Lattice Perspective on Strangeness and Quasi-Quarks

Rajiv V. Gavai
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Introduction

The Wróblewski Parameter

Quasi-quarks

Screening Lengths

Summary
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• Fluctuations in conserved charges, $B$, $Q$, as promising signals of QGP
  (Asakawa-Heinz-Müller, PRL ’00, Jeon-Koch PRL ’00).

• Quark Number Susceptibilities (QNS) measure these fluctuations. Lattice
  approach yields these directly from QCD. (Gottlieb et al, ’86,’87, .. , Gavai et al. ’89...)

• Ratios of the susceptibilities, $C_{K/L} ≡ \frac{\chi_K}{\chi_L} = \frac{\sigma_K^2}{\sigma_L^2}$ are robust variables in high T
  Phase: both theoretically and experimentally.

• Many aspects studied of QGP can be studied using these, e.g., the Wróblewski
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Nature of QGP Excitations

- Outstanding question in spite of long history of several investigations:
  - Equation of State: $T \geq 3 - 5T_c$ agrees with weak coupling schemes.
  - Quark Number Susceptibilities: Successful check on them.

- We address this directly using $C_{(KL)/L} = \frac{\langle KL \rangle - \langle K \rangle \langle L \rangle}{\langle L^2 \rangle - \langle L \rangle^2}$. Physically, create an excitation of quantum number $K$ and ask what else (like $L$) does it carry.

- Screening Masses: $T \geq 2T_c \Leftrightarrow$ Fermi gas of quarks?
  We explore continuum limit using overlap quarks.
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Assuming three flavours, $u$, $d$, and $s$ quarks, and denoting by $\mu_f$ the corresponding chemical potentials, the QCD partition function is

$$Z = \int DU \exp(-S_G) \prod_{f=u,d,s} \text{Det} M(m_f, \mu_f).$$

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Defining $\mu_B = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, baryon and isospin density/susceptibilities can be obtained as:

\cite{Gottlieb et al. '87, '96, '97, Gavai et al. '89}
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\[
n_i = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_i}, \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}, \quad i, j = 0, 3, u, d, s
\]
Similarly, Charge \((Q)\), Hypercharge \((Y)\), Strangeness \((S)\) susceptibilities can be defined. Higher order susceptibilities are defined by

\[
\chi_{fg\cdots} = \frac{T}{V} \frac{\partial^n \log Z}{\partial \mu_f \partial \mu_g \cdots} = \frac{\partial^n P}{\partial \mu_f \partial \mu_g \cdots}.
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These are Taylor coefficients of the pressure $P$ in its expansion in $\mu$. All of these can be written as traces of products of $M^{-1}$ and various derivatives of $M$; Evaluated using Gaussian Noise.

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♠ Theoretical Checks : Resummed Perturbation expansions, Dimensional Reduction, Models of (s)QGP, ..

♠ We (Gavai & Gupta, PR D ’02) have argued that

\[ \lambda_s = \frac{2\chi_s}{\chi_u + \chi_d}. \] (3)
Robustness of Ratios $C$
Here

1) $C_B/Q$ and $C_{(QY)}/Q$ at $T = 2T_c$ exhibited as a function of lattice spacing.
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2) Partially Quenched ⇔ Dynamical quarks of mass 0.1\( T_c \) on \( 16^3 \times 4 \), corresponding to \( m_\rho/T_c = 5.4 \) and \( m_\pi/m_\rho = 0.3 \)
Here:

1) $C_{B/Q}$ and $C_{(QY)/Q}$ at $T = 2T_c$ exhibited as a function of lattice spacing.

2) Partially Quenched $\Leftrightarrow$ Dynamical quarks of mass $0.1T_c$ on $16^3 \times 4$, corresponding to $m_\rho/T_c = 5.4$ and $m_\pi/m_\rho = 0.3$.

3) Valence light and strange quark masses: $m_{up}^{val}/T_c = 0.03$ and $m_{str}^{val}/T_c \simeq 0.75-1.0$. 

QCD in Extreme Conditions, RBRC, BNL, Upton, USA, August 1, 2006

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Top 7
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\[ C_{X/Q} \text{ as a function of } T/T_c \text{ for } X = B, S, Y \text{ and } I_3; \text{ For } X = S, C/2 \text{ shown.} \]
Some Robust Predictions for various fluctuations thus are:

\[ C_{X/Q} \] as a function of \( T/T_c \) for \( X = B, S, Y \) and \( I_3 \); For \( X = S, C/2 \) shown.

\[ C_{S/Q} \] and \( C_{B/Q} \) exhibit a large change in going from Hadronic phase to QGP.
Wróblewski Parameter
$\lambda_s$ vs $T/T_c$

$\gamma/\Delta m^2$ vs $T/T_c$

Nt = 4
Nf = 2 QCD

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- \( \lambda_s \approx 0.4 \) close to \( T_c \), in agreement with extractions from experiment (See, e.g., Cleymans, JPG 28 (2002) 1575.) and our own earlier result in Quenched QCD. Goes down at lower temperature.
Fluctuation-Dissipation Theorem, Kramers - Kröning relation & a relaxation time approximation \( \Rightarrow \) robust observable \( C_{s/u} \equiv \lambda_s \).

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Strongly dependent on \( m_s \) for \( T \leq T_c \). \( \chi_{BY}/\Delta_{us}^2 \), curves A, D and C with \( m_s/T_c = 0.1, 0.75 \) and 1, hint at kinematic effects in the shape of \( \lambda_s \).
Flavour Carriers: Quasi-quarks?

Flavour in quark sector assists in identification of relevant degrees of freedom. Excite one quantum number and look for magnitude of another.
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Koch, Majumder and Randrup (PRL 95,182301(2005)) introduced

\[ C_{BS} = -3C_{(BS)/S} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_s} , \]

to distinguish models of QGP excitations: \( C_{BS} \approx 2/3 \) for sQGP and unity for (ideal) quarks.
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Charge and Strangeness Correlation offers another similar possibility of being unity, if strangeness is carried by quarks:

\[ C_{QS} = 3 C_{(QS)}/S = 1 - \frac{2 \chi_{us} - \chi_{ds}}{\chi_{s}}. \]
• First Results on $C_{BS}$ and $C_{QS}$:
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\begin{center}
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=0.5\textwidth,
grid=major,
]
\addplot[only marks,mark options={scale=0.5},red,mark size=4pt] table [x=T/Tc, y=CXS, meta=errorbar] {data.txt};
\addplot[only marks,mark options={scale=0.5},blue,mark size=4pt] table [x=T/Tc, y=CXS, meta=errorbar] {data2.txt};
\addplot[only marks,mark options={scale=0.5},green,mark size=4pt] table [x=T/Tc, y=CXS, meta=errorbar] {data3.txt};
\end{axis}
\end{tikzpicture}
\end{center}

• Note that while both are different from unity below $T_c$, they become close to unity immediately above $T_c$:

$\Rightarrow$ Unit strangeness is carried by objects with baryon number $-1/3$ and charge $1/3$ near $T_c$. 
• Variation of $m_s/T_c$ between 0.1 and 1.0 does not alter the value for $T \geq T_c$, $\approx 1$, or the $T$-independence.
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• Natural Explanation of $T$-behaviour if Strange Excitations with Baryon Number become lighter at $T_c$.

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• Similar results in the light quark sector:
  From e.g., $C'_{(BU)/U}$ and $C'_{(QU)/U}$, or $C'_{(BD)/D}$ and $C'_{(QD)/D}$,
  $\Rightarrow u \ (d)$-flavour is carried by $B = 1/3$ and $Q = 2/3 \ (-1/3)$ objects.
• Interactions dress up quarks. Close to $T_c$ the coupling is presumably not weak, but these flavour linkages seem to persist $\Rightarrow$ quasi-quarks.
Screening Lengths

• Using overlap quarks, we obtained screening lengths for $T \geq 1.25T_c$ earlier for $N_t = 4$ & found better agreement for even $\pi$.

• Extend to larger $N_t$ to check whether continuum limit improves it further and closer to ideal gas of quarks.
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- Lattices used: $4 \times 10^2 \times 16$, $6 \times 14^2 \times 24$, $8 \times 18^2 \times 32$, $4 \times 12^3$, and $6 \times 14^3$.

- $\beta$ values: 6.0625, 6.3384 and 6.55, $\beta_c$ for $N_t = 8$, 12 and 16 respectively.

- Zolotarev Algorithm and Multi-Shift CG inversion used.
QCD in Extreme Conditions, RBRC, BNL, Upton, USA, August 1, 2006
Summary

- Ratios of Quark Number Susceptibilities, $C_{A/B}$ are robust variables. $C_{S/Q}$ and $C_{B/Q}$ exhibit a large change in going from Hadronic phase to QGP.
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- First full QCD results for the Wróblewski Parameter $\lambda_s$ are in agreement with RHIC and SPS results near $T_c$. Being robust observables, only small lattice cut-off effects expected.
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• First full QCD results for the Wróblewski Parameter $\lambda_s$ are in agreement with RHIC and SPS results near $T_c$. Being robust observables, only small lattice cut-off effects expected.

• Screening lengths exhibit excellent single cosh behaviour & very little $a$-dependence : $\pi$ continues to be $\sim 10\%$ below ideal gas value.