for QCD Critical Point

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Importance of Being Critical

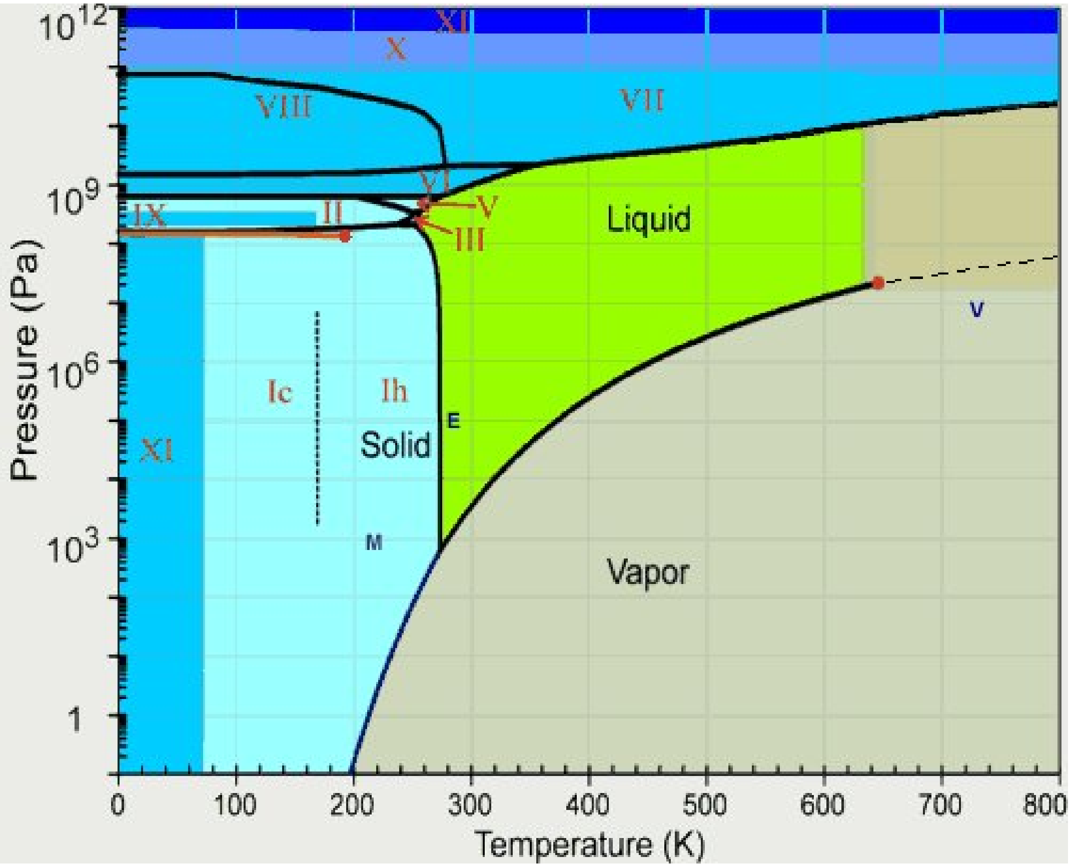
Lattice QCD Results

Searching Experimentally

Summary

Importance of Being Critical

Phase Diagram of Water



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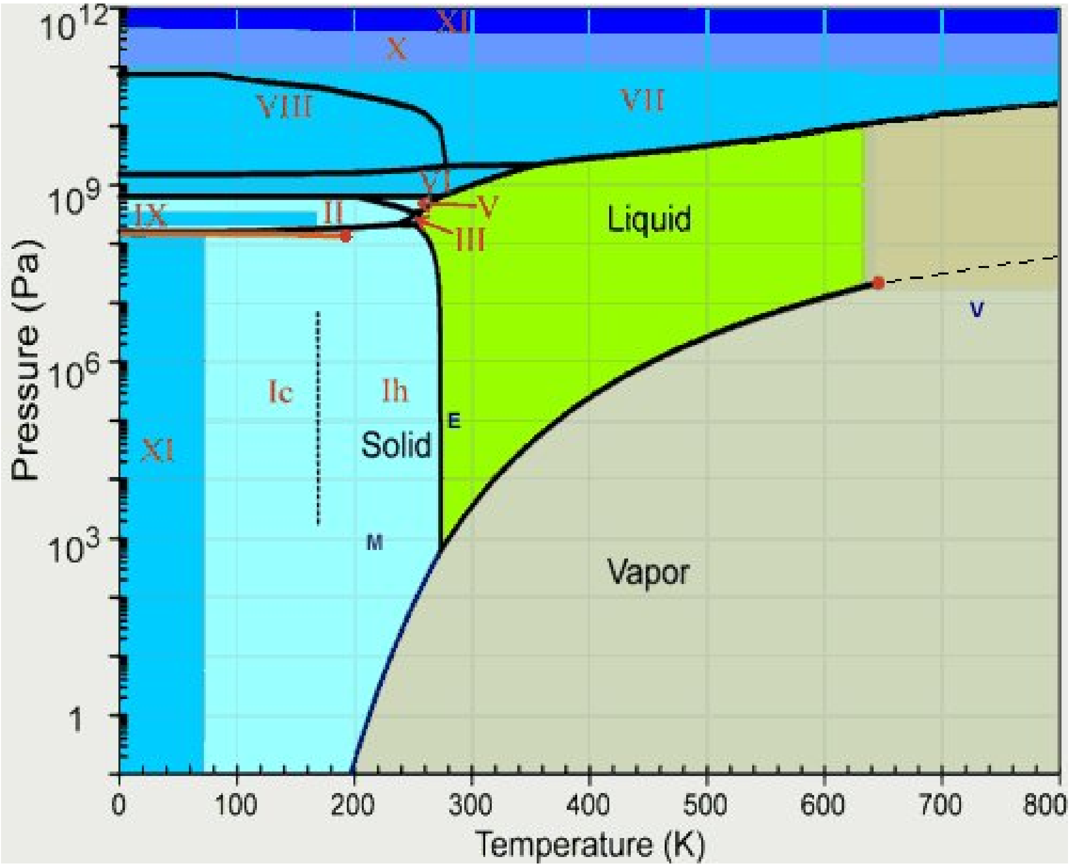

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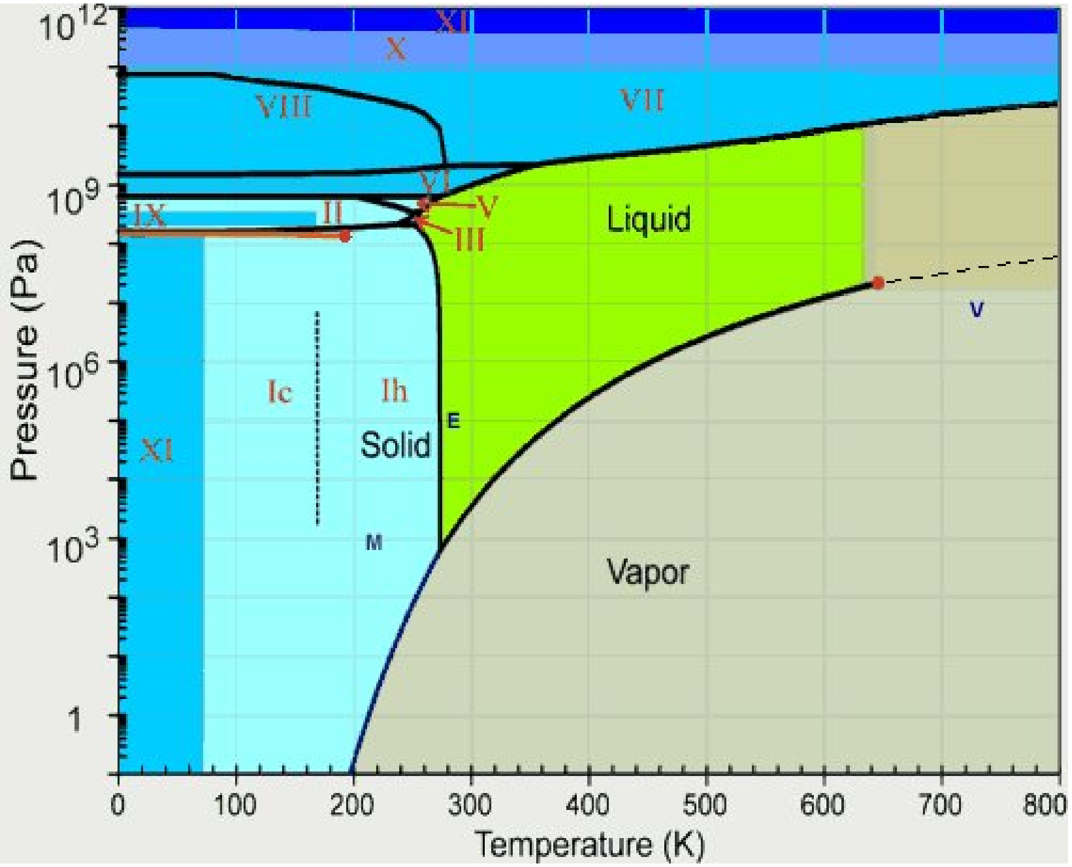

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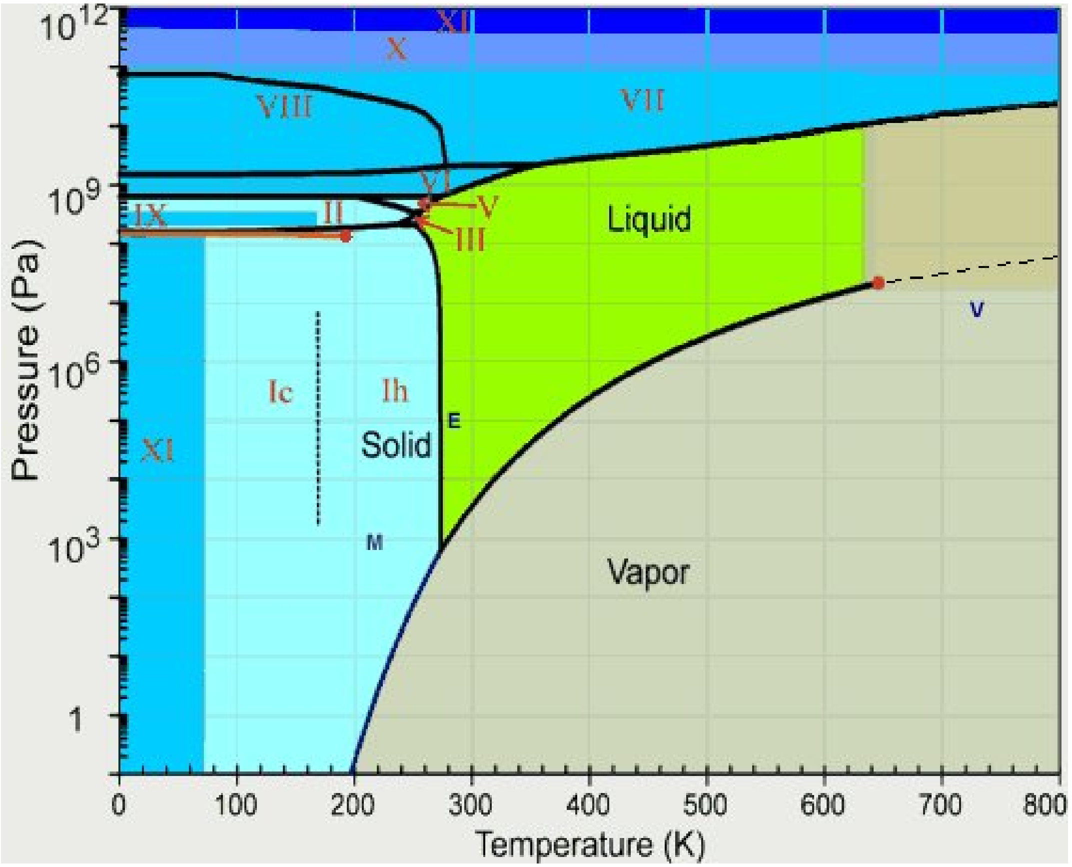

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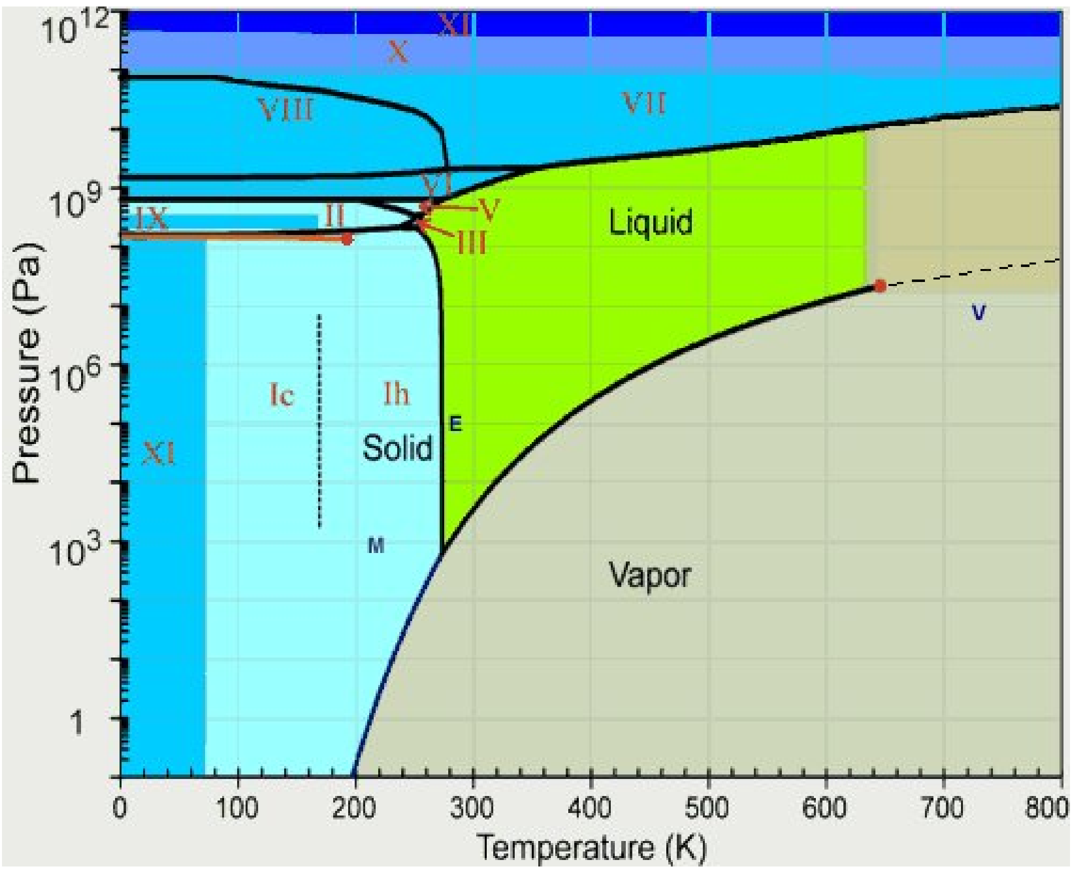

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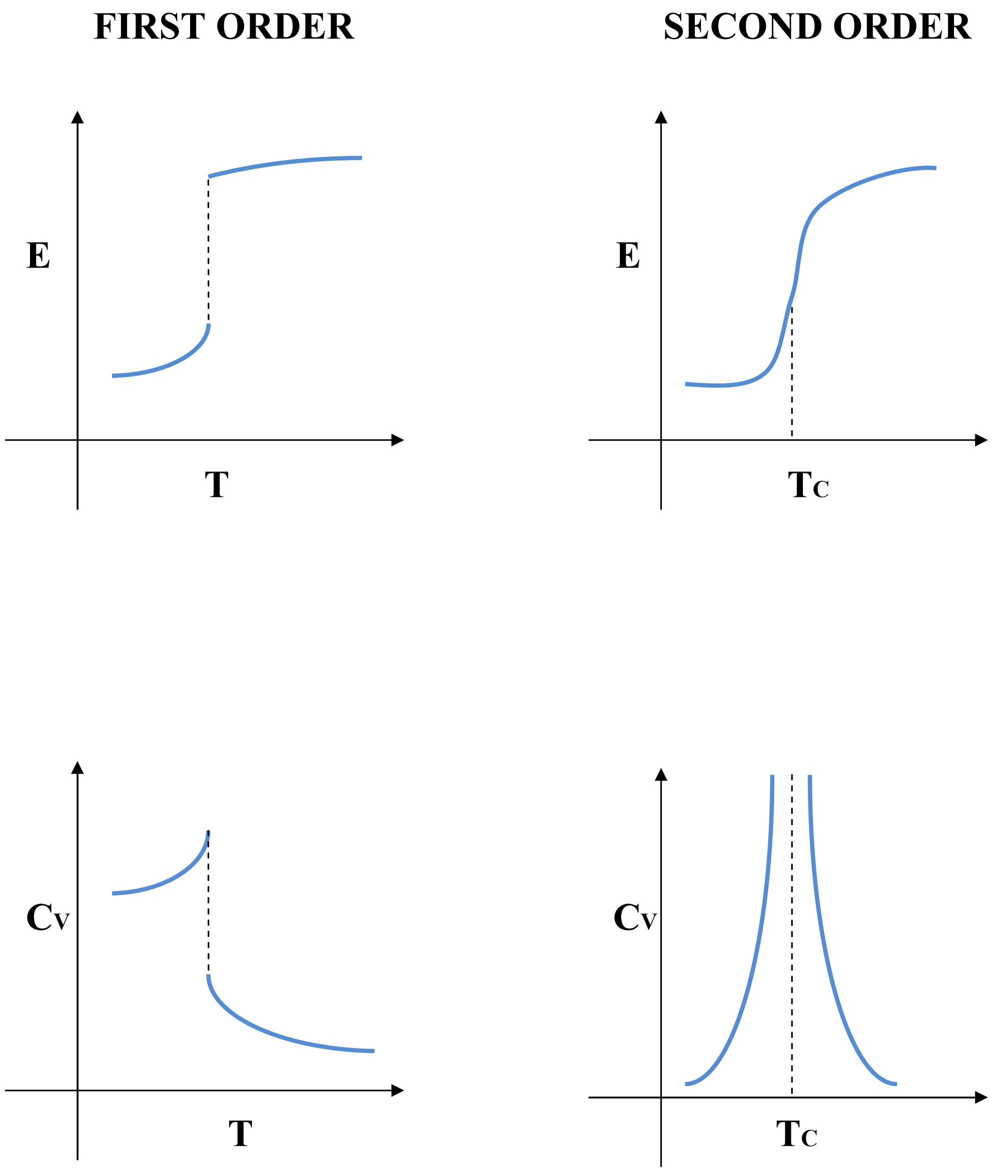
- Dielectric constant & Viscosity $\downarrow$.
- Many liquid fueled engines exploit such supercritical transitions.


FIRST ORDER
SECOND ORDER
E
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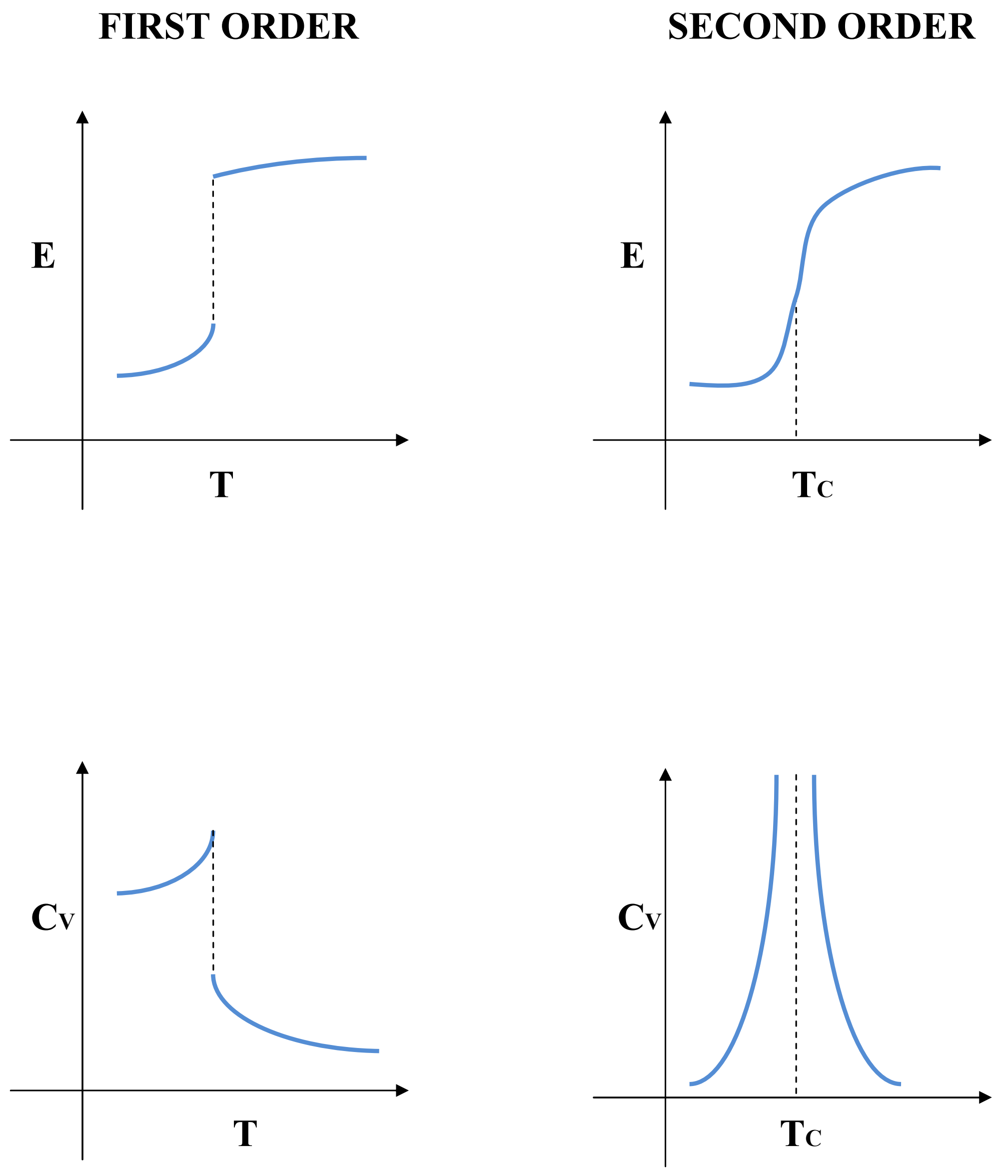
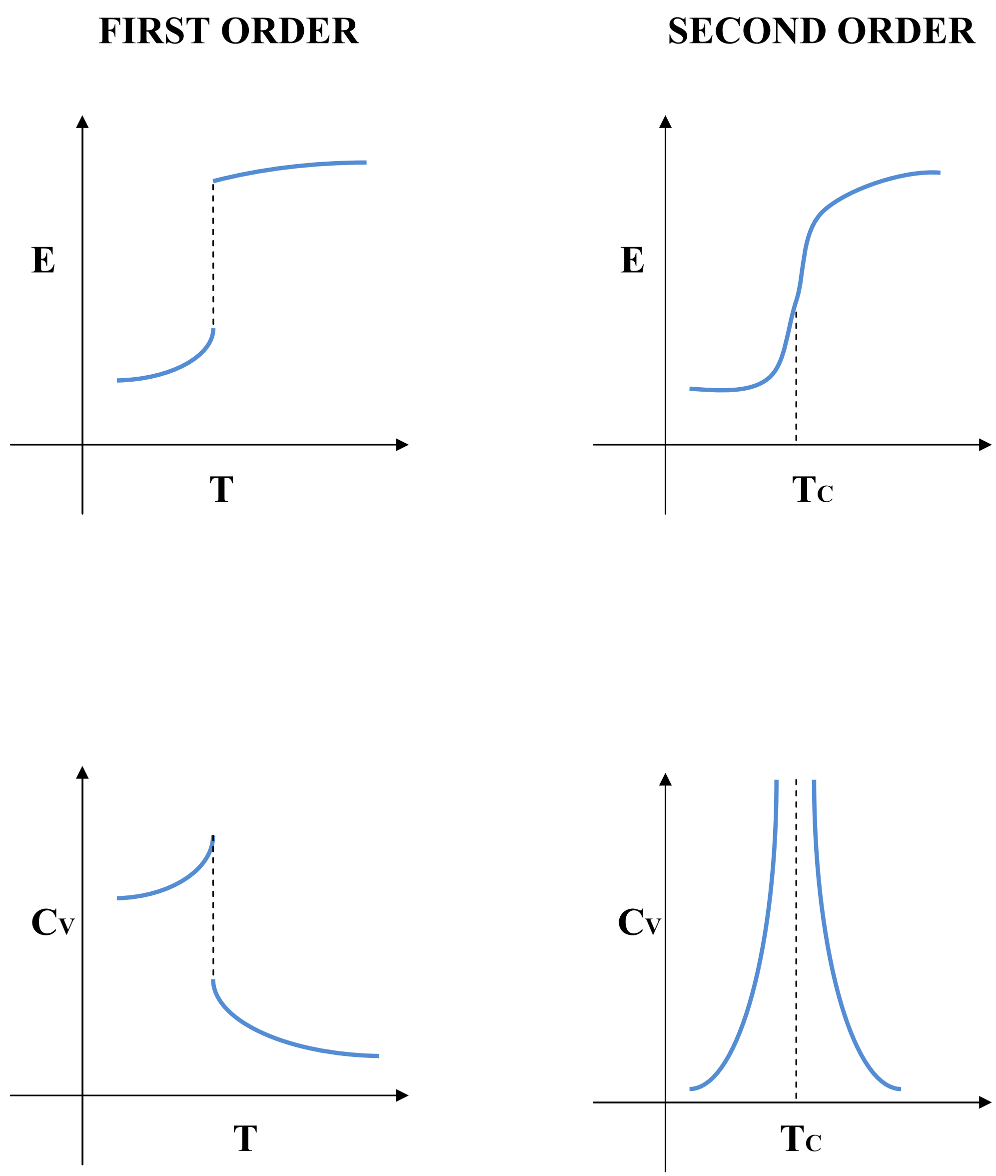


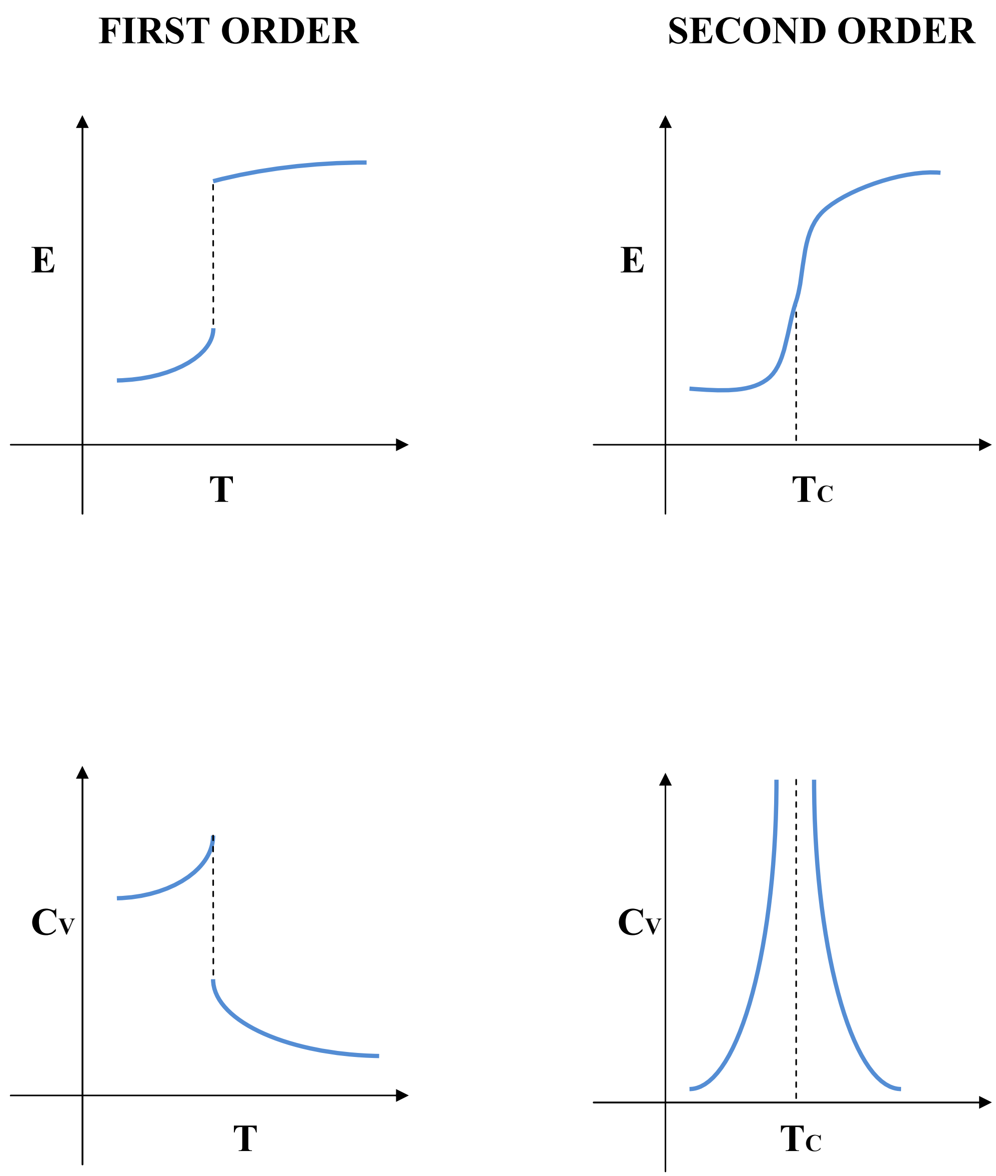
- Discontinuous ϵ – Nonzero Latent Heat– & finite C_v $\rightarrow$ First order PT.



- Discontinuous ϵ – Nonzero Latent Heat– & finite C_v $\longrightarrow$ First order PT.

- Continuous ϵ, & diverging C_v $\longrightarrow$ Second order PT.

- In(Finite) Correleation Length at 2nd (1st) Order transition.



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- In(Finite) Correleation Length at 2nd (1st) Order transition.
- "Cross-over" – mere rapid change in ϵ, with maybe a sharp peaked C_v.

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- For (N_f) massless particles, A or B do **not** change into each other: Chiral Symmetry $(SU(N_f) \times SU(N_f))$.

- Interactions can break the chiral symmetry dynamically, leading to effective masses for these particles.
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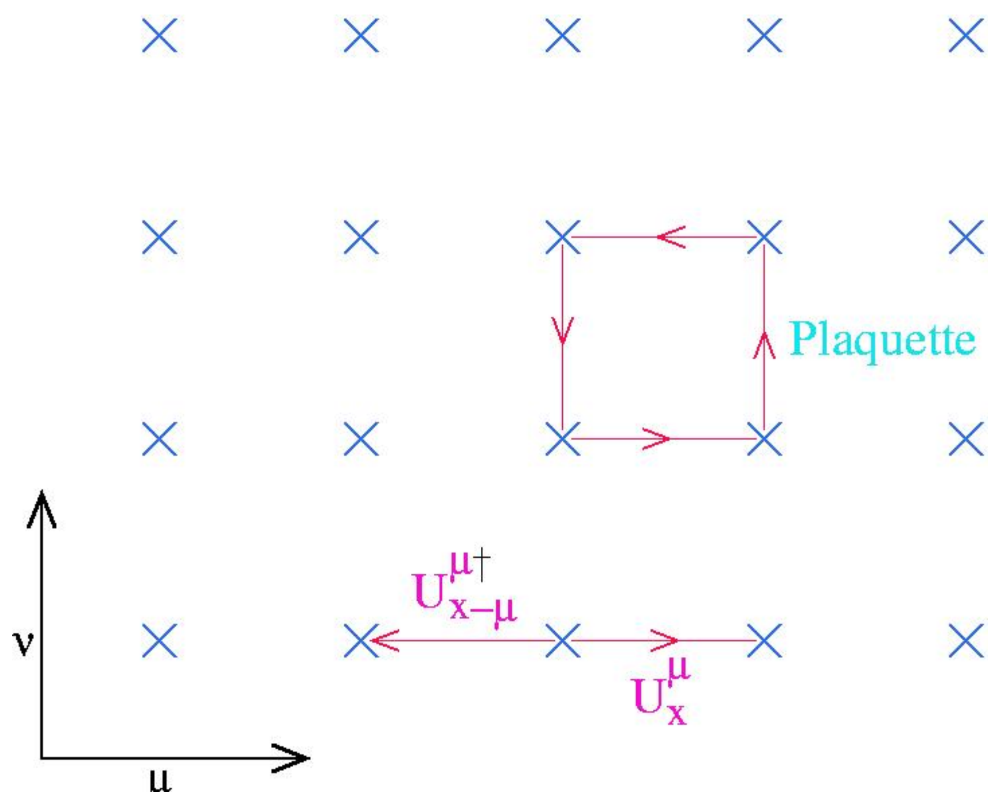
- New States at High Temperatures/Density expected on basis of models.
- Quark-Gluon Plasma is such a phase. It presumably filled our Universe a few microseconds after the Big Bang & can be produced in Relativistic Heavy Ion Collisions.
- Much richer structure in QCD : Quark Confinement, Dynamical Symmetry Breaking.. Lattice QCD should shed light on this all.

Basic Lattice QCD

- Discrete space-time : Lattice spacing a UV Cut-off.
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$



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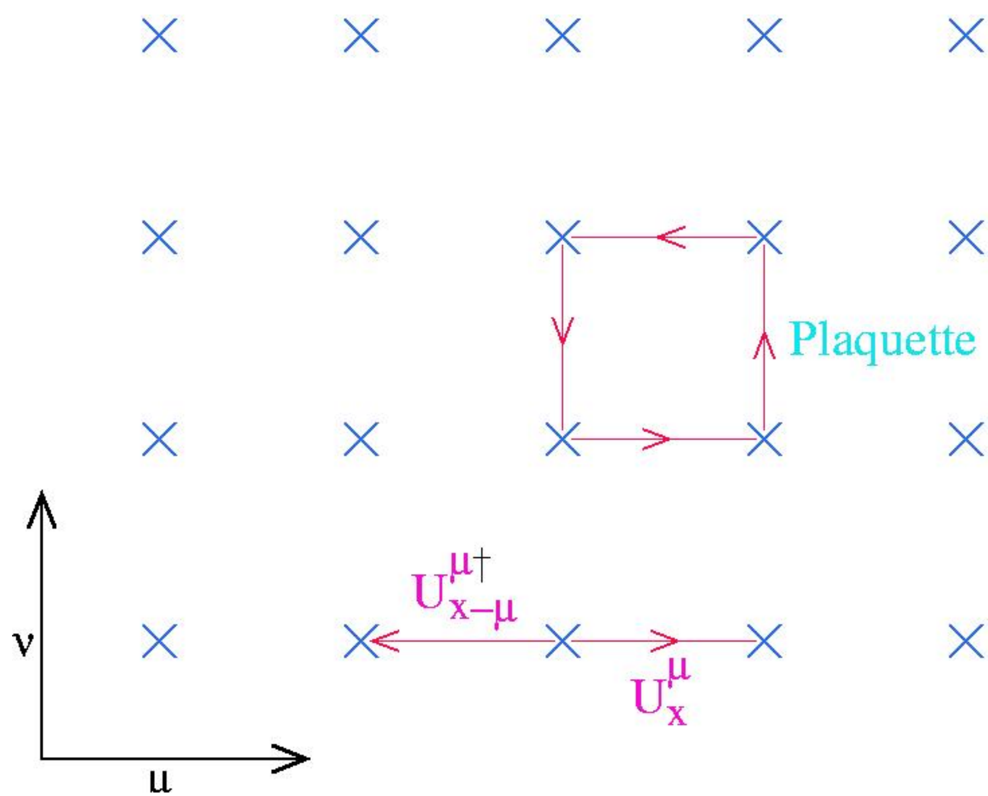
- Gauge invariance : Actions from Closed Wilson loops, e.g., plaquette.
- Fermion Actions : Staggered, Wilson, Overlap..



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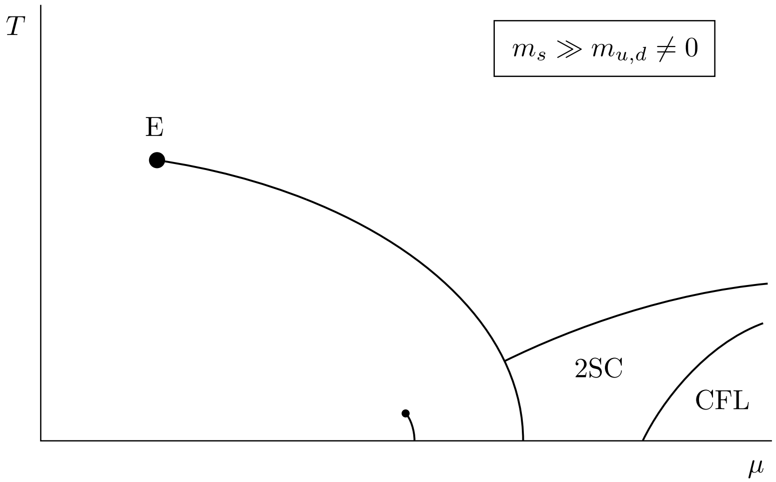
Expected QCD Phase Diagram



From Rajagopal-Wilczek Review

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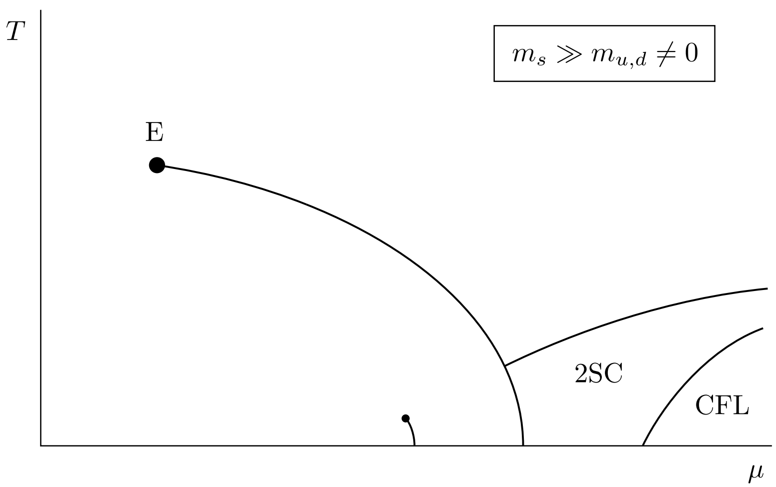

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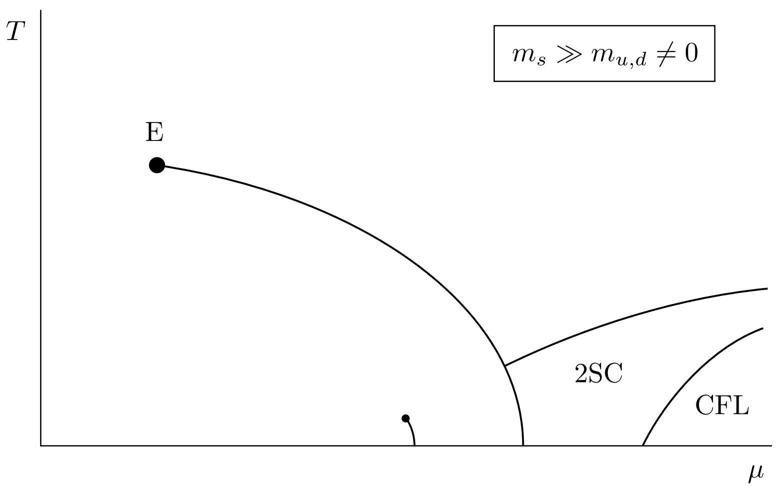

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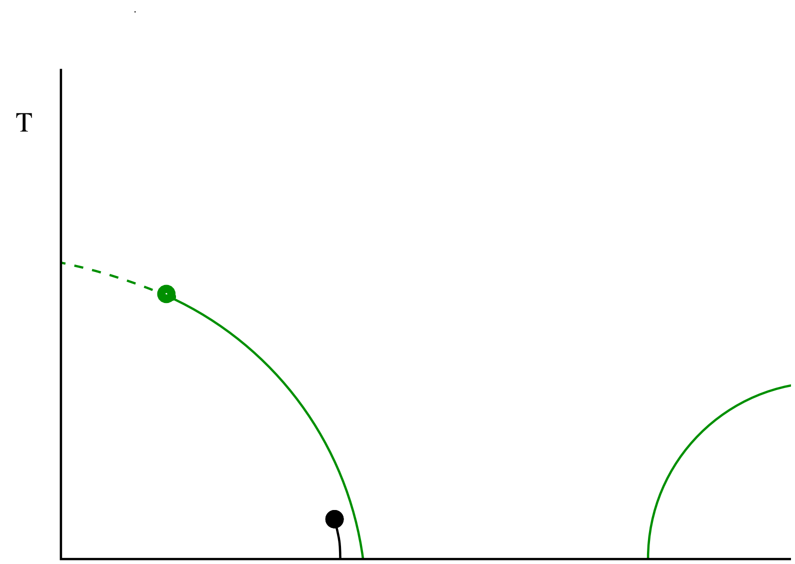

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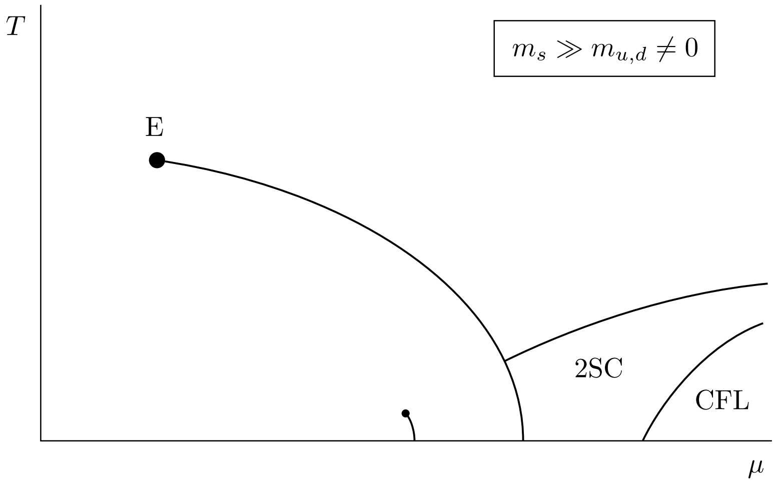

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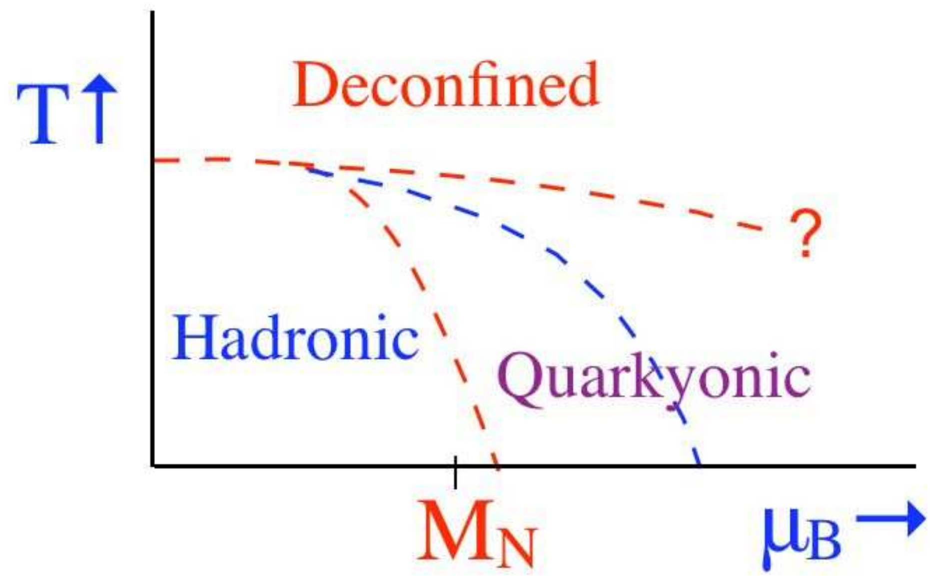

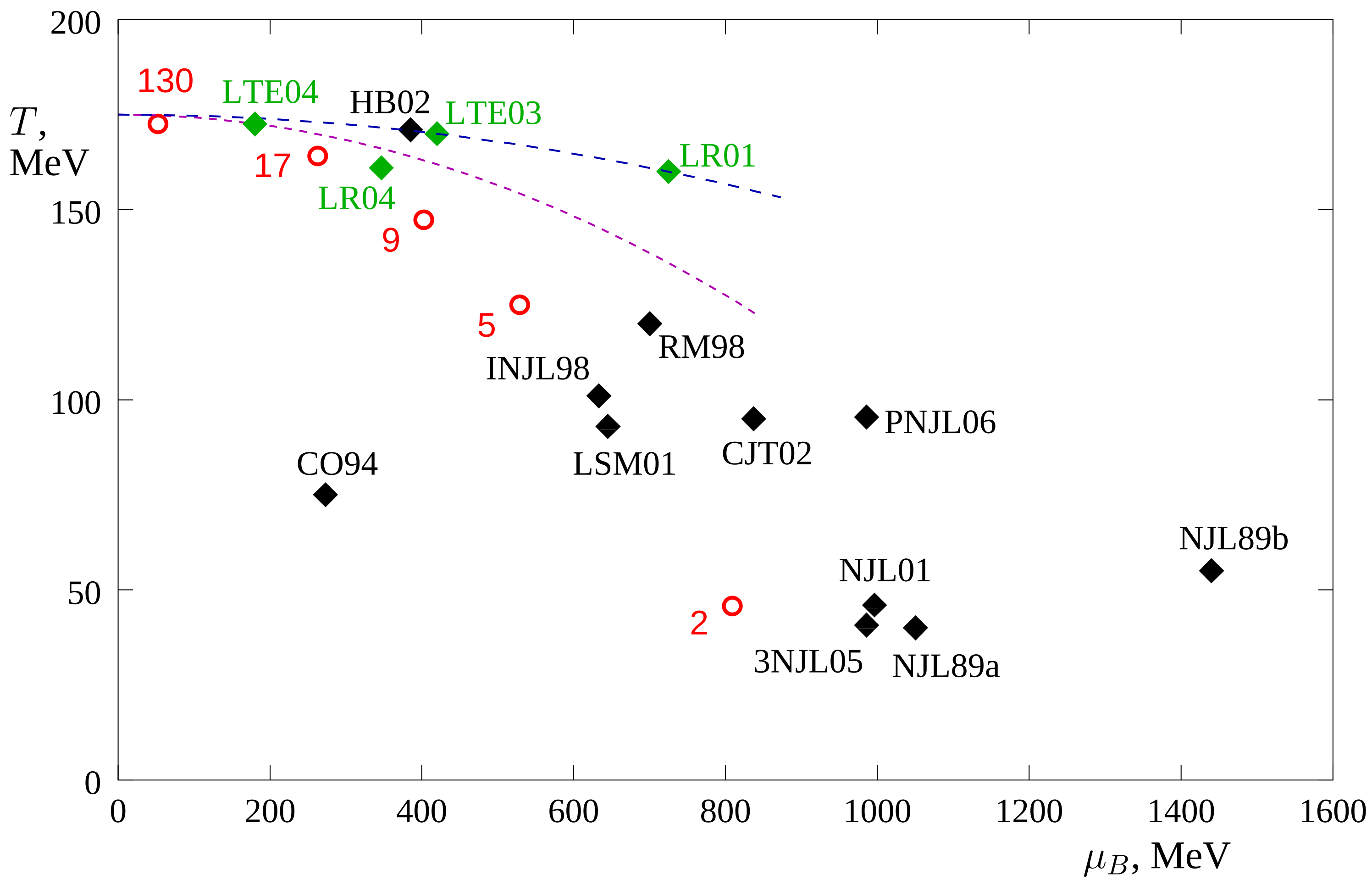

From M. Stephanov, Lattice 2007 Plenary.

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- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics.

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- Domain Wall or Overlap Fermions better. BUT Computationally expensive and introduction of μ unfortunately breaks chiral symmetry ! (Banerjee, Gavai & Sharma PRD 2008; PoS (Lattice 2008); PRD 2009)

The $\mu \neq 0$ problem

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det}\ M(m_f, \mu_f) \quad ,$$

and the thermal expectation value of an observable $\mathcal{O}$ is

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However, det M is a complex number for any $\mu \neq 0$: The Phase/sign problem

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- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

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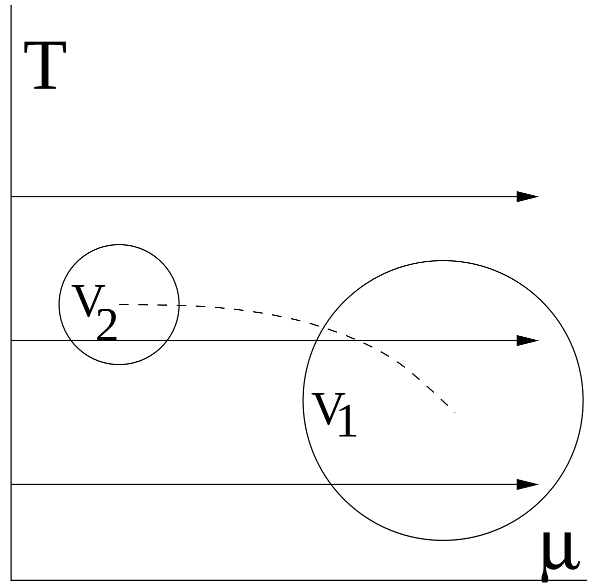

We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

How Do We Do This Expansion?

Canonical definitions yield various number densities and susceptibilities :

$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}$ and $\chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$.

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \qquad (1)$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence can be obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}}\right)^{1/n}$. We use both and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- Coefficients for the off-diagonal susceptibility, χ_{11}, can be constructed similarly.

- The ratio χ_{11}/χ_{20} can be shown to yield the ratio of widths of the measure in the imaginary and real directions at $\mu = 0$.

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Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.

- $m_\rho/T_c = 5.4 \pm 0.2$ and $m_\pi/m_\rho = 0.31 \pm 0.01$ (MILC)

- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)

- Lattice used : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, arXiv:0806.2233, PRD 2009). Needed to determine β_c. Our result ($\beta_c = 5.425(5)$) well bracketed by MILC for $m/T_c = 0.075$ and 0.15.

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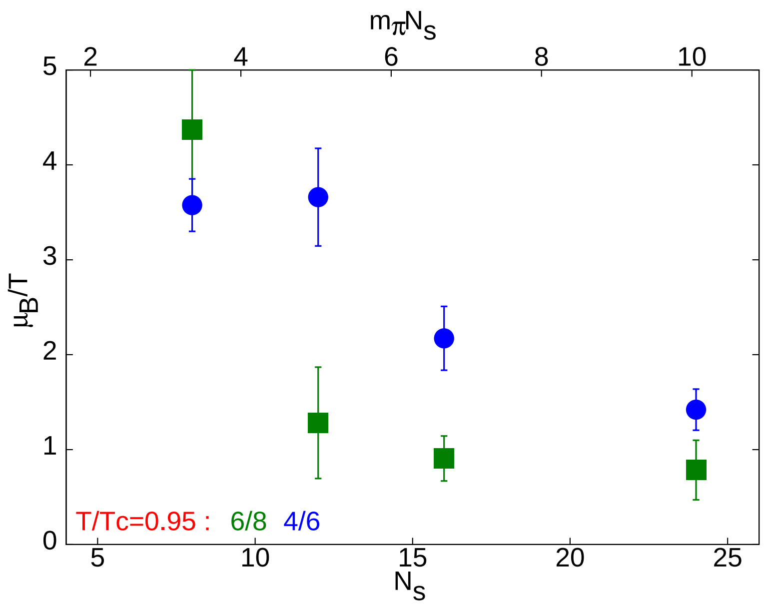
- New Simulations made at $T/T_c =$ 0.89(1), 0.92(1), 0.94(1), 0.97(1), 0.99 (1) 1.00(1), 1.21(1), 1.33(1), 1.48(3) and 1.92(5)
- Typical stat. 50-200 in max autocorrelation units.



$m_\pi N_S$
μ_B/T
T/Tc=0.95 : 6/8 4/6
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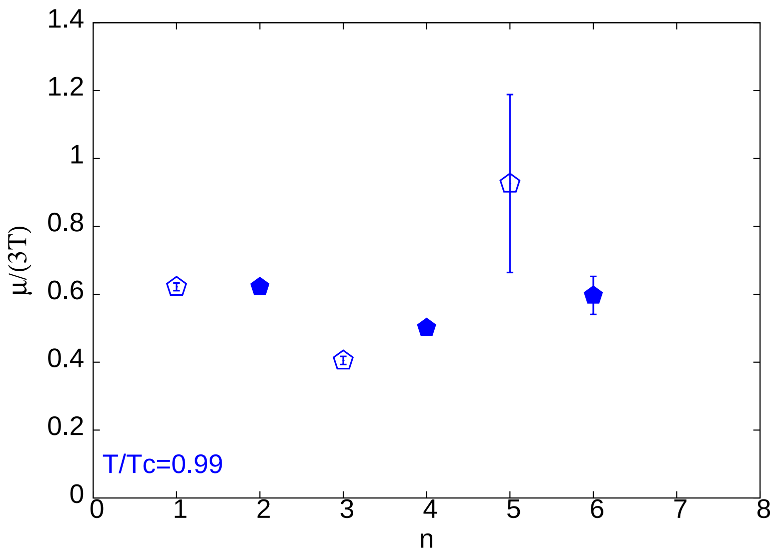
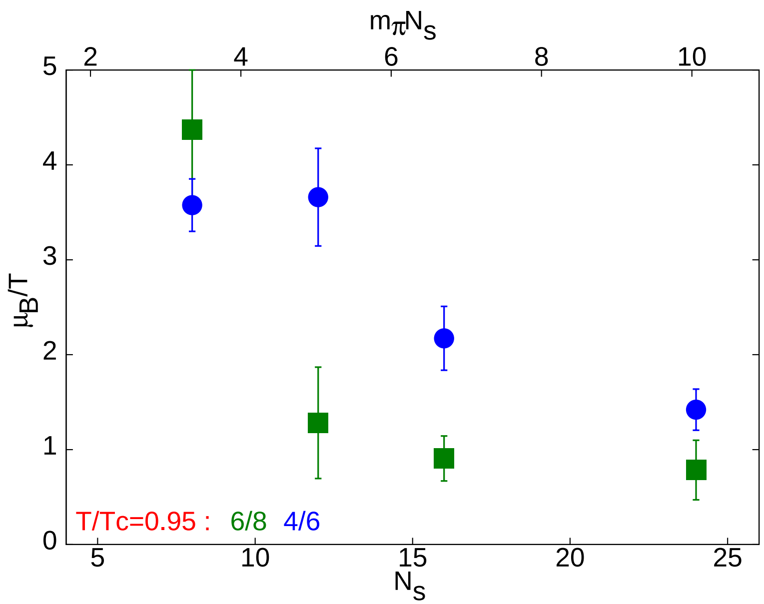


$\mu/(3T)$
T/Tc=0.99
n



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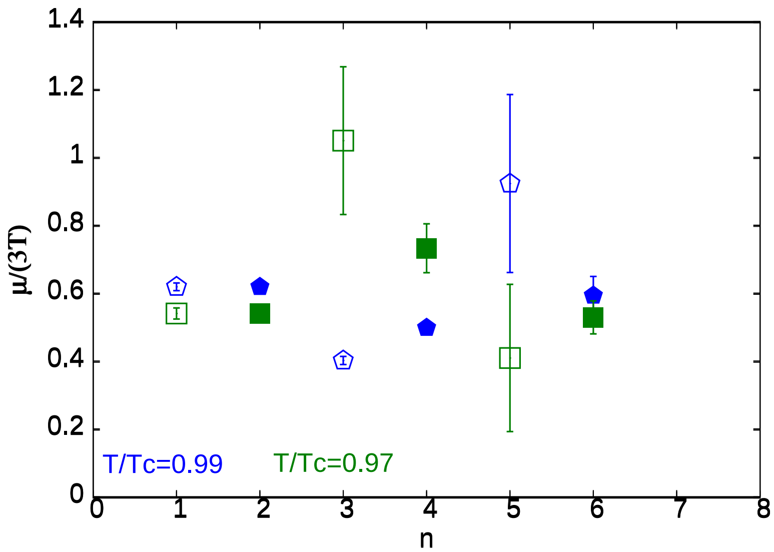
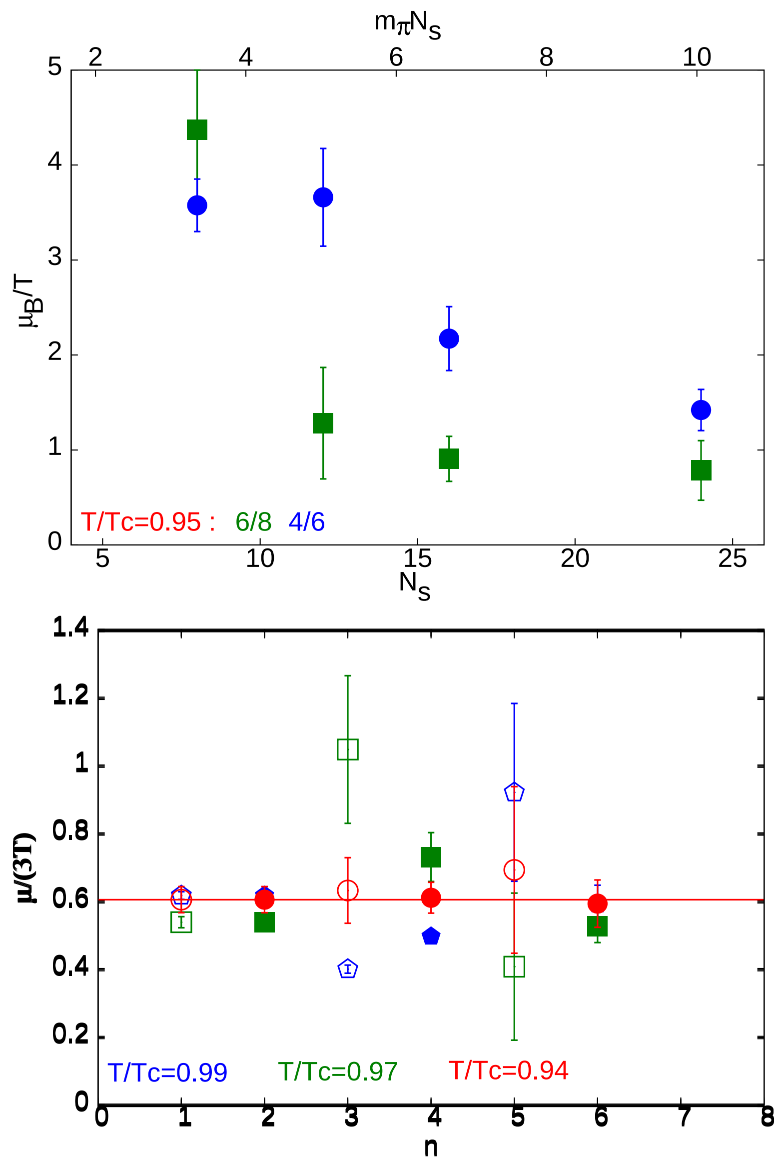
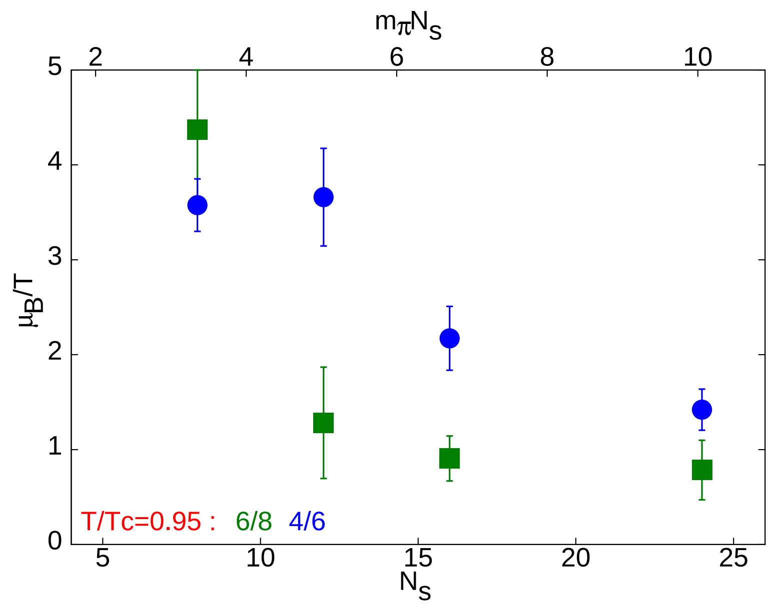
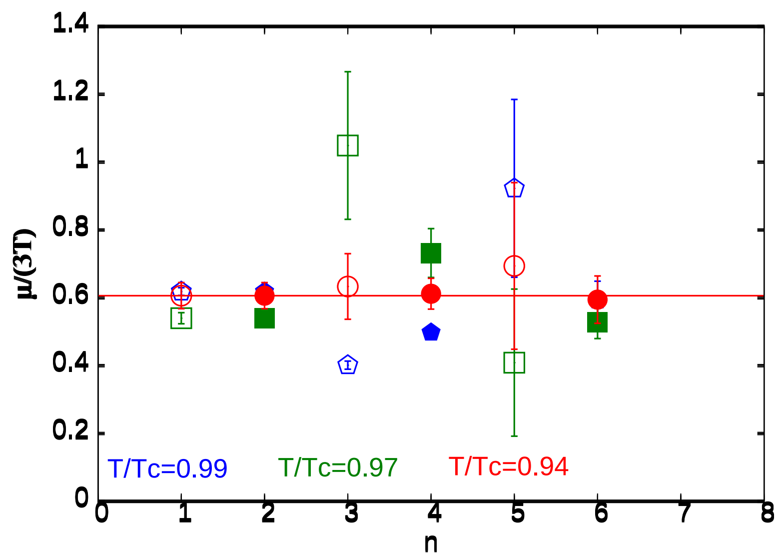
N_S



$\mu/(3T)$
T/Tc=0.99
T/Tc=0.97
n


$m_\pi N_S$
2 4 6 8 10
5 4 3 2 1 0
μ_B/T
T/Tc=0.95 : 6/8 4/6
5 10 15 20 25
N_S
1.4 1.2 1 0.8 0.6 0.4 0.2 0
μ/(3T)
T/Tc=0.99
T/Tc=0.97
T/Tc=0.94
0 1 2 3 4 5 6 7 8
n





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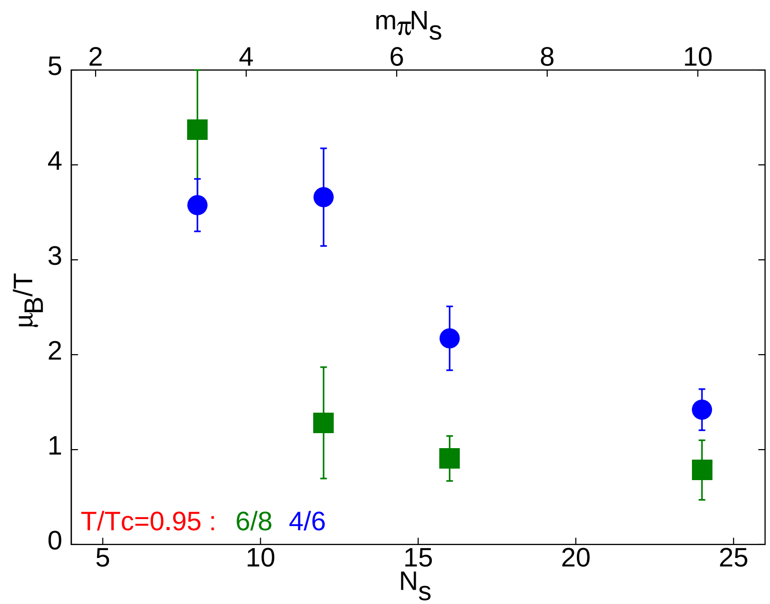

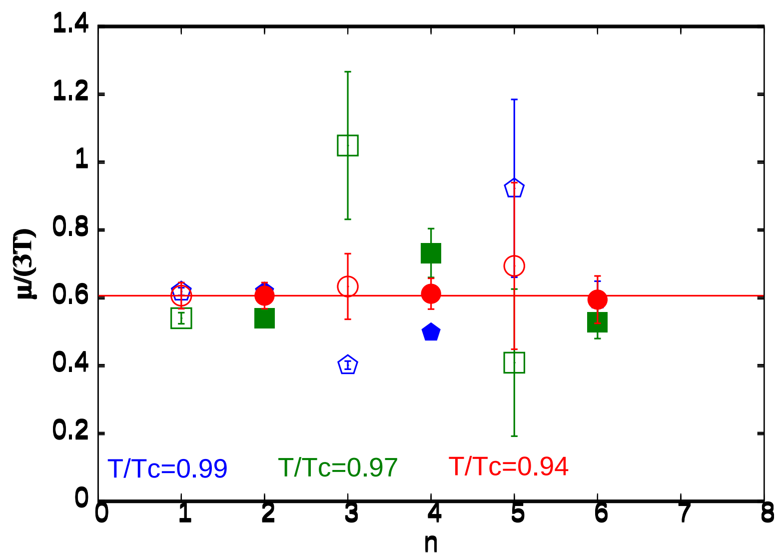

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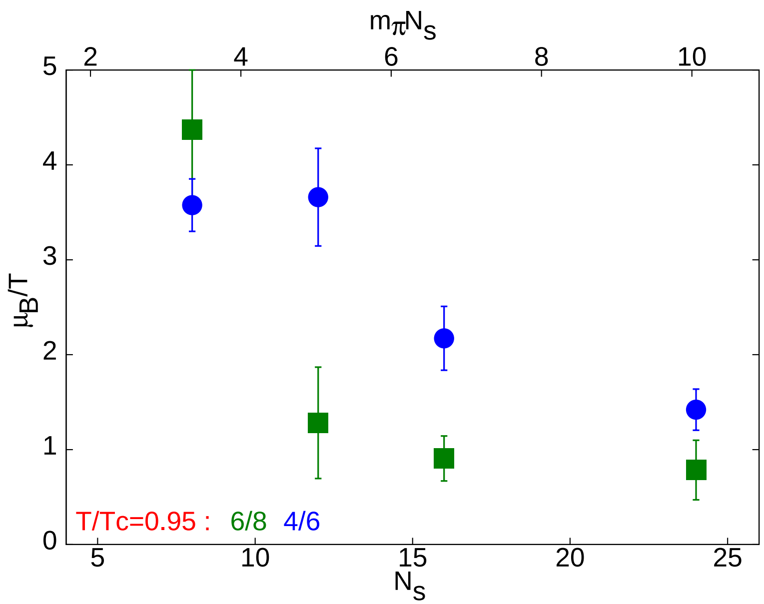

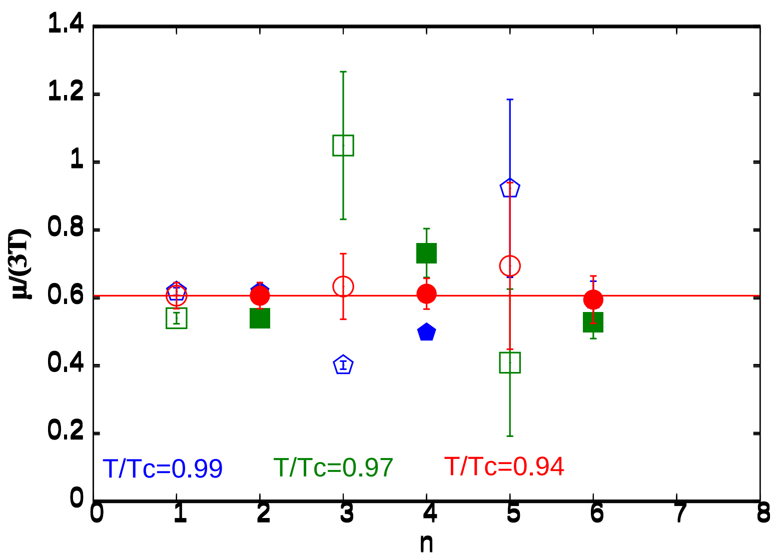

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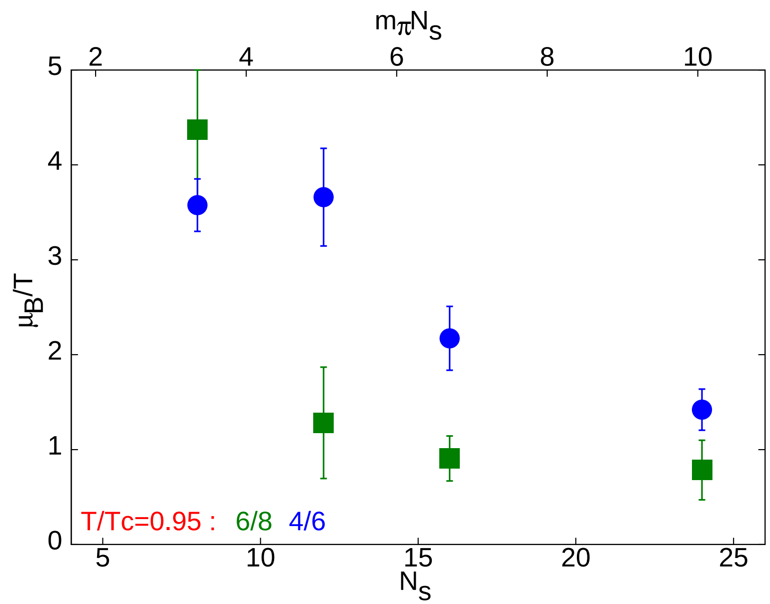

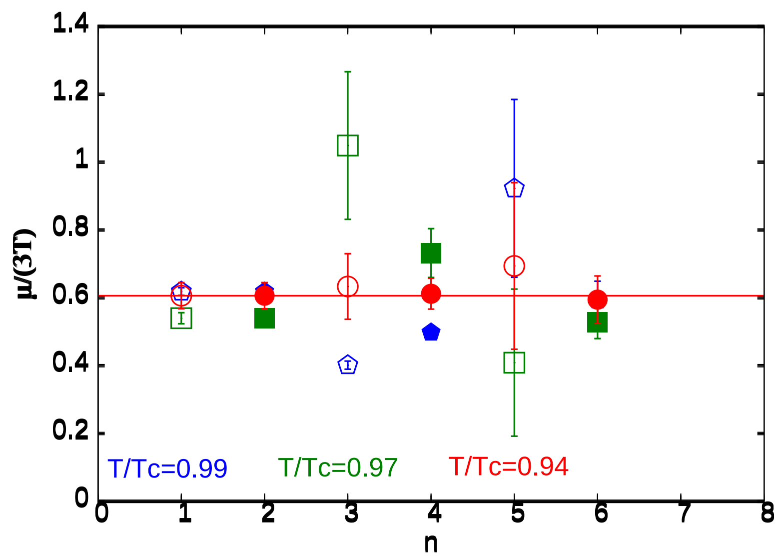

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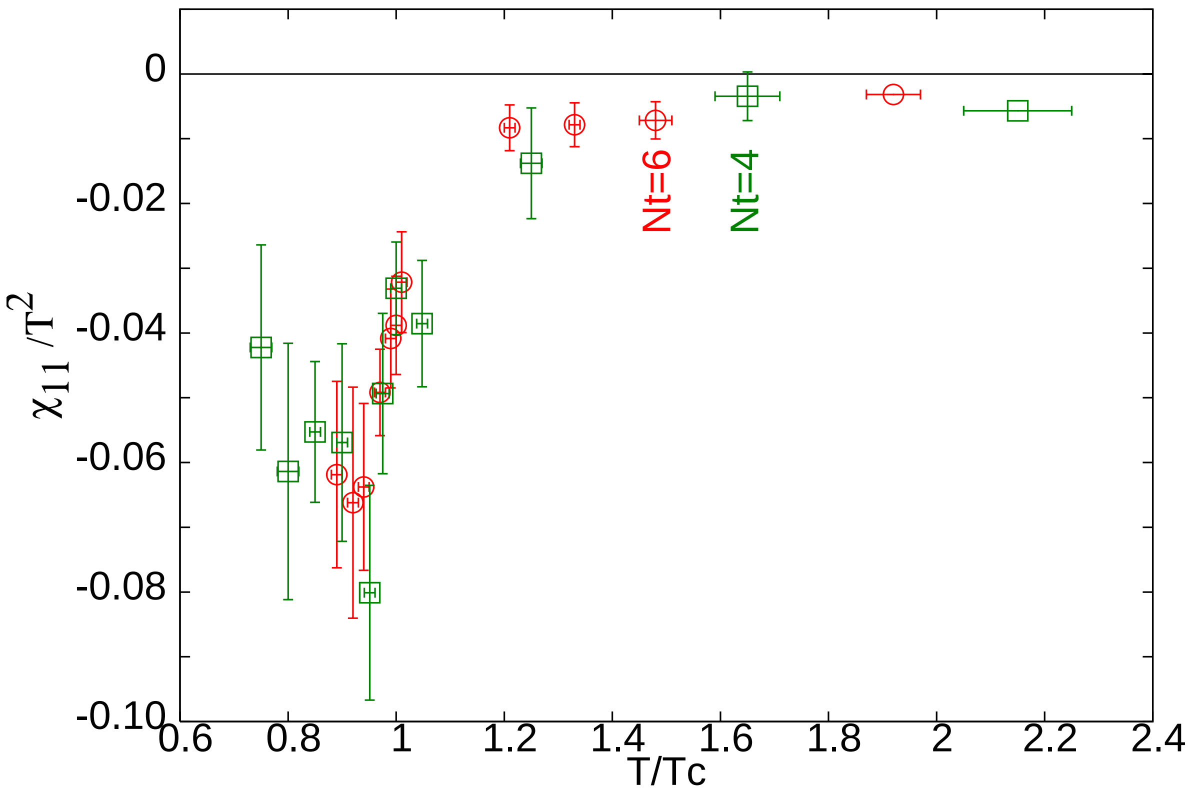
- Critical point shifted to smaller $\mu_B/T \sim 1 - 2$.

More Details

Measure of the seriousness of sign problem : χ_{11}; $N_t = 4$ & 6 agree.



Cross Check on μ^E/T^E

♠ Use Padé approximants for the series to estimate the radius of convergence.

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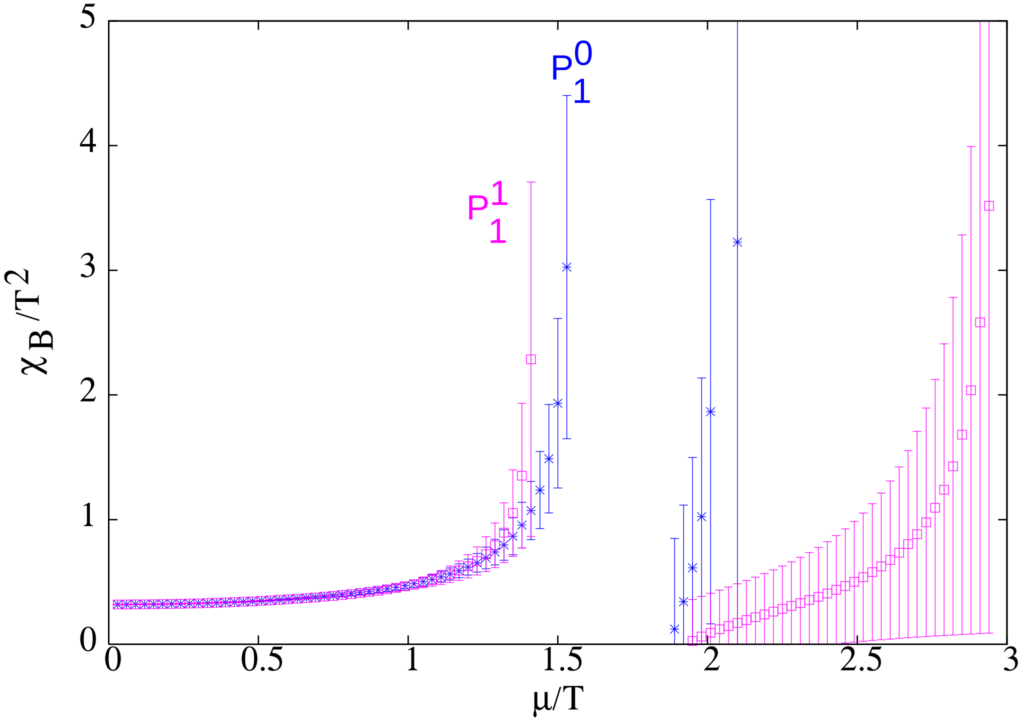

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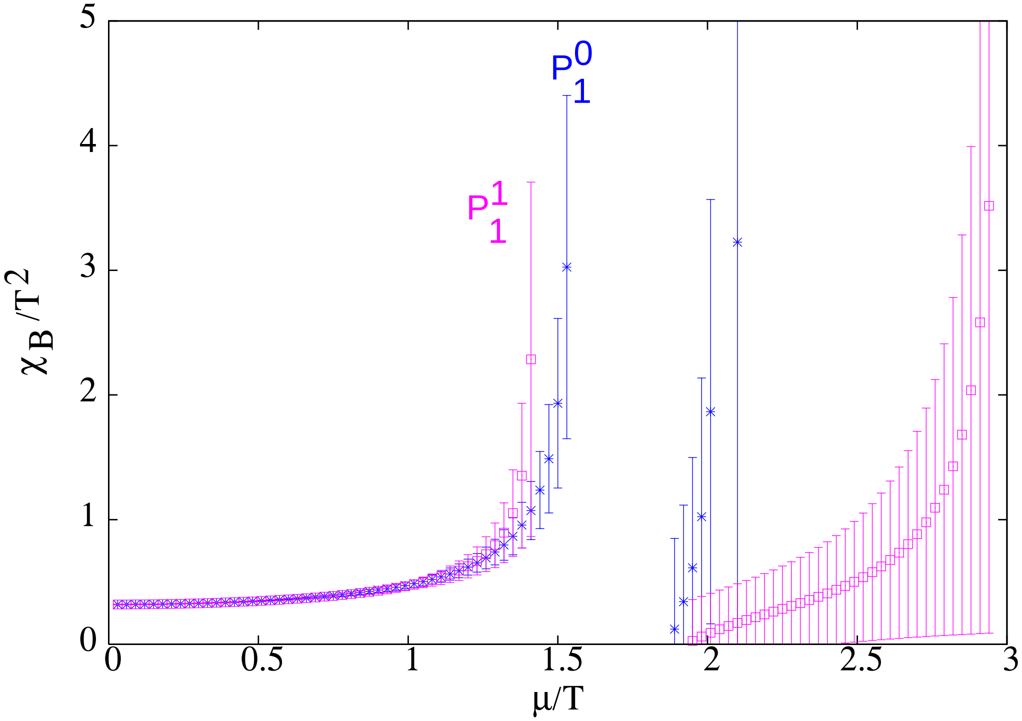
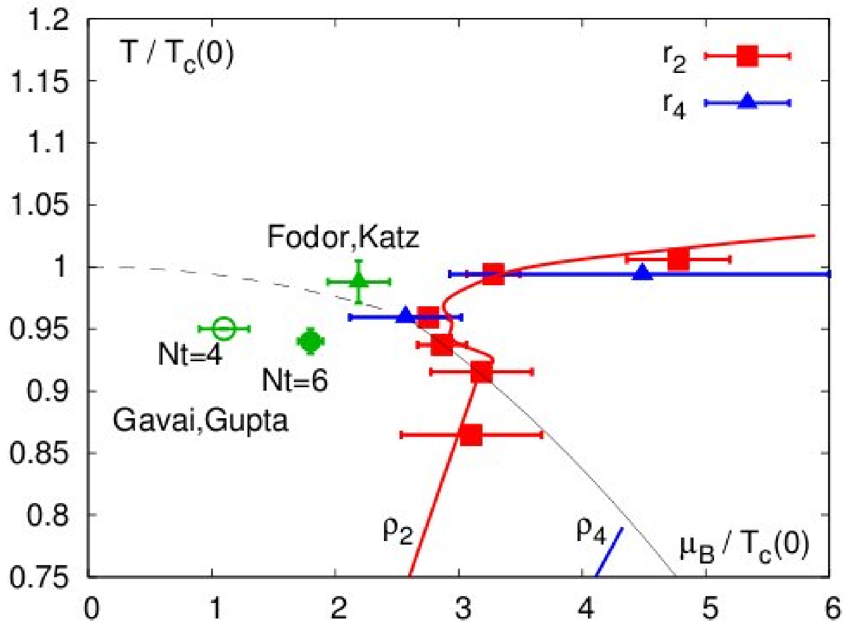


♡ Consistent Window with our other estimates.

Estimating $T_c(\mu_c)$ and μ_c/T

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)
- estimator for μ_c: $$\left(\frac{\mu_c(T)}{T_c(0)}\right)_n \equiv \rho_n = \frac{T}{T_c(0)}\sqrt{\frac{c_n}{c_{n+2}}}$$

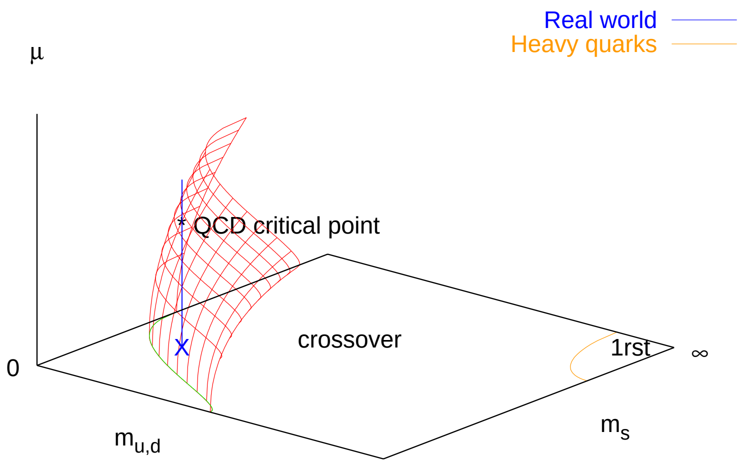


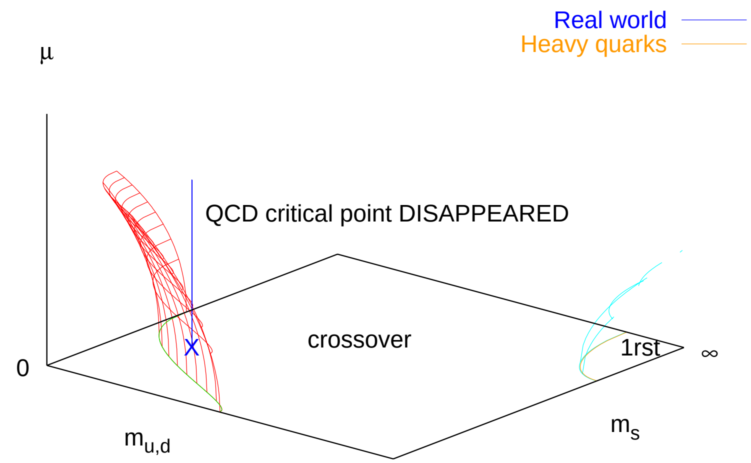
- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics

INT. Seattle 2008. F. Karsch – p. 20/3

Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811





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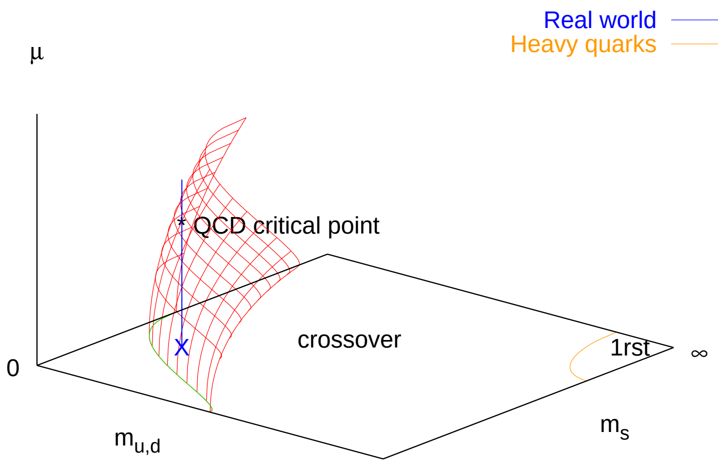

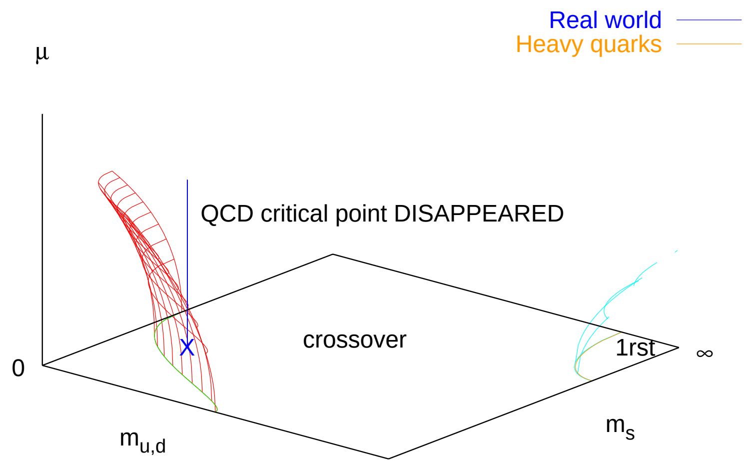

For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T_c}\right)^2 - 47(20)\left(\frac{\mu}{\pi T_c}\right)^4$, i.e., m_c shrinks with μ.

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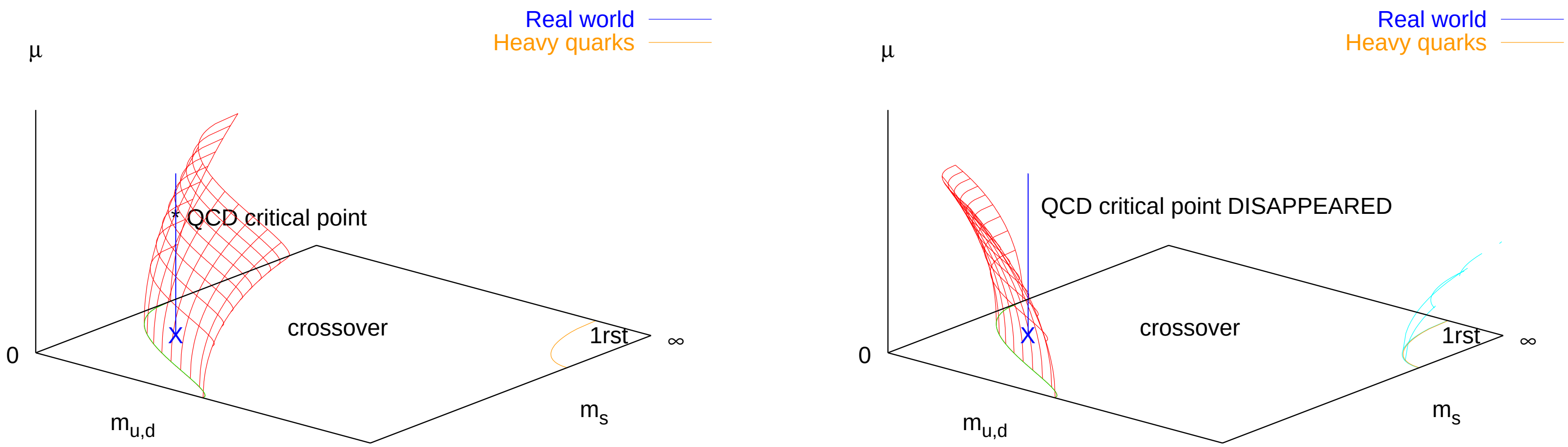

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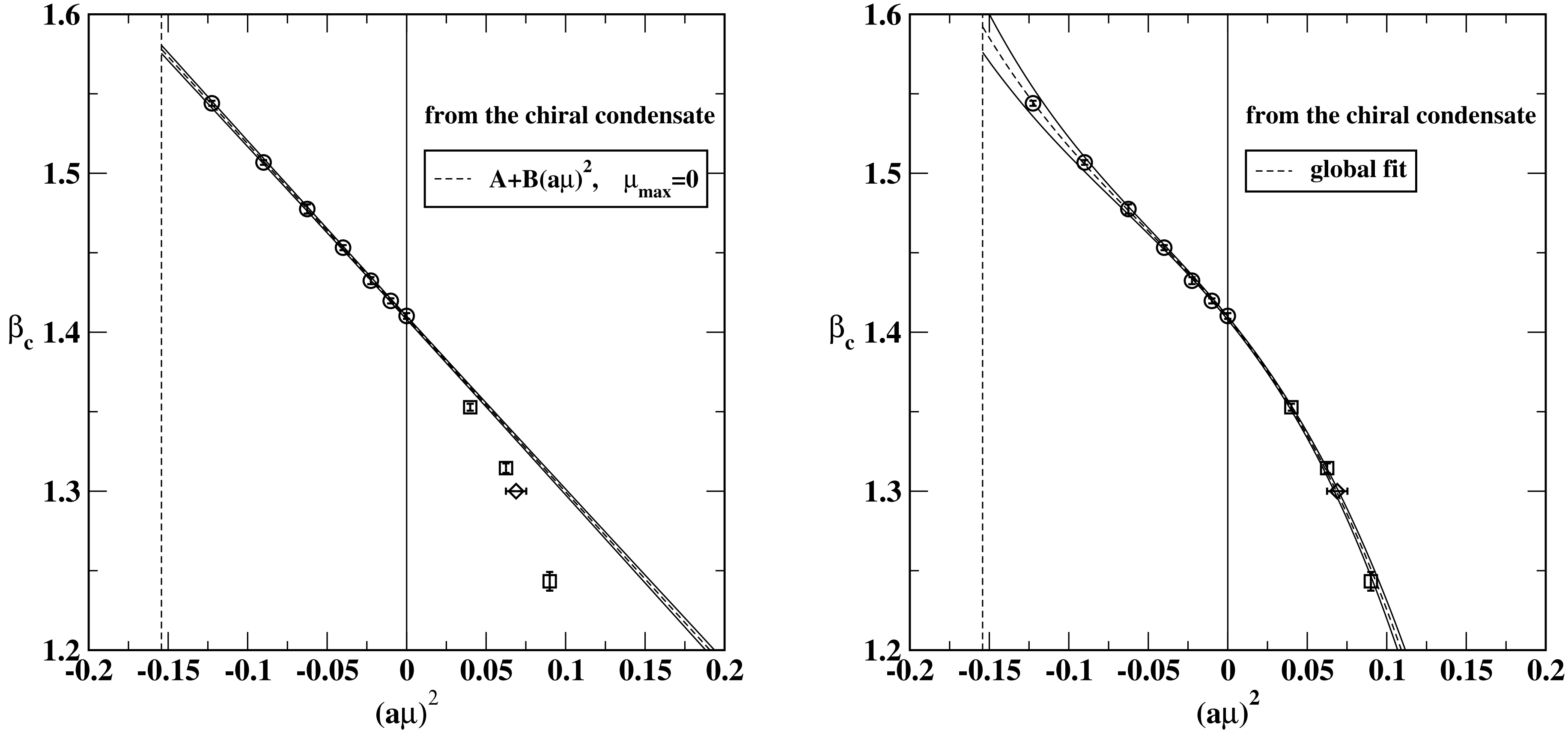
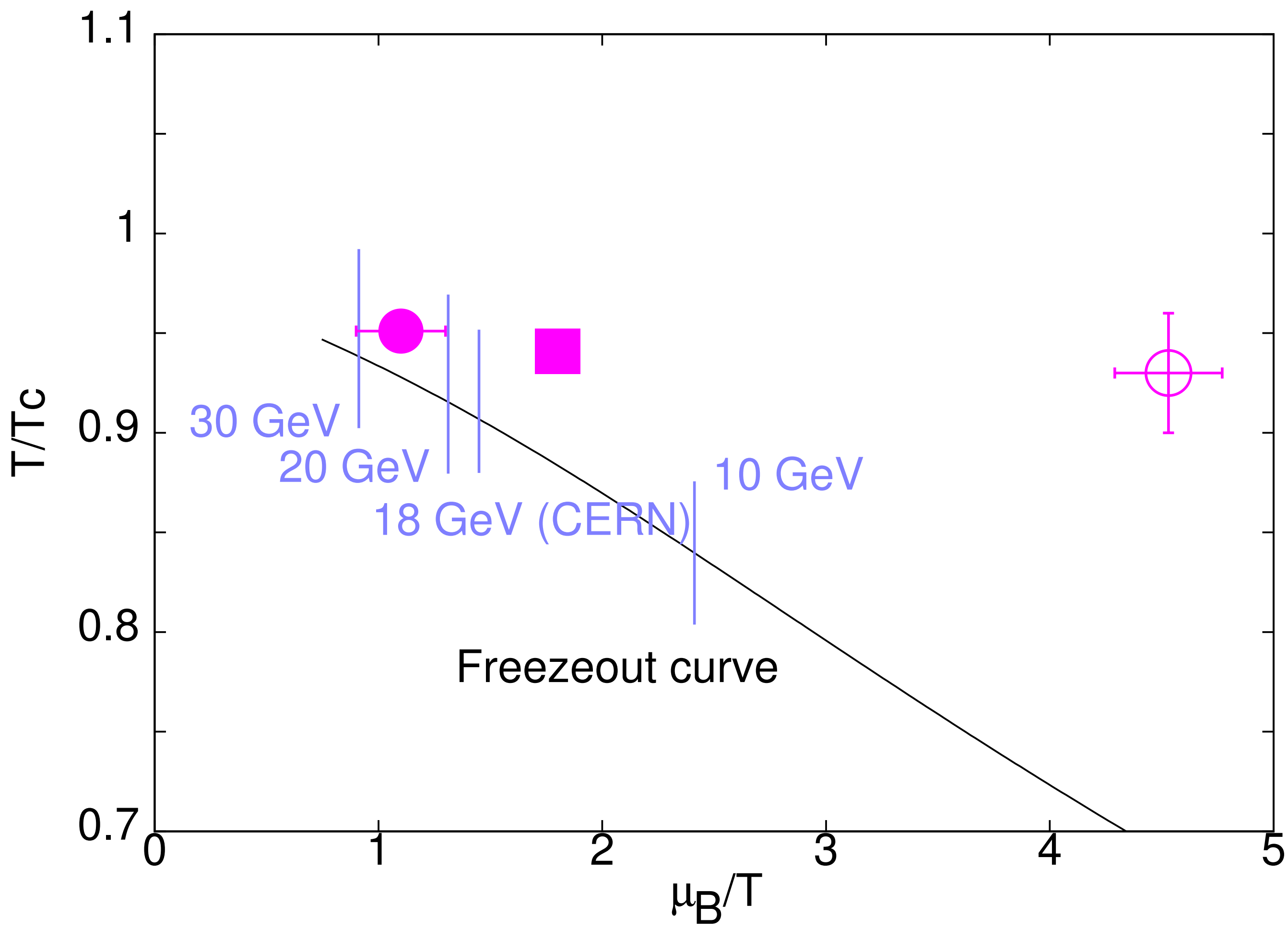
Problems : i) Positive coefficient for finer lattice (Philipsen, CPOD 2009), ii) Known examples where shapes are different in real/imaginary μ, “The Critical line from imaginary to real baryonic chemical potentials in two-color QCD”, P. Cea, L. Cosmai, M. D’Elia, A. Papa, PR D77, 2008




1.1
1
0.9
0.8
0.7
T/Tc
0
1
2
3
4
5
μB/T
30 GeV
20 GeV
18 GeV (CERN)
10 GeV
Freezeout curve

Searching Experimentally

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999)
- Look for nonmontonic dependence of the event-by-event fluctuations with colliding energy.

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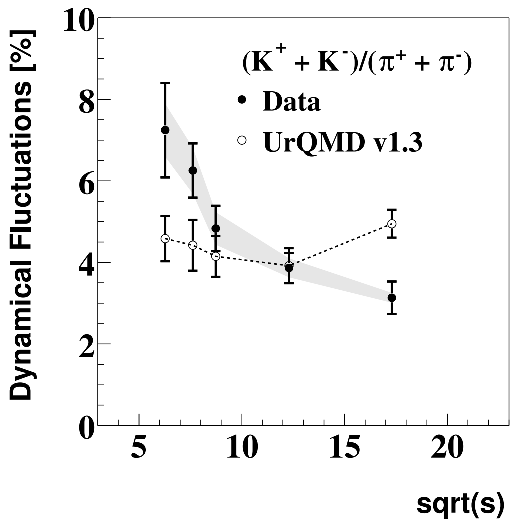

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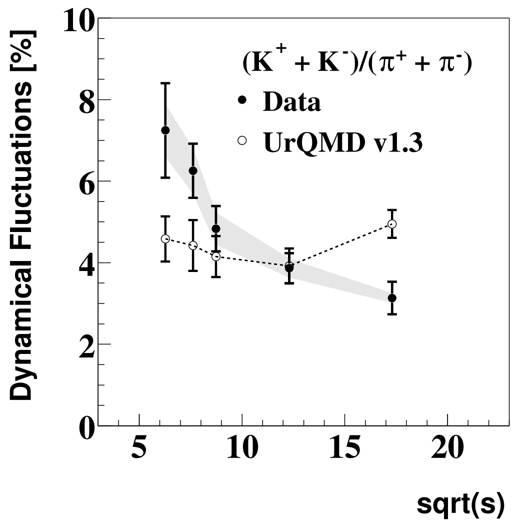

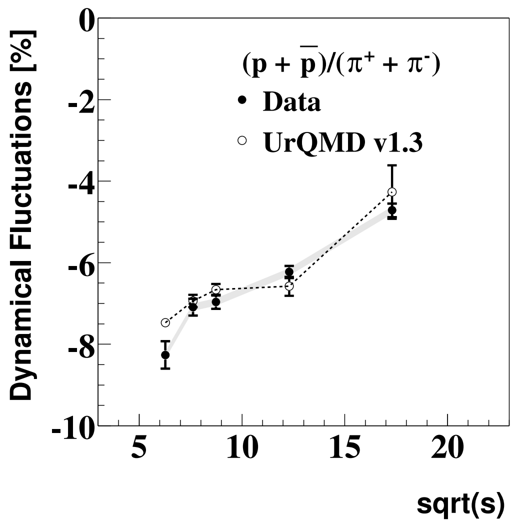

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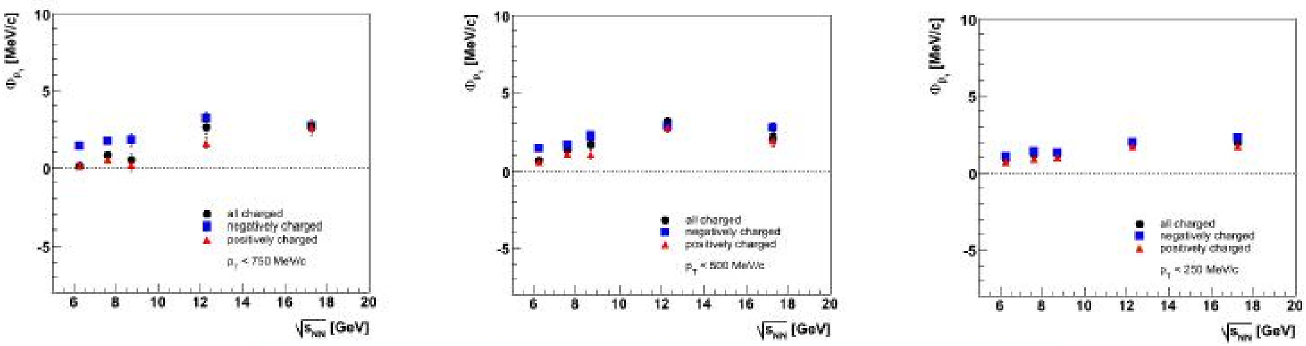
Fluctuations due to the critical point should be dominated by fluctuations of pions with $p_T \leq 500$ MeV/c

M. Stephanov, K. Rajagopal, E. V. Shuryak (Phys. Rev. **D60**, 114028, 1999): suggestion to do analysis with several upper p_T cuts

$p_T < 750$ MeV/c **$p_T < 500$ MeV/c** **$p_T < 250$ MeV/c**



No significant energy dependence of Φ_{PT} measure
also when low transverse momenta are selected.

Remark: predicted fluctuations at the critical point should result in $\Phi_{PT} \cong 20$ MeV/c, the effect of limited acceptance of NA49 reduces them to $\Phi_{PT} \cong$ **10 MeV/c**

- Proton number fluctuations (Hatta-Stephenov, PRL 2003)

- Neat idea : directly linked to the baryonic susceptibility which ought to diverge at the critical point. Since diverging ξ is linked to σ mode, which cannot mix with any isospin modes, expect χ_I to be regular.

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- Isentropic trajectories focus at the critical point (Asakawa-Nonaka, PRC 2005).

- This leads to the emission of high p_T particles at earlier times. (Asakawa-Bass-Nonaka-Müller, INT 2008 workshop).

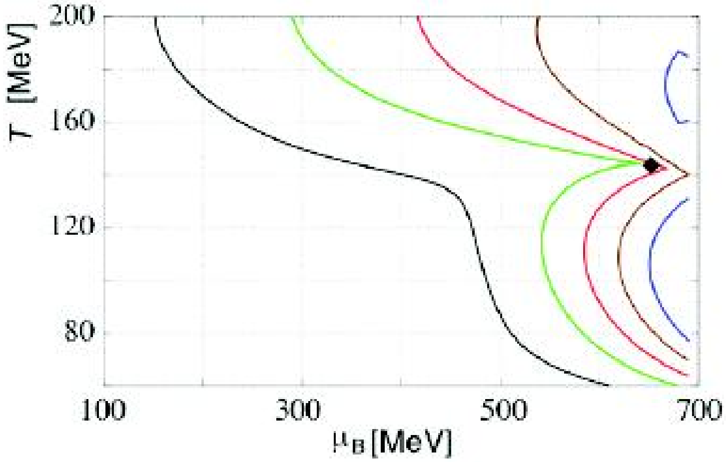
- Note this is NOT a fluctuations signal but model (EoS) dependent ?

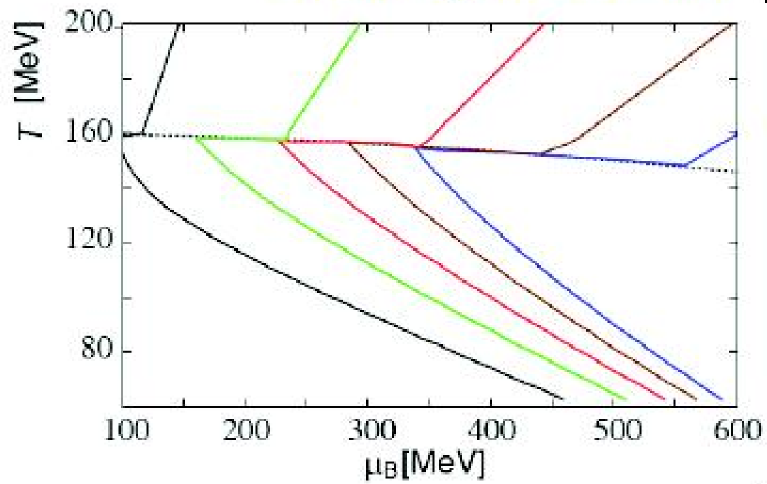
Focusing Effect

- **Isentropic trajectories on T- μ_B plane**

With QCD critical point

Bag Model +
Excluded Volume Approximation
(No Critical Point)
= Usual Hydro Calculation



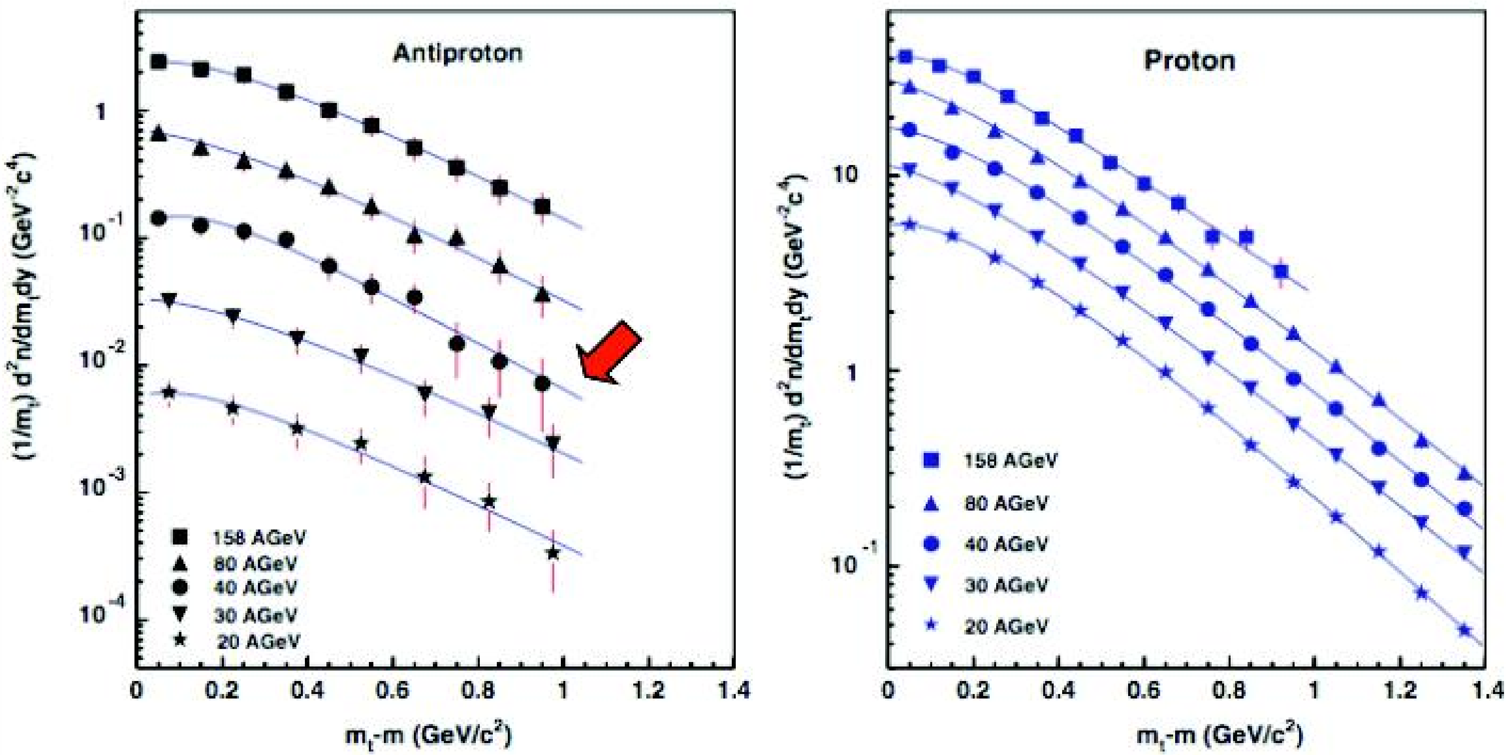


Focused

Not Focused

Chiho NONAKA

QCD Critical Point?



steeper $\overline{p}$ spectra at high P_T NA49, PRC73,044910(2006)

Chiho NONAKA

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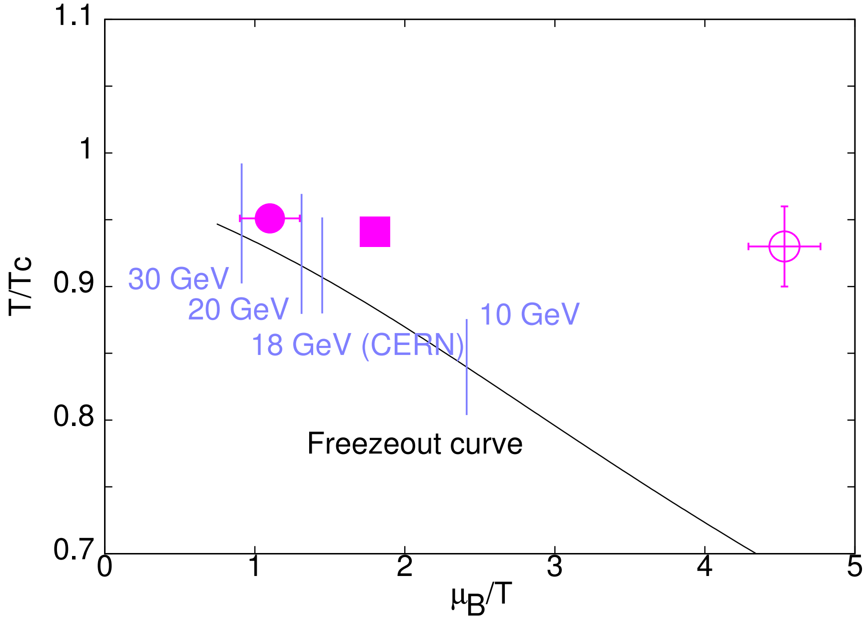

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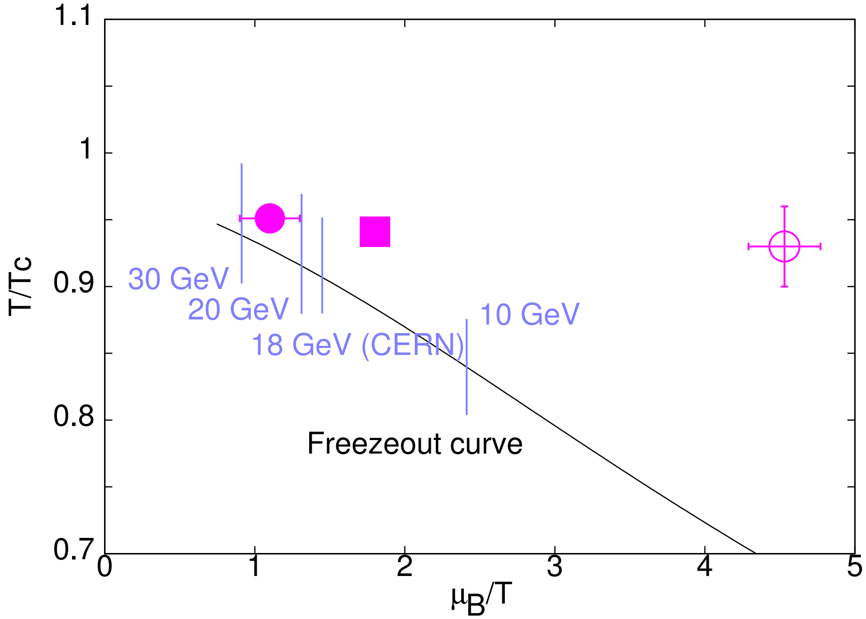

So far no signs of a critical point in the experimental results at CERN. Will RHIC deliver it for us ?