Charm Flow at PHENIX : A New Milestone for Lattice QCD ?

*Rajiv V. Gavai**
T. I. F. R., Mumbai & Universität Bielefeld

* *With Debasish Banerjee, Saumen Datta & Pushan Majumdar, arXiv:1109.5738 , submitted to Phys. Rev. D*

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Introduction : Anisotropic Flow

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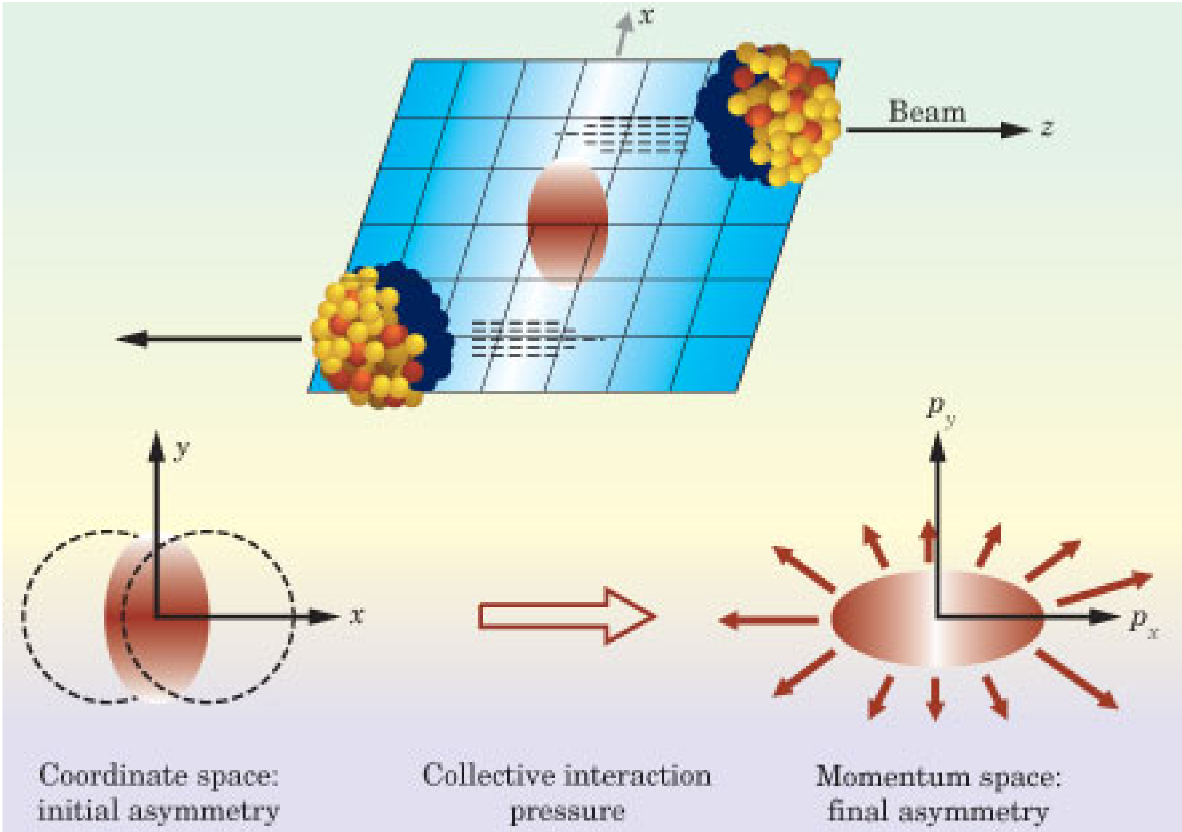
- Obtained from asymmetric collisions of two nuclei, with their centres not aligned.



•

$$v_2(y, p_T) = \frac{\int d\phi \ dN/(p_T dP_T d\phi dy) \ \cos(2\phi)}{\int d\phi \ dN/(p_T dP_T d\phi dy)} \tag{1}$$

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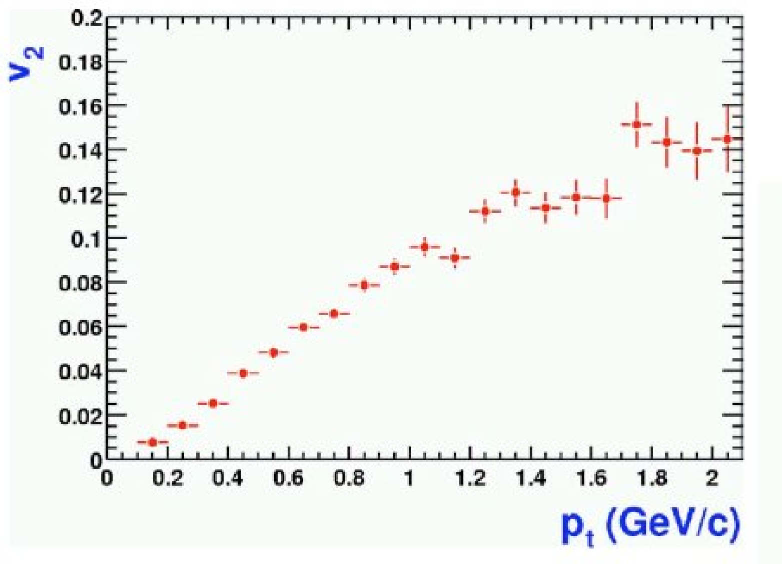
- (STAR Collaboration, Ackermann et al., PRL 86 (2001) 402.)



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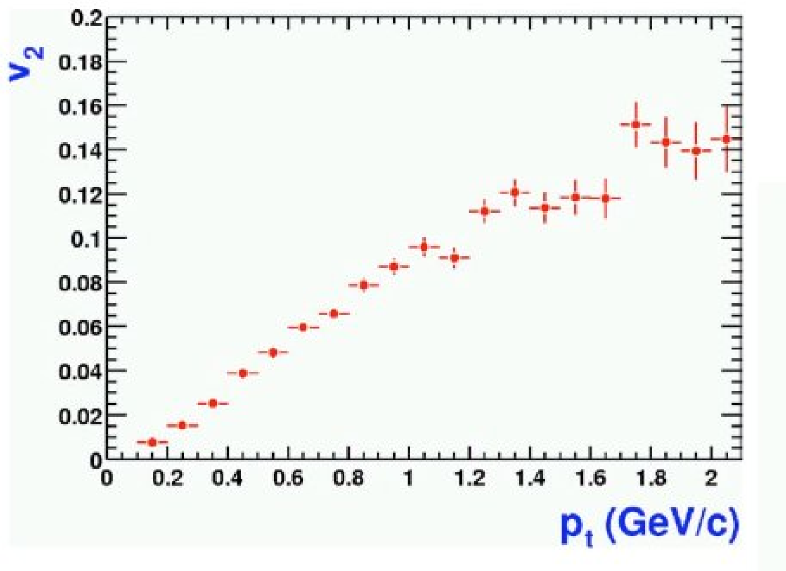
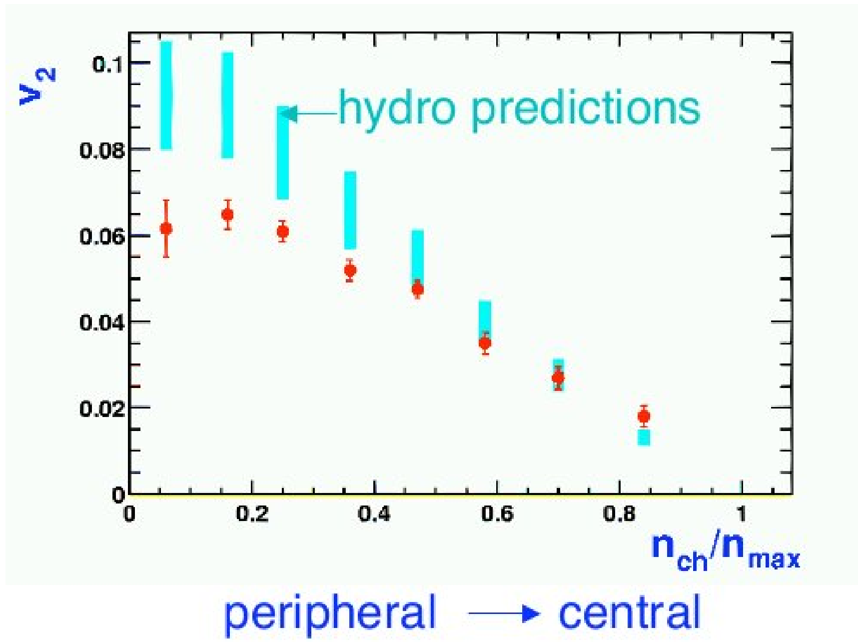
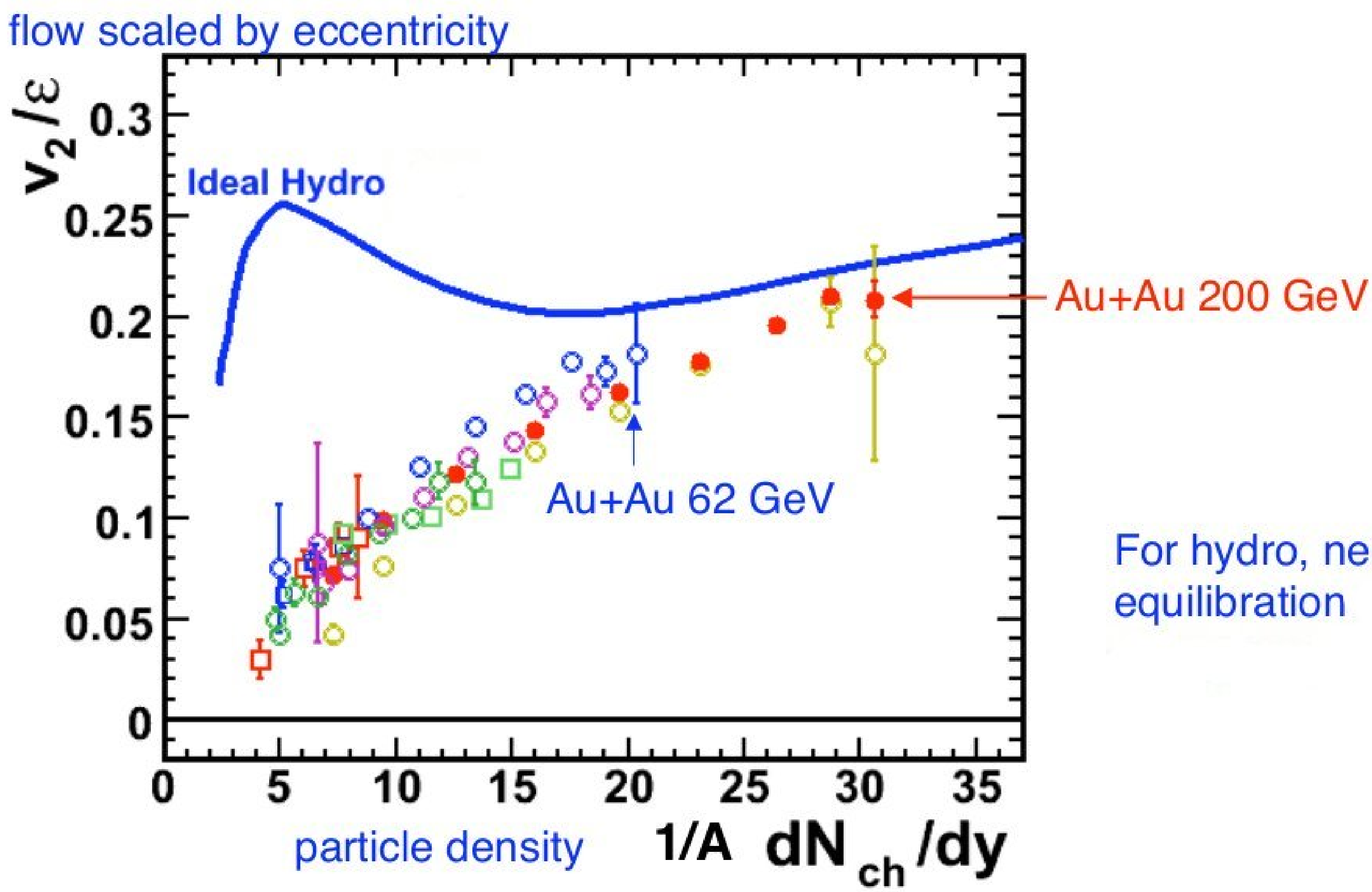




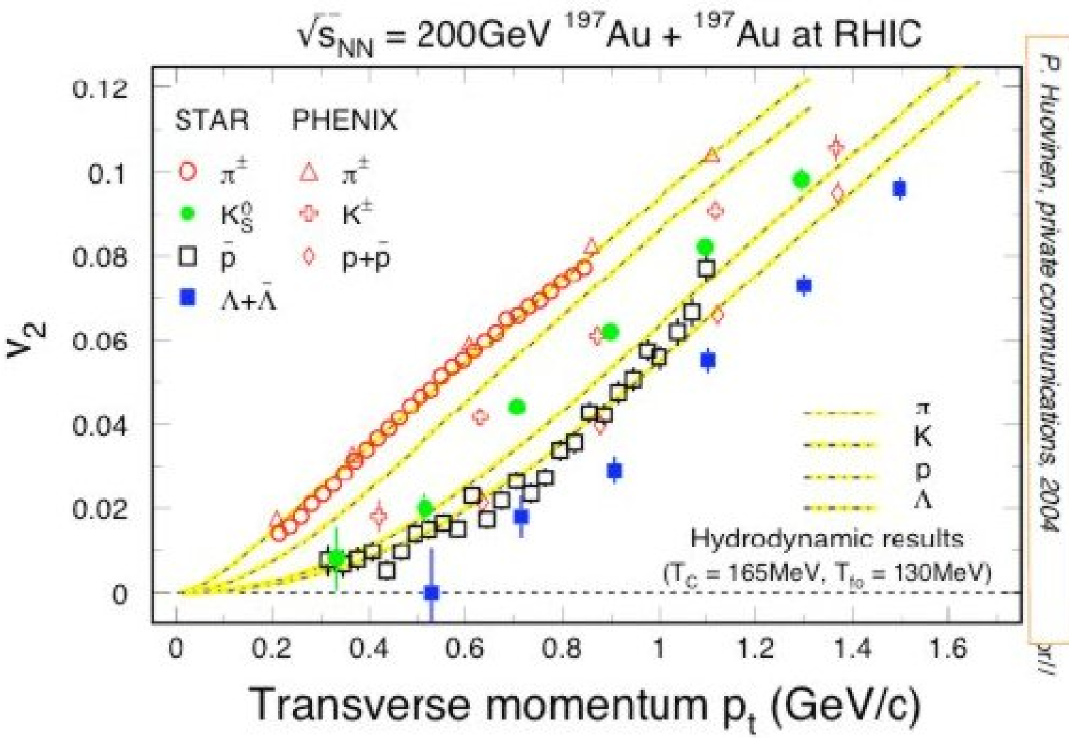
• Good agreement with ideal hydro: Suggesting early thermalization and perfect fluid and many more interesting things.



For hydro, need early equilibration

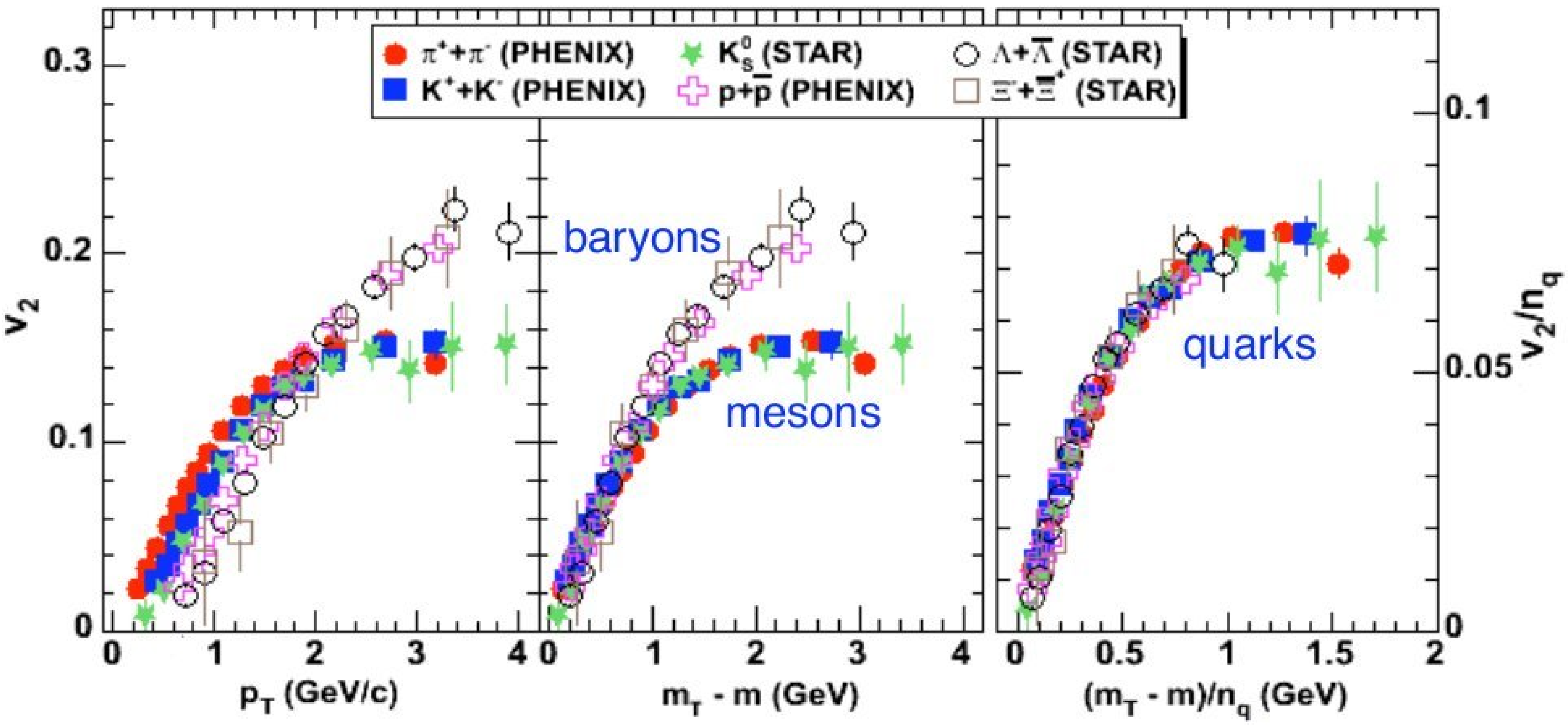
(S. Voloshin, QM06, JPG 31 (2007) S883 & Hydro Curve: Kolb-Sollfrank-Heinz, PRC 62 (2000) 054909.)

v_2 at Low p_T Region



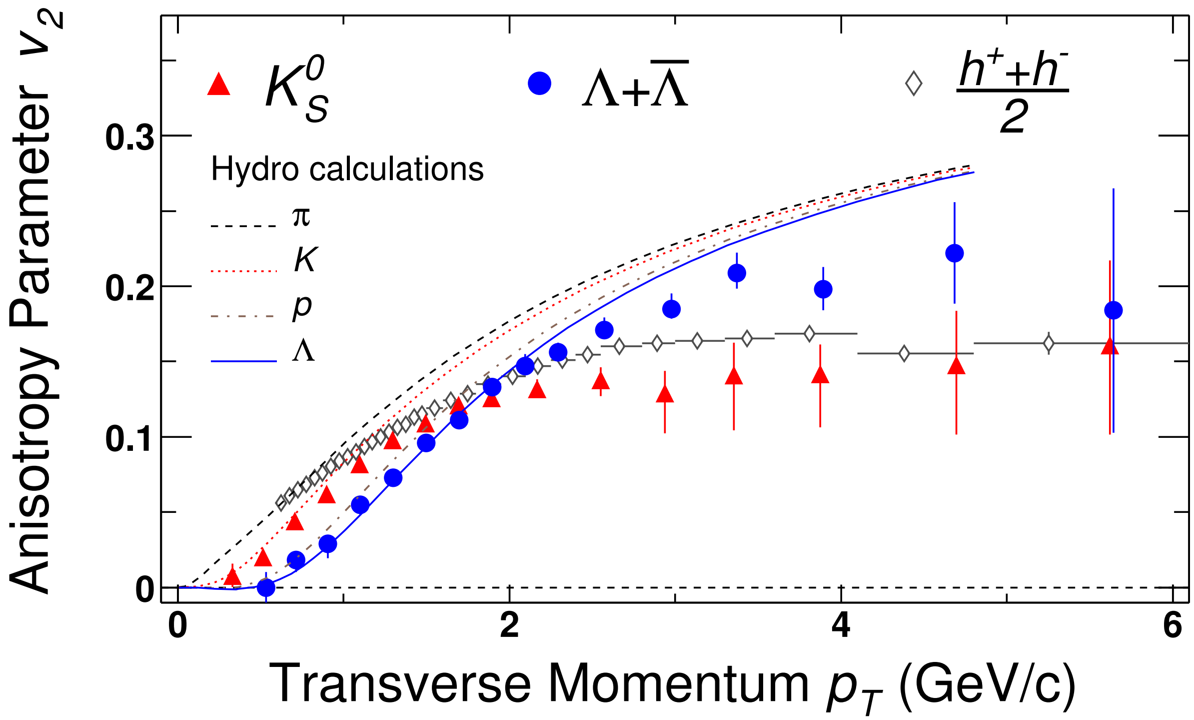
(STAR Collaboration, JPG 31 (2005) S437 & P. Huovinen.)

- Mass Pattern as expected by Hydrodynamics Models. Quantitative agreement depends on the equation of state.



(S. Voloshin, QM02, STAR PRL 95 (2005) & PHENIX PRL 98 (2007))

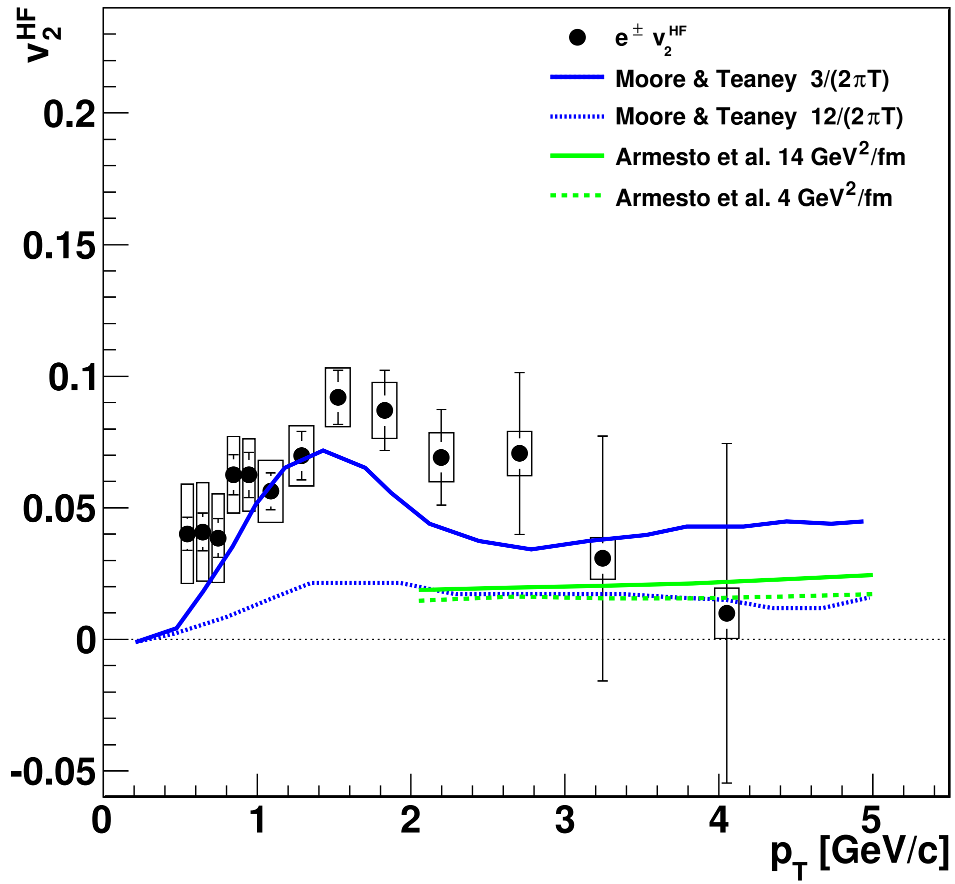
- v_2 scales as number of quarks. Thus, hadrons appear to follow the 'underlying' quark flow as Recombination Model would suggest.



(STAR Collaboration, Adams et al., PRL 92 (2004) 052302.)

♡ Minimum Bias Au+Au Collisions at 200 GeV/c : Strangeness flows like normal hadrons.

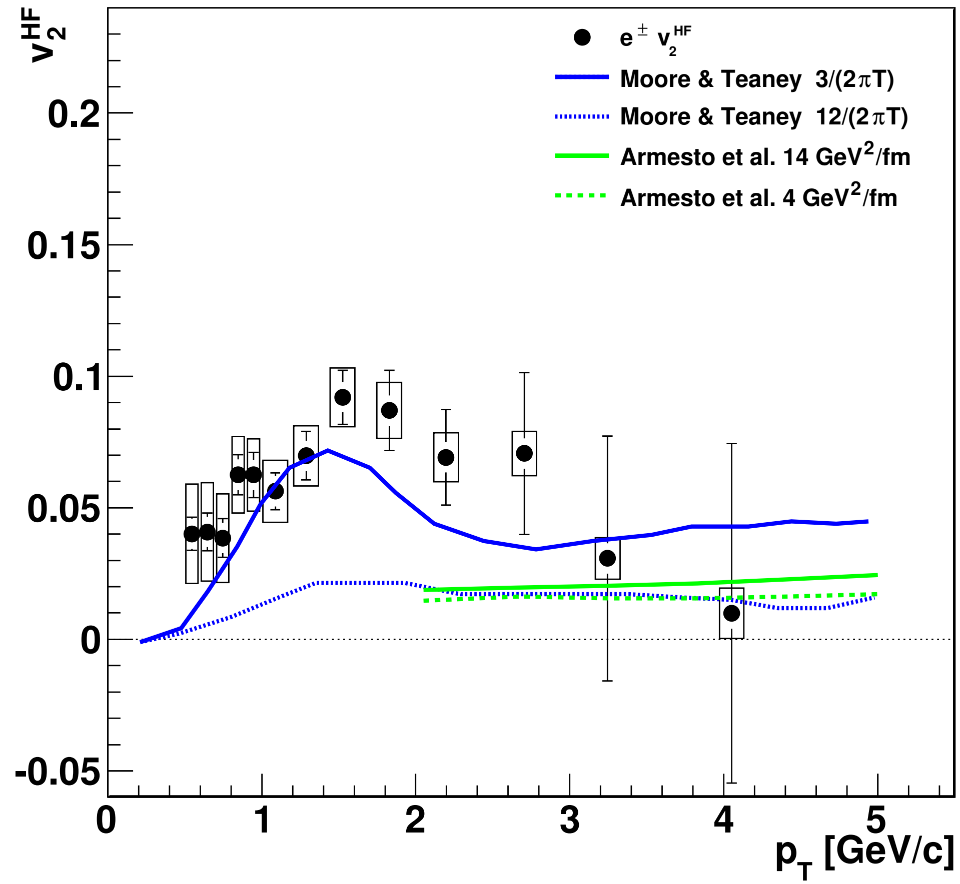
• Naıvely expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.



(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

- Naıvely expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.

- In models (Moore-Teaney, PRC 71, 2005), heavy quark diffusion coefficients governs its elliptic flow **and** suppression.



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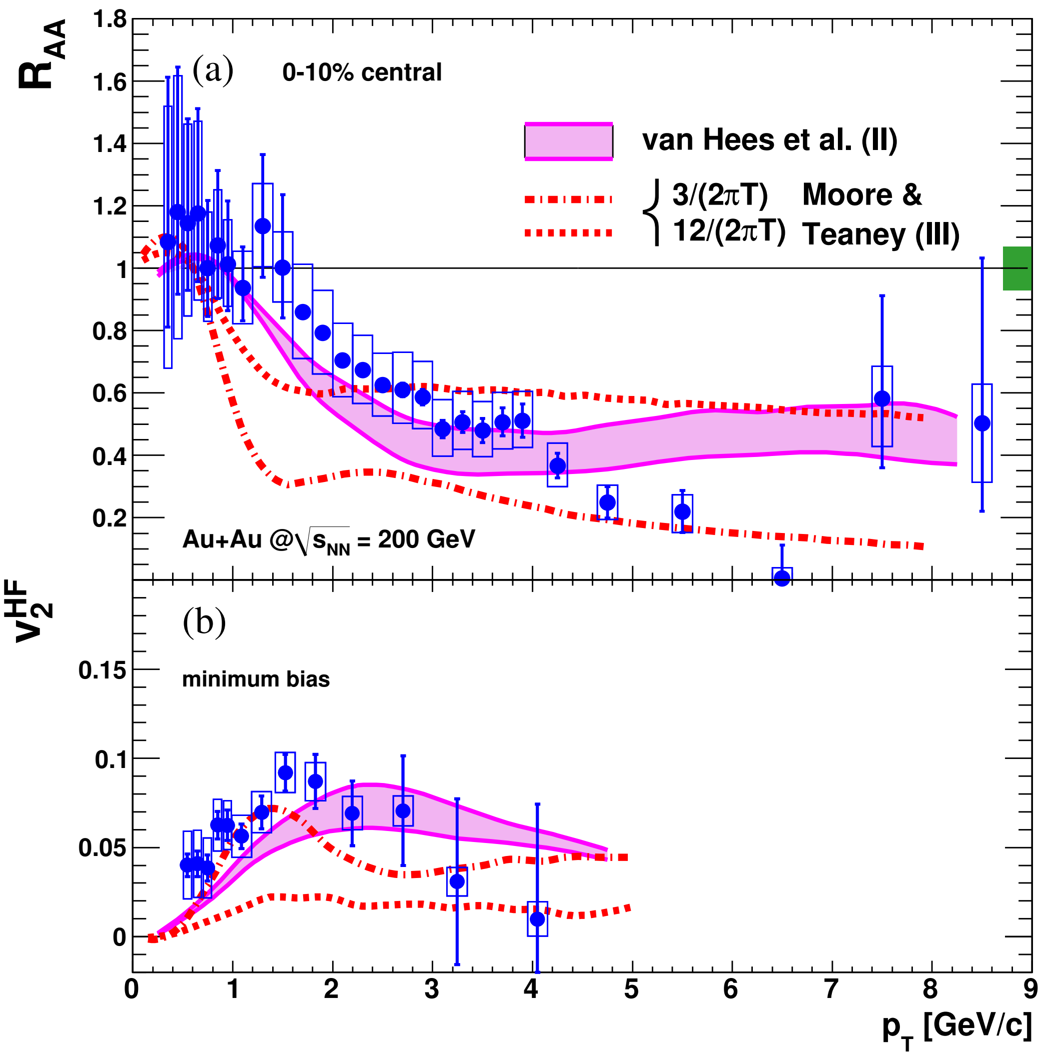

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- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.

- **Smaller** $D \simeq 3/2\pi T$ seems required by data.

- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.

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- Other models, e.g. van Hees-Greco-Rapp, seem to suggest the same: Heavy Quark Diffusion coefficient is much smaller than perturbative estimates.
- Is it non-perturbative ? Strong coupling models — AdS/CFT based — do lead to values in the desired range under "suitable" assumptions [Casalderrey-Solana & Teaney (2006), Gubser(2007)]
- Can Lattice QCD shed some light on the Charm Flow ?

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.

- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

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$$\frac{dp_i}{dt} = -\eta_D \, p_i + \xi_i(t) \qquad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t') \tag{2}$$

- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time, $\kappa = \frac{1}{3} \int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t) \xi_i(0) \rangle$.

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- Diffusion constant D can be found to be $2T^2/\kappa$ with $\eta_D = \kappa/2MT$.

- Moore-Teaney also showed that an initial power-law (LO pQCD) transverse momentum distribution of a heavy Q in an expanding QGP at $T_0 = 300$ MeV by $T_f = 165$ MeV approximates a thermal one **provided** $D \leq 3/2\pi T$, assuming an ideal Bjorken expansion of the plasma.

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- Casalderrey-Solana & Teaney (PRD 74 (2006) 085012) suggested to obtain κ from a correlator of the (colour) force exerted on a heavy Q by the (deconfined & coloured) medium.

- Caron-Huot, Laine & Moore (JHEP 0904, 053) provided a suitable definition for κ for a lattice evaluation: The force acting on the heavy quark is given by $M \; dJ^i/dt$, where $J^\mu(\vec{x},t) = \bar{\psi}(\vec{x},t)\gamma^\mu\psi(\vec{x},t)$ is the conserved current for the heavy quark.

- Using Heavy Quark Effective Theory, they narrowed it down to studying $G_E^{\rm Lat}(\tau) = -\frac{1}{3L} \sum_{i=1}^{3} \left\langle {\rm Re~tr} \left[U(\beta,\tau)\ E_i(\tau,\vec{0})\ U(\tau,0)\ E_i(0,\vec{0}) \right] \right\rangle$, where L is the Polyakov loop.

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- The spectral function, $\rho(\omega)$, is obtained from the $G_E(\tau)$, as usual, by

$$G_E(\tau) \ = \ \int_0^\infty \frac{d\omega}{\pi} \, \rho(\omega) \, \frac{\cosh \omega(\tau - \frac{1}{2T})}{\sinh \frac{\omega}{2T}} \, . \qquad (3)$$

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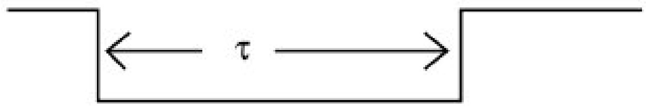
- Then momentum diffusion coefficient $\kappa = \lim_{\omega \to 0} \frac{2T}{\omega}\, \rho(\omega)$.

- They also suggested a suitable discrete version for Lattice QCD : $E_i(\vec{x},\tau) = U_i(\vec{x},\tau)\, U_4(\vec{x}+\hat{i},\tau) - U_4(\vec{x},\tau)\, U_i(\vec{x}+\hat{4})$.

- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.

$$G^i_{E,\mathrm{num}}(\tau) \;=\; C^i(\tau+1) + C^i(\tau-1) - 2C^i(\tau)$$

$$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t+\tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4).$$



Graphical Representation of $C(\tau)$.

Our Lattice Results

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- We attempted $N_\tau = 12$, 16, 20 and 24 for quenched QCD. Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.

- For the same size error on $G(10)[G(3)]$ on $N_\tau = 20$ lattices, it was found to be $\sim 2500[200]$ times more efficient: Very crucial in getting κ.

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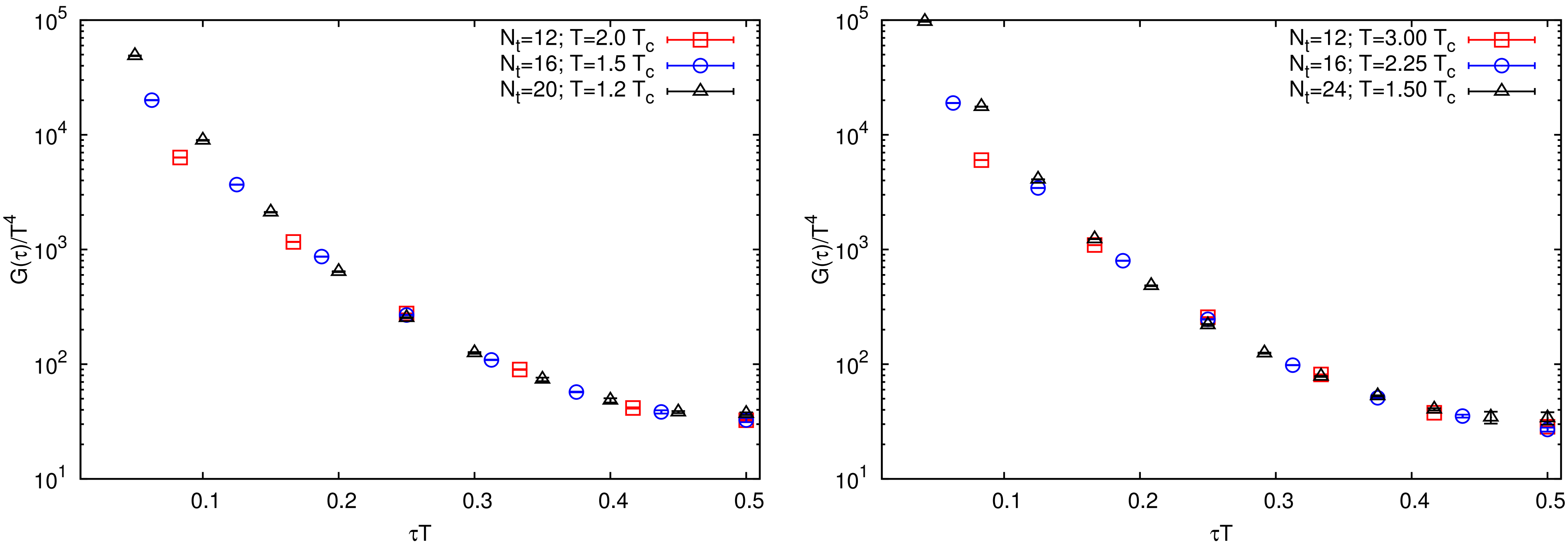
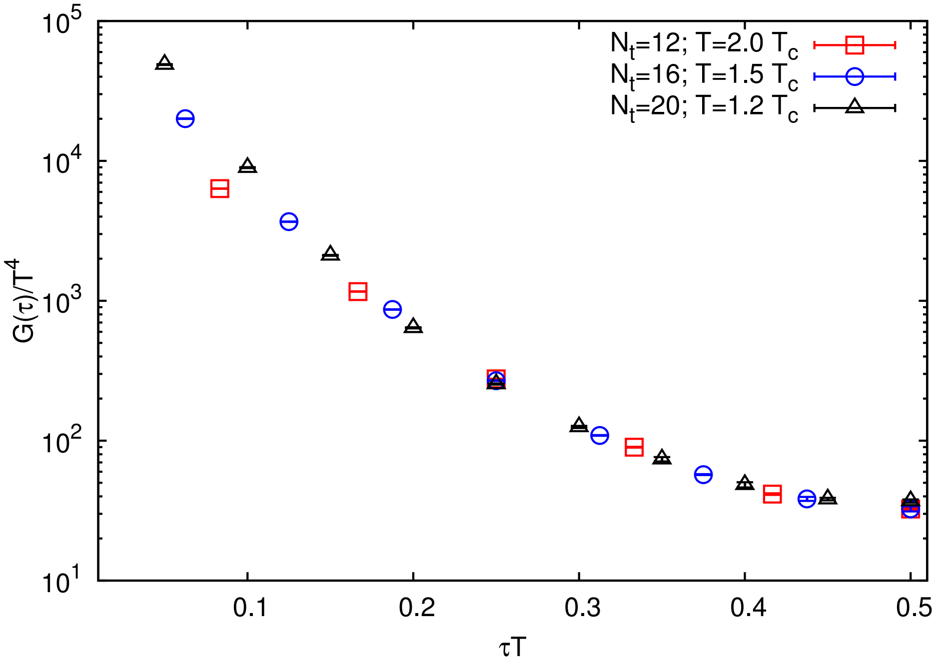
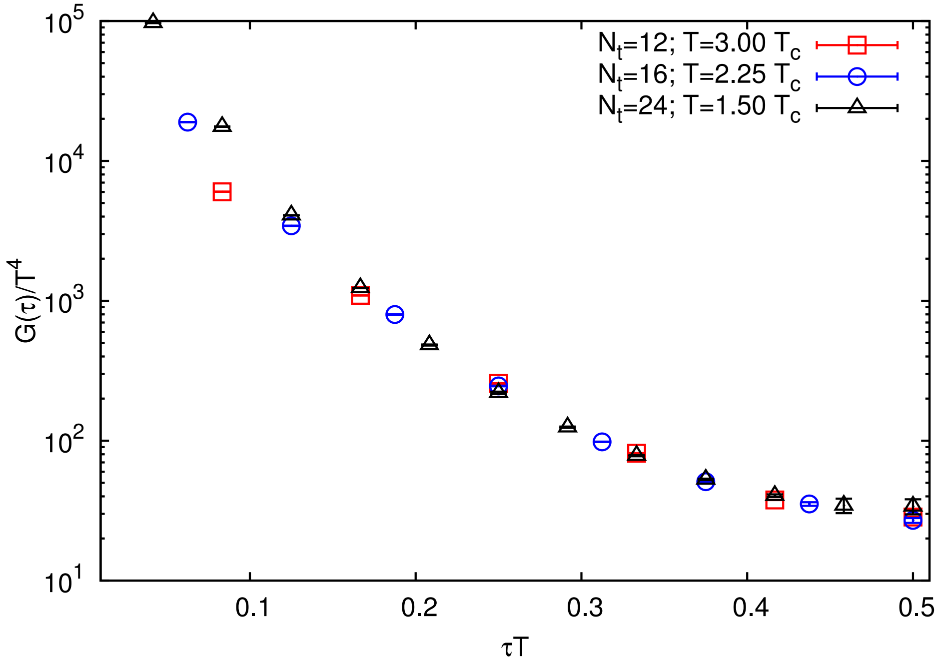
- Spatial volumes are such that $N_s \geq 2N_\tau$.

- Couplings were chosen suitably to make simulations at $T/T_c = 1.04$, 1.09, 1.24, 1.5 and 1.96 for the two largest N_τ.

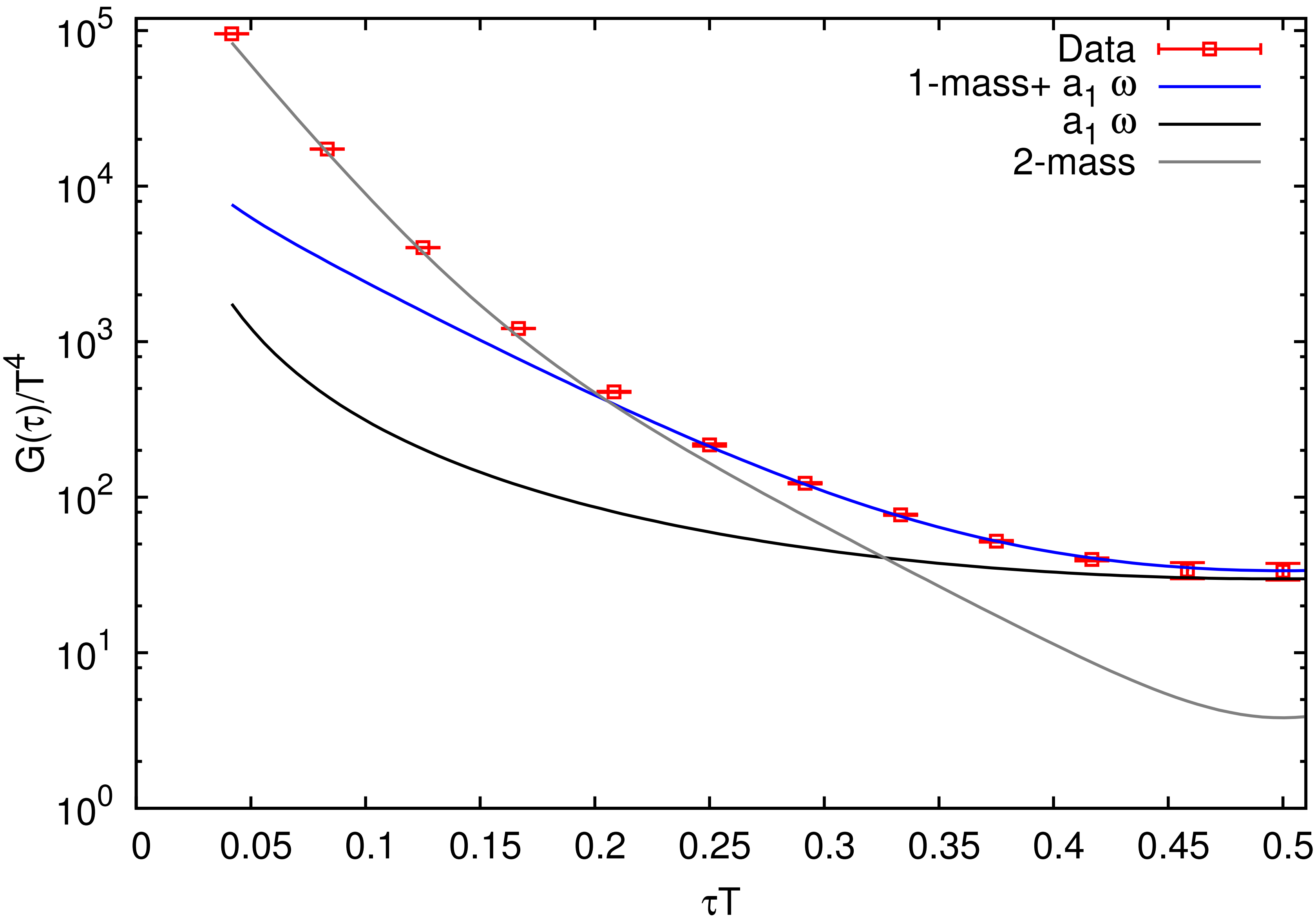
- Typical Statistics : Few hundred Independent Configurations, with a few thousand multilevel updates.


N_t=12; T=2.0 T_c
N_t=16; T=1.5 T_c
N_t=20; T=1.2 T_c
N_t=12; T=3.00 T_c
N_t=16; T=2.25 T_c
N_t=24; T=1.50 T_c
$G(\tau)/T^4$
τT
10^5
10^4
10^3
10^2
10^1
0.1
0.2
0.3
0.4
0.5





- Large τ region shows scaling.
- Low τ region, on the other hand, has only lattice artifacts.



Data
1-mass+ a1 ω
a1 ω
2-mass
G(τ)/T4
τT
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
10^5
10^4
10^3
10^2
10^1
10^0

Extracting D

- Getting to the spectral function ρ, an ill-posed problem, has attracted a lot of attention. Many methods can be tried.

- We use an $ansatz$ for ρ, obtain G from it, and then fit in the large τ range $[N_\tau/4, N_\tau/2]$

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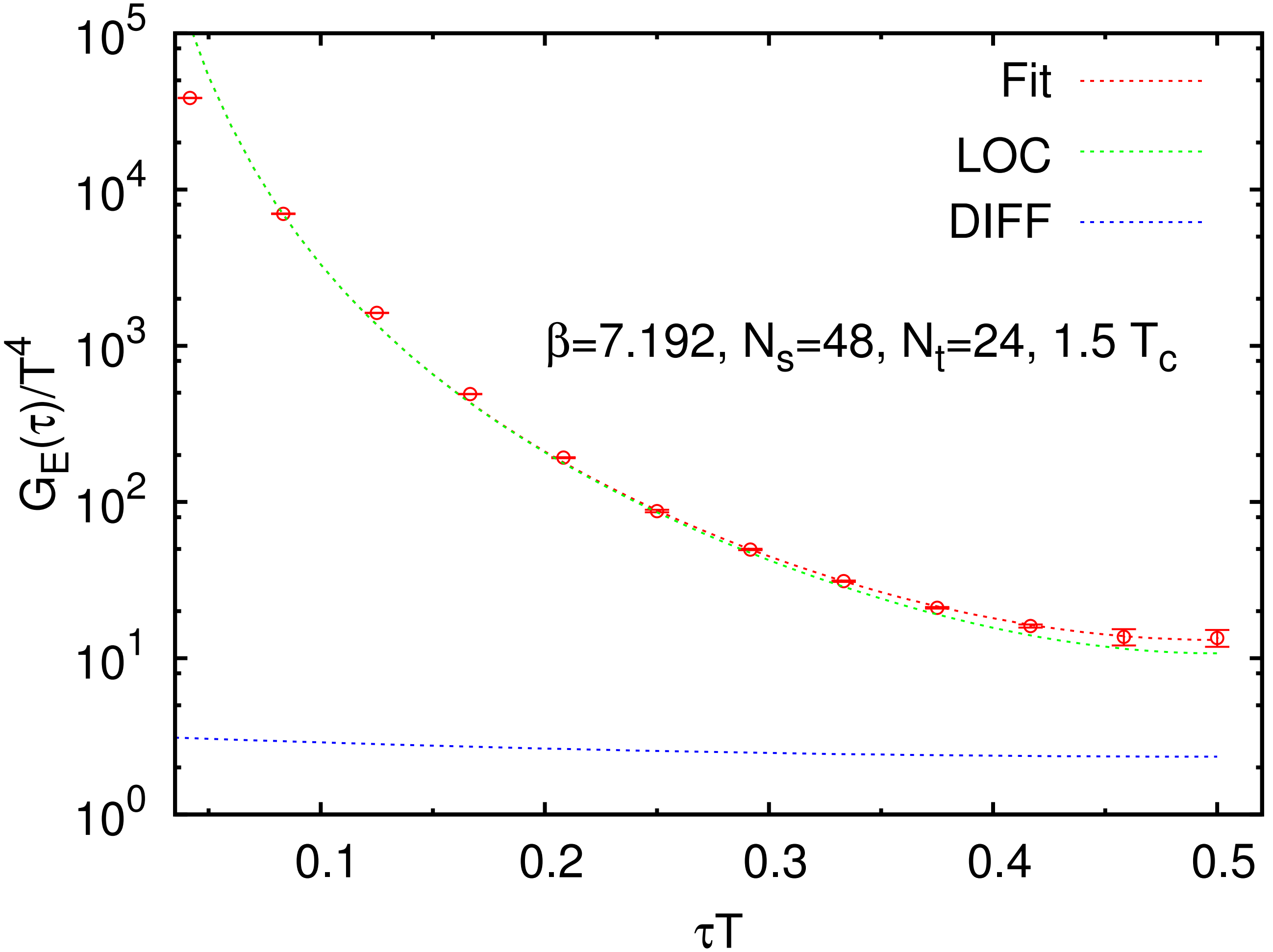
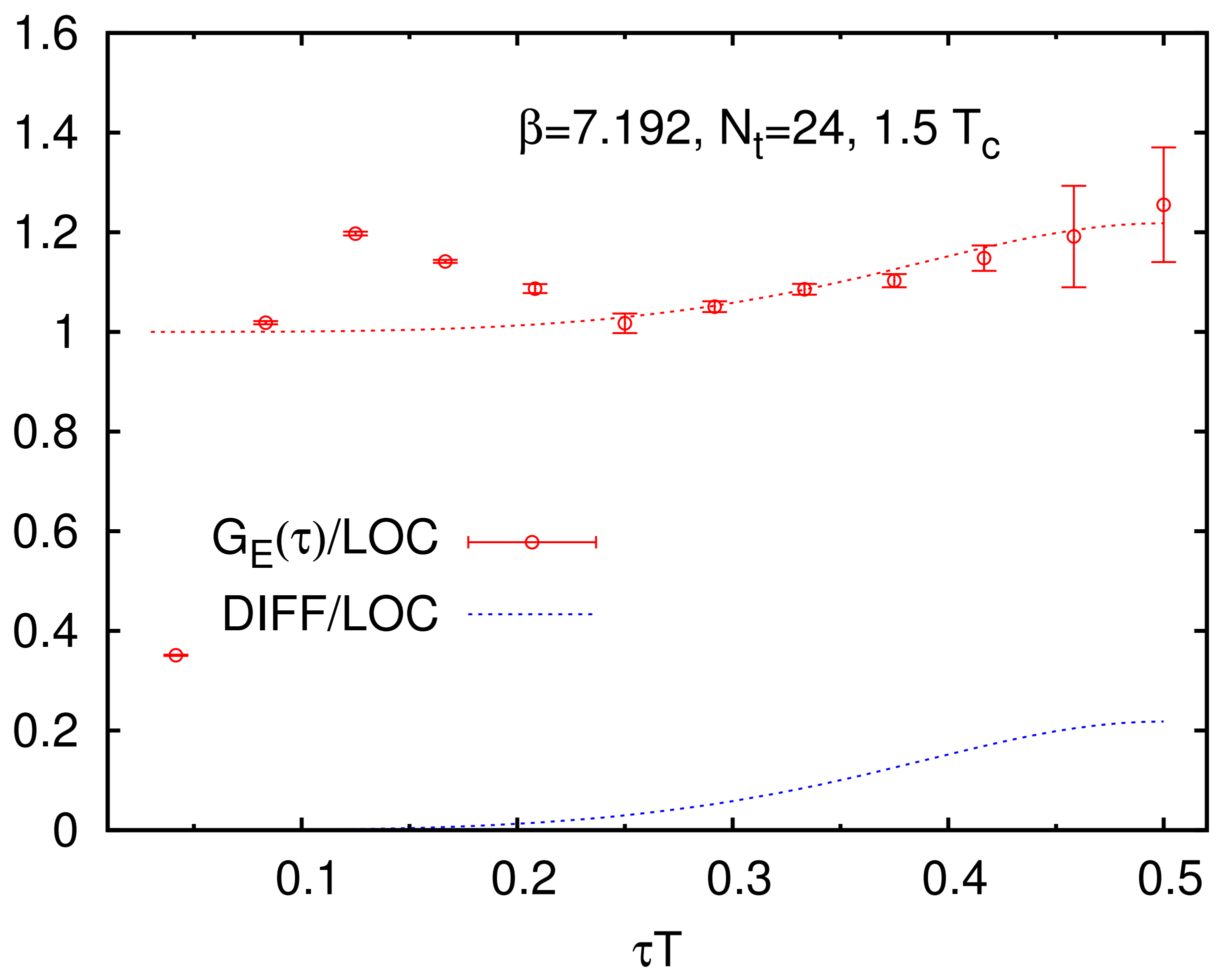
- $\rho(\omega) \ = \ a\omega\,\Theta(\omega - \Lambda) \ + \ b\omega^3$
  First term is the due to the expected DIFFusion constant, and the second is motivated by leading perturbation theory (LOC)

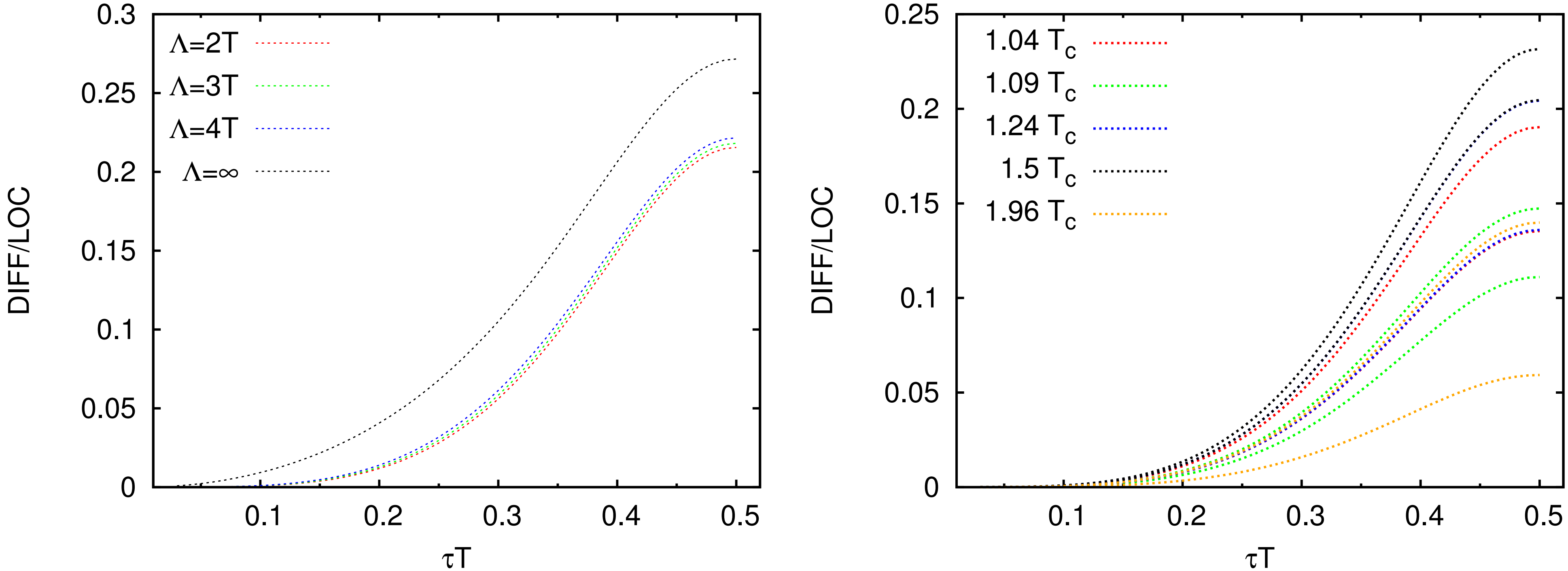
- $\Lambda = 3T$ used; varied from 2 to ∞ for systematic error.



♠ Contribution of the two terms shown as DIFF and LOC.



♠ Comparing the DIFF fit with the data after eliminating the LOC.



♠ Variation of a with the cut-off Λ and the temperature.

♠ Our fit parameter $a \rightsquigarrow \kappa$ modulo the renormalization factor for the electric fields.

♠ We use the tadpole factor. It is $\sim$ 1.2 as evaluated from our plaquette values.

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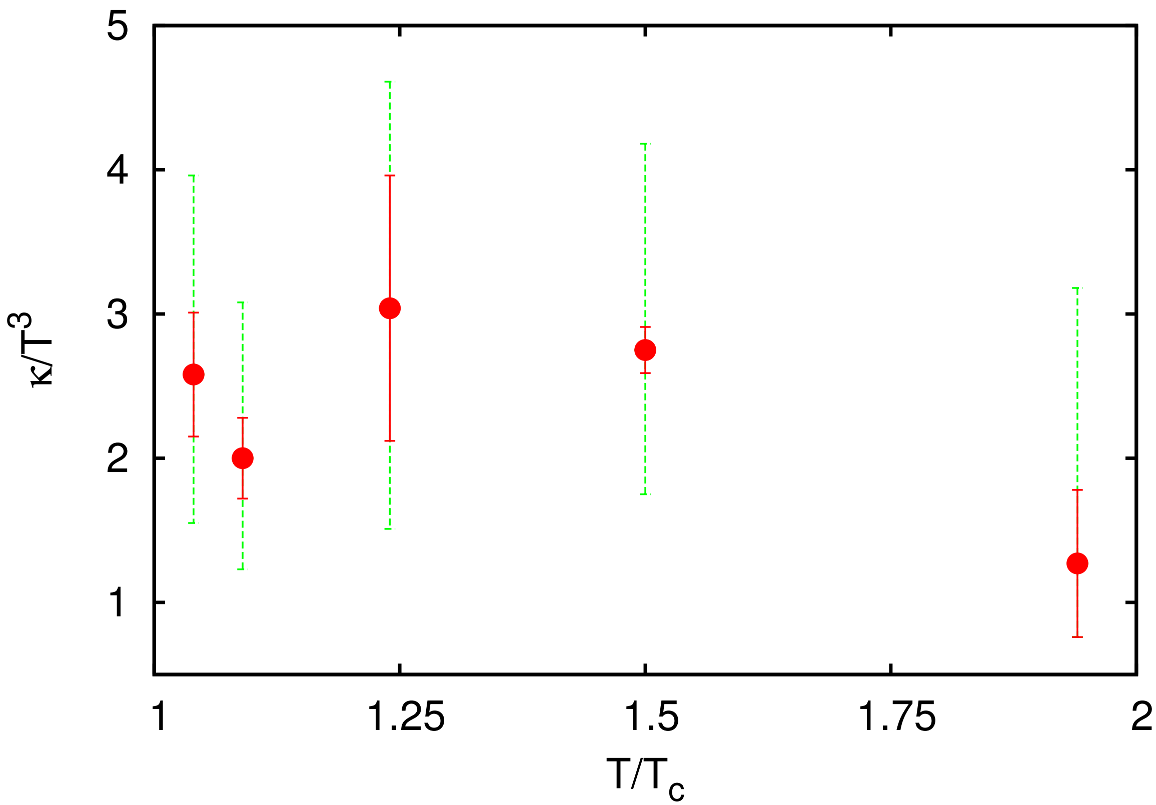
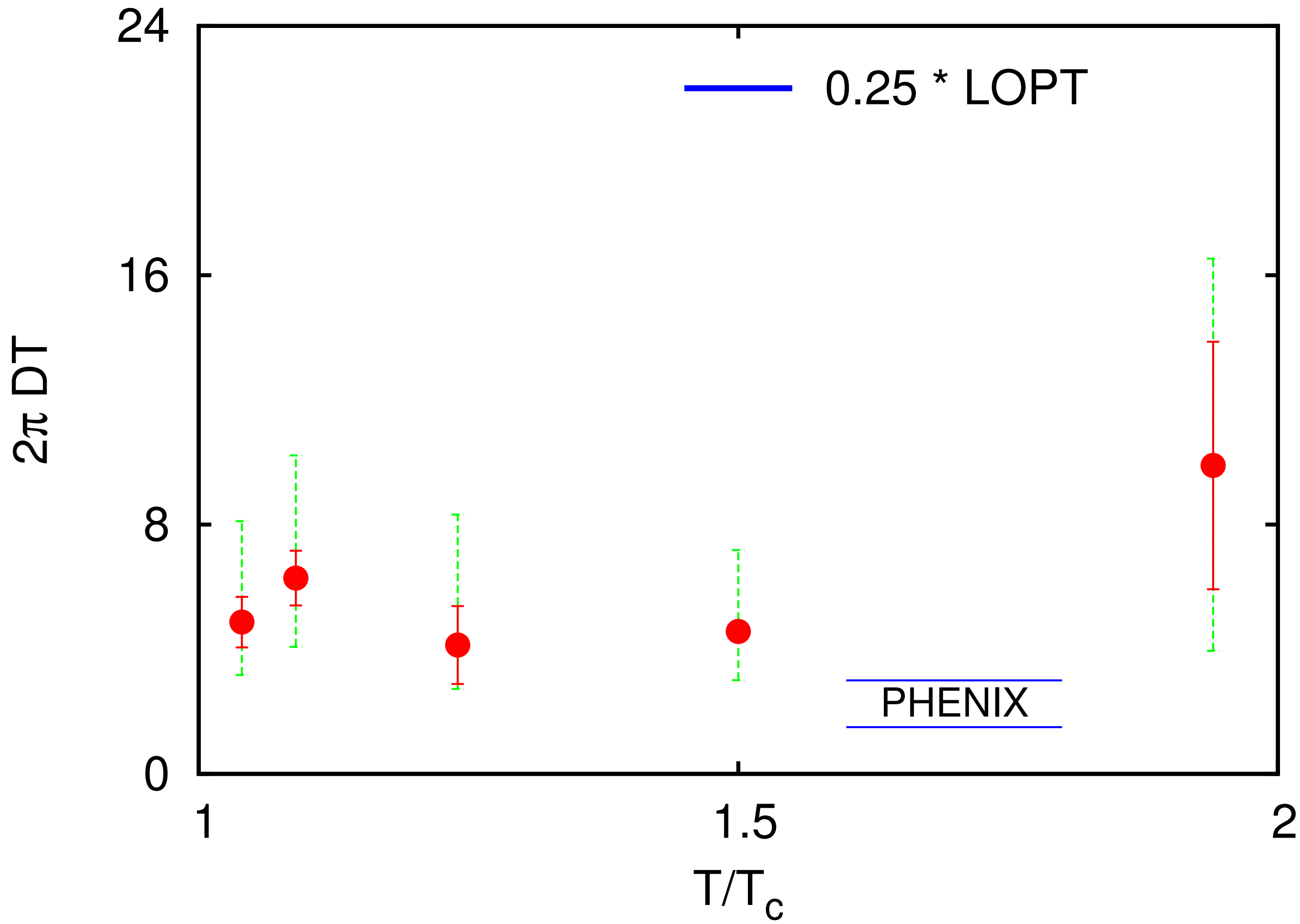
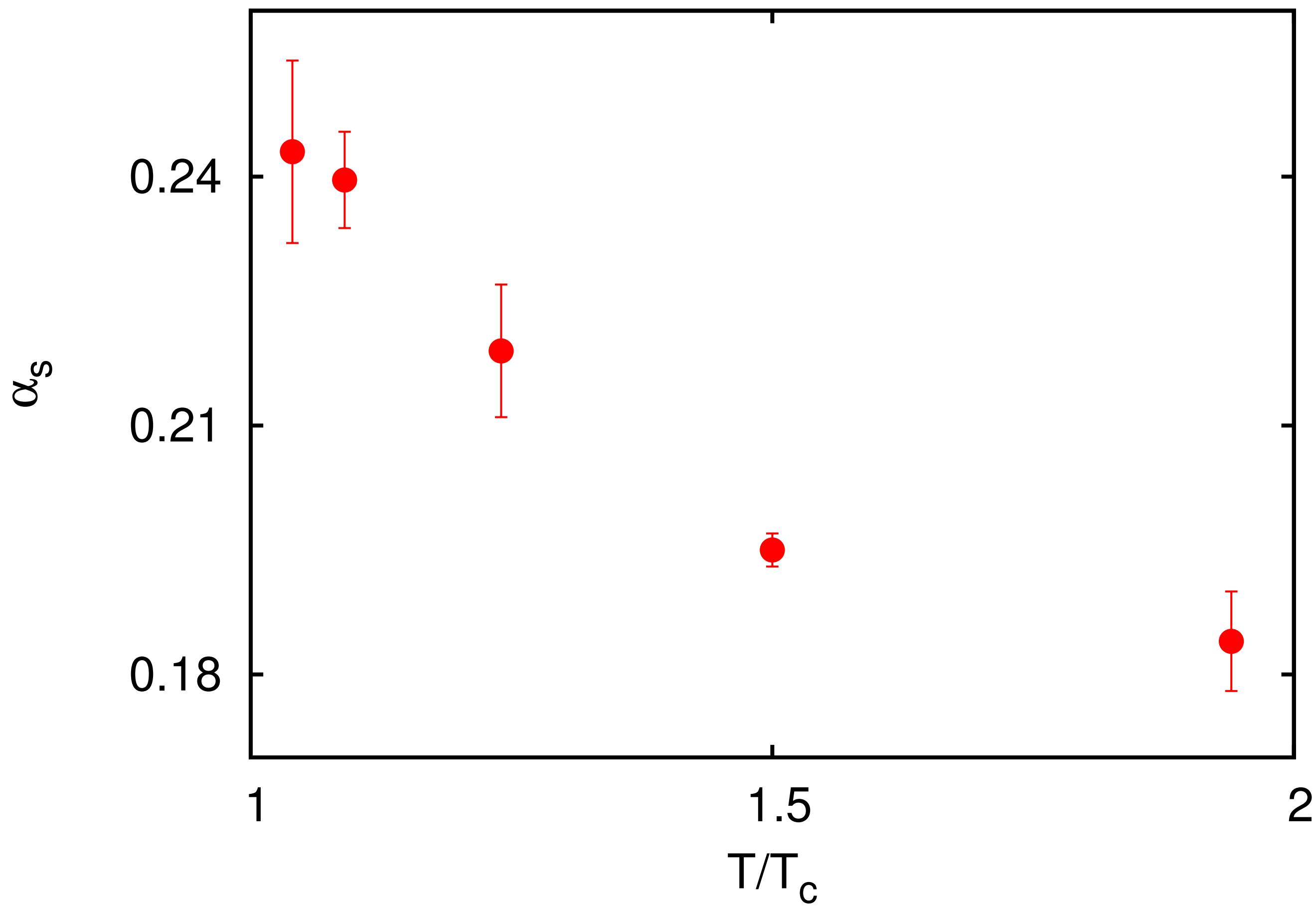


♠ Multiplying by T, obtain D, the quantity used by Moore-Teaney and PHENIX.



♡ In agreement with preliminary Bielefeld estimates (Ding et al. 1107.0311; Francis et al. 1109.3941).

♠ The ω^3 term comes with g^2. Use as a scheme to define α_s non-perturbatively.



♡ In agreement with other similar estimates (Ding et al. PRD 83 (2011) 034504).

J/ψ: Flows or not ?

♣ The diffusion coefficient D results from $colour$ interactions. Expect it to be zero for the colourless J/ψ, leading to very small flow for it due to its large mass.

♢ But the thermal charm may be in abundance and may also obey the n_q-scaling.

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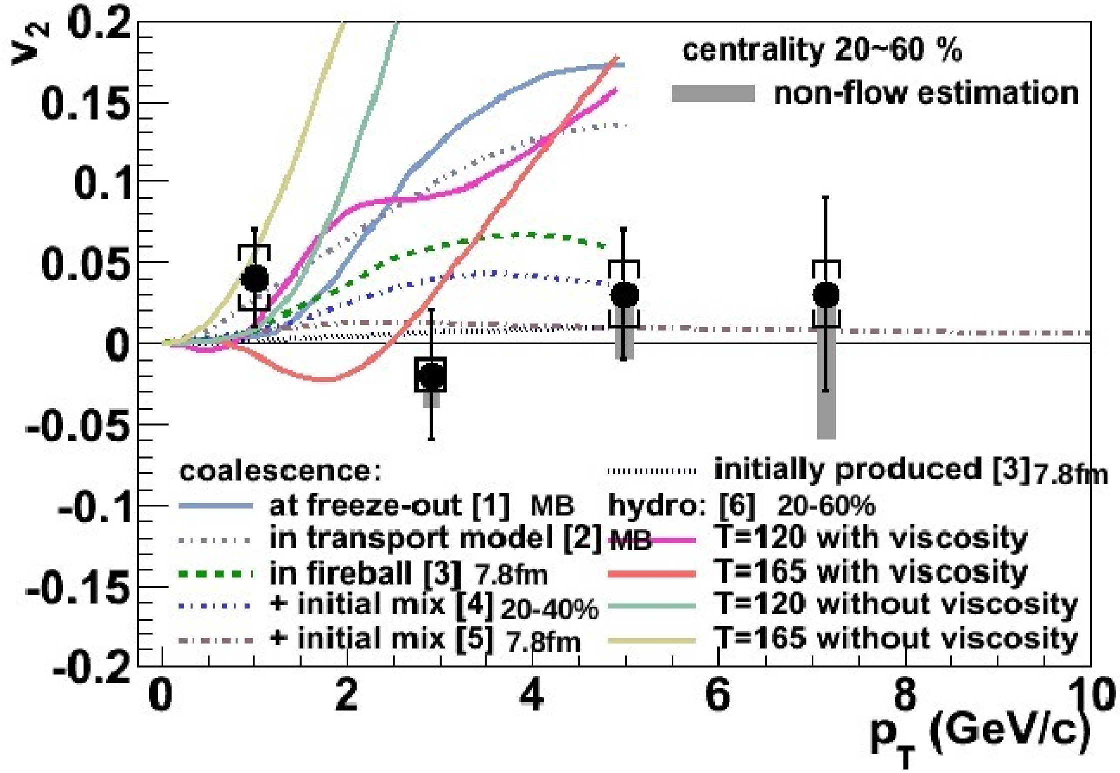
♢ But the thermal charm may be in abundance and may also obey the n_q-scaling.

♠ If thermal charm 'recombines' to produce many J/ψ, then one expects J/ψ to flow still.

♡ The STAR collaboration presented results for J/ψ flow in the recent Quark Matter 2011.

J/ψ elliptic flow v_2



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[6] U. Heinz, C. Shen, priviate communication.

Consistent with zero

Disfavor coalescence from thermalized charm quarks

Zebo Tang, USTC QM2011

Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.

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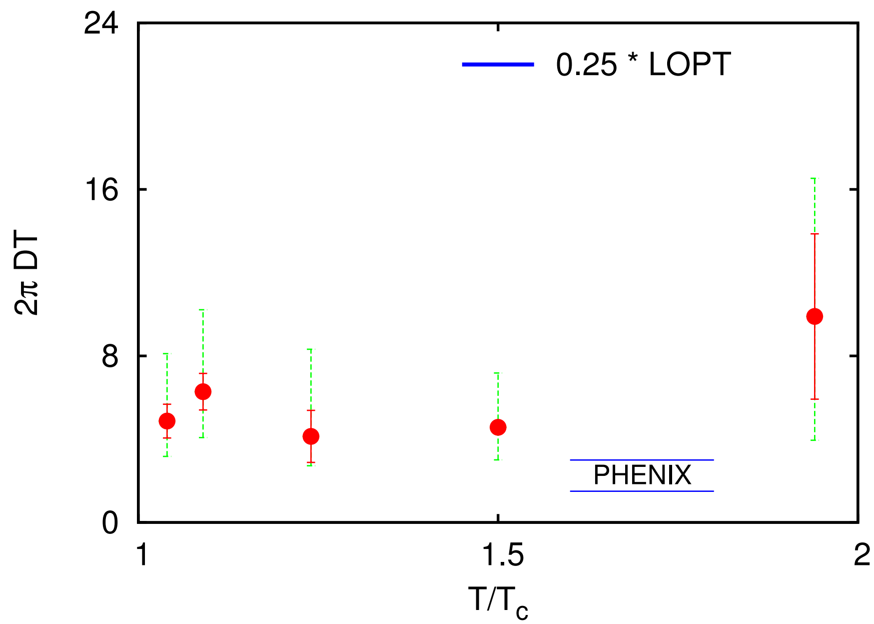

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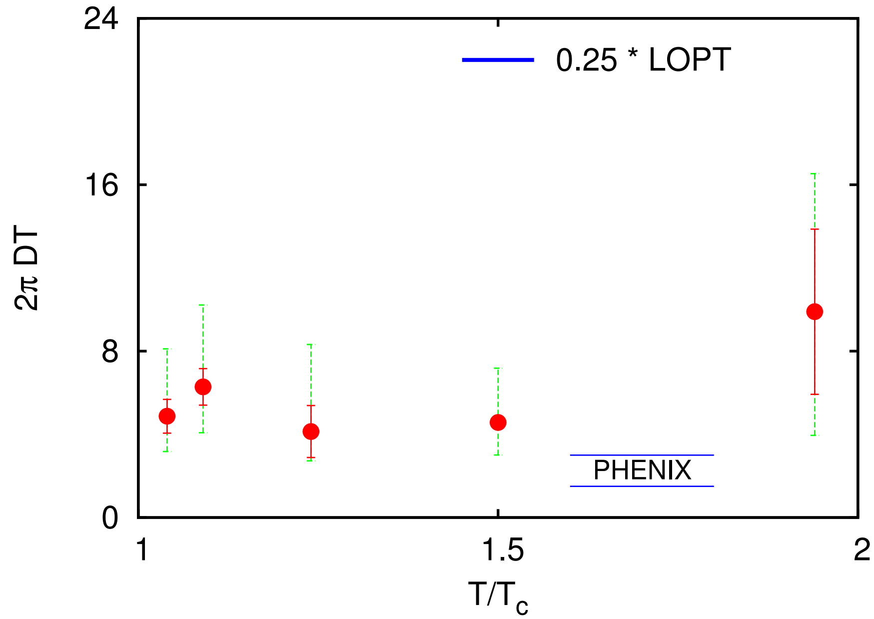
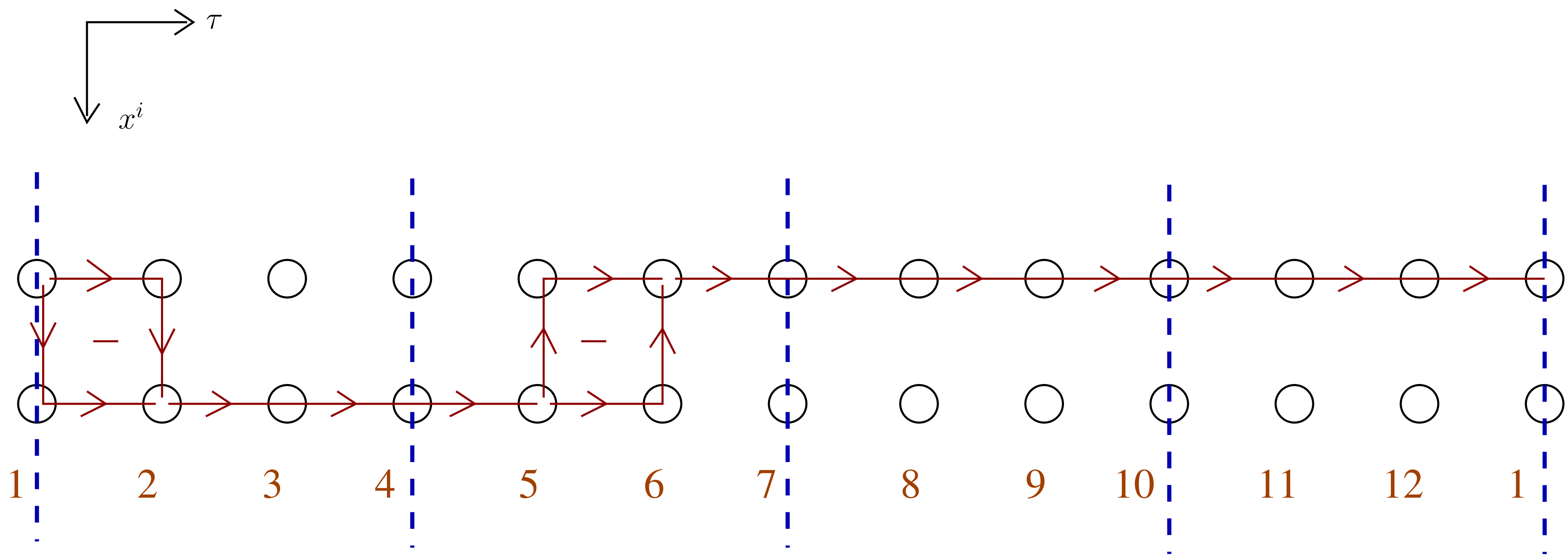
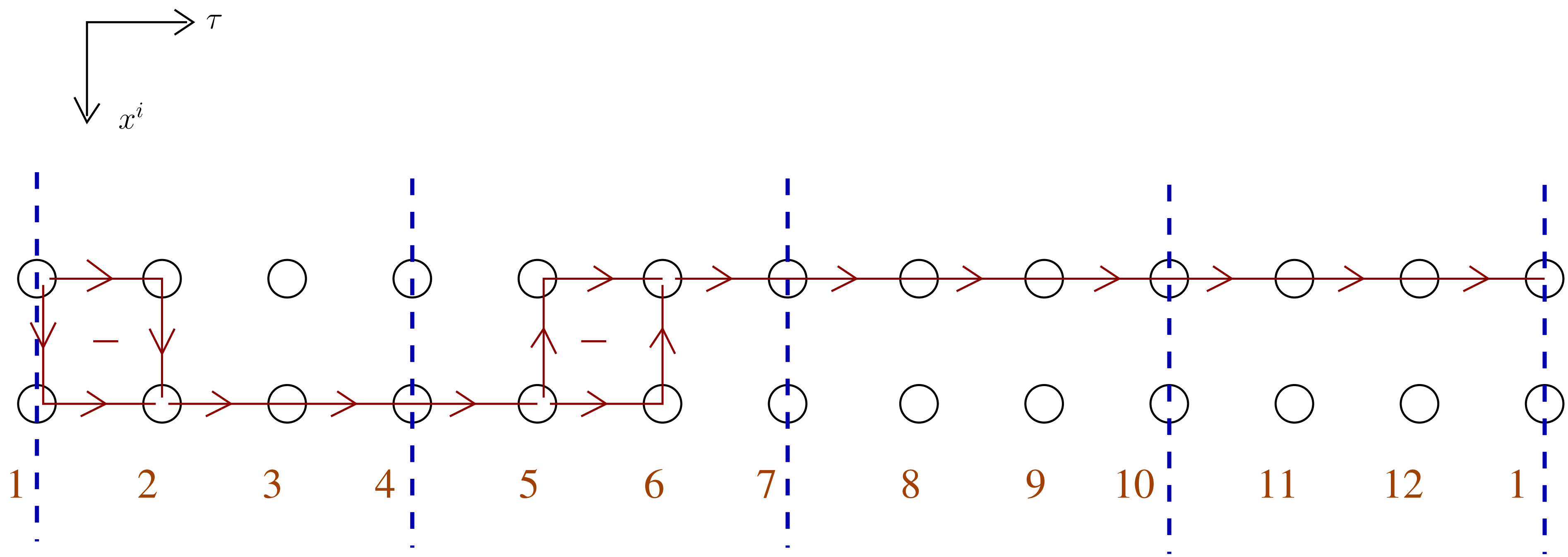


It would be interesting to see if DT vs. T/T_c exhibits similar flavour independence as the pressure.



τ
x^i
−
−
1
2
3
4
5
6
7
8
9
10
11
12
1



β	6.76	6.80	6.90	7.192	7.255
N_τ	20	20	20	24	20
T/T_c	1.04	1.09	1.24	1.5	1.96

Table 1: List of lattices on which diffusion coefficients were extracted, and the temperatures.