_n = \frac{T}{T_c(0)}\sqrt{\frac{c_n}{c_{n+2}}}$$

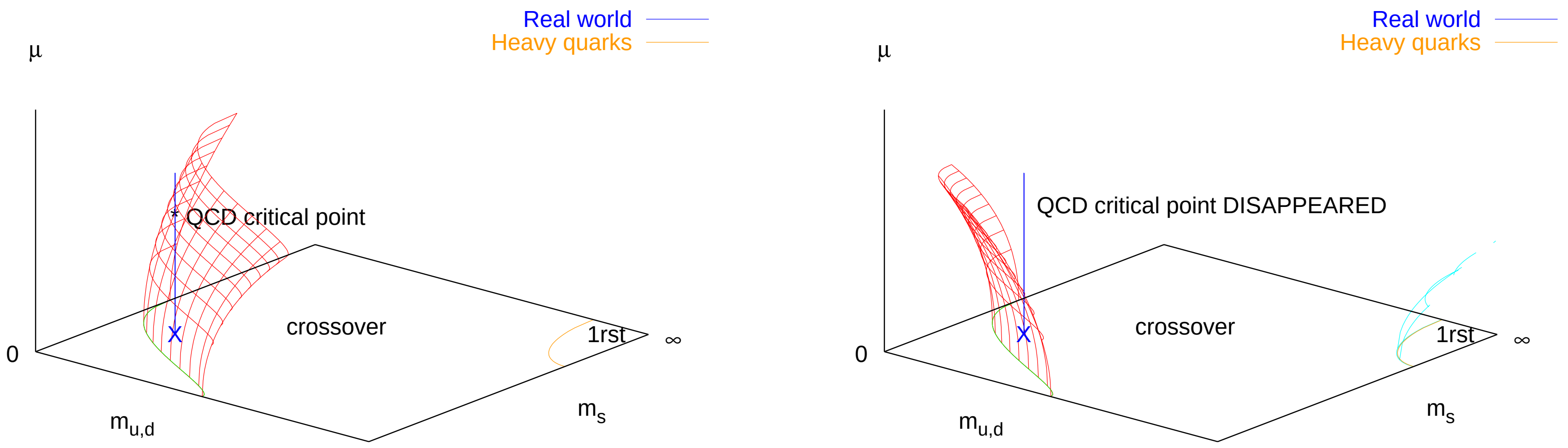


- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics

INT. Seattle 2008. F. Karsch – p. 20/3

Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811



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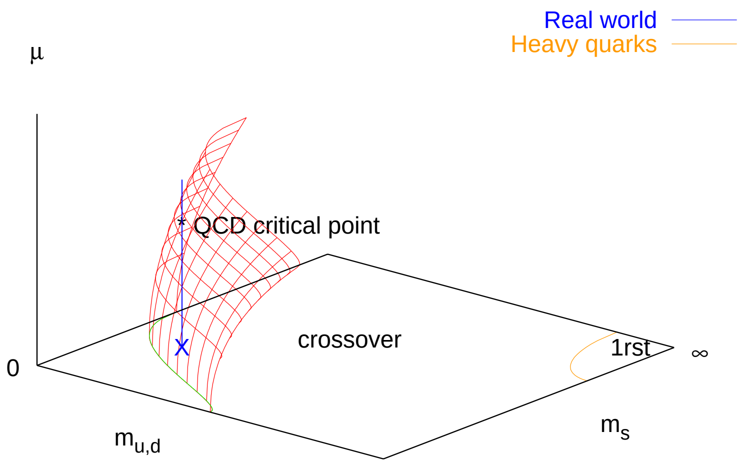

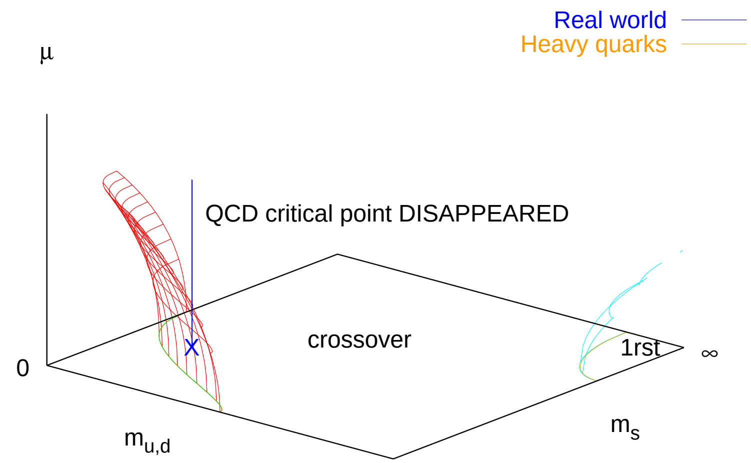

For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T_c}\right)^2 - 47(20)\left(\frac{\mu}{\pi T_c}\right)^4$, i.e., m_c shrinks with μ.

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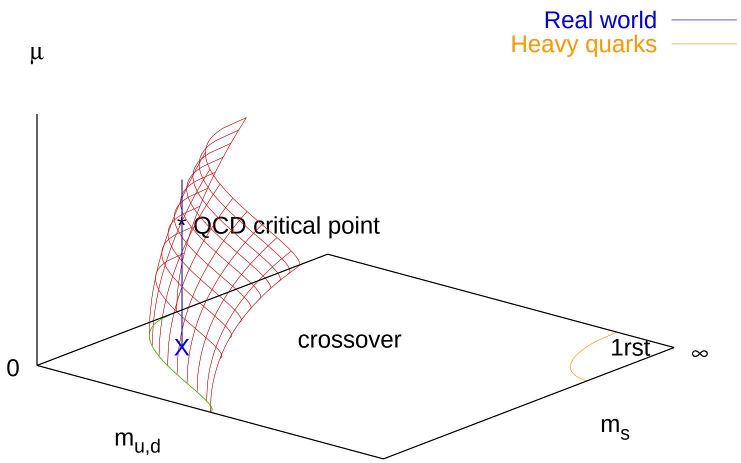

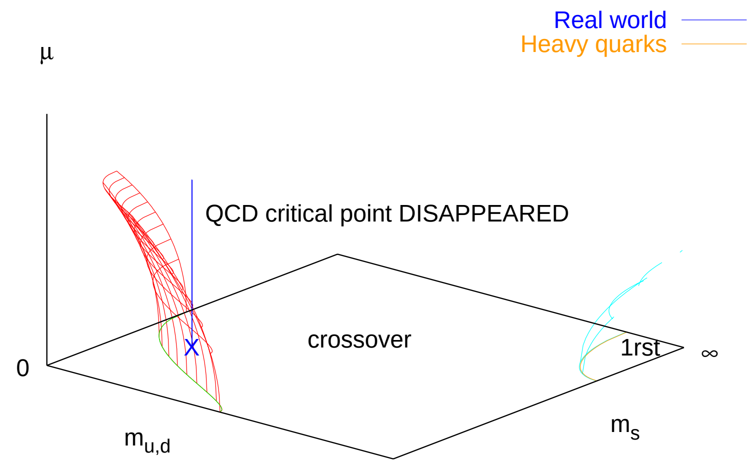

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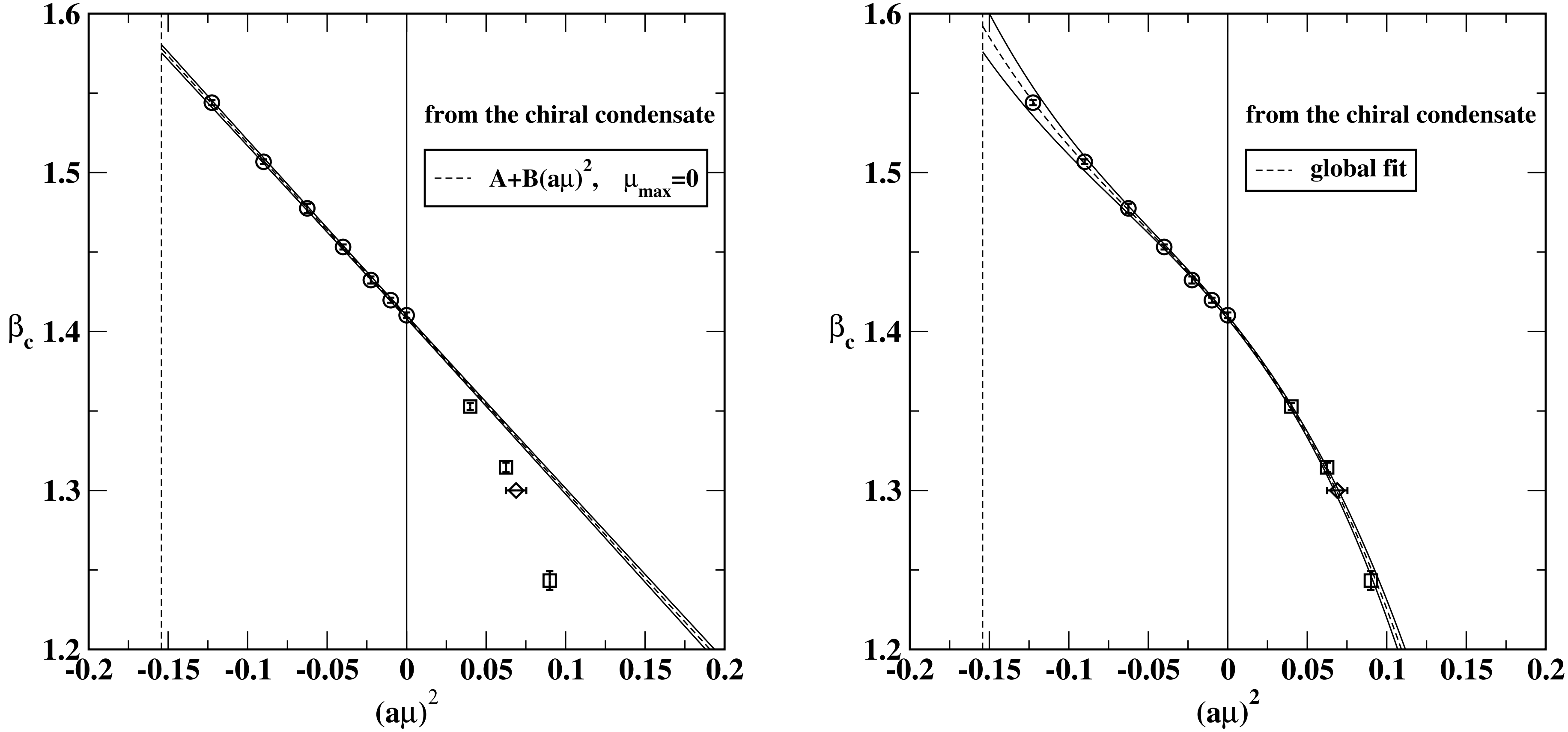
Problems : i) $N_f = 3 \rightarrow$ Anomaly and Staggered quarks ? ii) Known examples where shapes are different in real/imaginary μ,

"The Critical line from imaginary to real baryonic chemical potentials in two-color QCD", P. Cea, L. Cosmai, M. D'Elia, A. Papa, PR D77, 2008



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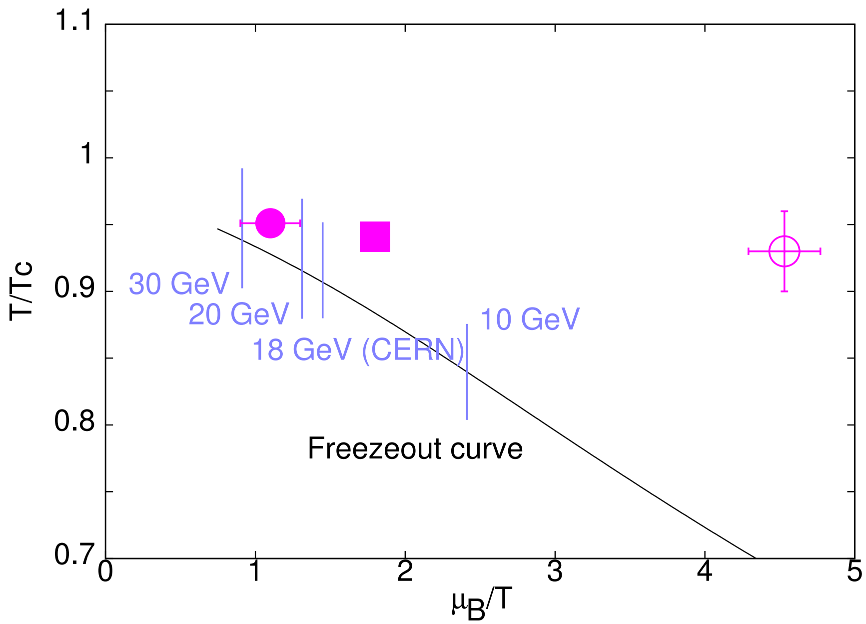

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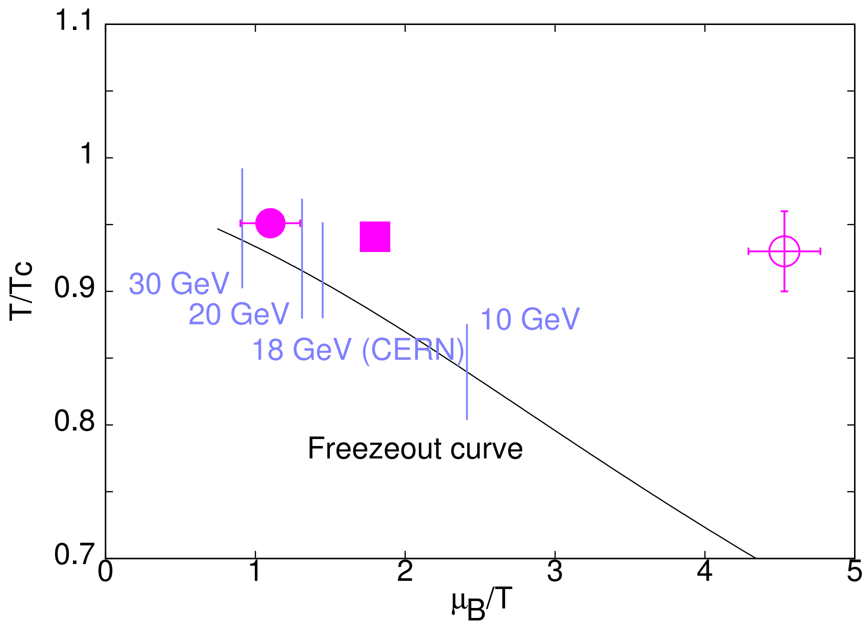
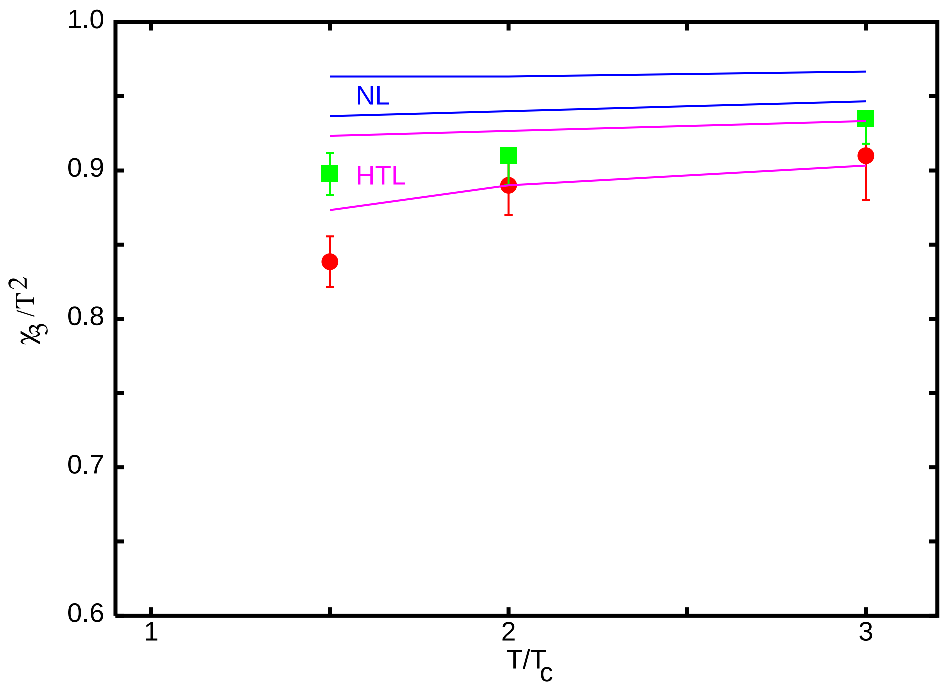


So far no signs of a critical point in the experimental results.
Will RHIC-scan deliver it for us ? or wait for CBM/FAIR ?

The continuum susceptibility vs. T (in quenched QCD) agrees better (Gavai & Gupta PRD 2002 & PRD 2003) :



Naik action (Squares) and Staggered action (circles)

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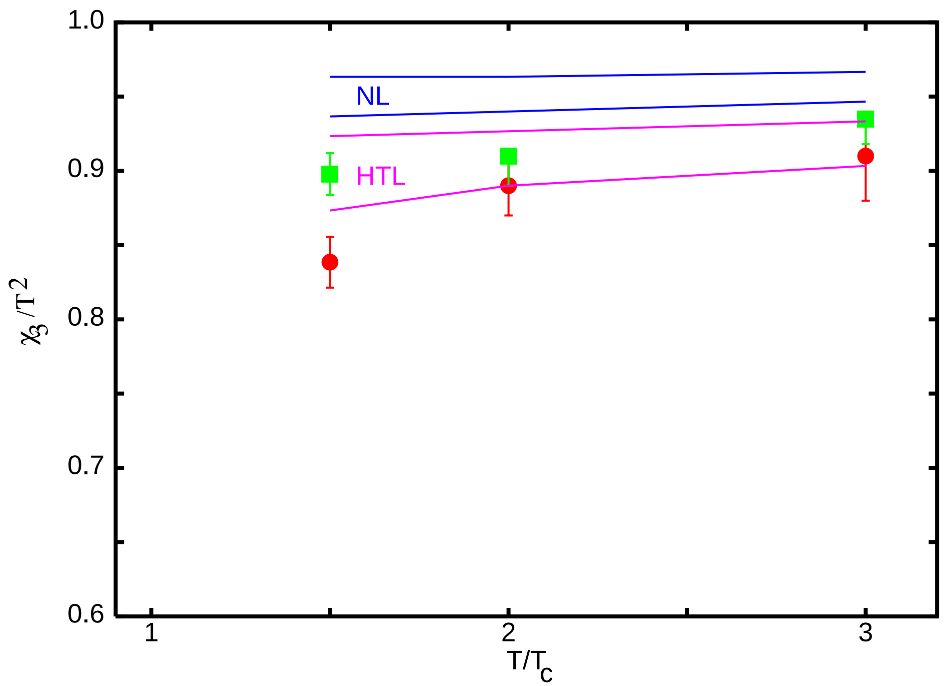

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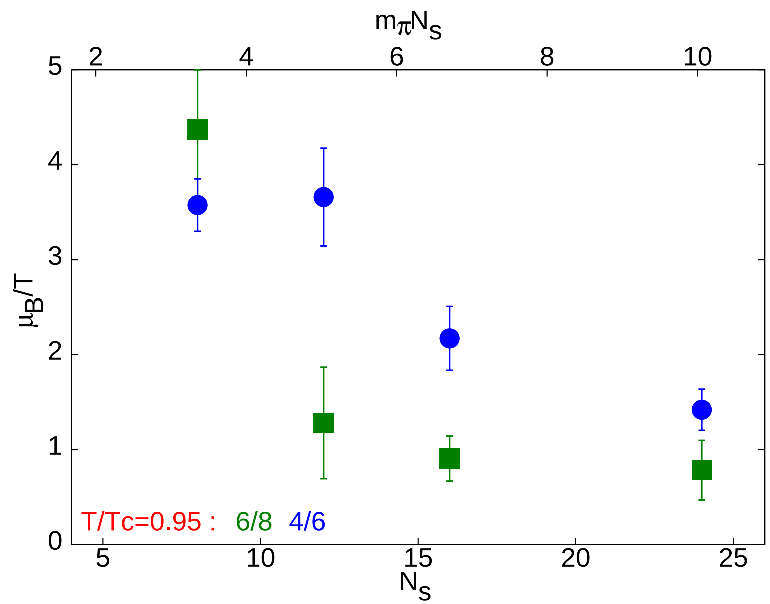
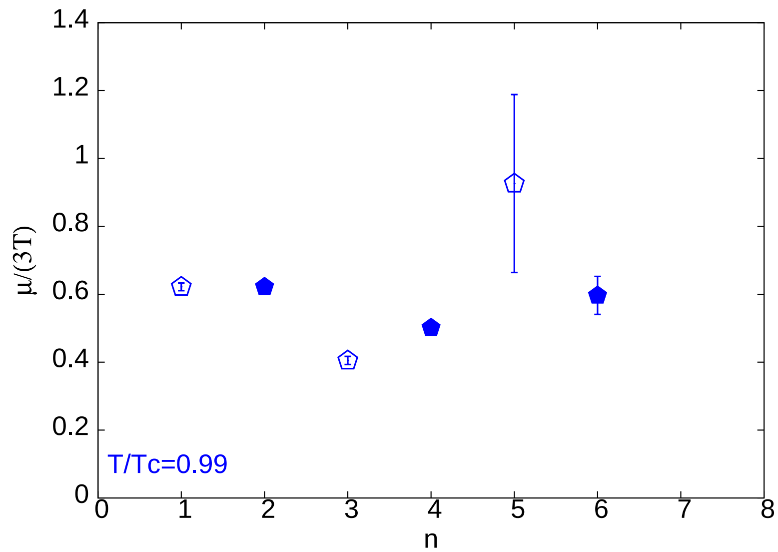
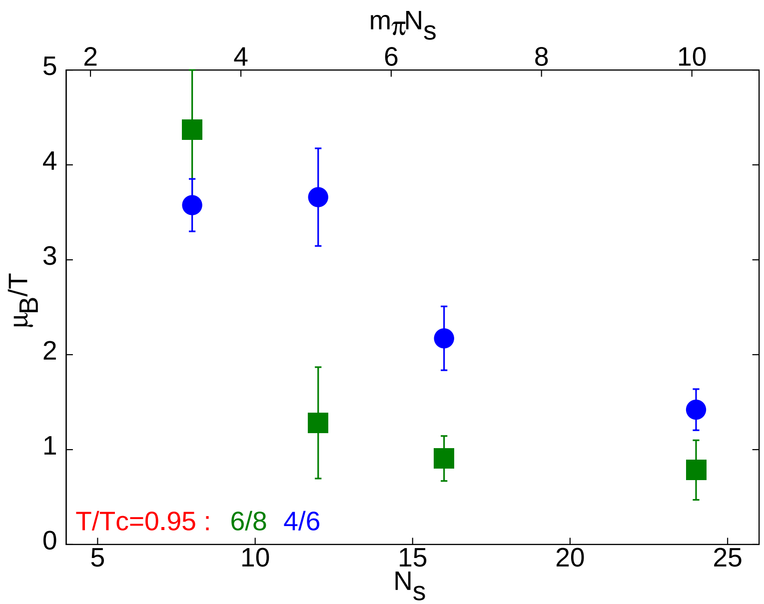


$m_\pi N_S$
μ_B/T
T/Tc=0.95 : 6/8 4/6
N_S



μ/(3T)
T/Tc=0.99
n



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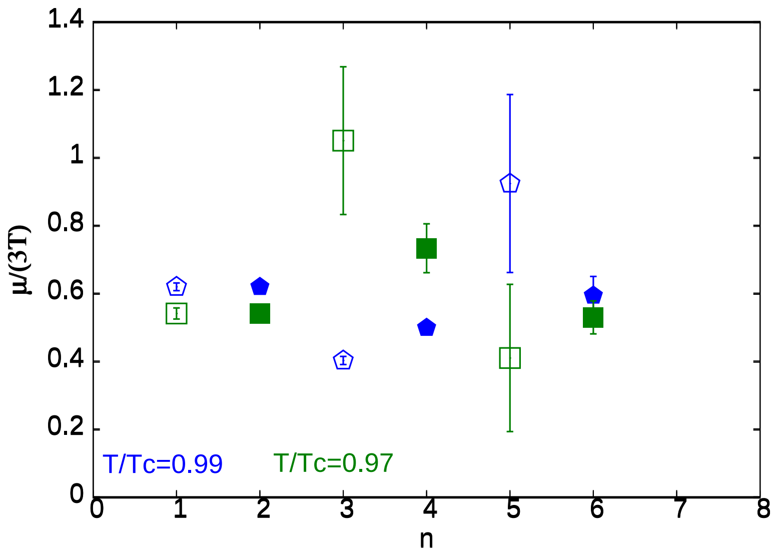
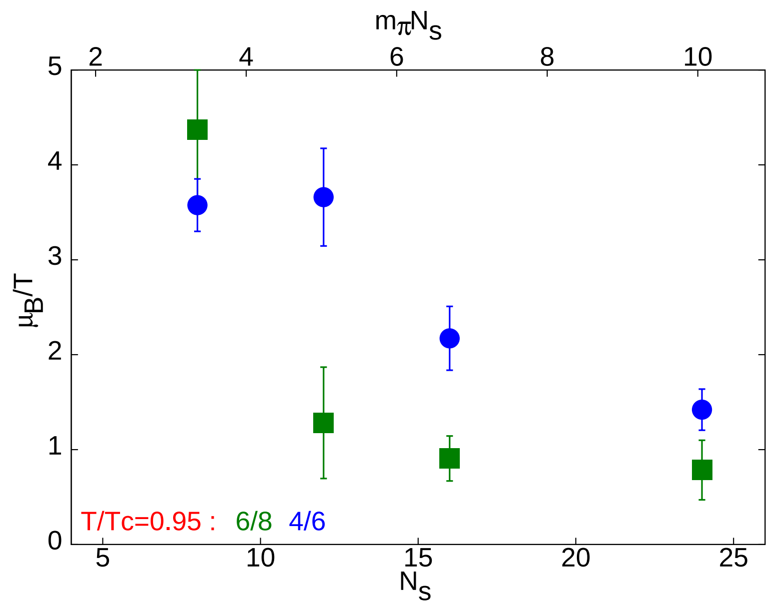
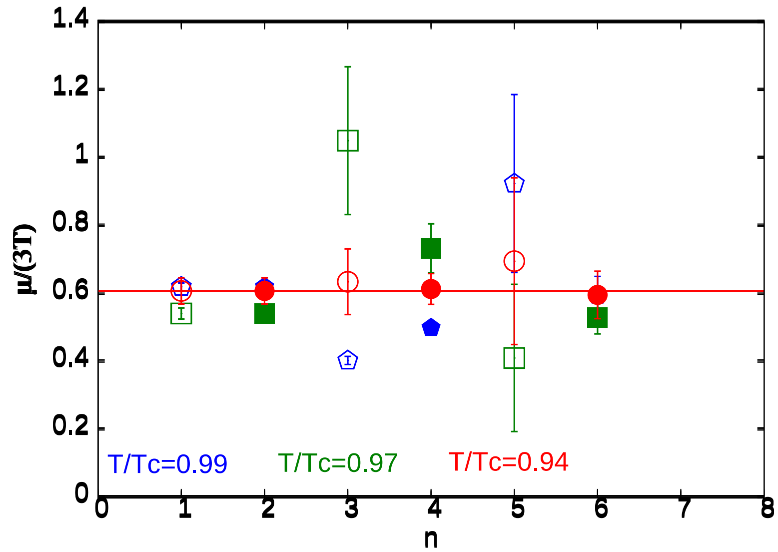


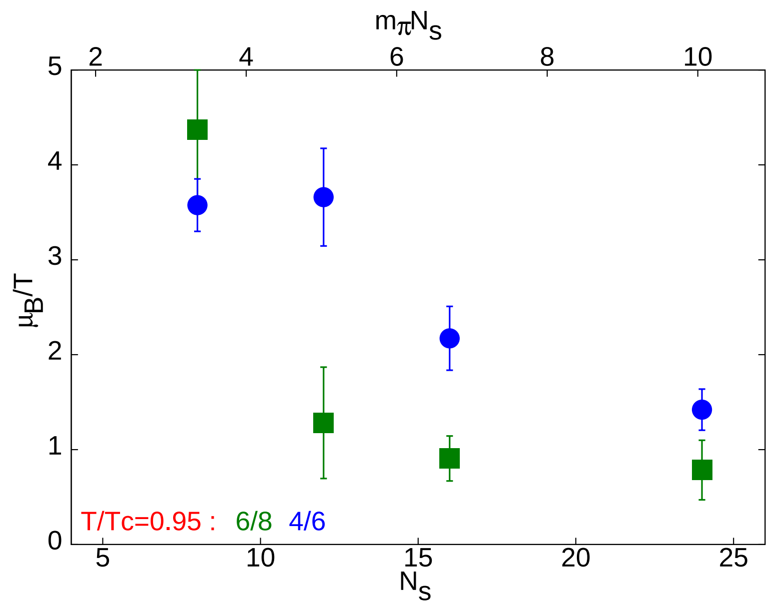
μ/(3T)
T/Tc=0.99
T/Tc=0.97
n

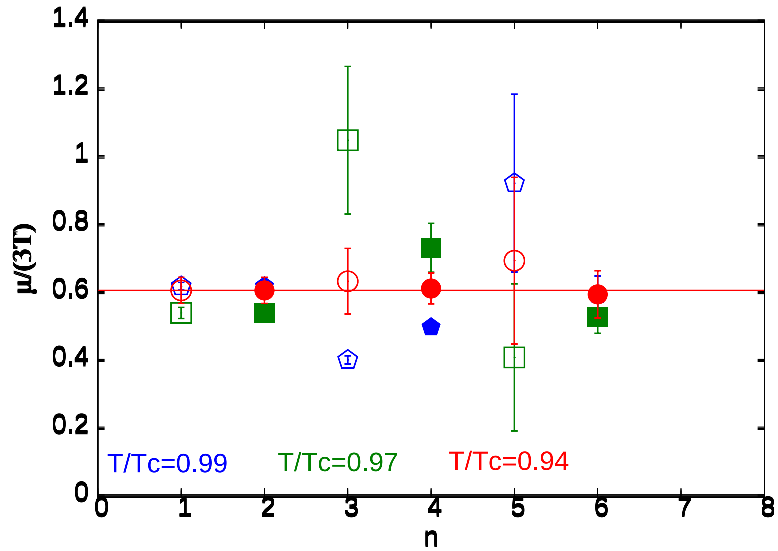


$m_\pi N_S$
2
4
6
8
10
5
4
3
2
1
0
μ_B/T
T/Tc=0.95 : 6/8 4/6
5
10
15
20
25
N_S



1.4
1.2
1
0.8
0.6
0.4
0.2
0
μ/(3T)
T/Tc=0.99
T/Tc=0.97
T/Tc=0.94
0
1
2
3
4
5
6
7
8
n





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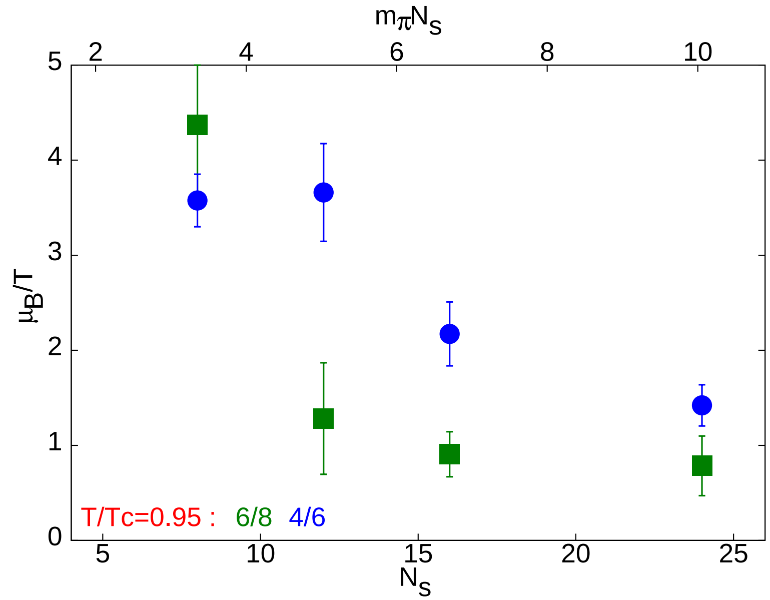

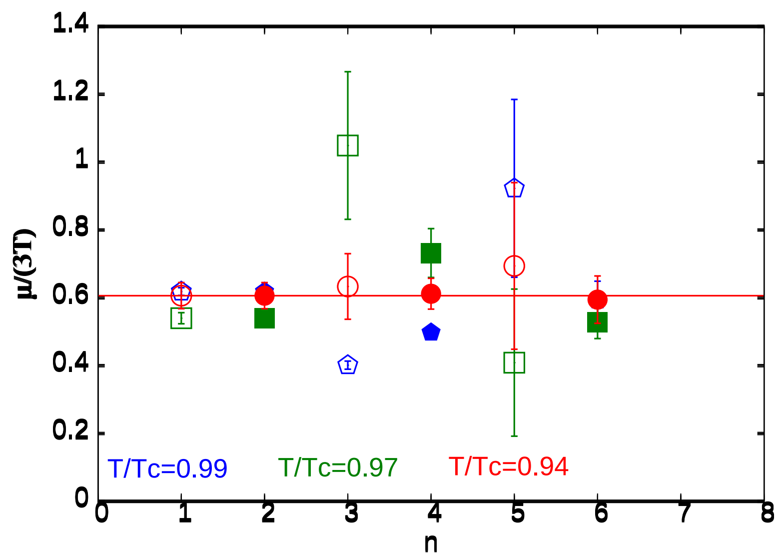

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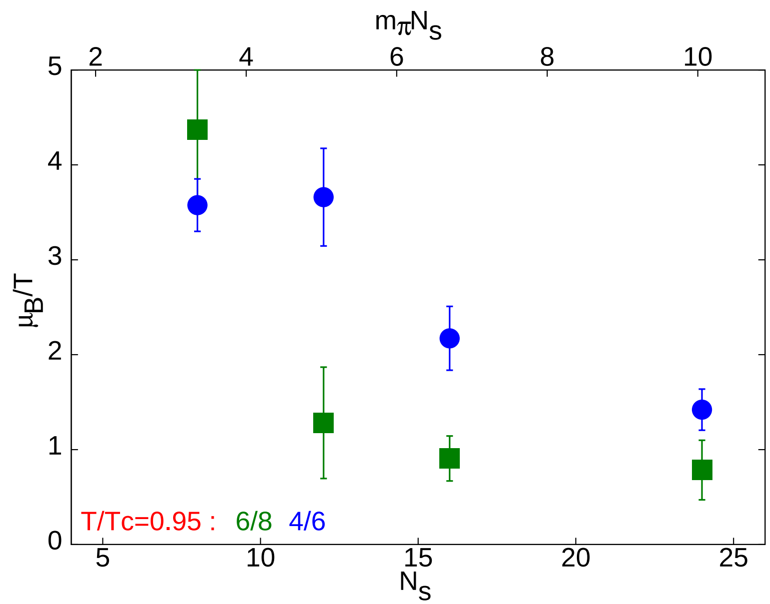

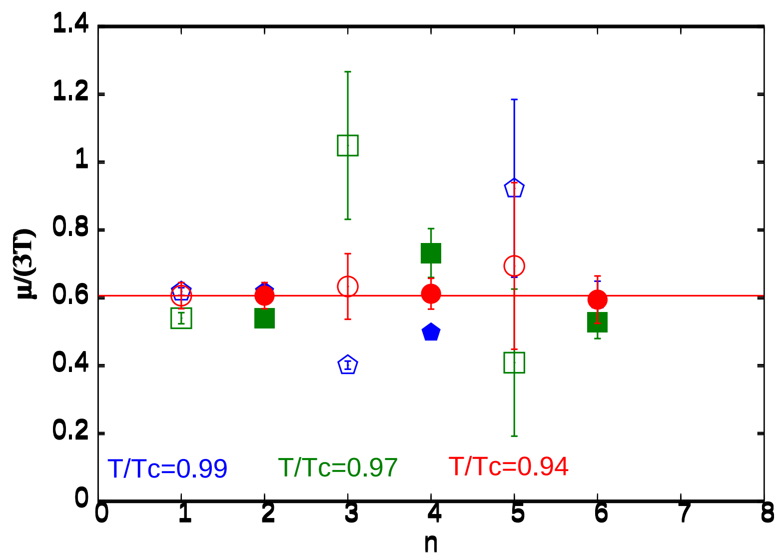

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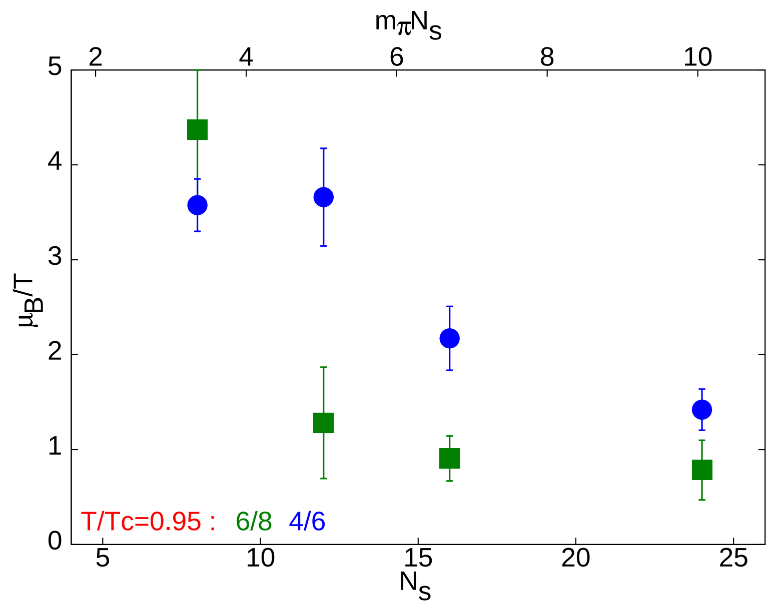

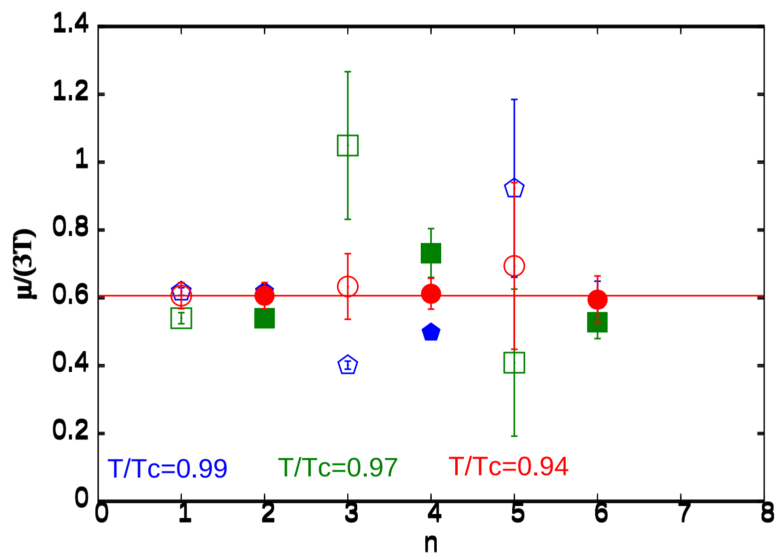

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- Critical point shifted to smaller $\mu_B/T \sim 1 - 2$.

Searching Experimentally

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999)
- Look for nonmontonic dependence of the event-by-event fluctuations with colliding energy.

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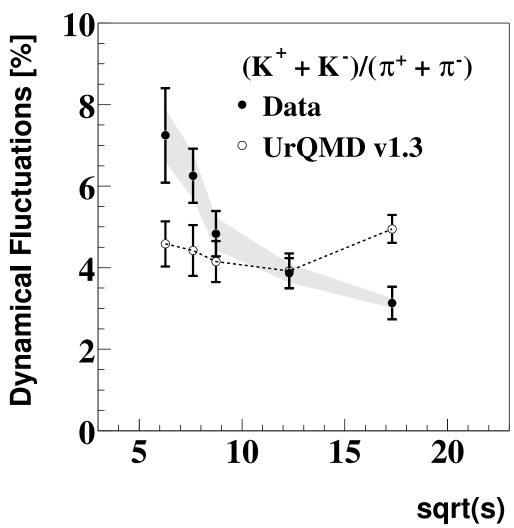

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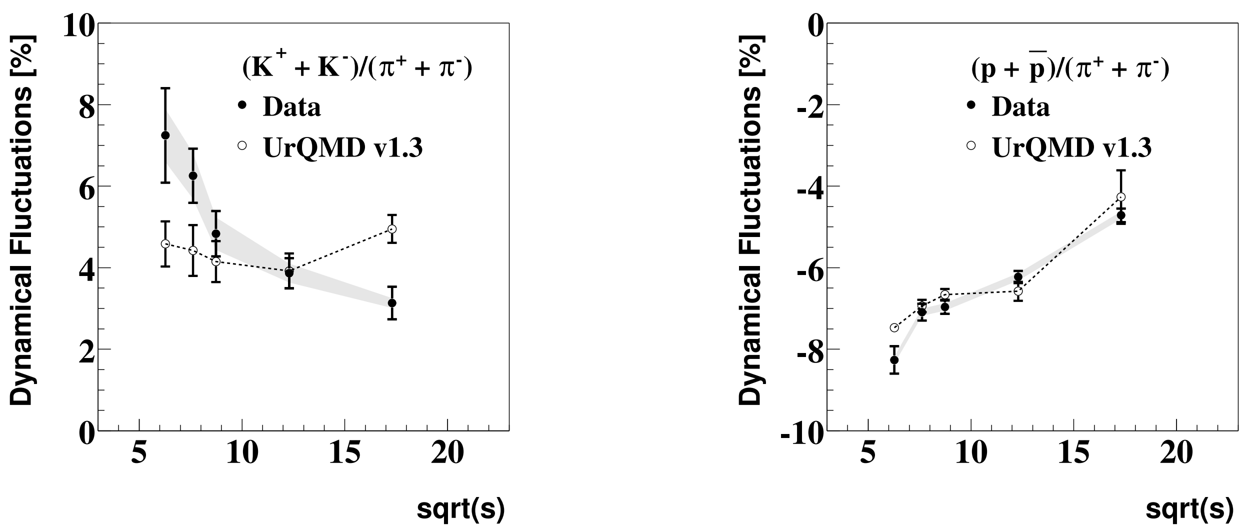

- Fluctuations in mean p_T of low p_T pions.

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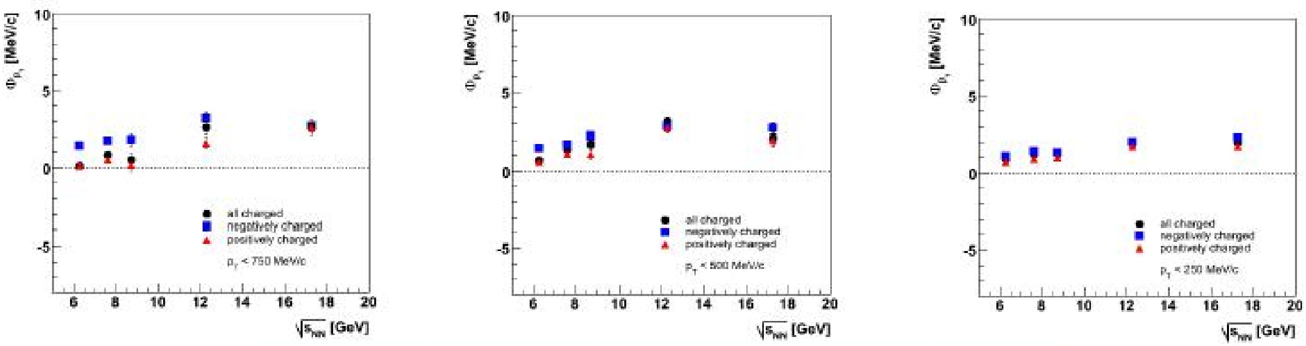
Fluctuations due to the critical point should be dominated by fluctuations of pions with $p_T \leq 500$ MeV/c

M. Stephanov, K. Rajagopal, E. V. Shuryak (Phys. Rev. **D60**, 114028, 1999):

suggestion to do analysis with several upper p_T cuts

$p_T < 750$ MeV/c **$p_T < 500$ MeV/c** **$p_T < 250$ MeV/c**



No significant energy dependence of Φ_{PT} measure
also when low transverse momenta are selected.

Remark: predicted fluctuations at the critical point should result in $\Phi_{PT} \cong 20$ MeV/c, the effect of limited acceptance of NA49 reduces them to $\Phi_{PT} \cong$ **10 MeV/c**

- Proton number fluctuations (Hatta-Stephenov, PRL 2003)
- Neat idea : directly linked to the baryonic susceptibility which ought to diverge at the critical point. Since diverging ξ is linked to σ mode, which cannot mix with any isospin modes, expect χ_I to be regular.

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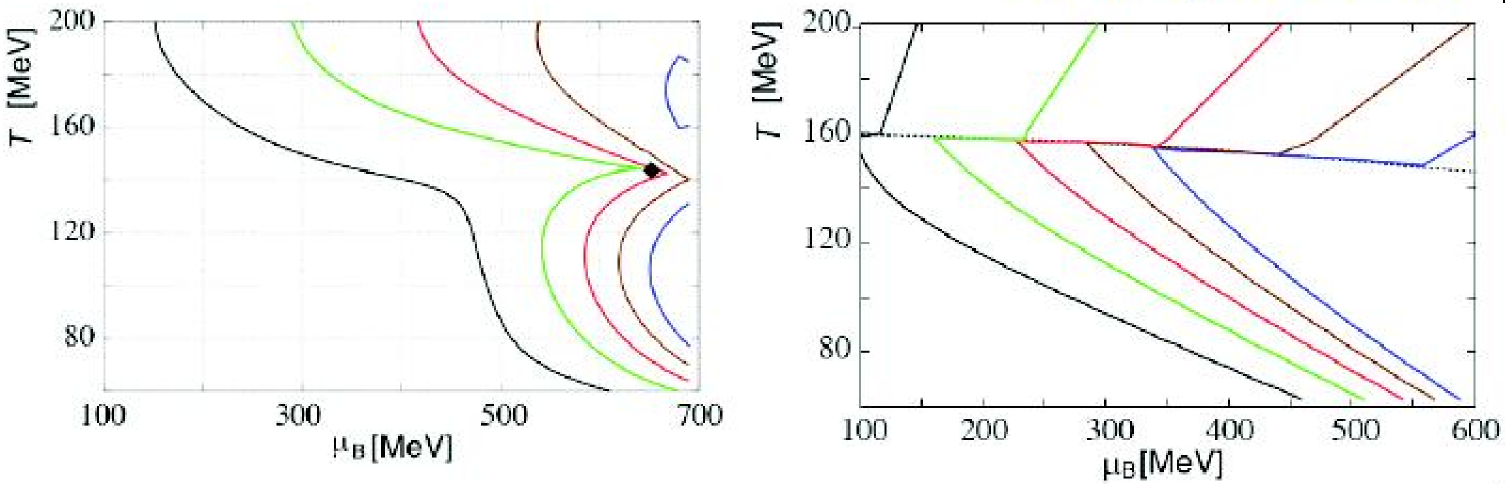
- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be proton number fluctuations only.
- Isentropic trajectories focus at the critical point (Asakawa-Nonaka, PRC 2005).
- This leads to the emission of high p_T particles at earlier times. (Asakawa-Bass-Nonaka-Müller, INT 2008 workshop).
- Note this is NOT a fluctuations signal but model (EoS) dependent ?

Focusing Effect

- **Isentropic trajectories on T- μ_B plane**

With QCD critical point

Bag Model +
Excluded Volume Approximation
(No Critical Point)
= Usual Hydro Calculation

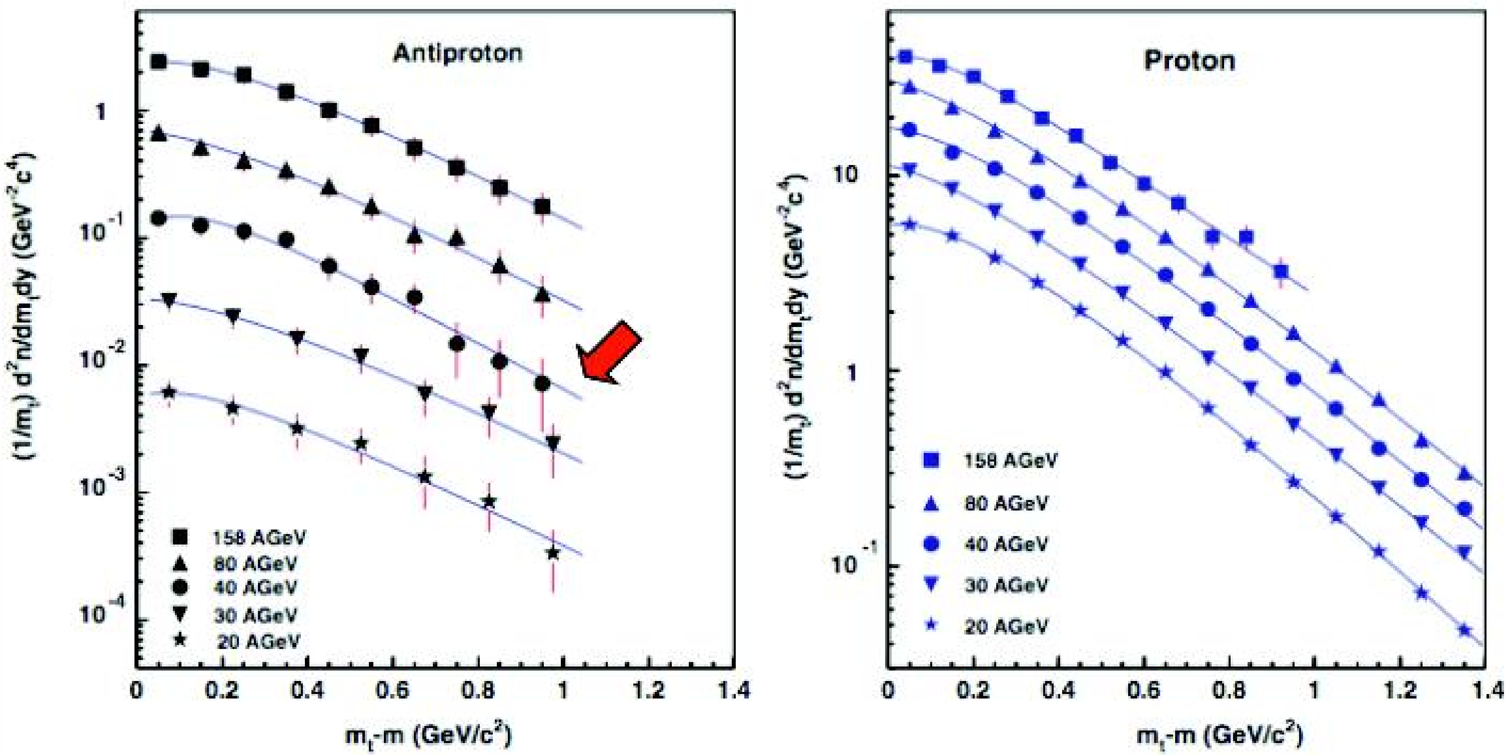


Focused

Not Focused

Chiho NONAKA

QCD Critical Point?



steeper $\overline{p}$ spectra at high P_T NA49, PRC73,044910(2006)

Chiho NONAKA