Present Status of Lattice QCD

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Introduction

EOS, Speed of Sound

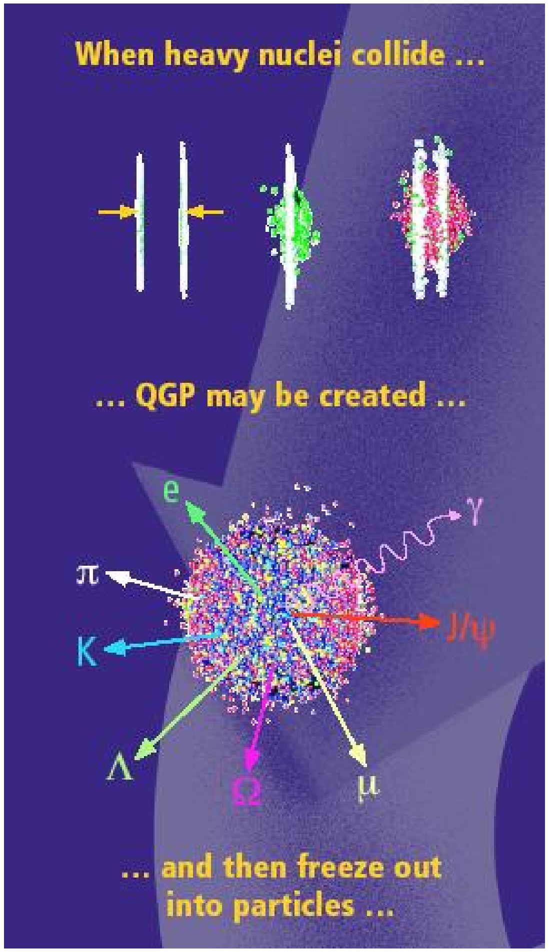
J/ψ suppression

QCD Phase Diagram

Summary

Introduction

♠ Quest for Quark-Gluon Plasma : Heavy Ion Collisions at SPS, RHIC and LHC.



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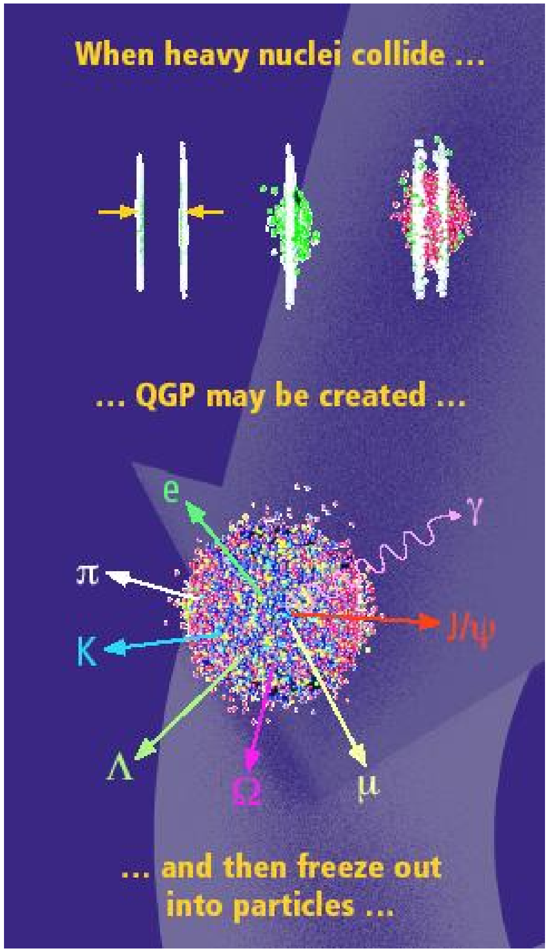

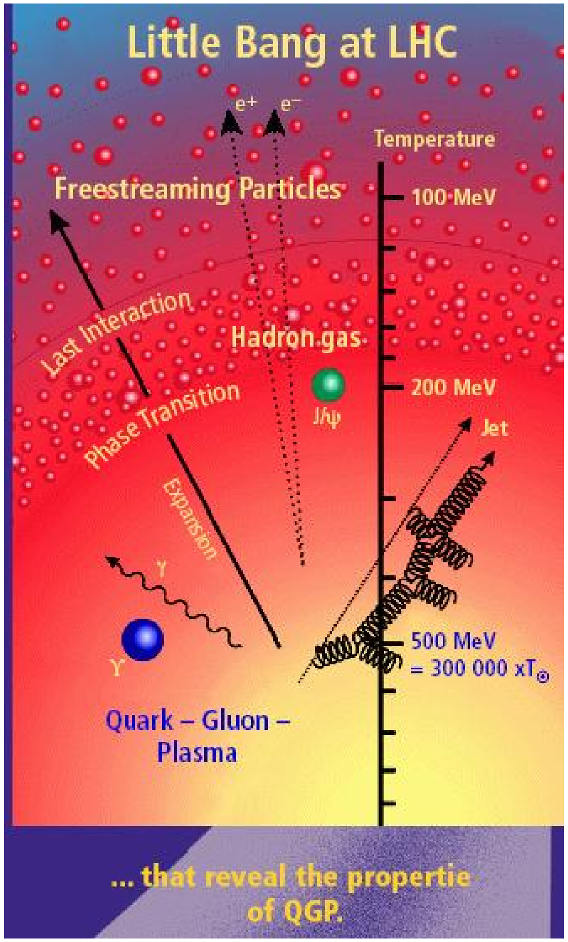

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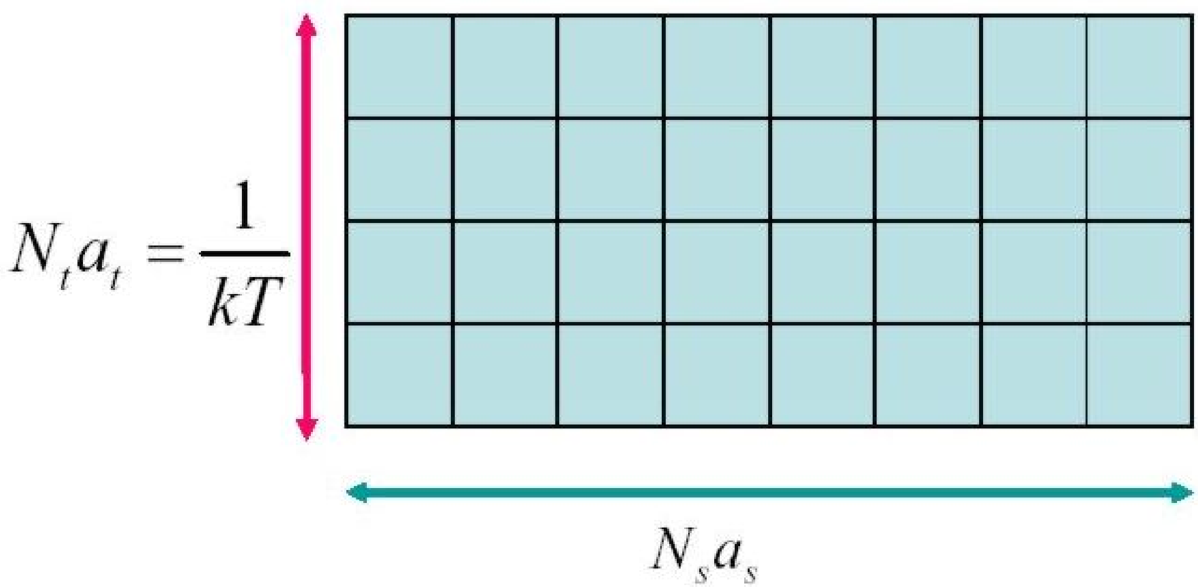

Need $N_s \gg N_t$ for thermodynamic limit and large N_t for continuum limit.

- The Transition Temperature $T_c \sim 170$ MeV (Bielefeld & CP-PACS, Tsukuba 2001) and Equation of State (EOS), have been predicted by lattice QCD.

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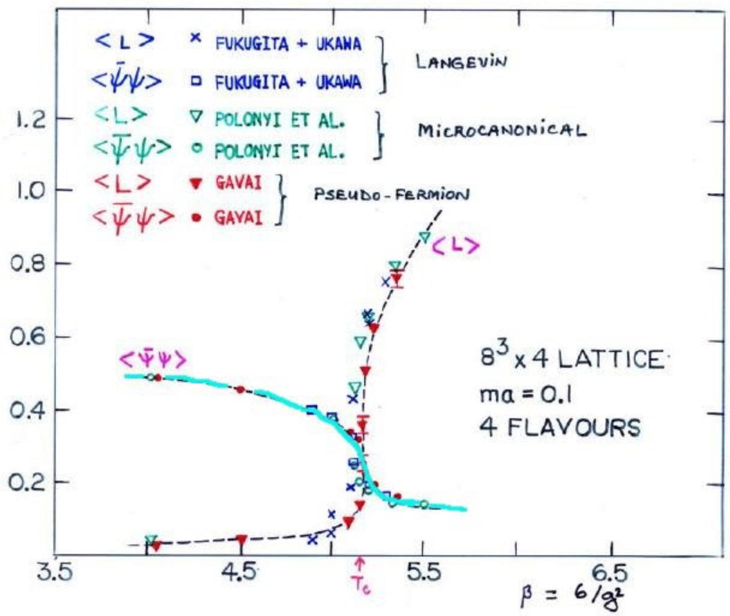

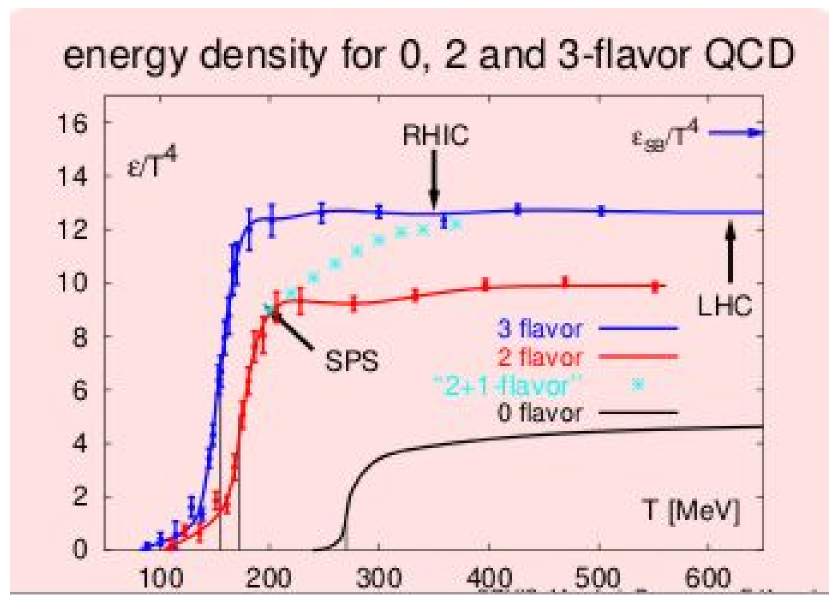

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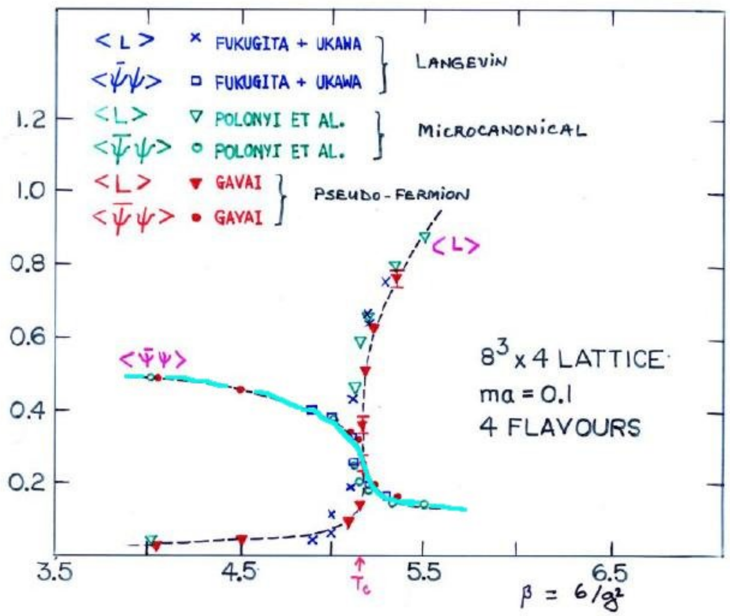

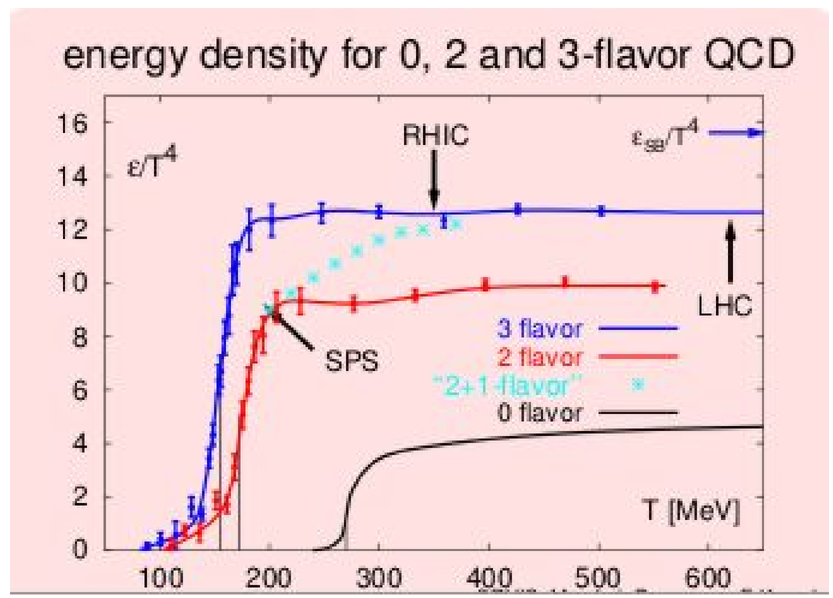

- Other quantities, notably strangeness enhancement in Heavy Ion Physics, the Wróblewski Parameter λ_s (RVG & Sourendu Gupta PR D 2002) have also been predicted by lattice QCD.

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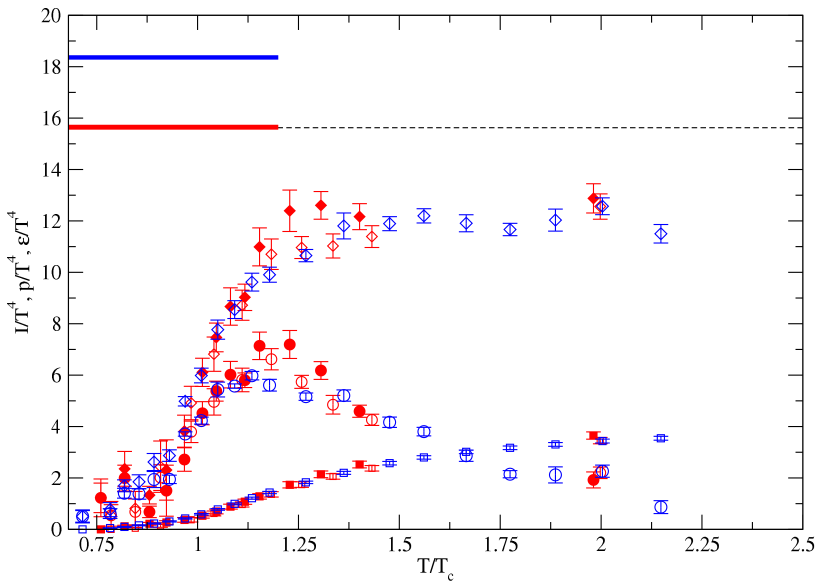
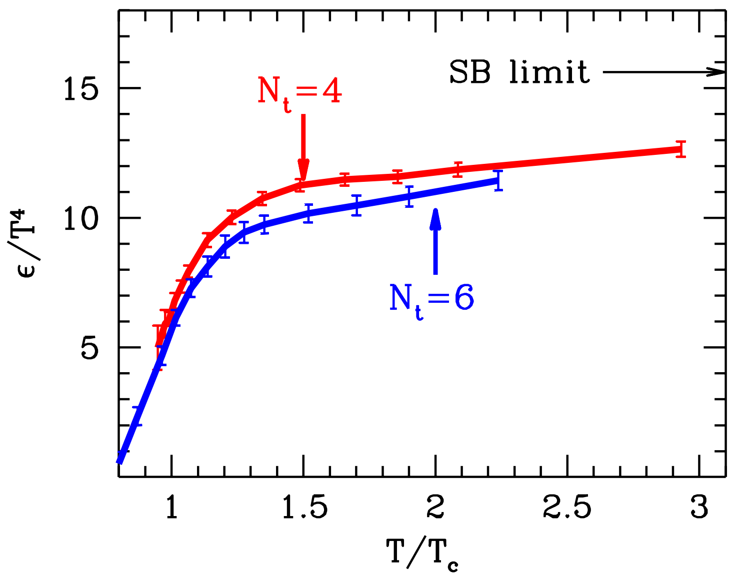
- Lot of activity in Model Building to explain Lattice QCD results: Quasiparticle models, Hadron Resonance Gas, Quarkonia from Lattice $Q\bar{Q}$ potential, sQGP and coloured states...

EoS, Speed of Sound

- Recent results for EoS : N_t=6, Smaller quark masses.





Bernard et al., MILC hep-lat/0509053;

Aoki et al., hep-lat/0510084.

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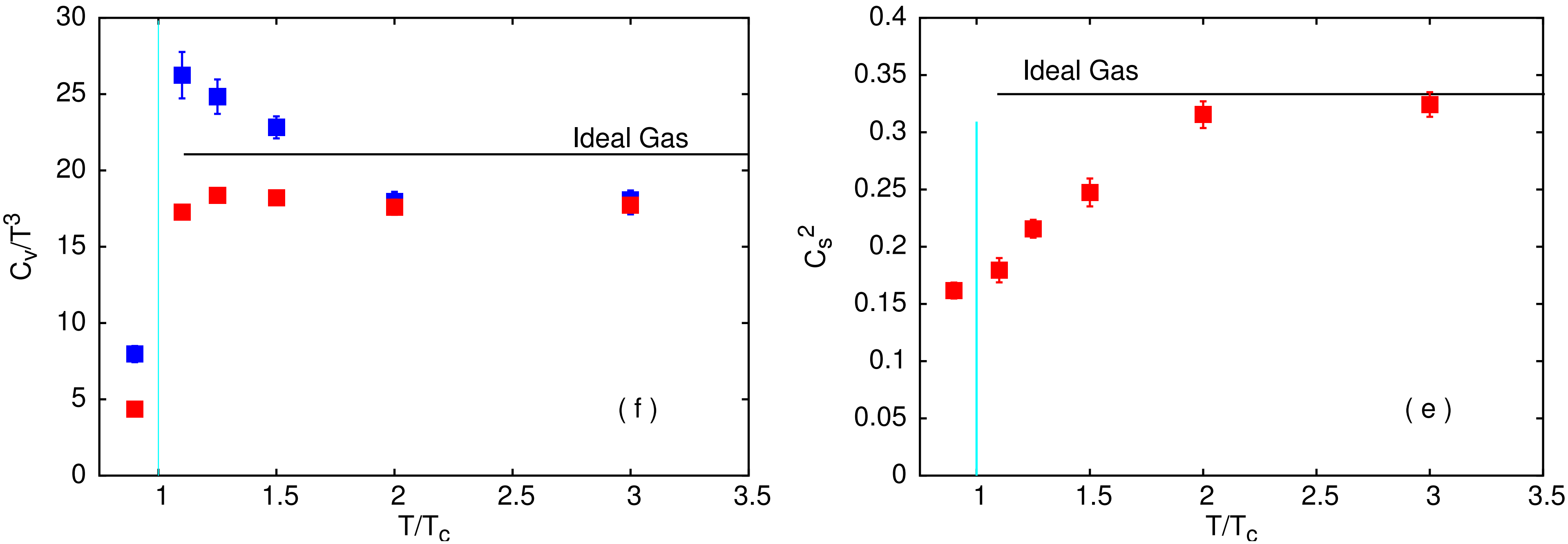
- Using lattices with 8, 10, and 12 temporal sites ($38^3 \times 12$ and 38^4 lattices) and with statistics of 0.5-1 million iterations, ϵ, P, s, C_s^2 and C_v obtained in continuum.



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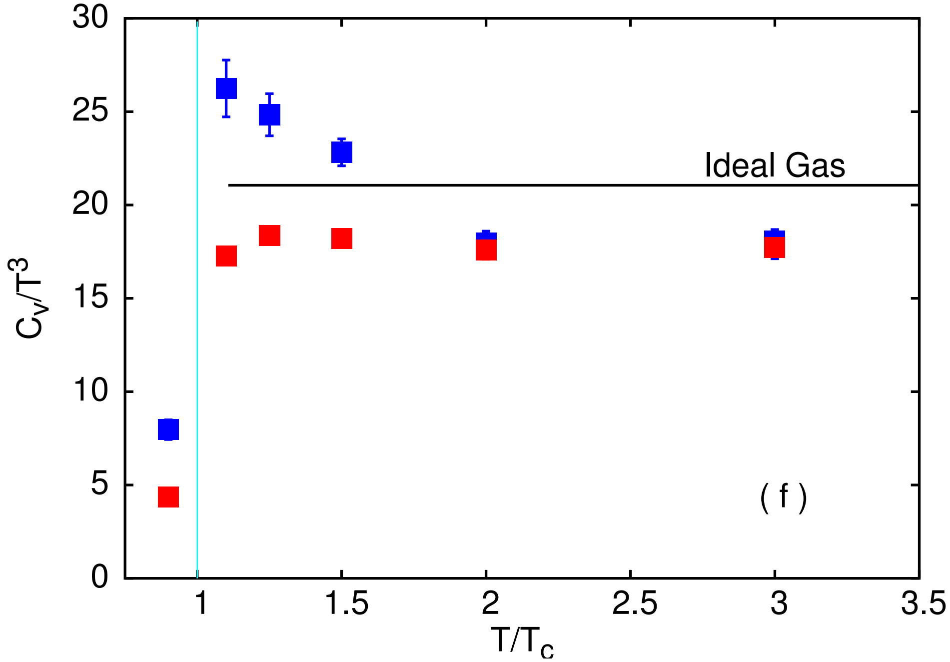

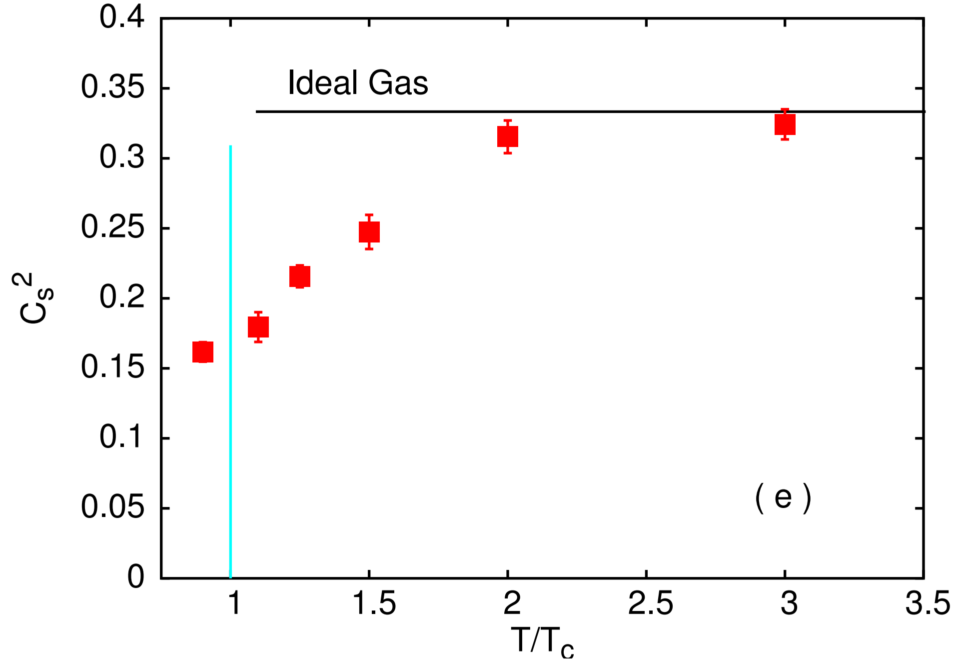

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♠ $C_v \sim 4\epsilon$ for $2T_c$ but No Ideal Gas limit.

♠ Specific heat $\Longleftrightarrow$ fluctuations in p_T ?

♠ C_s^2 closer to Ideal Gas limit; Any structure near T_c ??

- Entropy agrees with strong coupling SYM prediction (Gubser, Klebanov & Tseytlin, NPB '98, 202) for $T = 2 - 3T_c$ but fails at lower T, as do various weak coupling schemes : $\frac{s}{s_0} = f(g^2 N_c)$, where $f(x) = \frac{3}{4} + \frac{45}{32}\zeta(3)x^{-3/2} + \cdots$ and $s_0 = \frac{2}{3}\pi^2 N_c^2 T^3$.

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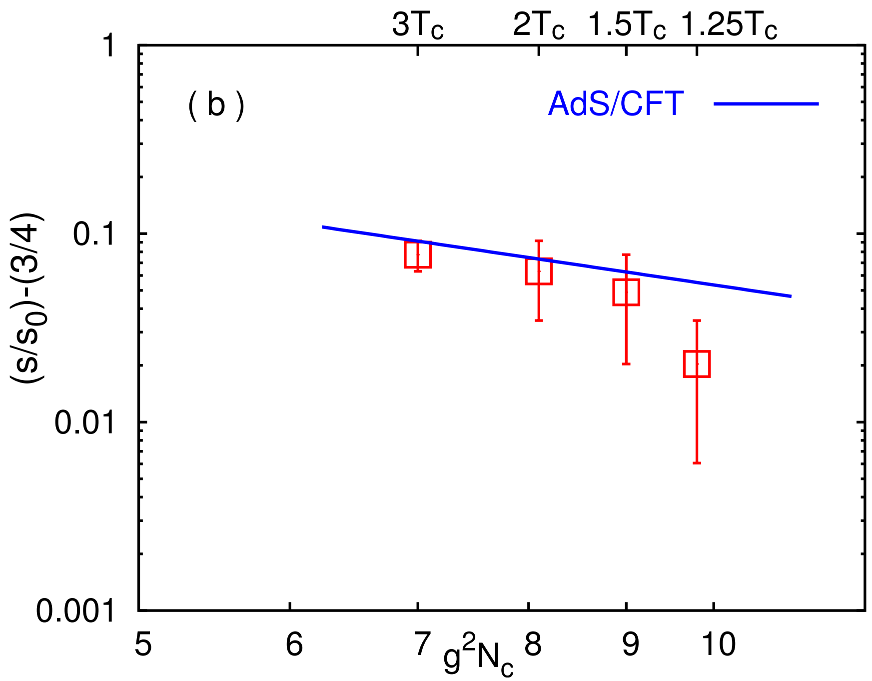

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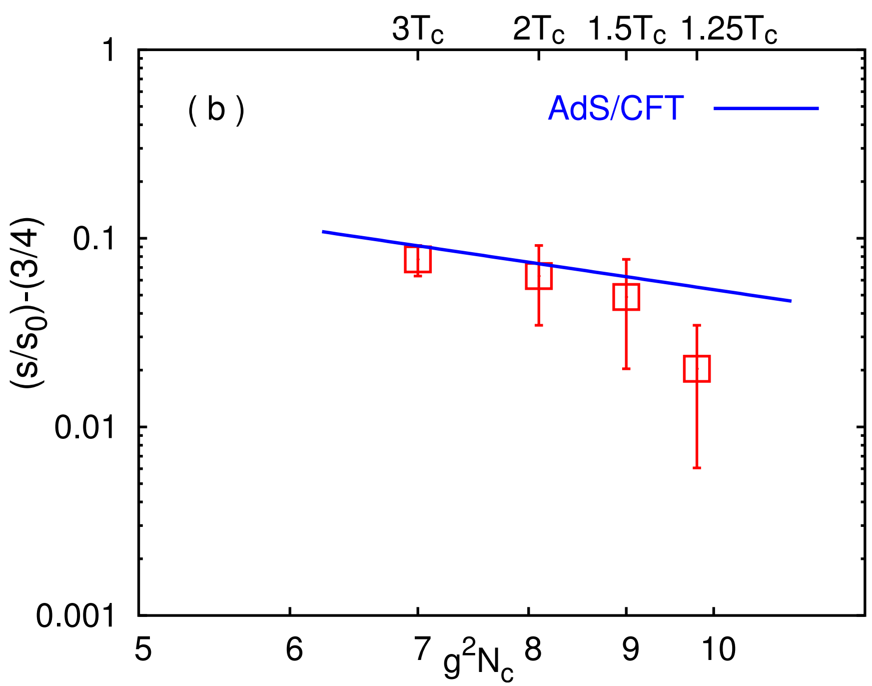
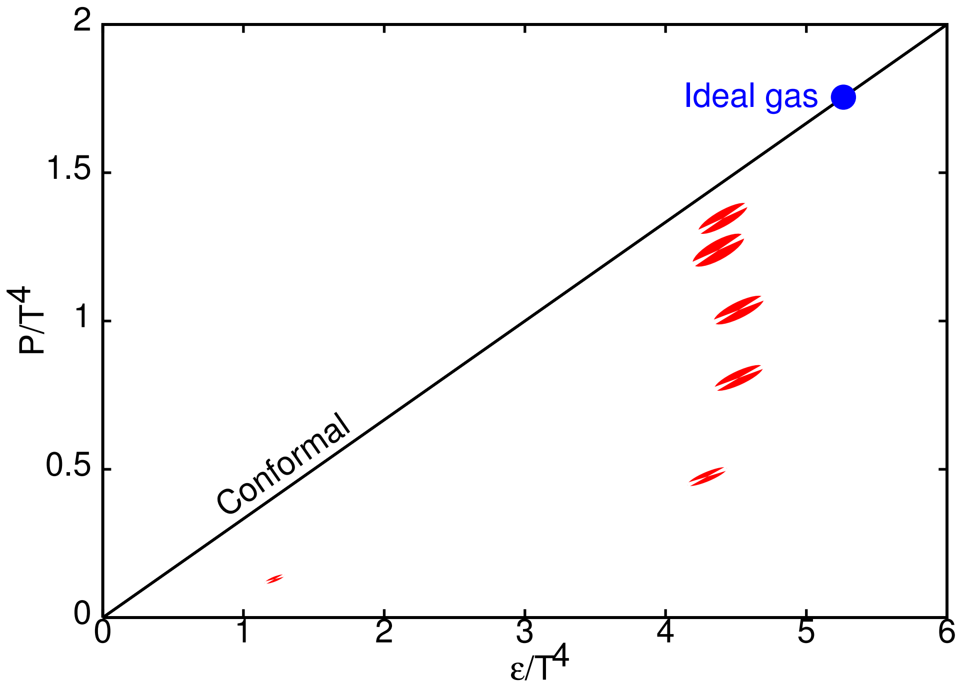
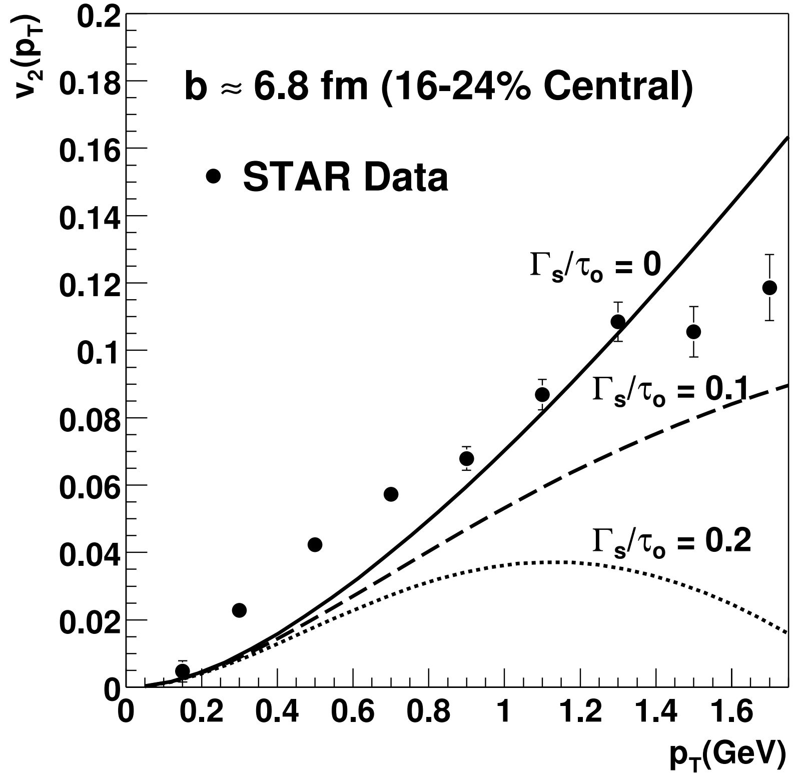




QGP - (Almost) Perfect Liquid



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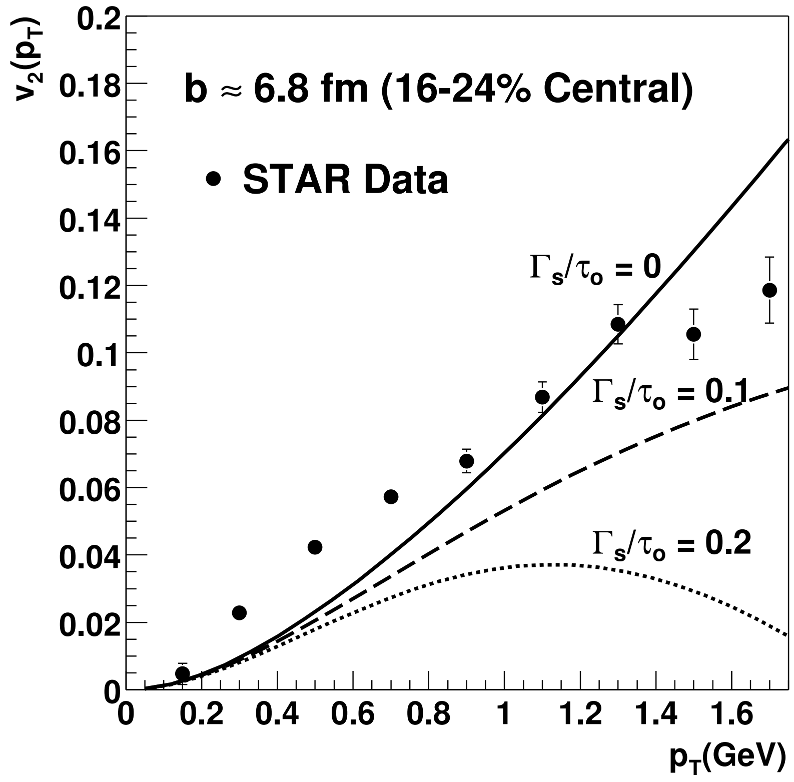

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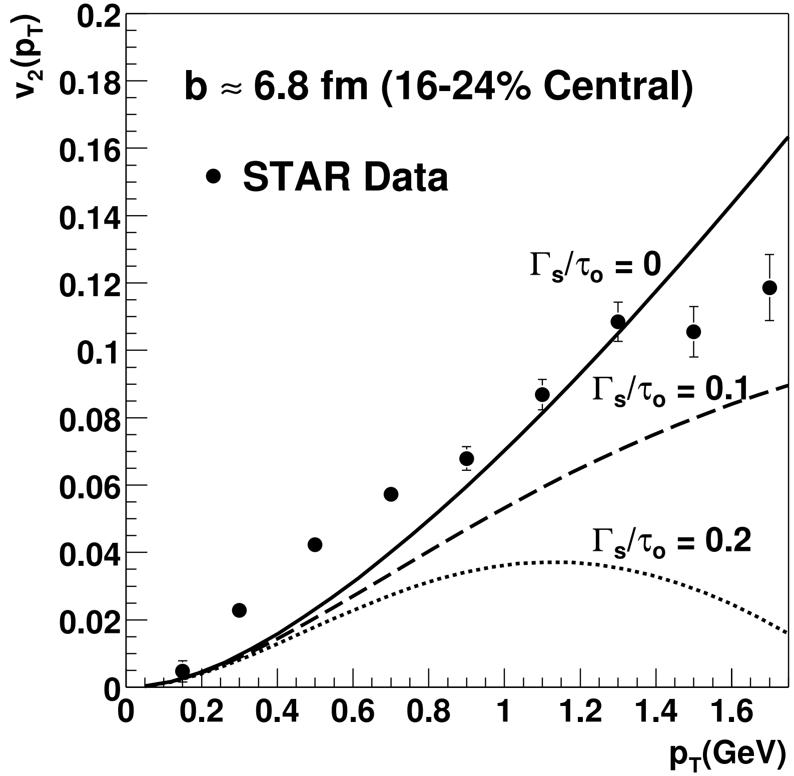

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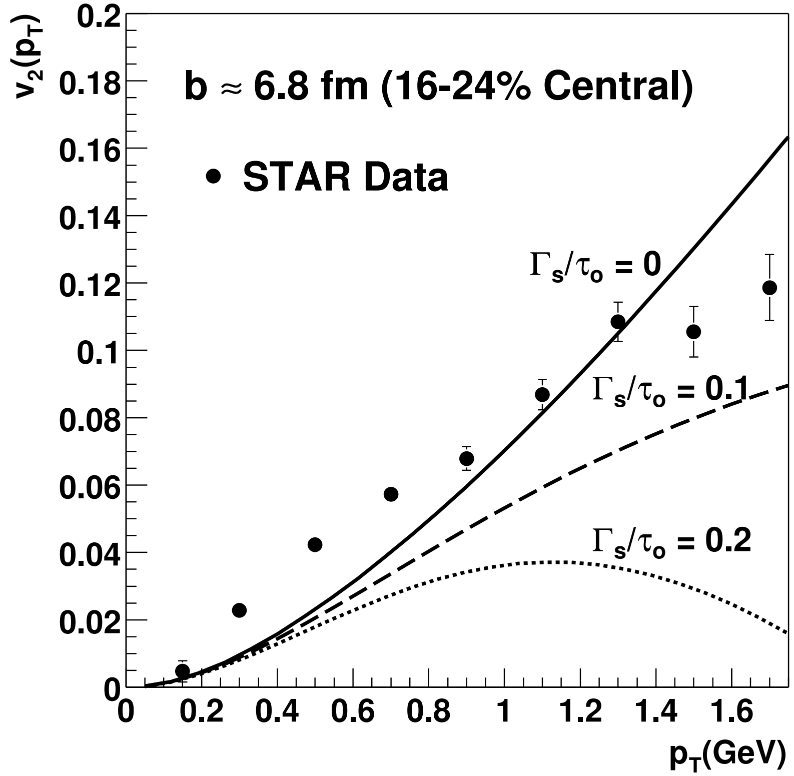

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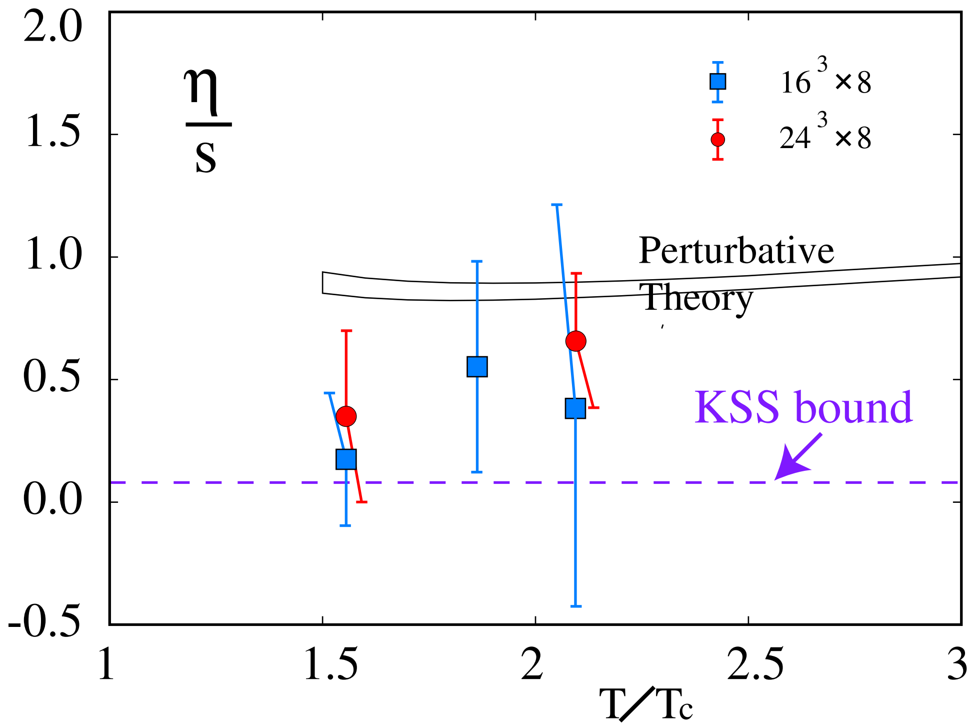
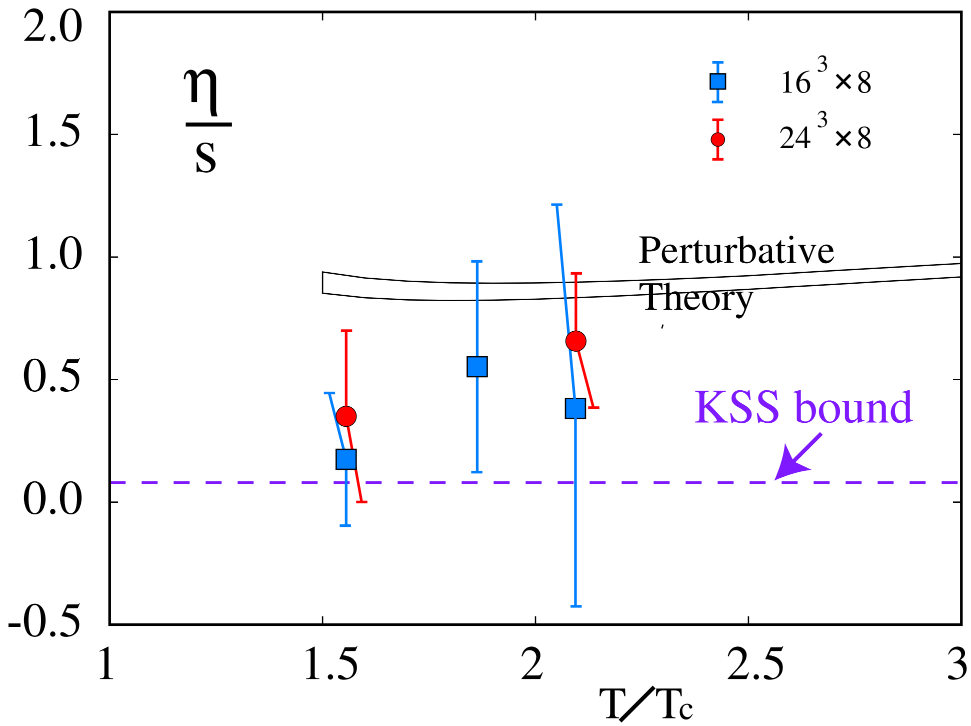
Perturbation theory $\Rightarrow$ Large η/s
Small $\eta/s \longrightarrow$ Strongly Coupled Liquid.



Nakamura and Sakai, PRL 94 (2005).



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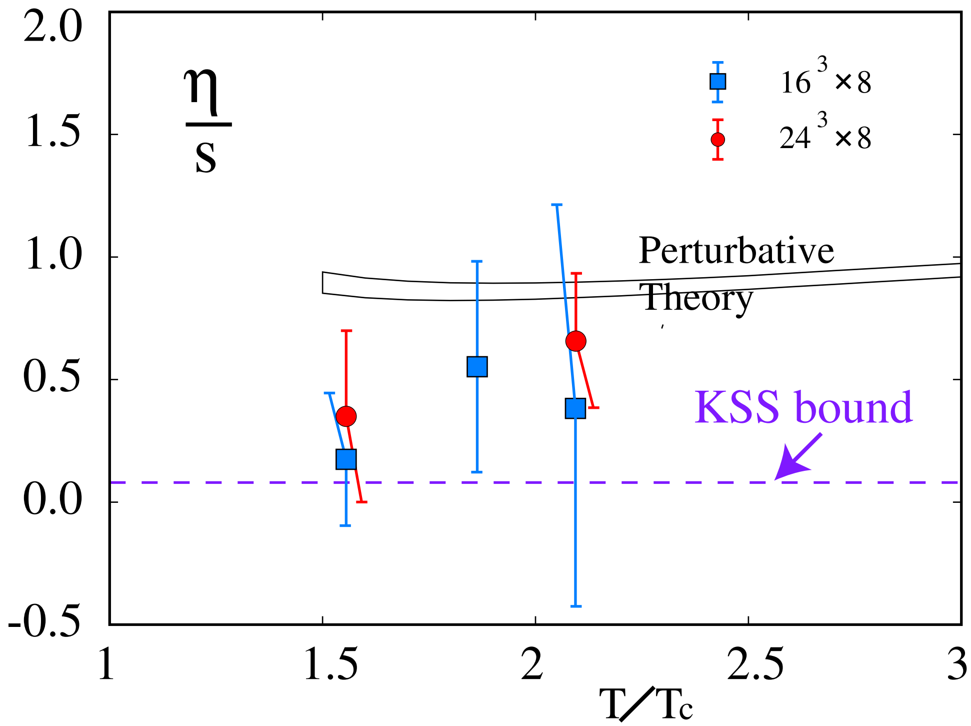

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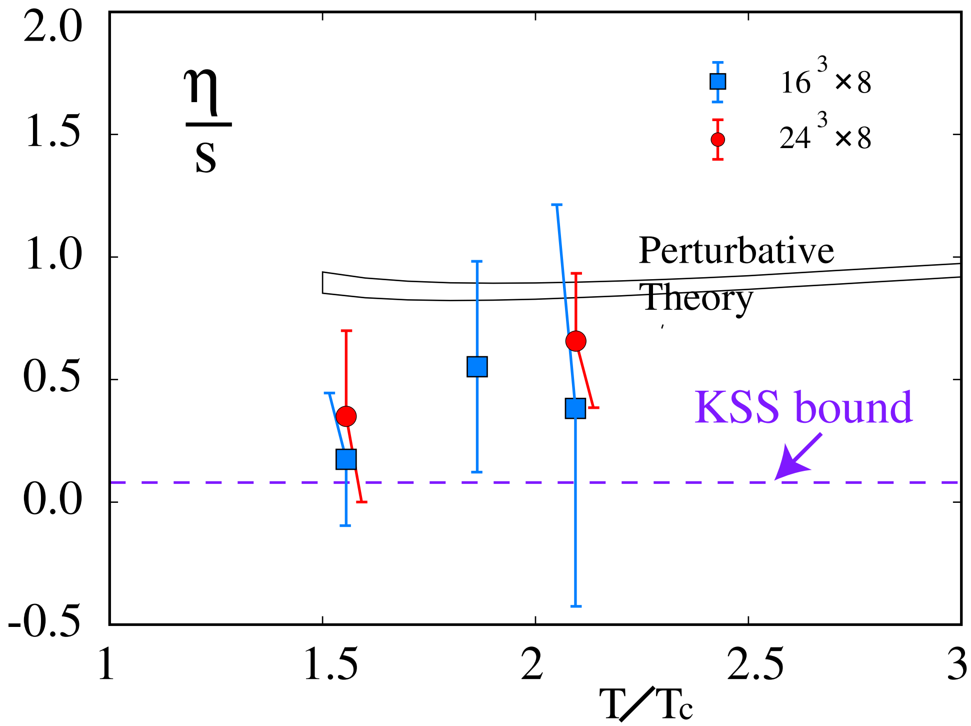

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- Continue them to get Retarded ones ⇝ Shear, Bulk Viscosities.
- Larger lattices and inclusion of dynamical quarks in future.

Anomalous J/ψ Suppression : CERN NA50 results

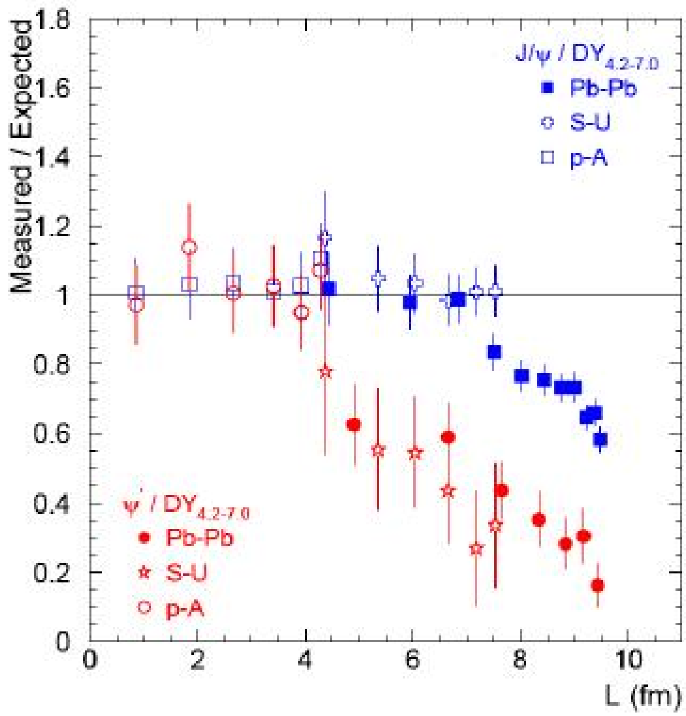
♠ Matsui-Satz idea — J/ψ suppression as a signal of QGP.
♠ Deconfinement $\rightsquigarrow$ Screening of coloured quarks,
which cannot bind.

Anomalous J/ψ Suppression : CERN NA50 results

Expected = Glauber absorption model

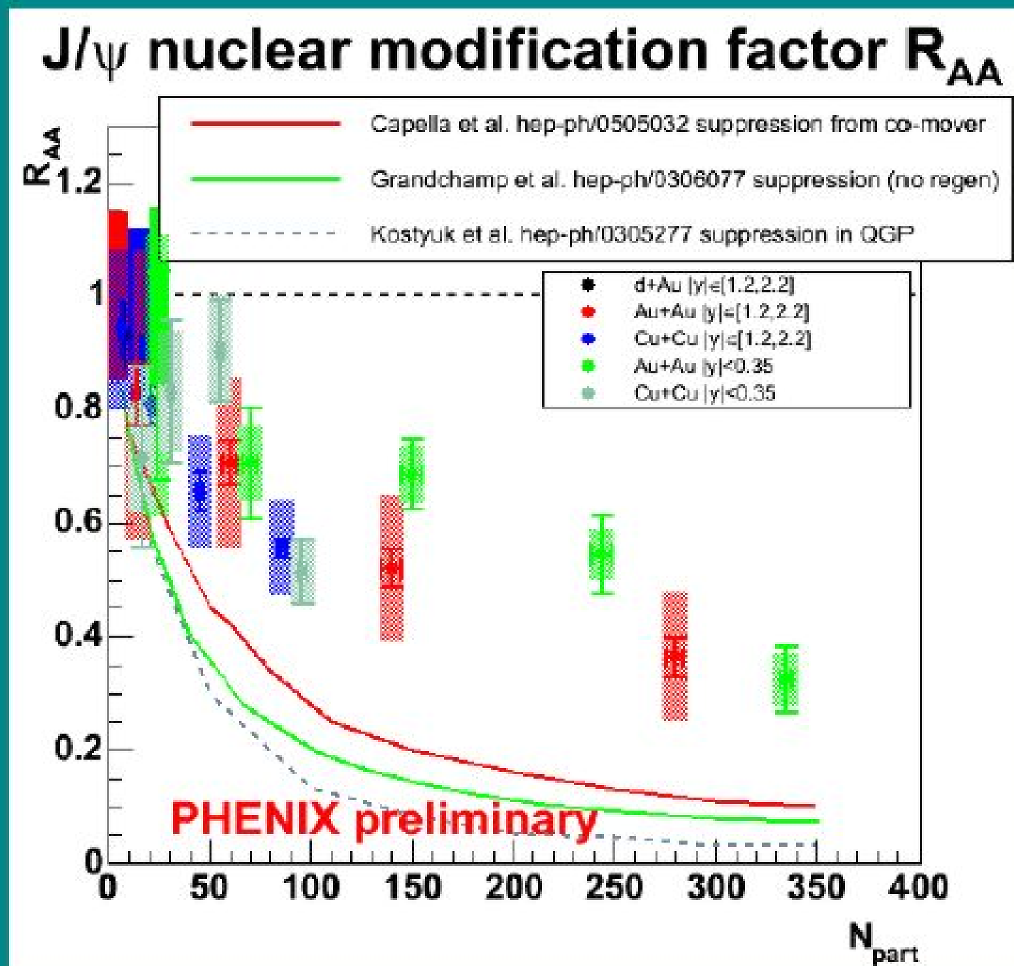
$\sigma_{abs}(J/\psi) = 4.18 \pm 0.35$ mb

$\sigma_{abs}(\psi') = 7.60 \pm 1.12$ mb



- **S-U** and **peripheral Pb-Pb (J/ψ)/DY** results follow the absorption curve extrapolated from p-A measurements.
- **Pb-Pb central** collisions show an **anomalous (J/ψ)/DY suppression** with respect to p-A behaviour.
- **ψ'/DY** behaviour is the same in **S-U** and **Pb-Pb** interactions and not compatible with the one observed in p-A collisions.
- **ψ' anomalous suppression** sets in earlier than the J/ψ one.

System-Size Dependence



Models that were successful in describing SPS data fail to describe data at RHIC

- too much suppression -

J/ψ Suppression

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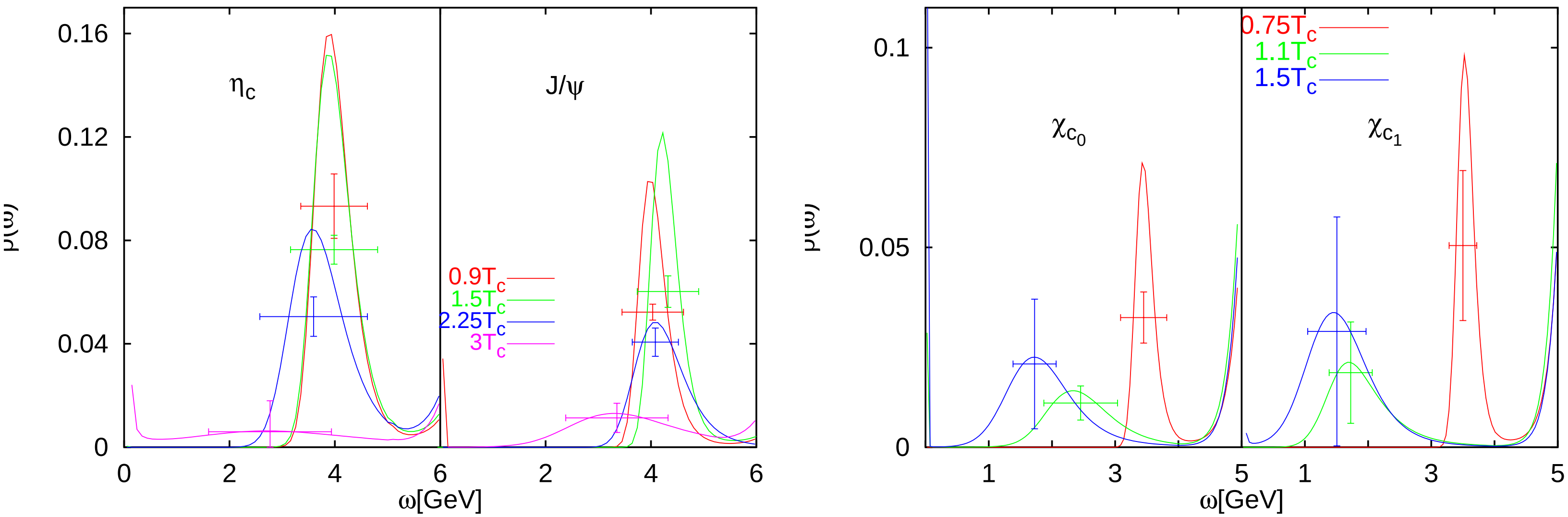
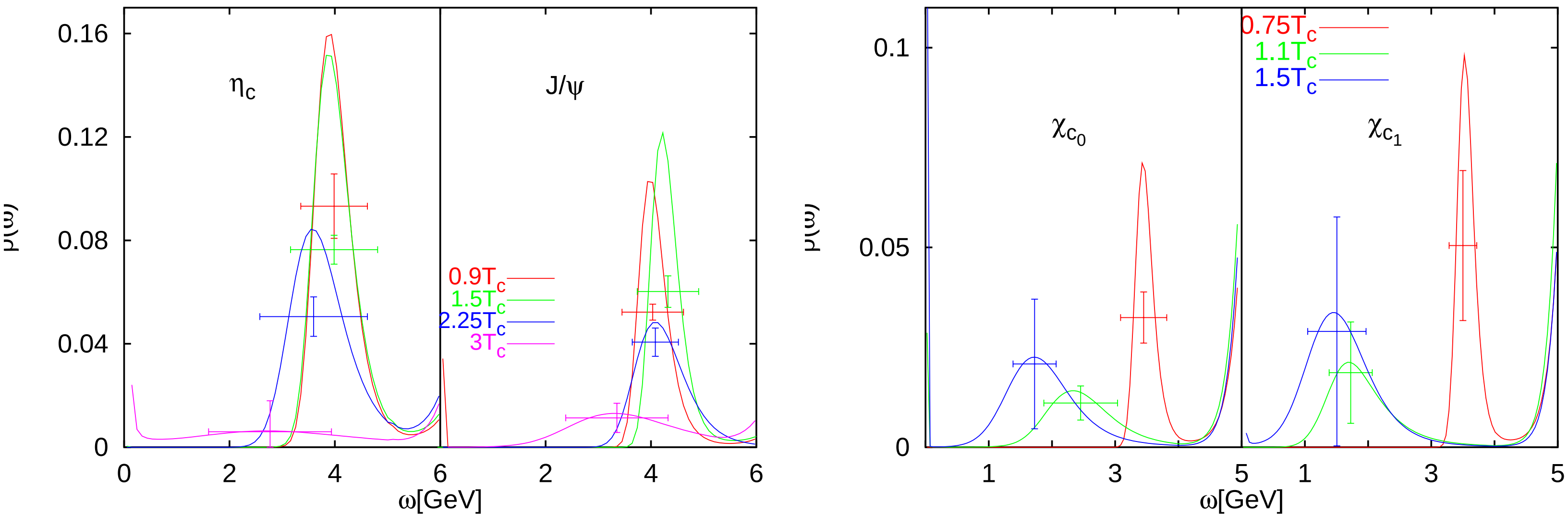
- Caution : nonzero temperature obtained by making temporal lattices shorter : $48^3 \times 12$ to $64^3 \times 24$ Lattices used. (S. Datta et al., Phys. Rev. D 69, 094507 (2004).)



♠ χ_c seems to indeed dissolve by 1.1T_c, however, J/ψ and η_c persist up to 2.25 T_c and are gone at $3T_c$; Similar results by Asakawa-Hatsuda and Matsufuru.



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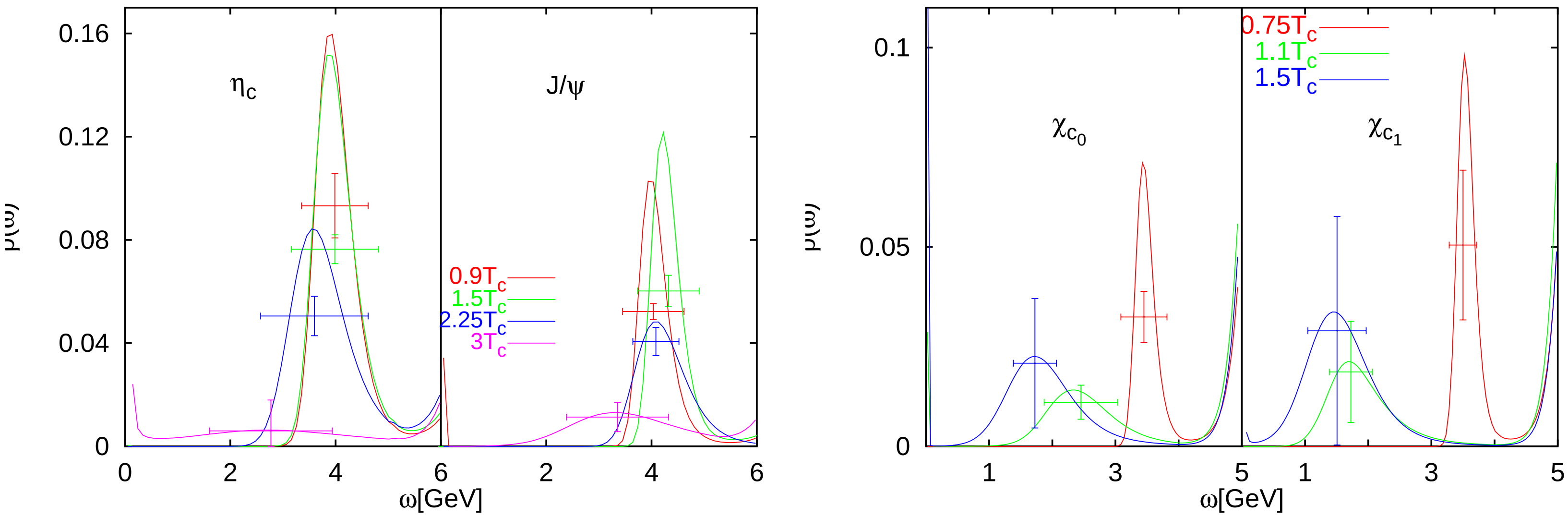

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♠ No Significant Effect of inclusion of dynamical fermions ?

Quarkonia moving in the Heat Bath

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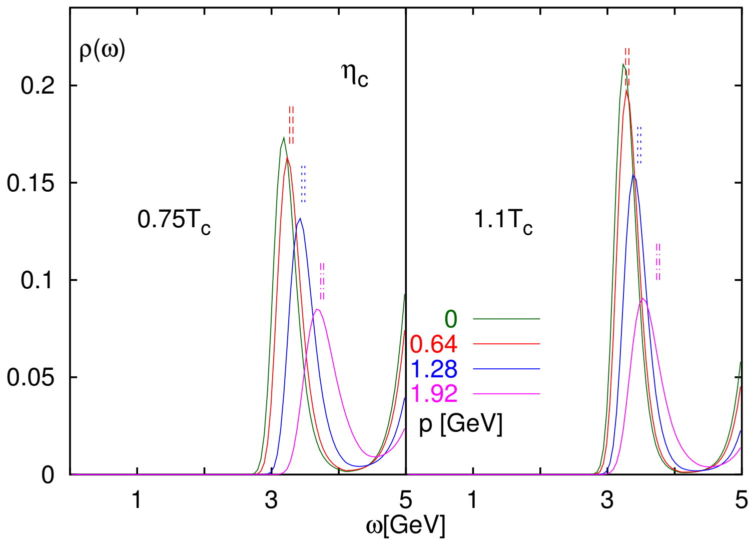

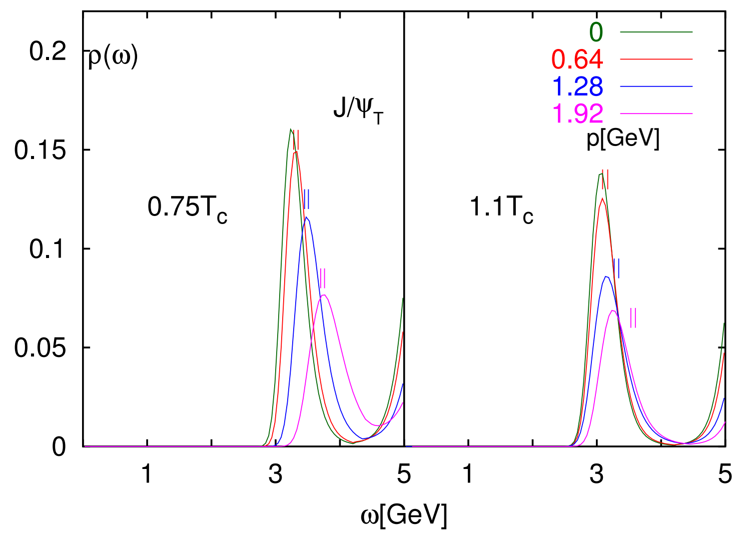

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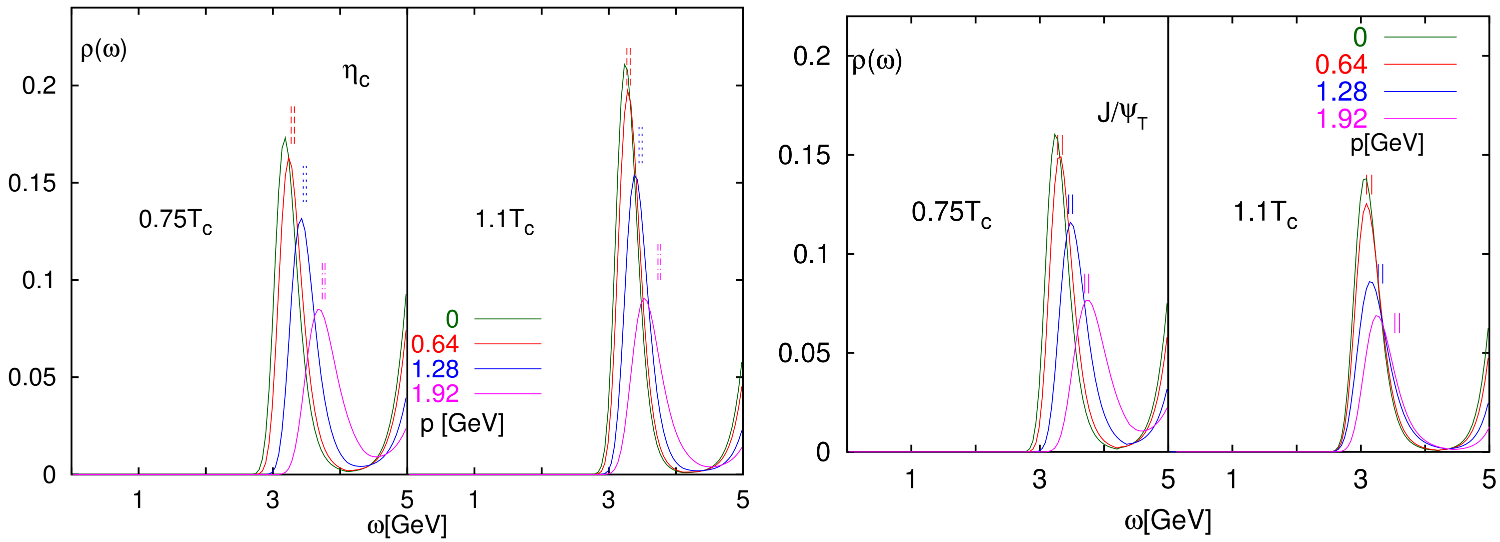
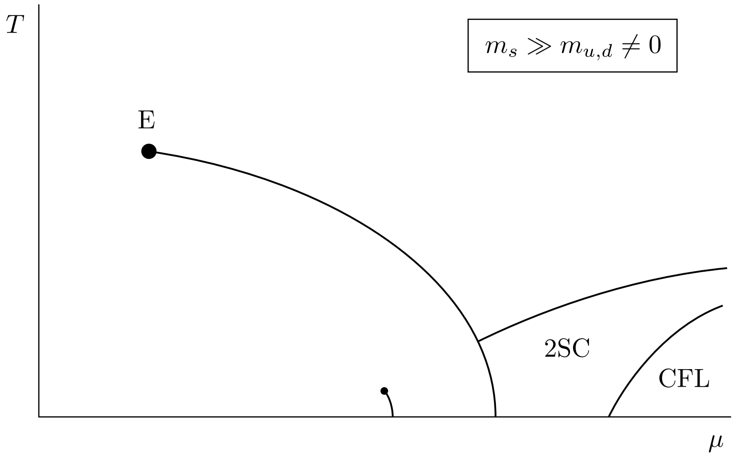


♠ Both J/ψ and η_c do show this trend.

♠ The effect is significant at both 0.75 and $1.1T_c$.

QCD Phase Diagram

Expected QCD Phase Diagram and Lattice Approaches to unravel it.



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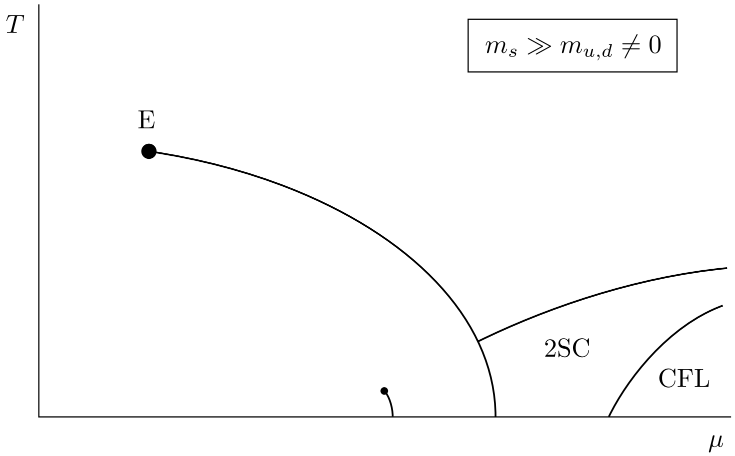

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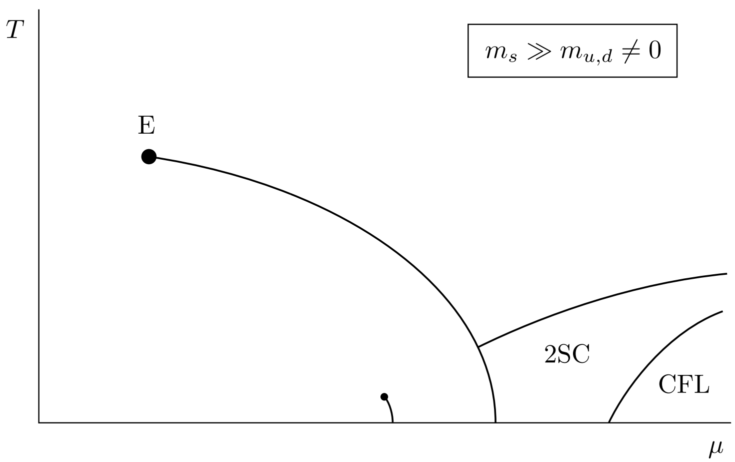

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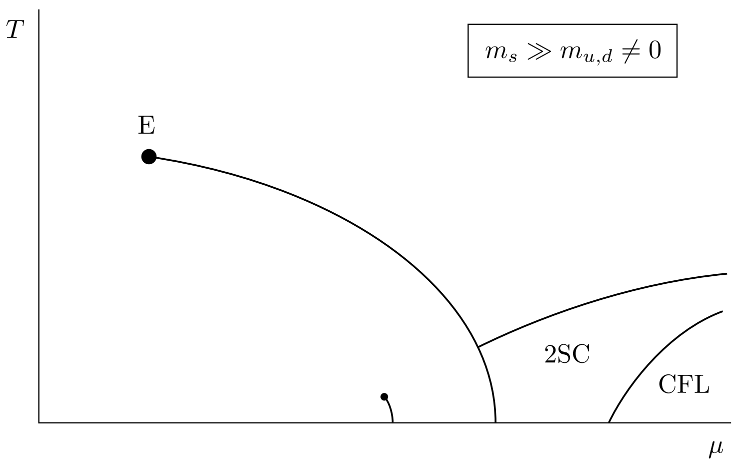

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- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).

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- Reweighting reasonable for only small μ ? (Ejiri 2004)

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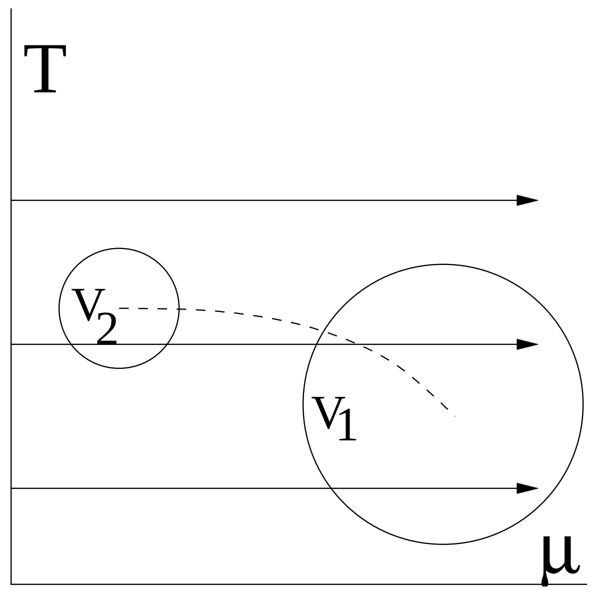
- E.g., exp$[\Delta S]$ factor makes this exponentially tough for re-weighting.
- Discretization errors propagate in an unknown manner in re-weighting.
- Reweighting reasonable for only small μ ? (Ejiri 2004)



We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

How Do We Do This Expansion?

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \prod_f \mathrm{Det}\ M(m_f, \mu_f) \quad .$$

Canonical definitions then yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \qquad (2)$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence can be obtained from these using $\sqrt{\left|\frac{\chi_B^n}{\chi_B^{n+2}}\right|}$ or $\left[\left|\frac{\chi_B^0}{\chi_B^n}\right|\right]^{\frac{1}{n}}$. We use terms up to 8th order in μ, i.e., estimates from 2/4, 4/6 and 6/8 terms.

- Coefficients for the off-diagonal susceptibility, χ_{11}, can be constructed similarly.

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How Do We Do This Expansion?

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Our Simulations & Results

- Lattice used : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24
- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm with traj. length of 1 MD time on $N_s = 8$, scaled $\propto N_s$ on larger ones.
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- Using the strange and u-d susceptibilities, ratio χ_s/χ_u can be obtained.

We use $m/T_c = 0.03$ for u, d and $m/T_c = 1$ for s quark;
At each T, ratio of χ's $\quad \rightarrow \quad \lambda_s(T)$.

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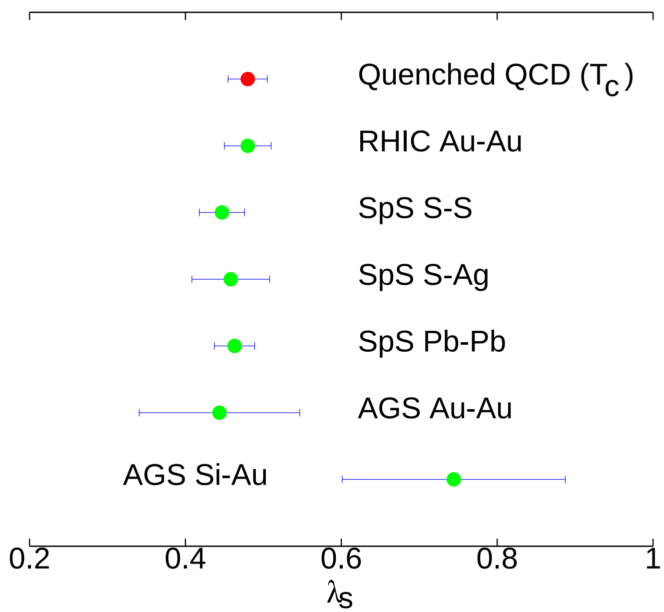

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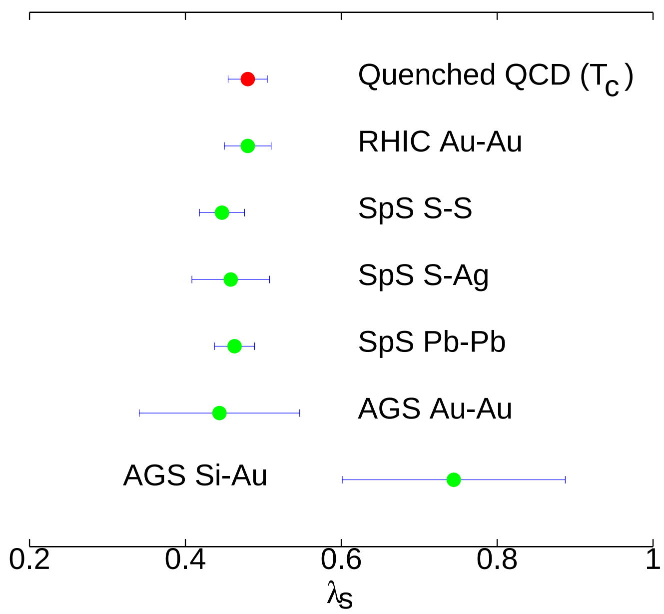

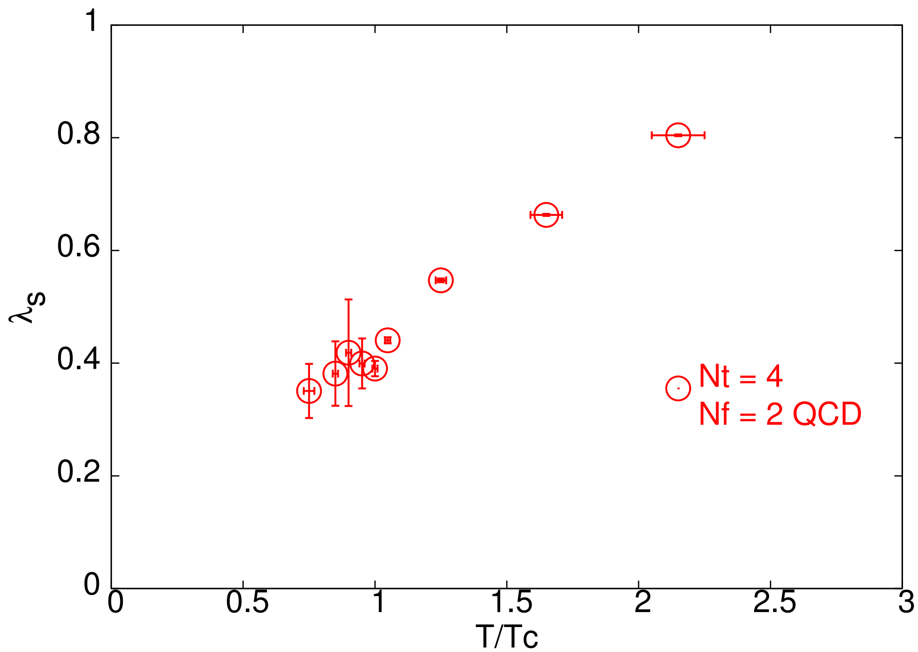

Baryon-Strangeness Correlation

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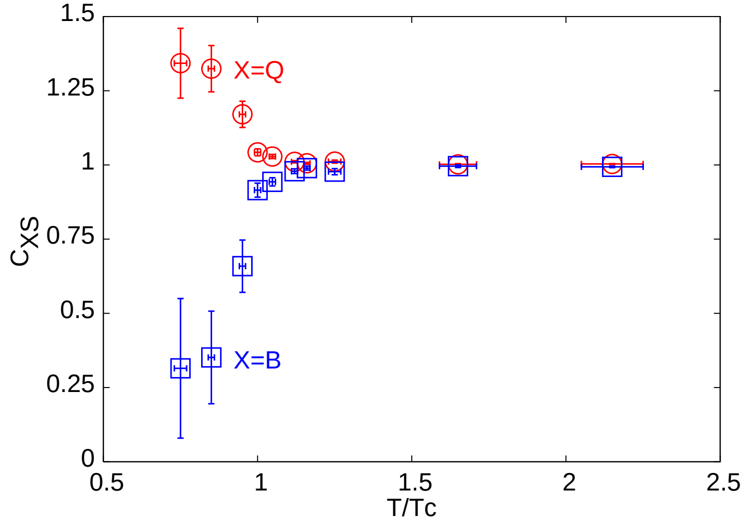

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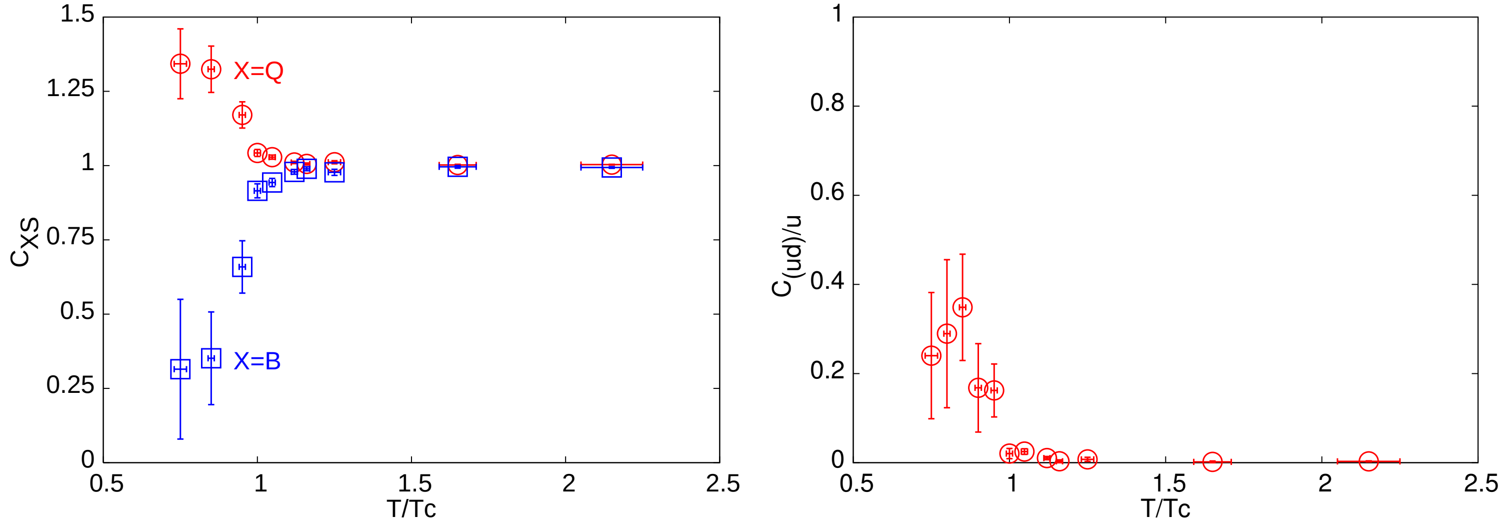
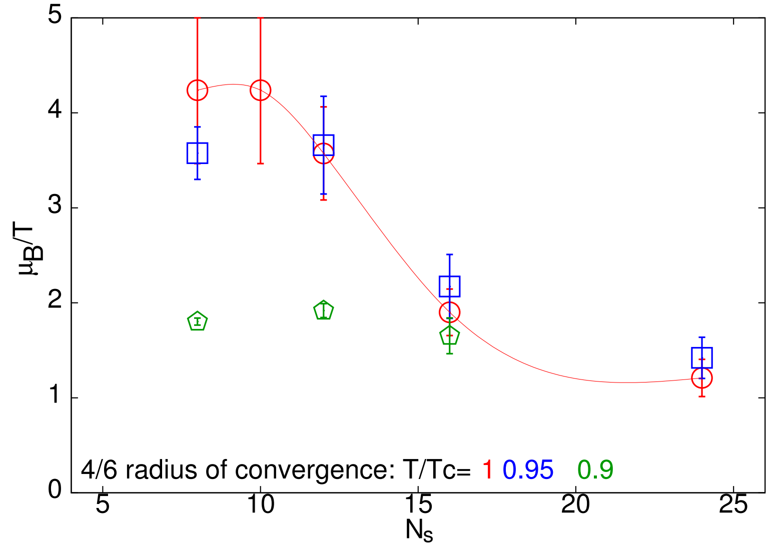
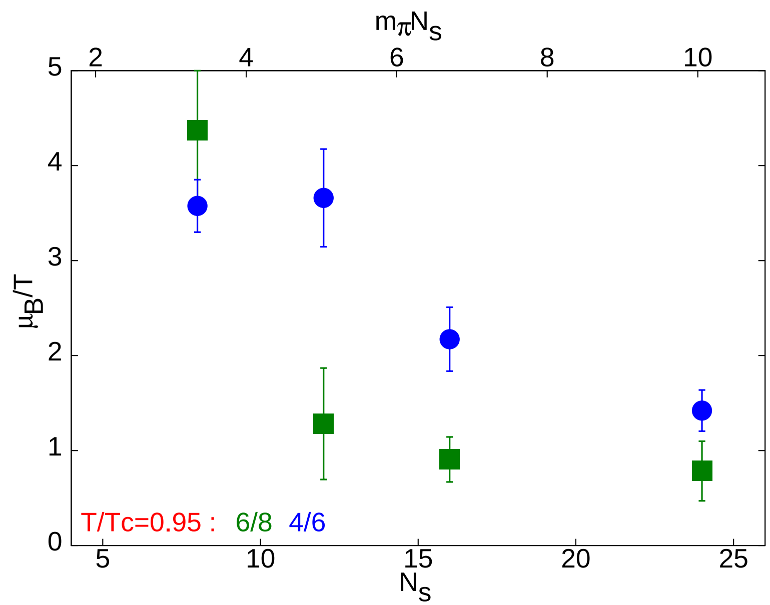



μB/T
Ns
4/6 radius of convergence: T/Tc= 1 0.95 0.9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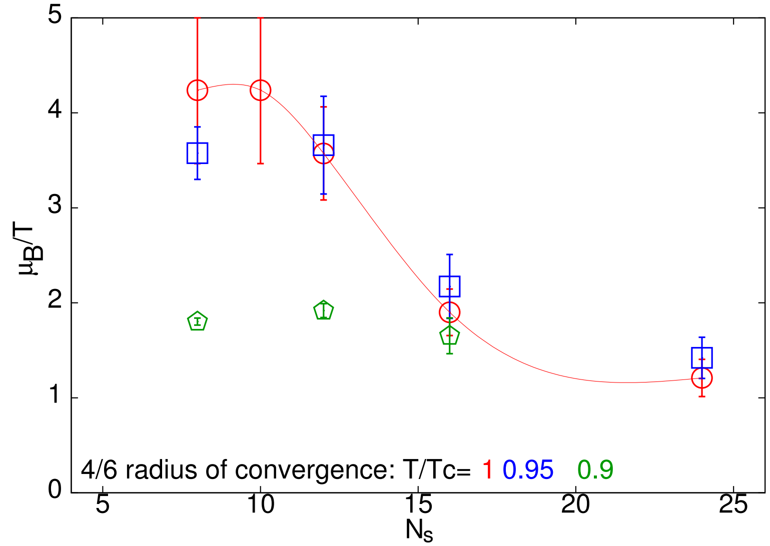

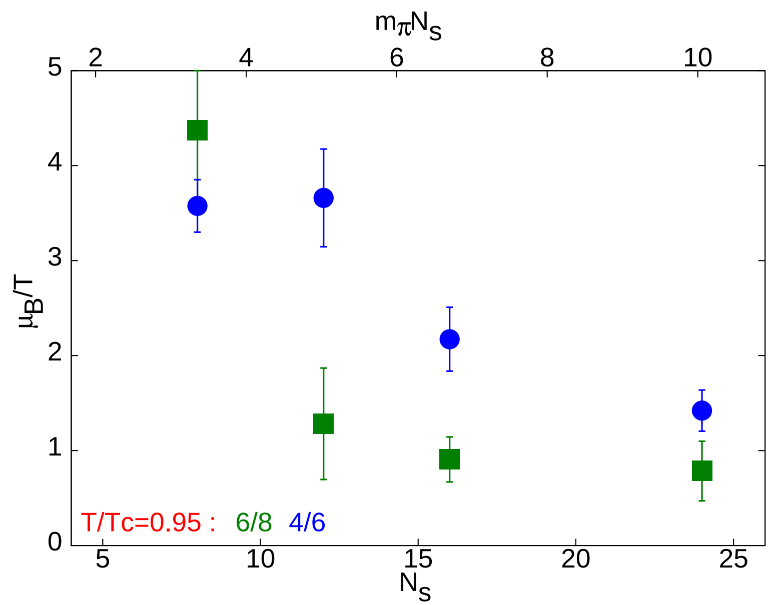

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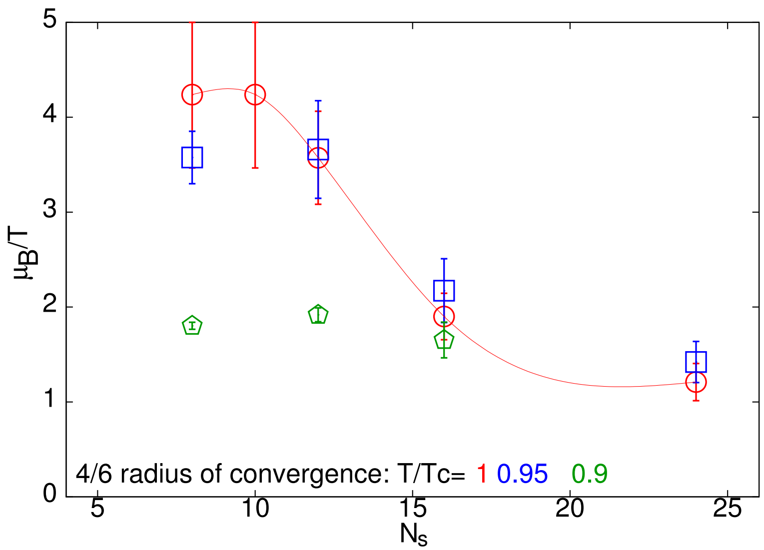

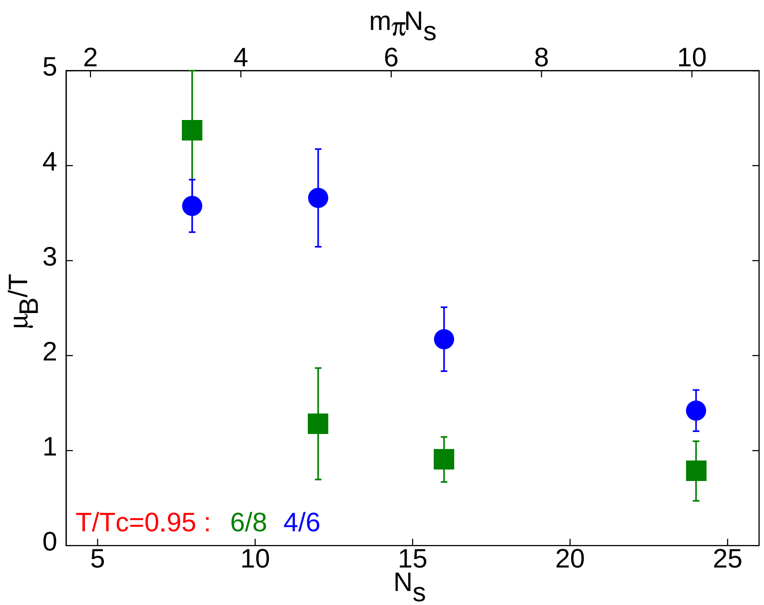

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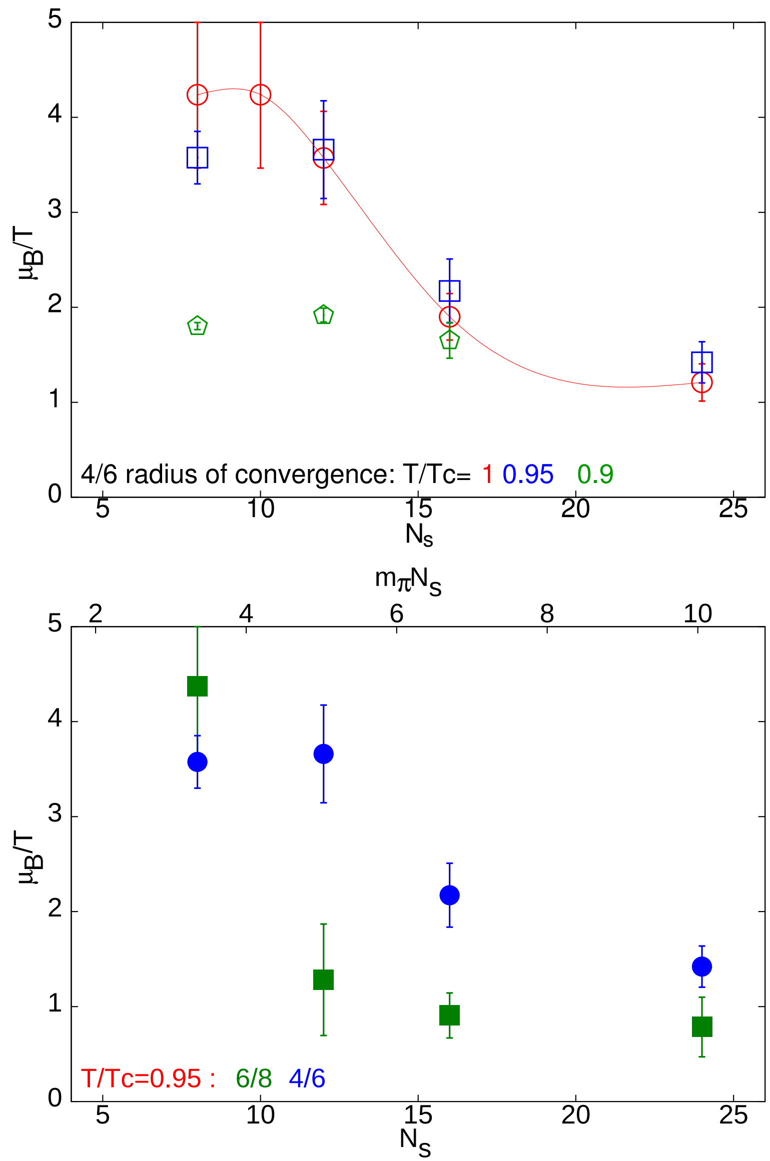

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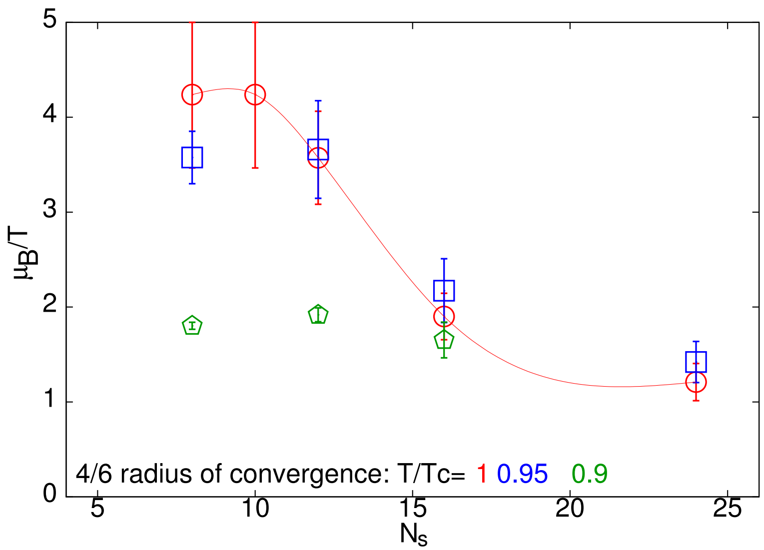

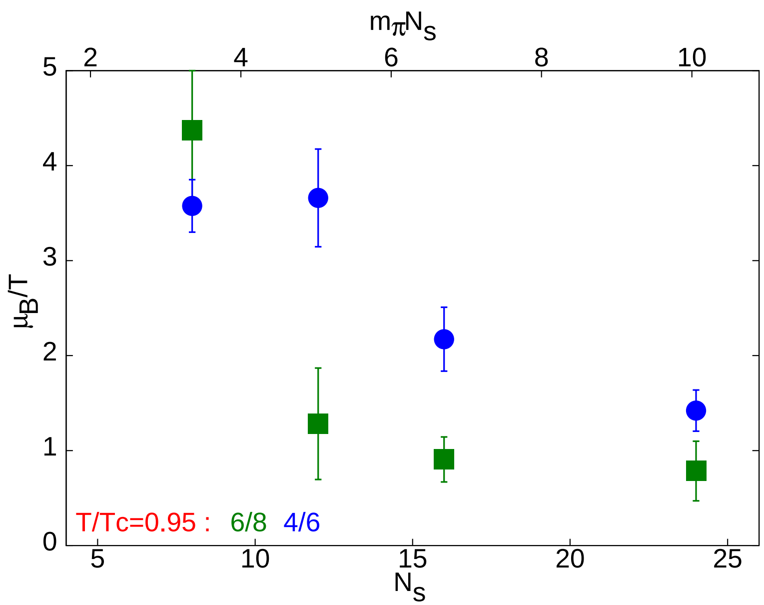

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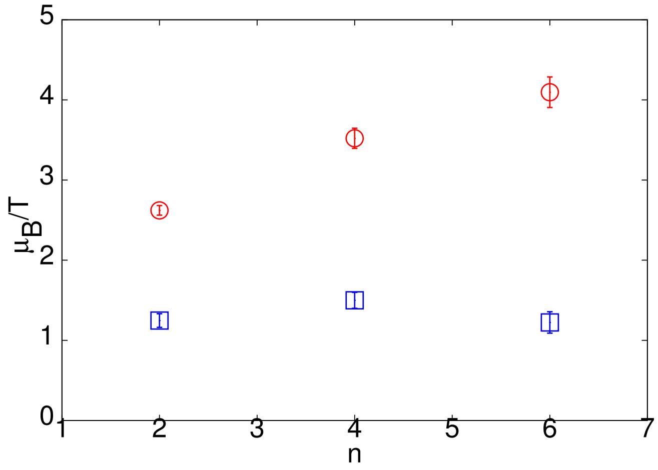

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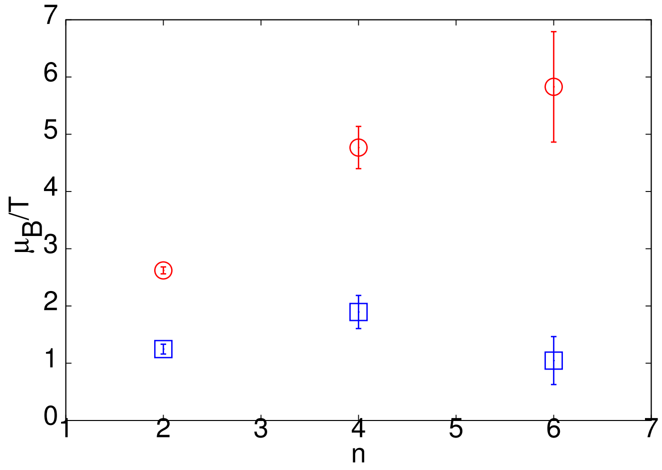

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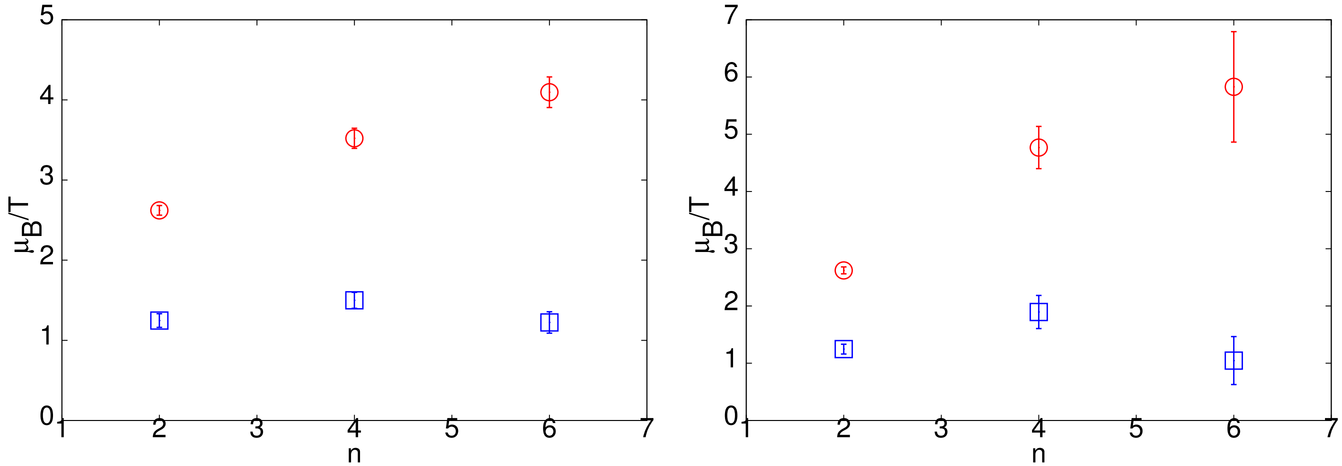

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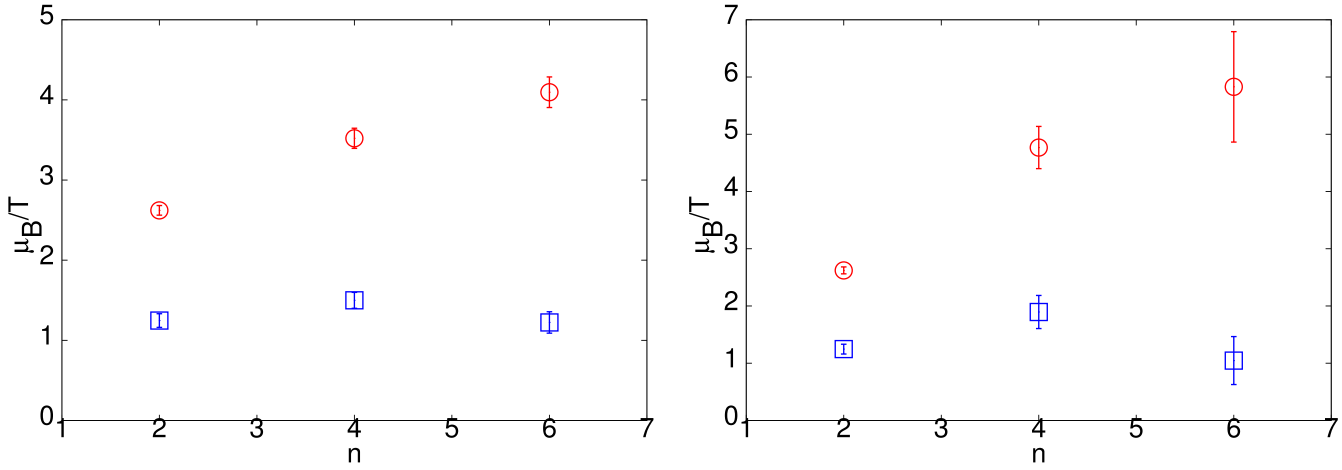

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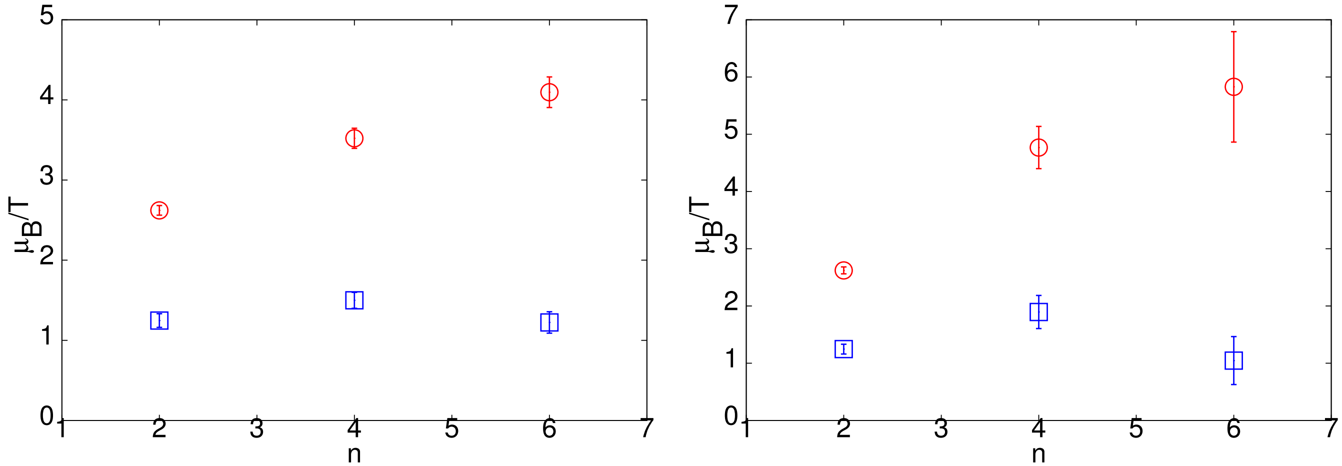

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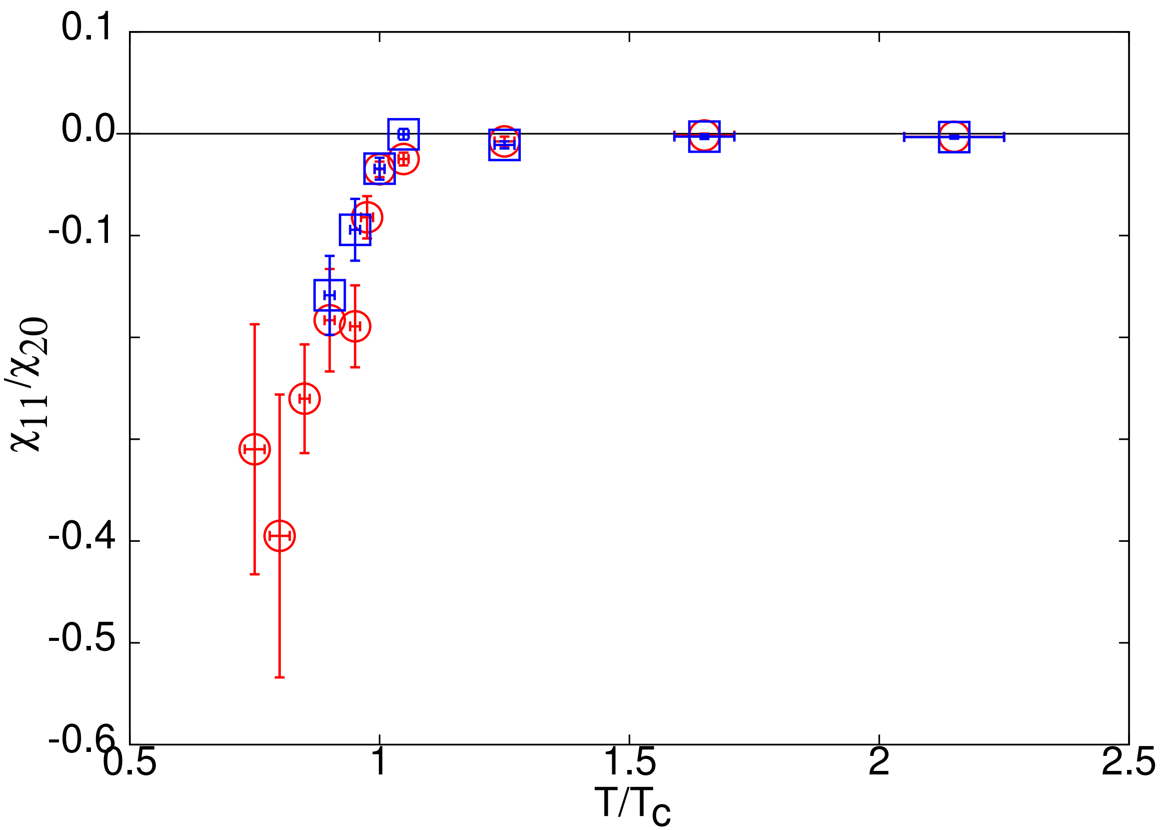
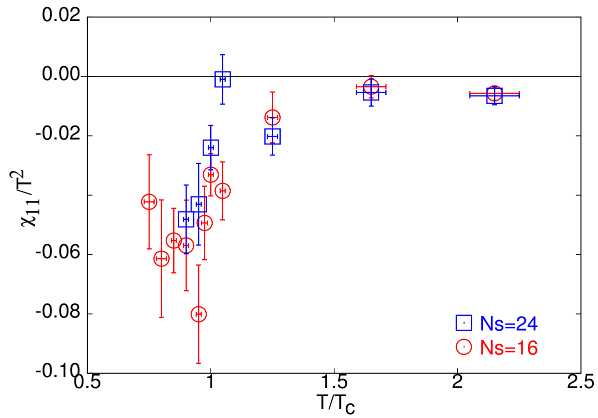
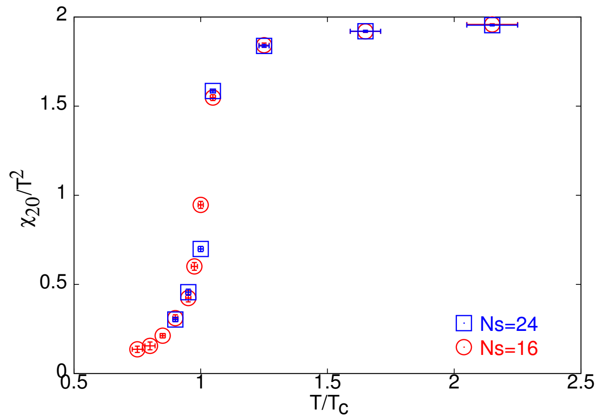
More Details

Measure of the seriousness of sign problem : Ratio χ_{11}/χ_{20}







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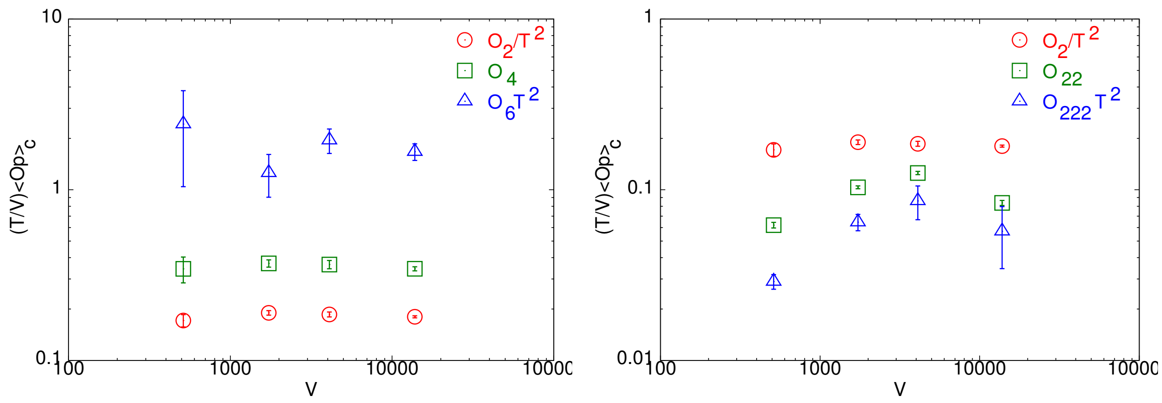
♠ We had earlier suggested to obtain more pairs of diverging terms by taking larger N_f.

♠ E.g. $T/V\langle\mathcal{O}_{22}\rangle_c$ should be finite as it is a combination of Taylor Coeffs.



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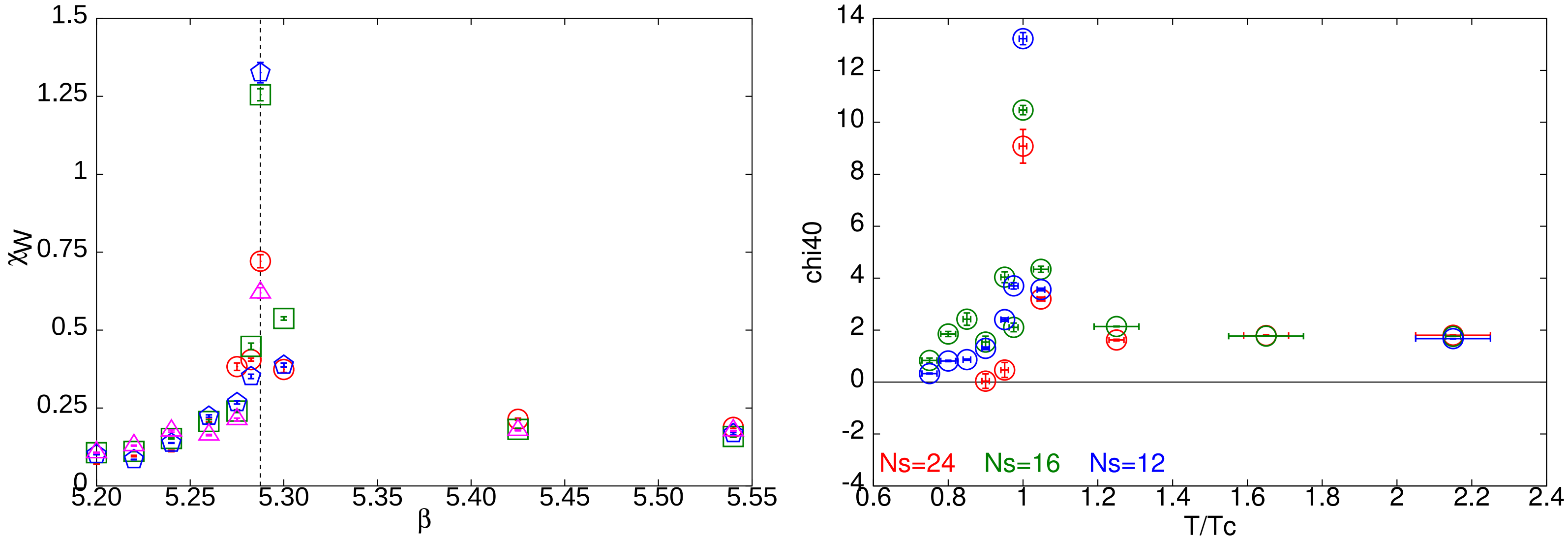

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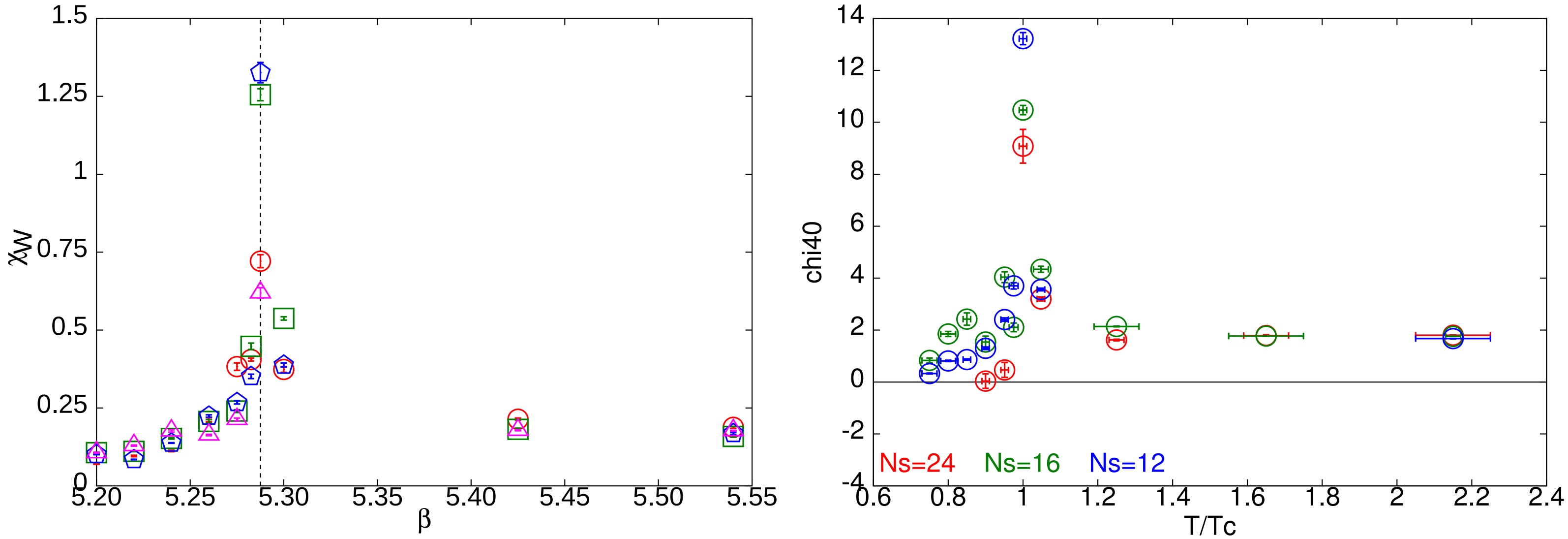

♠ Similar behaviour in higher order terms as well.

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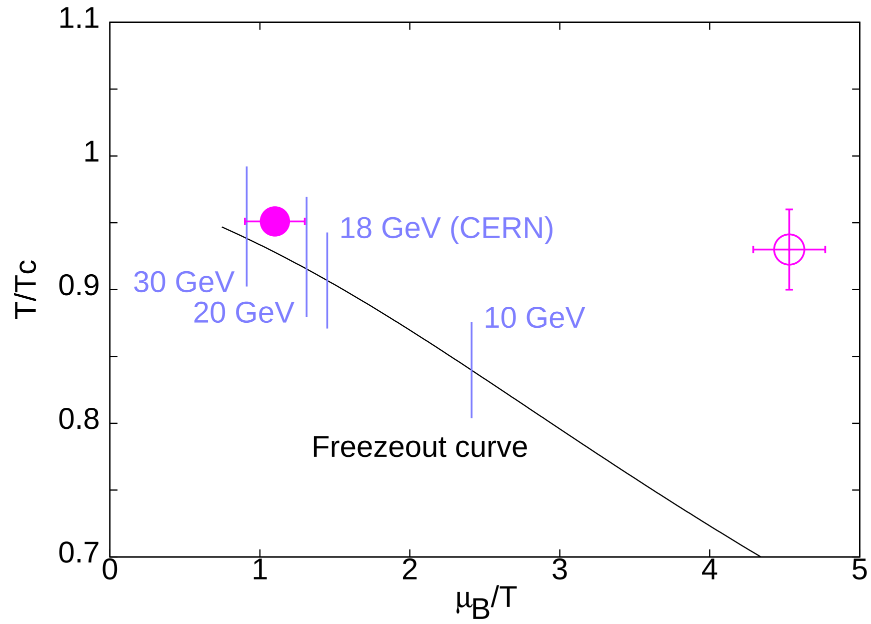

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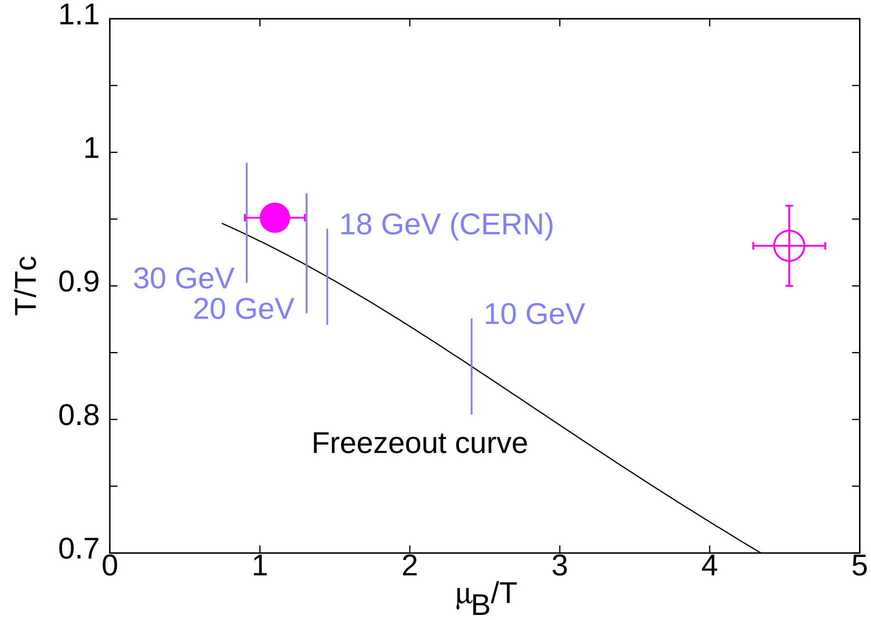

Heavy Ion Collisions in CERN Geneva, and BNL, New York, have seen tell-tale signs of QGP : Many surprises already and more excitement likely to come.

m_ρ/T_c	m_π/m_ρ	m_N/m_ρ	$N_s m_\pi$	flavours	T^E/T_c	μ_B^E/T^E
5.372 (5)	0.185 (2)	—	1.9–3.0	2+1	0.99 (2)	2.2 (2)
5.12 (8)	0.307 (6)	—	3.1–3.9	2+1	0.93 (3)	4.5 (2)
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5.5 (1)	0.70 (1)	—	15.4	2	—	—

Table 1: Summary of critical end point estimates— the lattice spacing is $a = 1/4T$. N_s is the spatial size of the lattice and $N_s m_\pi$ is the size in units of the pion Compton wavelength, evaluated for $T = \mu = 0$. The ratio m_π/m_K sets the scale of the strange quark mass.

Results are sequentially from Fodor-Katz '04, Fodor-Katz '02, Gavai-Gupta, de Forcrand- Philipsen and Bielefeld-Swansea.