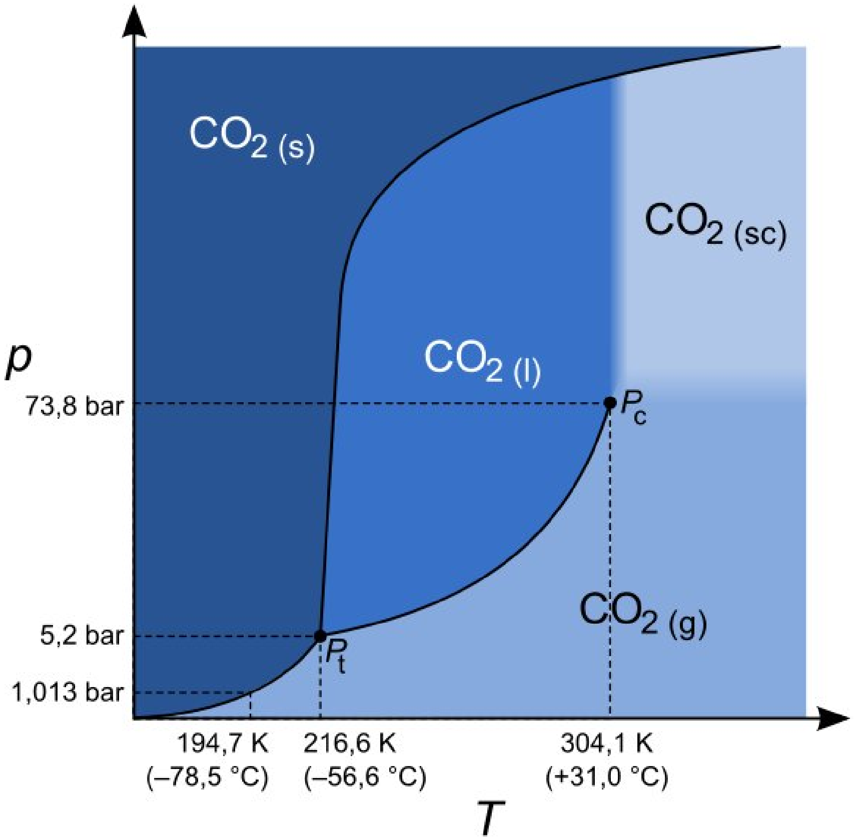
Supercomputers in Aid of QCD Critical Point Search

Rajiv V. Gavai
T. I. F. R., Mumbai, India

Introduction



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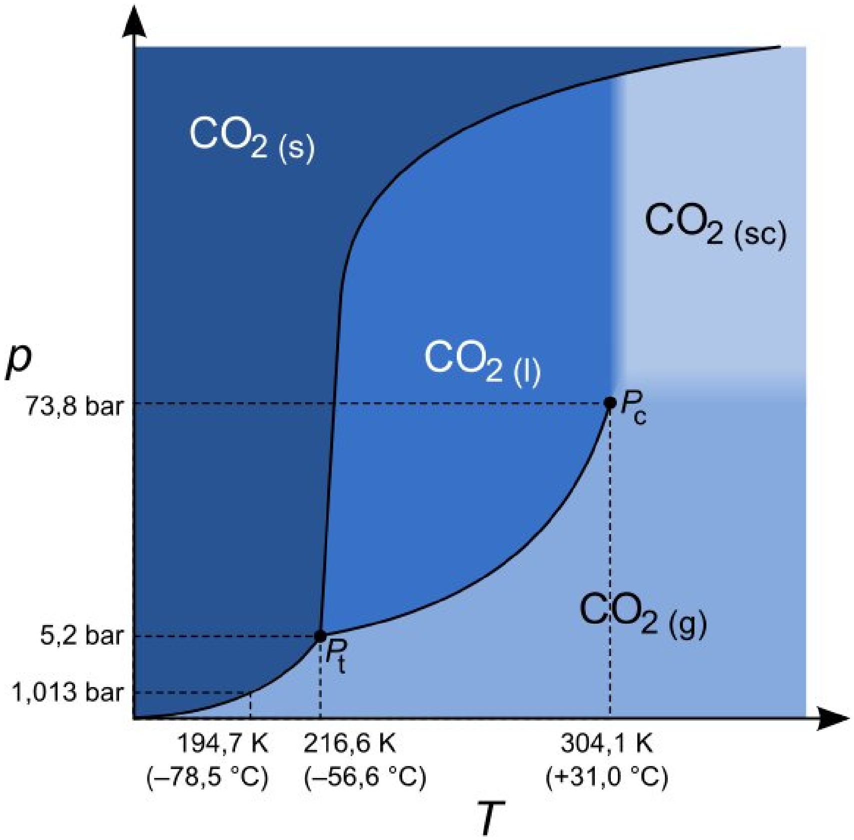

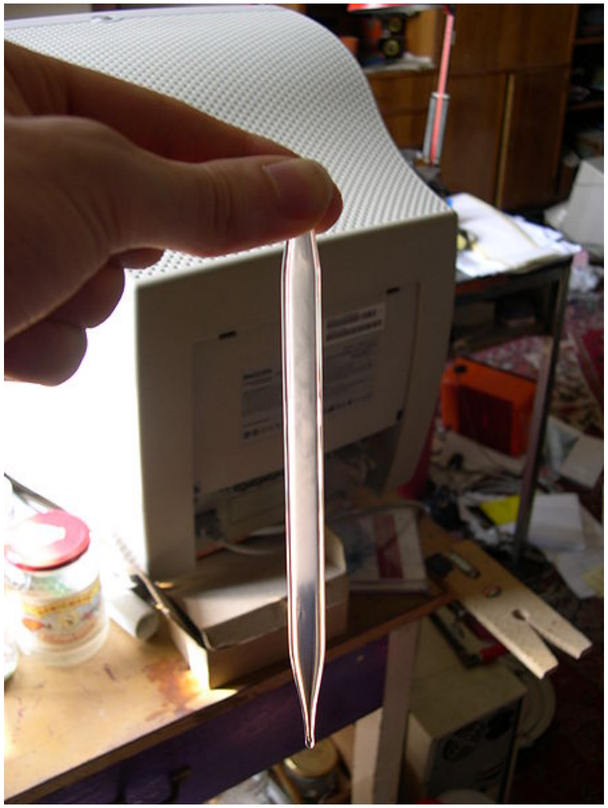

From Wikipedia

♣ What about a similar phase diagram for strongly interacting matter ?

♣ Quantum Chromo Dyanmics (QCD) is the theory for strong interactions. Can it make predictions for new phases for such matter ?

♠ Is there a QCD Critical Point in T-μ_B plane?

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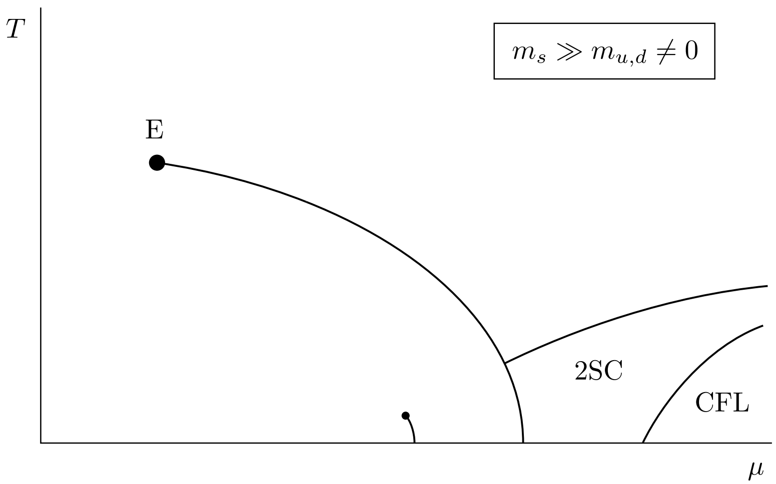

From Rajagopal-Wilczek Review

- Search for its location using *ab initio* methods
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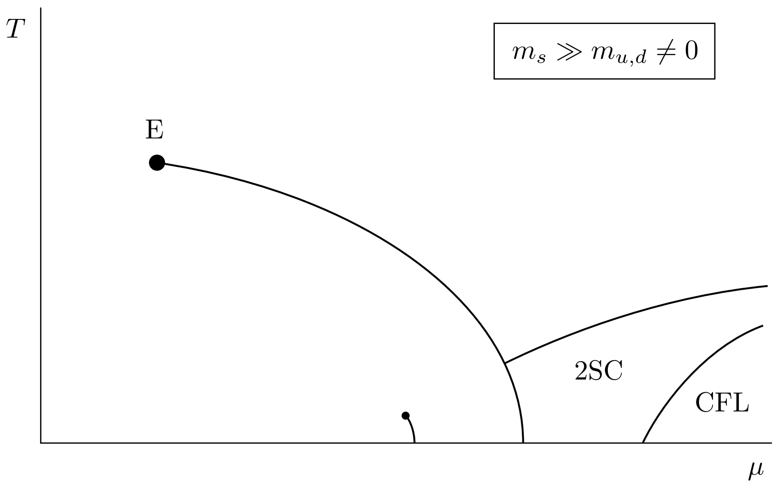
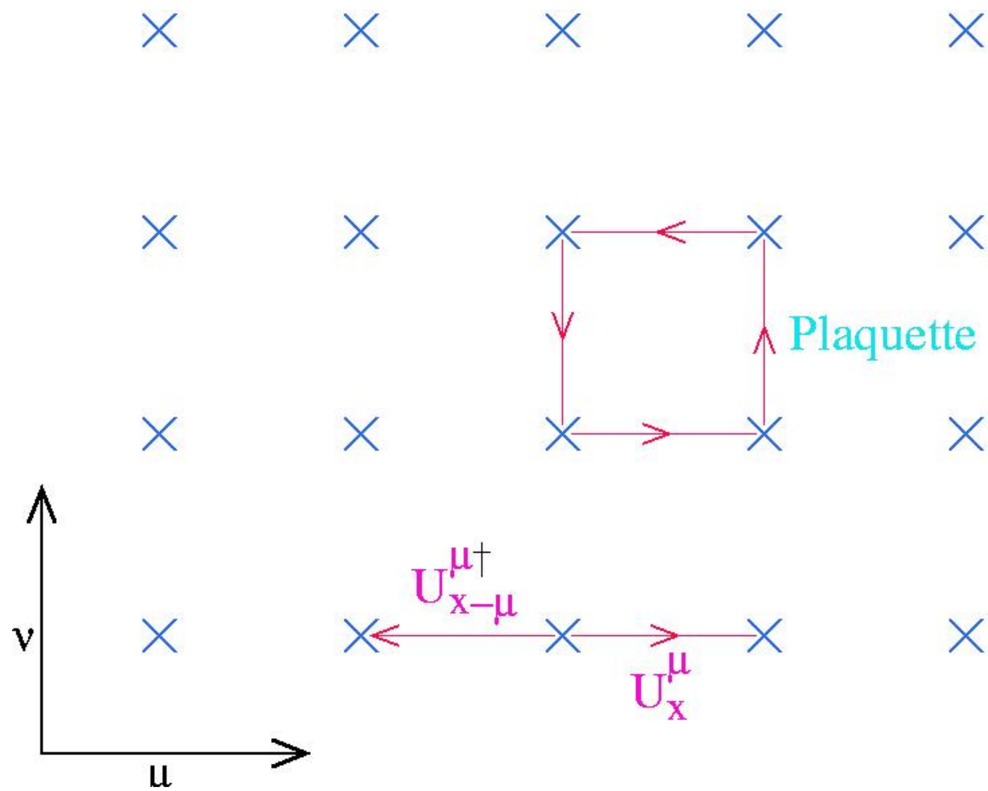


From Rajagopal-Wilczek Review

- Search for its location using *ab initio* methods
- Search for it in the experiments RHIC, FAIR,...
- What hints can Lattice QCD investigations provide ?

Basic Lattice QCD

- Discrete space-time : Lattice spacing a UV Cut-off.
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$



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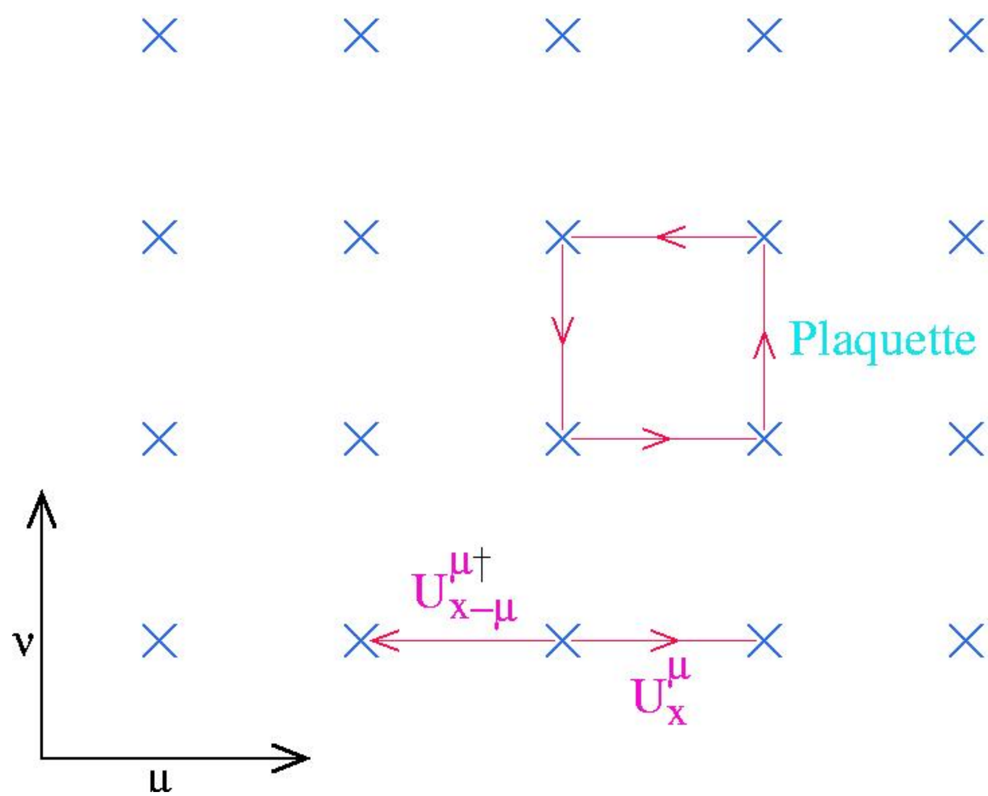
- Discrete space-time : Lattice spacing a UV Cut-off.
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$
- Gauge invariance : Actions from Closed Wilson loops, e.g., plaquette.
- Fermion Actions : Staggered, Wilson, Overlap..



The $\mu \neq 0$ problem : The Measure

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det}\ \ M(m_f, \mu_f) \quad ,$$

and the thermal expectation value of an observable $\mathcal{O}$ is

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A few million dimensional integral $\longrightarrow$ Monte Carlo Simulations to evaluate $\langle \mathcal{O} \rangle$.

Simulations can be done IF Det $M > 0$ for any set of $\{U\}$ as probabilistic methods are used. Even then, it is not easy as M is $\sim$million dimensional too.

However, det M is a complex number for any $\mu \neq 0$: The Phase/sign problem

The $\mu \neq 0$ problem : The Measure

The Phase/sign problem :

Lattice Approaches in the past decade —

- Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
- Imaginary Chemical Potential (Ph. de Forcrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).
- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, PoS LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

How Do We Do This Expansion?

Canonical definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \qquad (1)$$

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- We construct the series for baryonic susceptibility from this expansion. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^n}\right)^{1/n}$. We use both these definitions.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8^{th} order. Need 20 inversions of up to 250K $\times$ 250K matrix $(D+m)$ on $\sim$ 500 vectors for a single measurement.

- 10th & even 12th order may be possible : Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2010) which save up to 60 % computer time.

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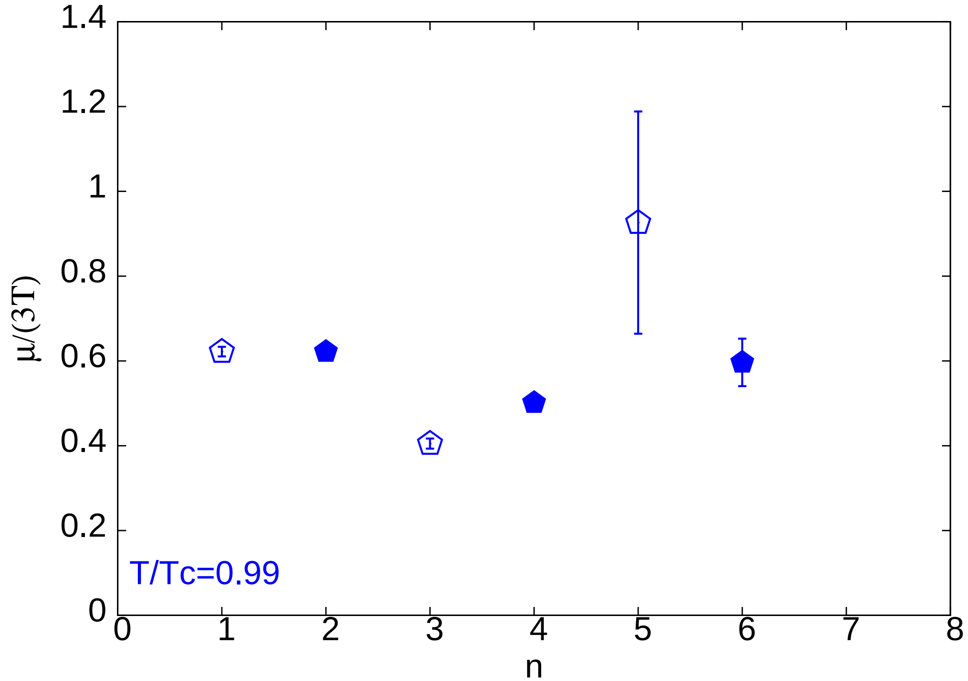
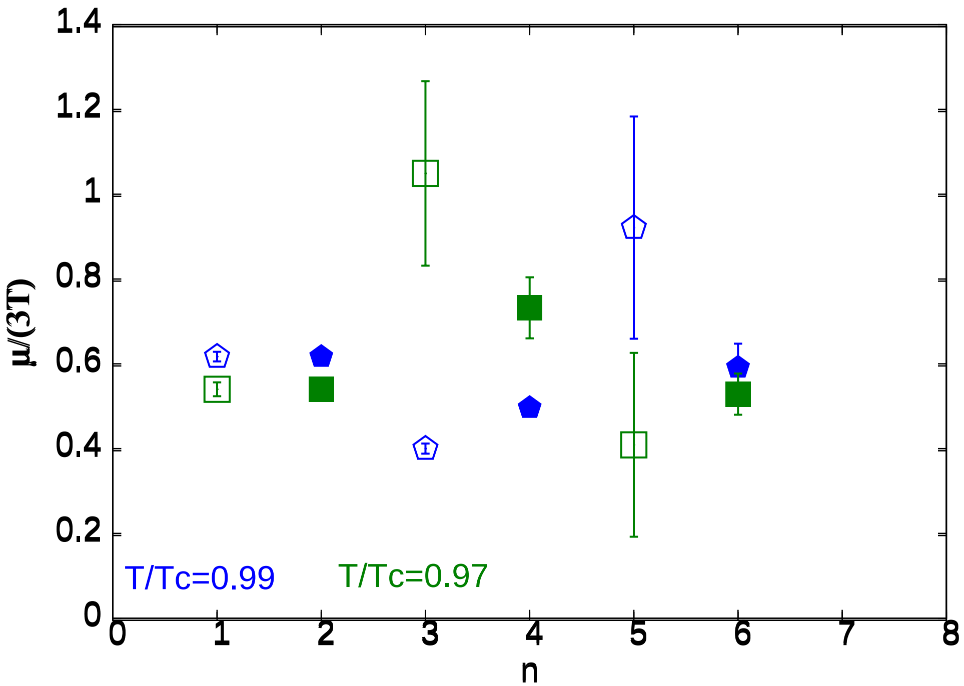
How Do We Do This Expansion?

CRAY X1 of I L G T I , T I F R, Mumbai

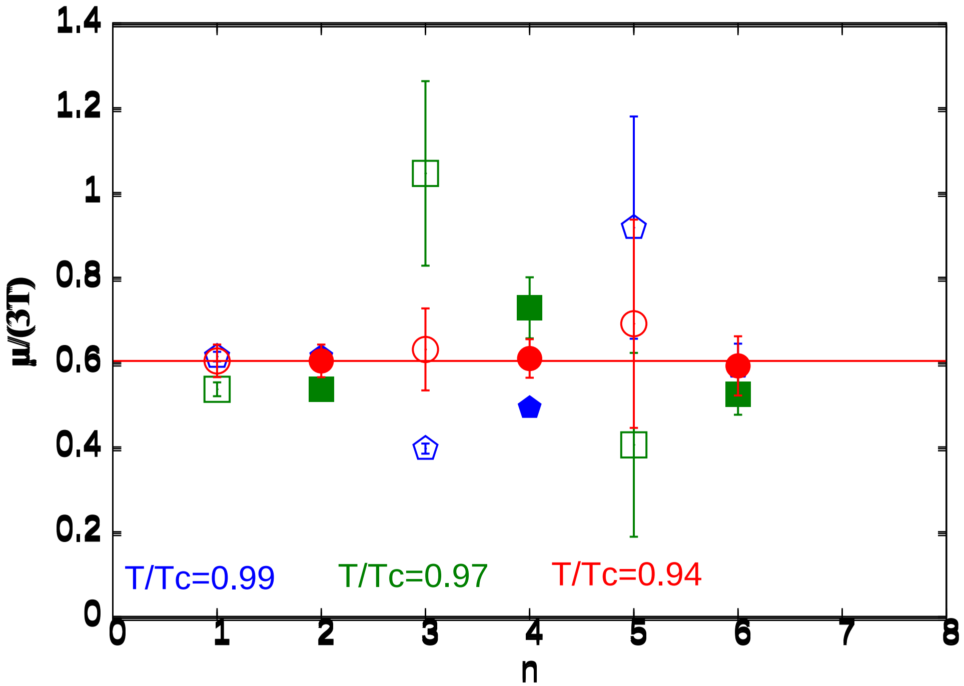
Lattice QCD Results

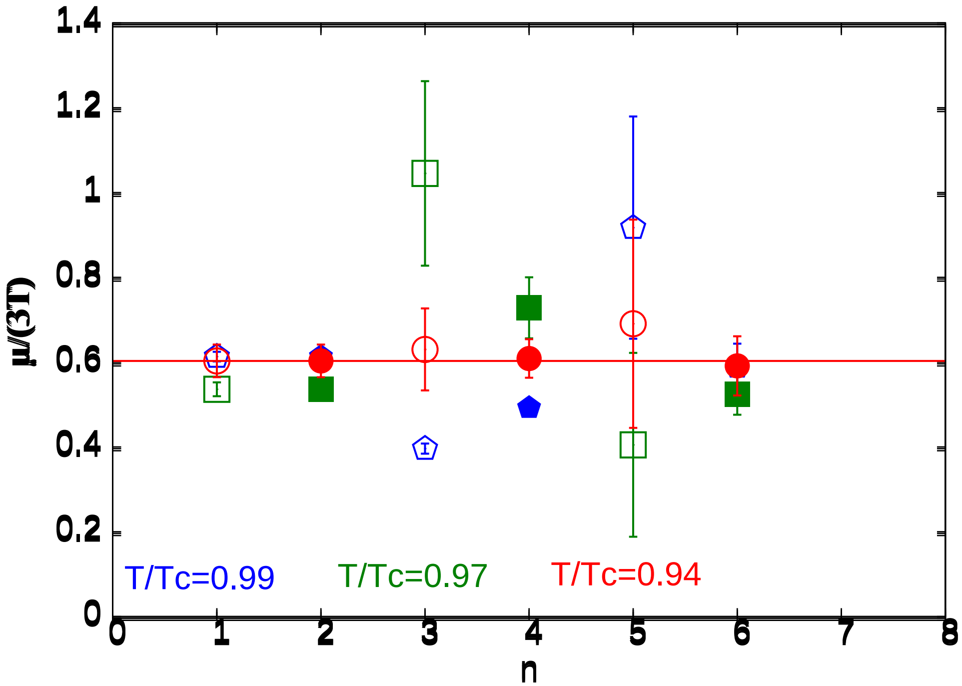
- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- $m_\pi =$ 230 MeV.
- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)
- Finer Lattice : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009). We determined β_c. Our result ($\beta_c = 5.425(5)$) well bracketed by MILC for $m/T_c = 0.075$ and 0.15.
- Our Simulations made for 0.89 $\leq T/T_c \leq$ 1.92. Typical stat. 50-200 in autocorrelation units.


μ/(3T)
n
T/Tc=0.99
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
1
2
3
4
5
6
7
8



μ/(3T)
n
T/Tc=0.99
T/Tc=0.97


1.4
1.2
1
0.8
0.6
0.4
0.2
0
µ/(3T)
0
1
2
3
4
5
6
7
8
n
T/Tc=0.99
T/Tc=0.97
T/Tc=0.94

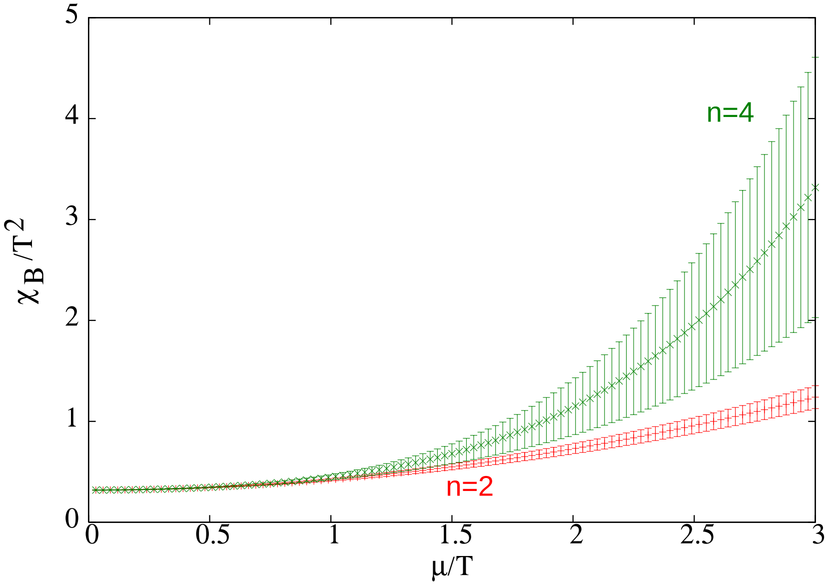


- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: $\downarrow$ to 1.1(1)

- Critical point at $\mu_B/T \sim 1 - 2$.

Cross Check on μ^E/T^E

♠ Use the series directly to construct χ_B for nonzero $\mu \longrightarrow$ smooth curves with no signs of criticality.



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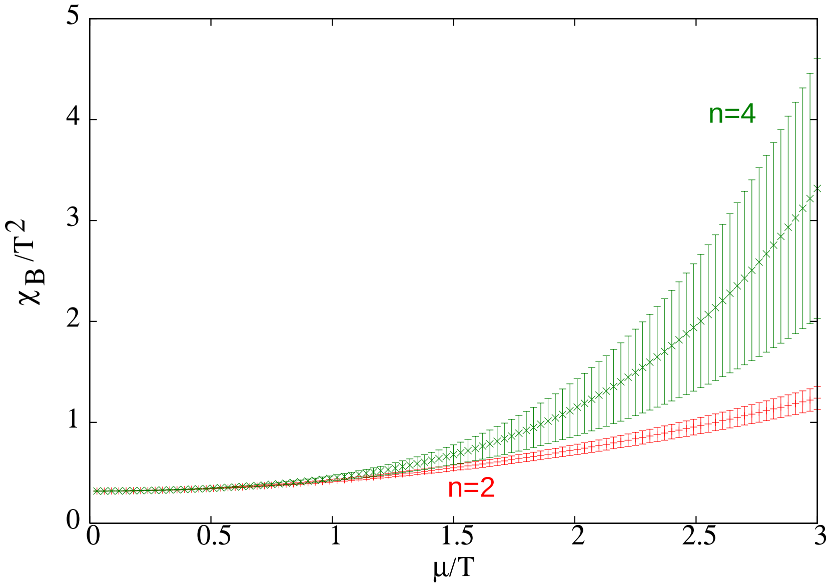
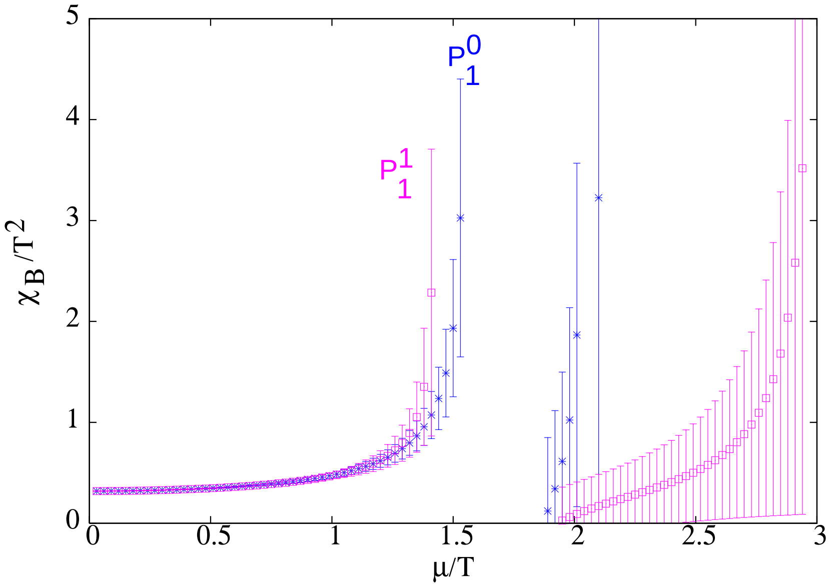


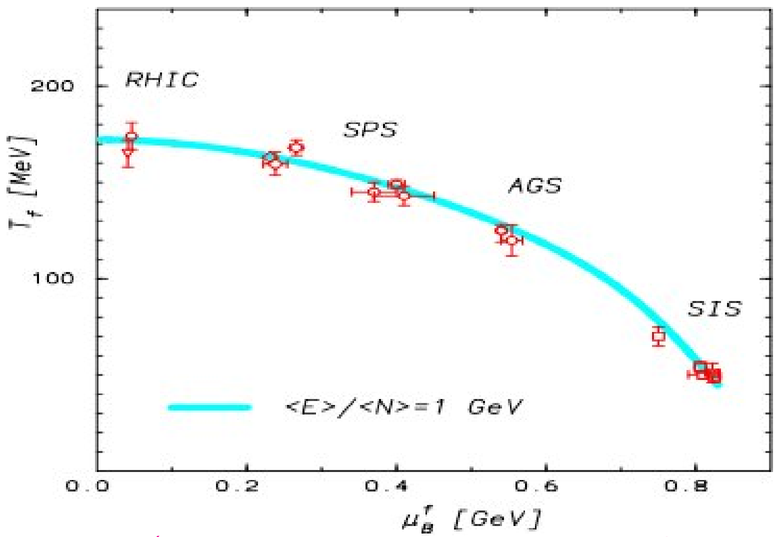


♠ Use Padé approximants for the series to estimate the radius of convergence.

♡ Consistent Window with our other estimates.

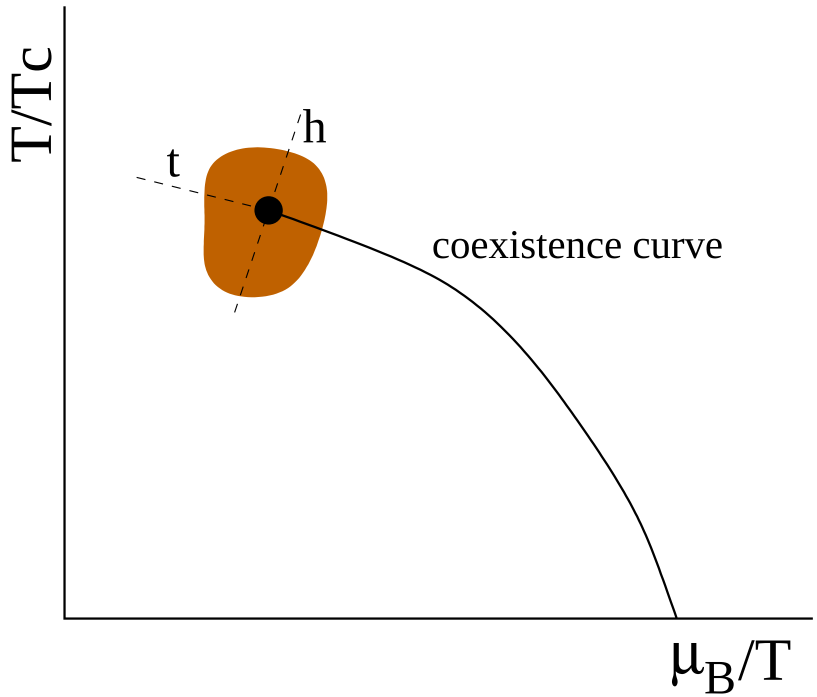
Lattice predictions along the freezeout curve

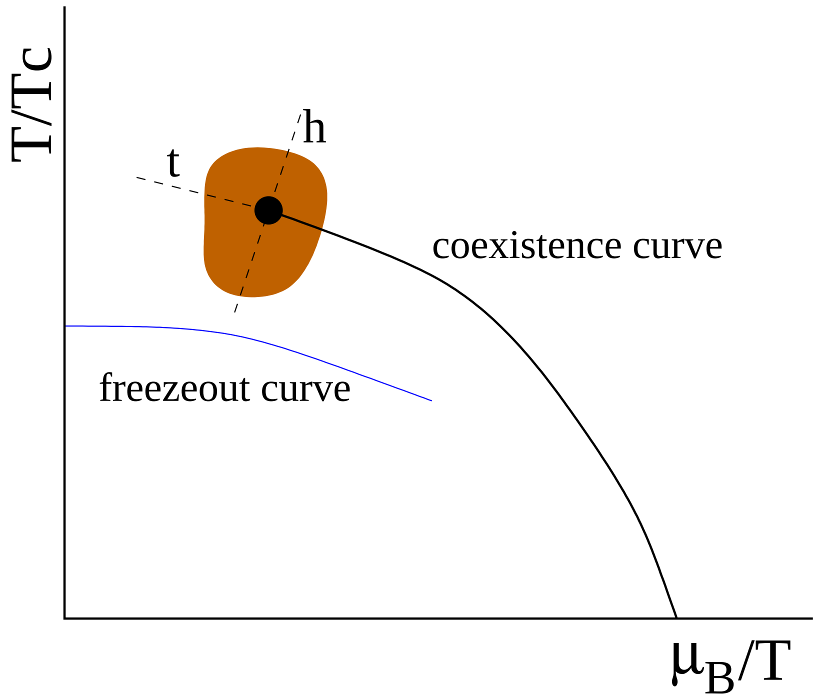
- Hadron yields data from BNL and CERN experiments lead to a freezeout curve in the T-μ_B plane. (Andronic,et al 2009 ; Oeschler, et al, 2009, Cleymans-Redlich, 1998)

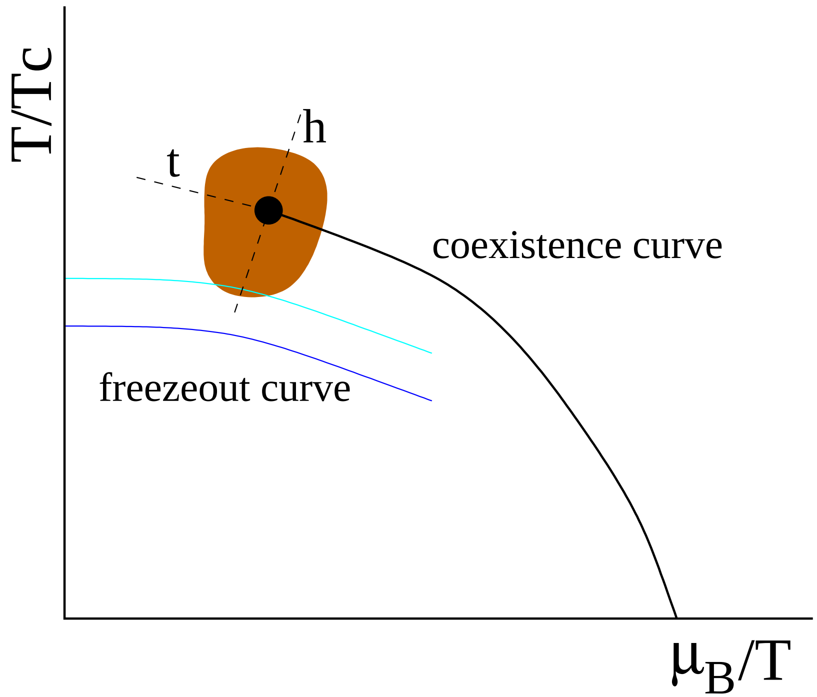


(From Braun-Munzinger, Redlich and Stachel nucl-th/0304013)

- Key point : Freeze-out curve, based soled on data on hadron yields, gives the (T, μ) accessible as a function of $\sqrt{s}$ in heavy-ion experiments.


T/Tc
h
t
coexistence curve
μB/T


T/Tc
h
t
coexistence curve
freezeout curve
μ_B/T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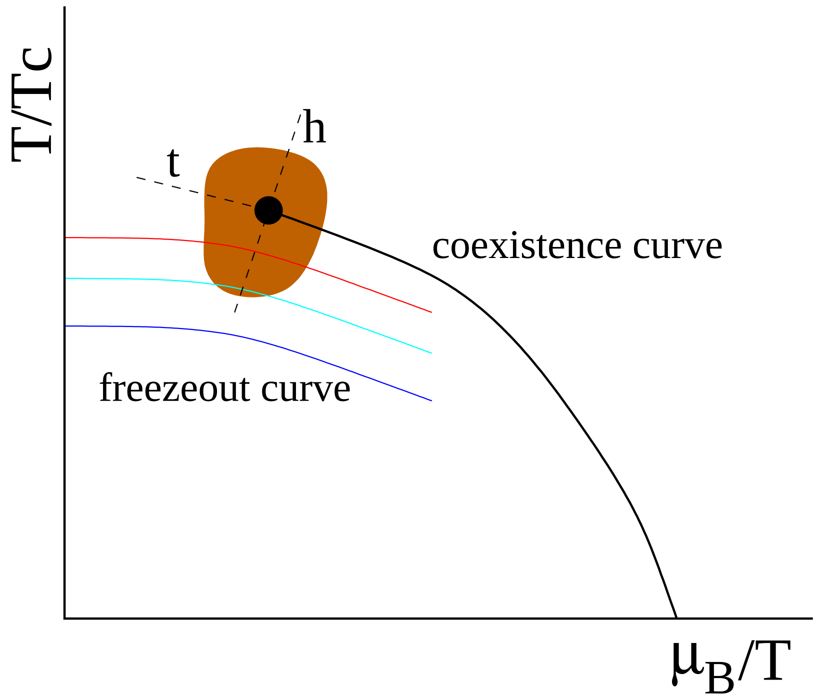
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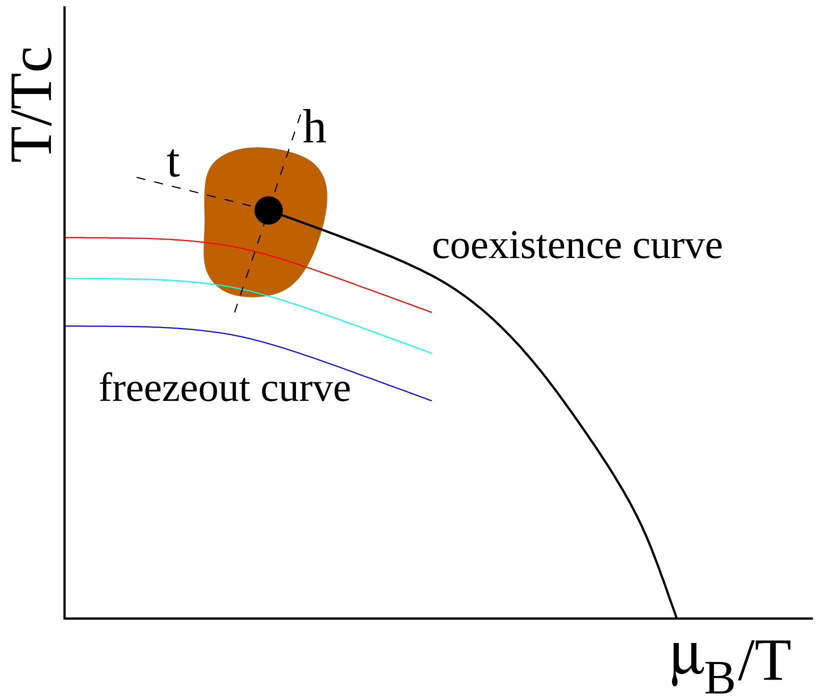
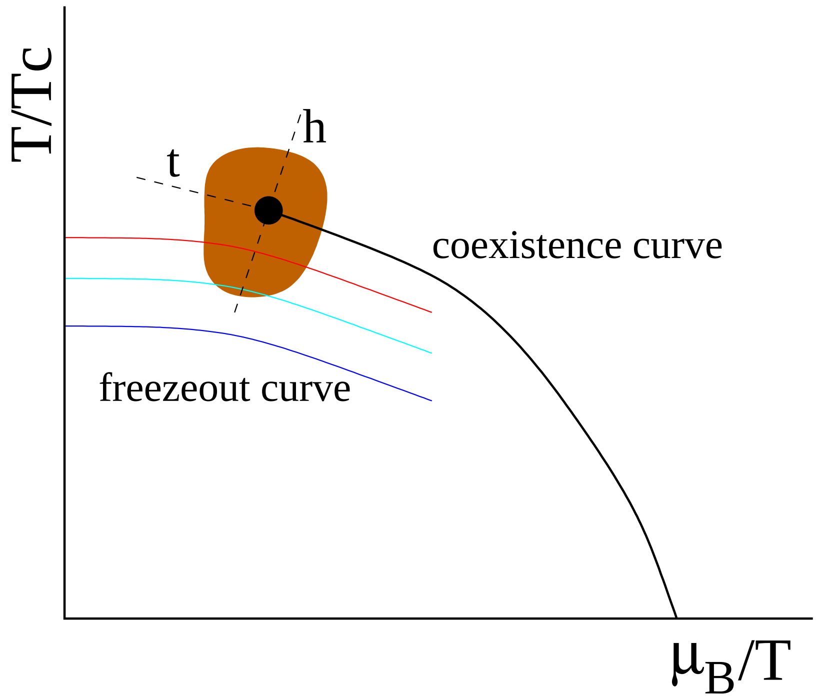


- Use the freezeout curve computed from hadron abundances to relate $(T, \mu_B)$to $\sqrt{s}$ and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)



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- Define $m_1 = \frac{T\chi^{(3)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)}$, $m_3 = \frac{T\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)}$, and $m_2 = m_1 m_3$ (Gupta, arXiv : 0909.4630) and use the Padè method to construct them.

- Near the critical point, $\chi_B \sim |\mu - \mu_E|^{\delta}$. Thus the ratios of successive NLS, m_i, should diverge in the critical region as well.

- Spatial Volume cancels out in these ratios $\Longrightarrow$ Suitable for experiments who can use their favourite proxy for it.

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- Defining $z = \mu_B/T$, and denoting by r_{ij} the estimate for radius of convergence using χ_i, χ_j, one has

$$m_1 = \frac{2z}{r_{24}^2}\Big[1 + \Big(\frac{2r_{24}^2}{r_{46}^2} - 1\Big)z^2 + \Big(\frac{3r_{24}^2}{r_{46}^2 r_{68}^2} - \frac{3r_{24}^2}{r_{46}^2} + 1\Big)z^4 + \mathcal{O}(z^6)\Big] \ .$$

- Similar series expressions for m_2 and m_3. Resum these by Padè ansatz :

$$m_1 = zP_1^1(z^2; a, b), \qquad m_3 = \frac{1}{z}P_1^1(z^2; a', b')$$

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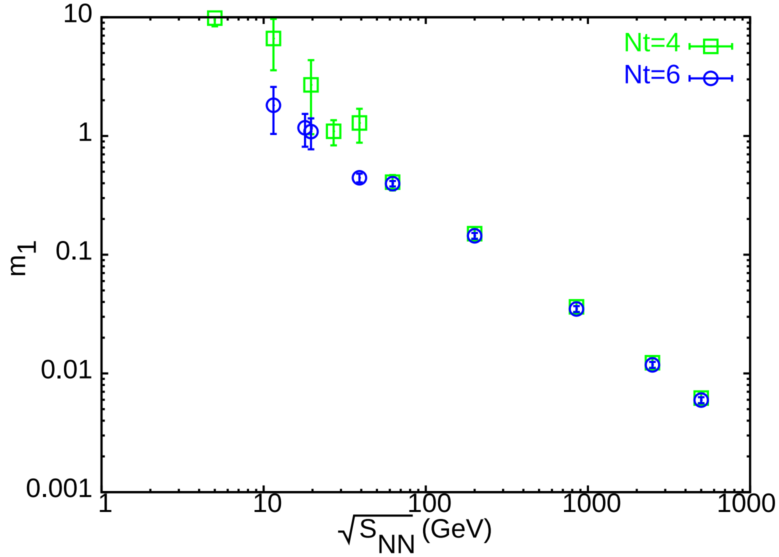
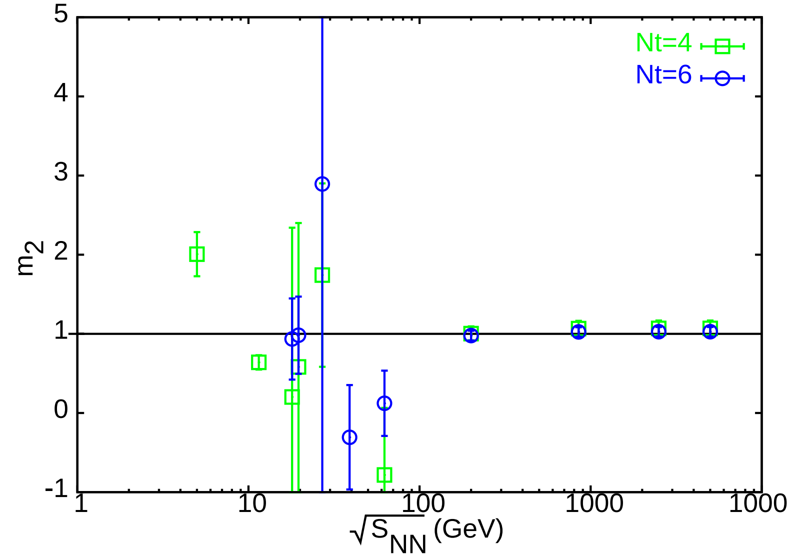
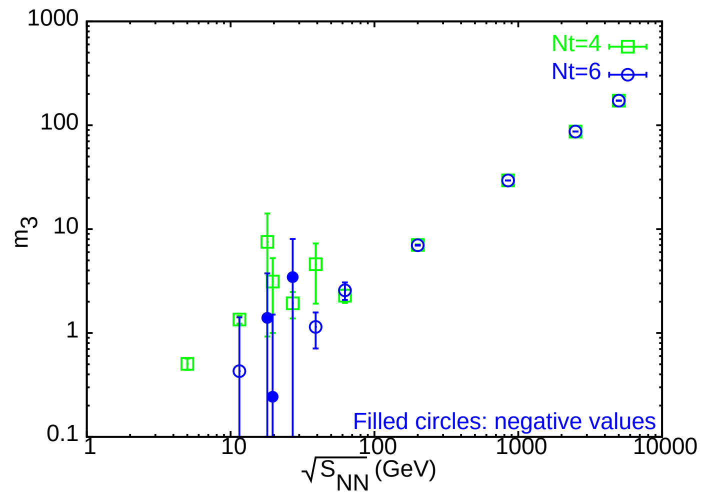
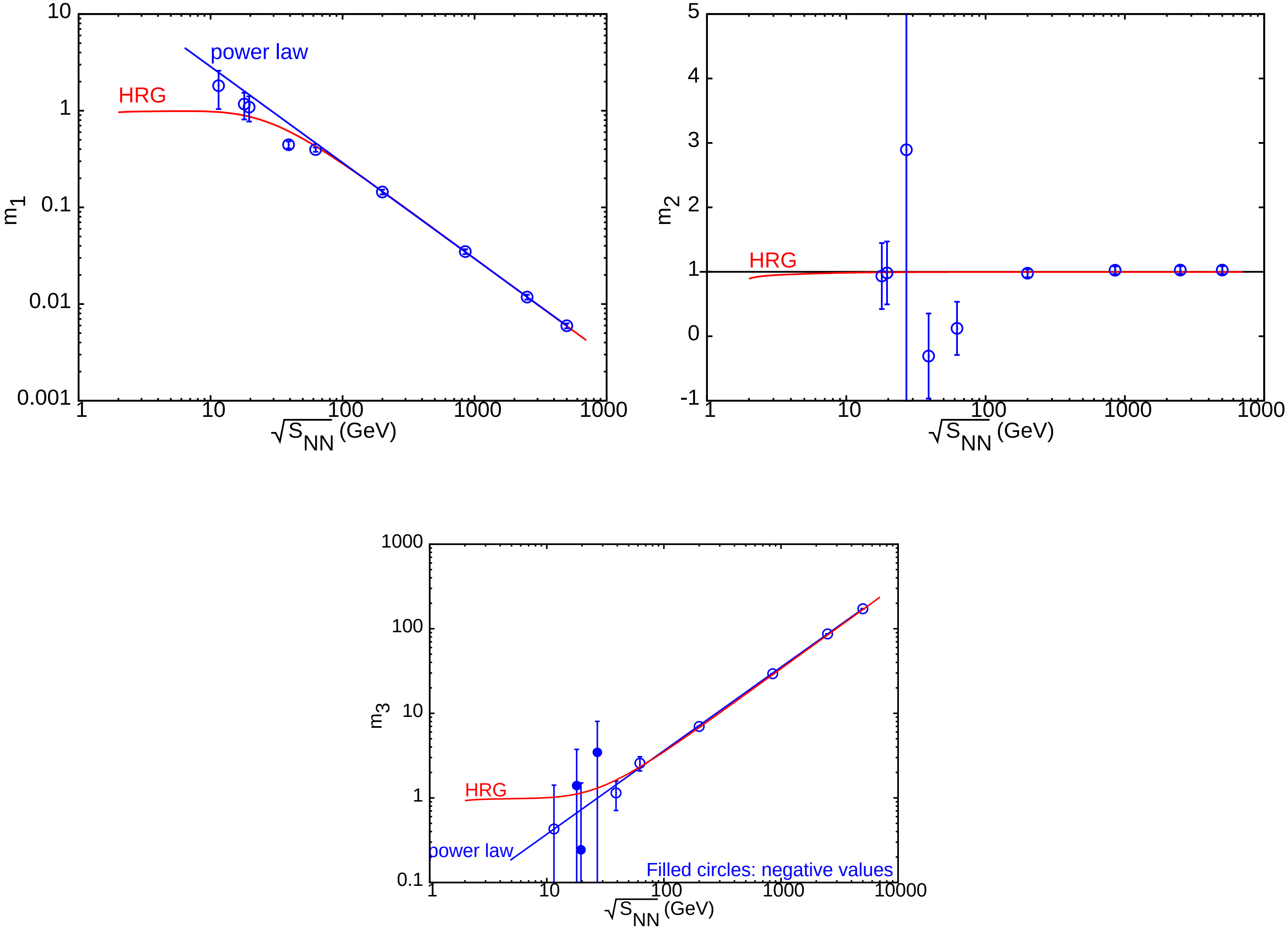

Nt=4
Nt=6
m1
10
1
0.1
0.01
0.001
1
10
100
1000
1000
√SNN (GeV)


Nt=4
Nt=6
m2
5
4
3
2
1
0
-1
1
10
100
1000
1000
√SNN (GeV)


Nt=4
Nt=6
m3
1000
100
10
1
0.1
1
10
100
1000
10000
√SNN (GeV)
Filled circles: negative values


power law
HRG
m_1
$\sqrt{S_{NN}}$ (GeV)
HRG
m_2
$\sqrt{S_{NN}}$ (GeV)
HRG
power law
Filled circles: negative values
m_3
$\sqrt{S_{NN}}$ (GeV)

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- Note that even in this smooth region, an experimental comparison is exciting : Direct Non-Perturbative test of QCD in hot and dense environment.

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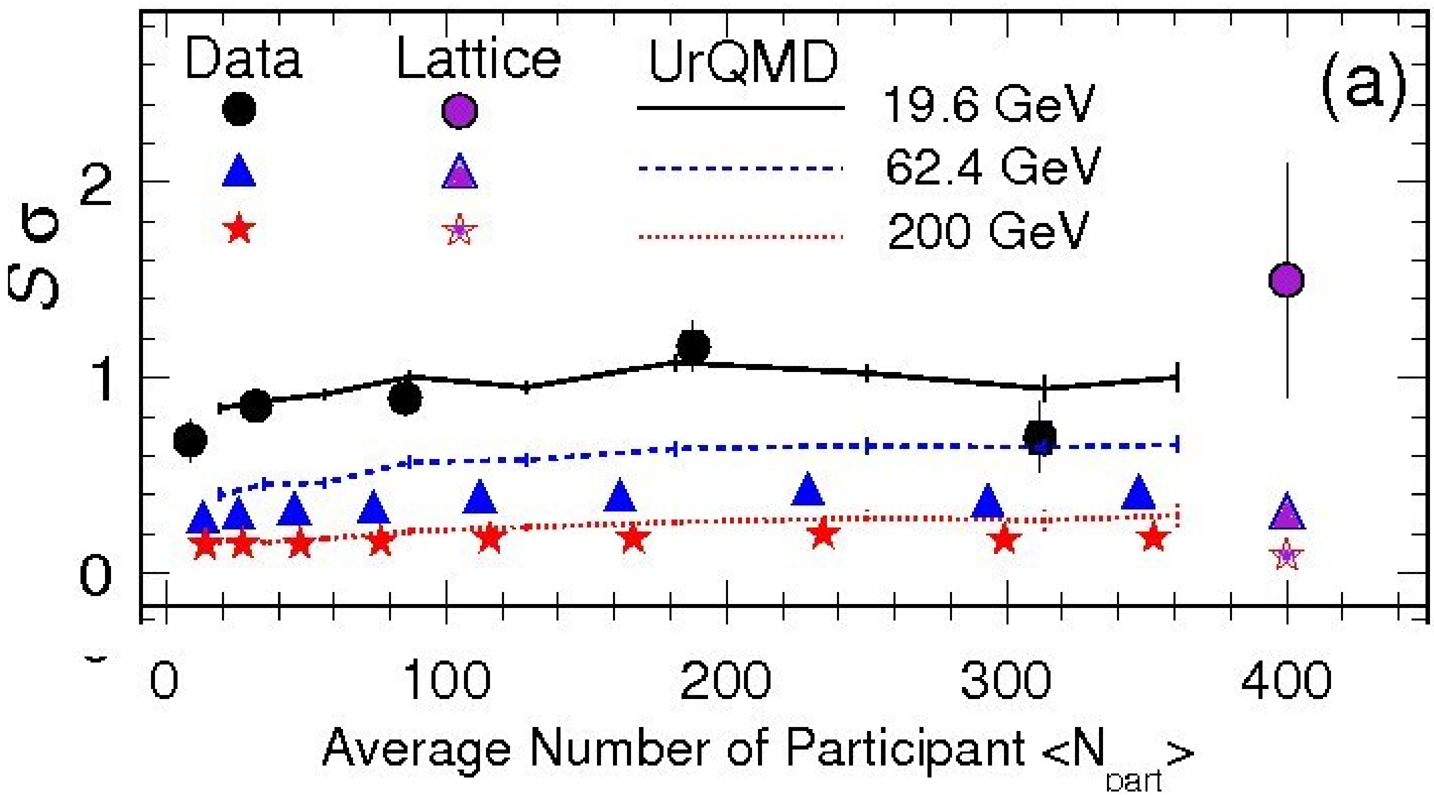
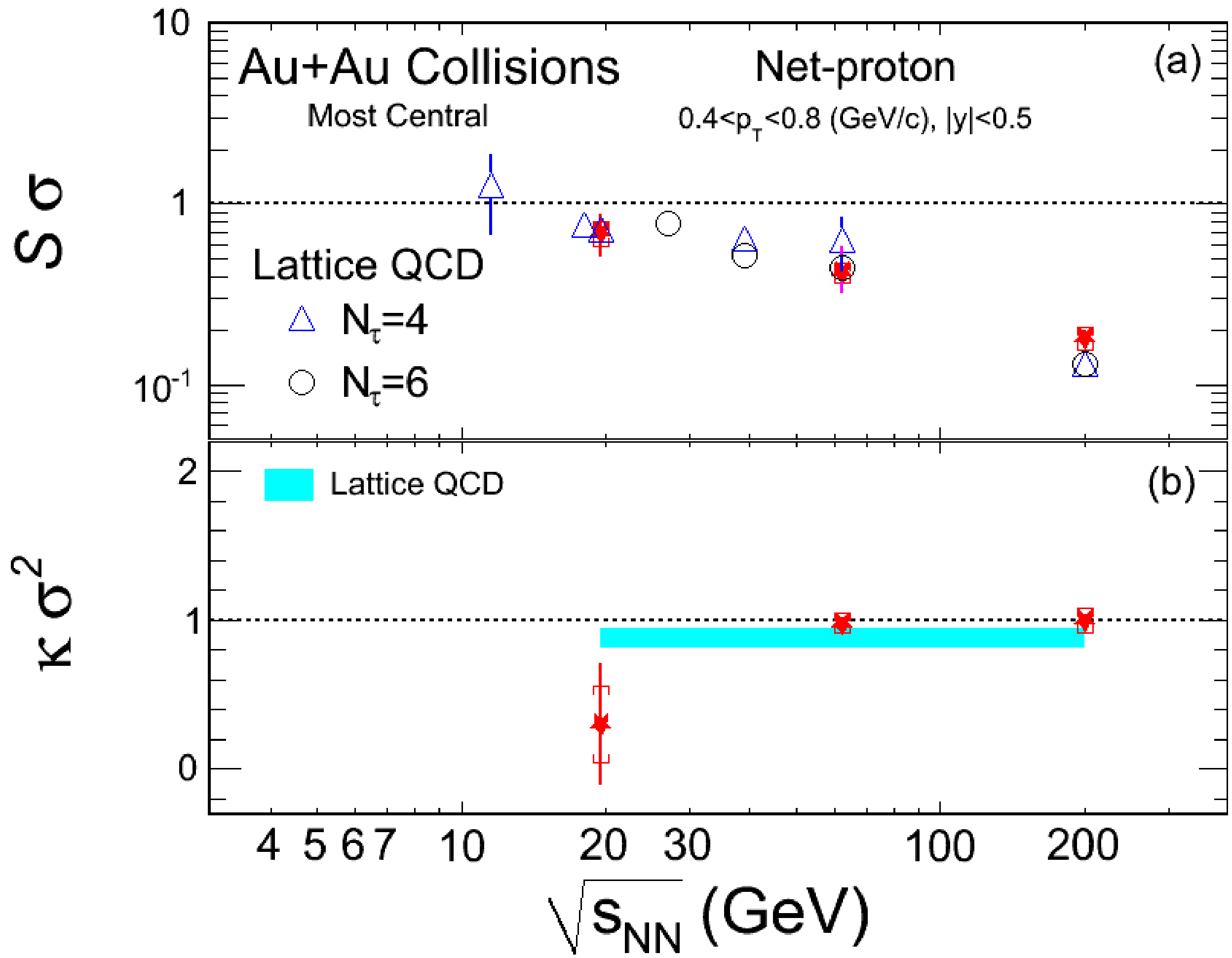
- Our estimated critical point suggests non-monotonic behaviour in all m_i, which would be accessible to the low energy scan of RHIC BNL !
- Proton number fluctuations (Hatta-Stephenov, PRL 2003): Diverging ξ at critical point is linked to σ mode which cannot mix with any isospin modes $\Rightarrow \chi_I$ to be regular.
- Leads to a ratio χ_Q:χ_I:χ_B = 1:0:4
- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be proton number fluctuations only.



Aggarwal et al., STAR Collaboration, arXiv : 1004.4959

- Reasonable agreement with our lattice results. Where is the critical point ?



H. Ritter, STAR Collaboration, ICPAQGP, Goa, 2010.

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

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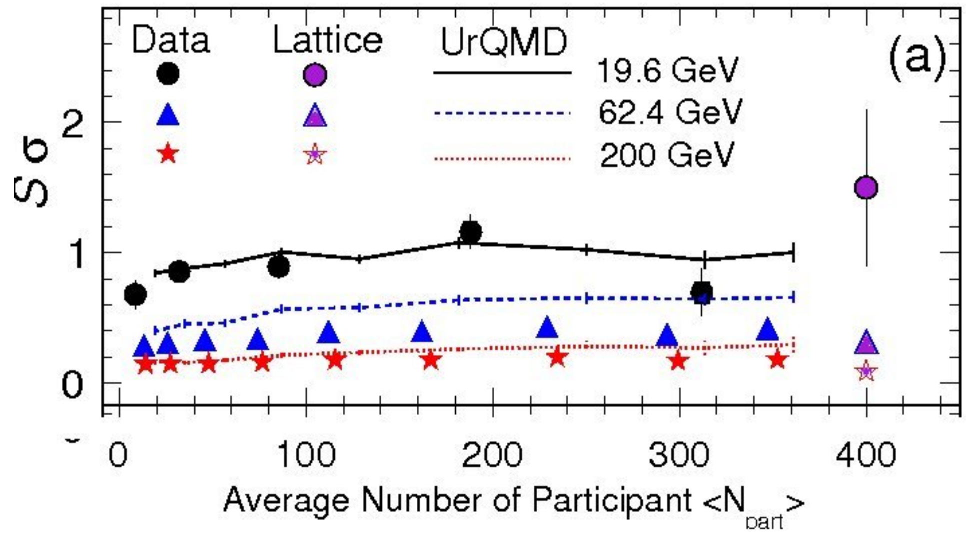

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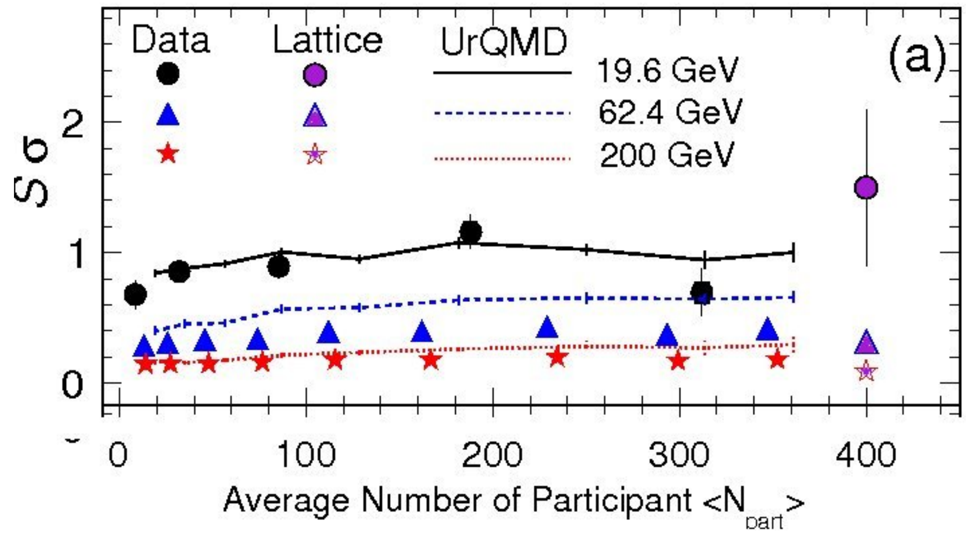

So far no signs of a critical point in the experimental results at CERN.
Will RHIC energy scan deliver it for us ? and/or Will it be FAIR ?

The critical endpoint (II)

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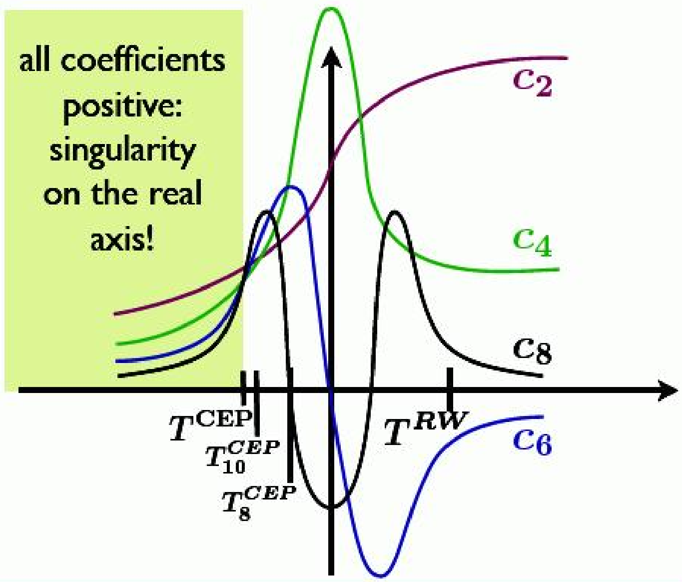
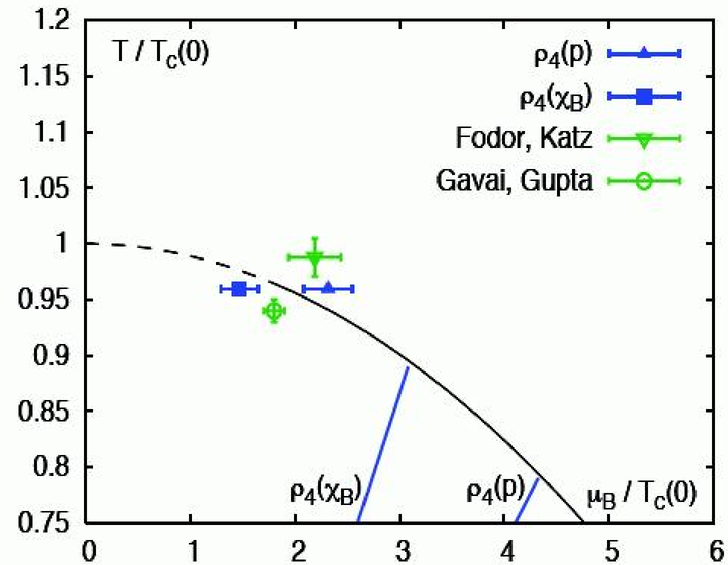
method for locating of the CEP:

- determine largest temperature where all coefficients are positive → T^{CEP}
- determine the radius of convergence at this temperature → μ^{CEP}

$$p = c_0 + c_2\,(\mu_B/T)^2 + c_4\,(\mu_B/T)^4 + \cdots$$

$$\chi_B = 2c_2 + 12c_4\,(\mu_B/T)^2 + 30c_6\,(\mu_B/T)^4 + \cdots$$





first non-trivial estimate of T^{CEP} by c_8
second non-trivial estimate of T^{CEP} by c_{10}

$$\rho_n(p) = \sqrt{c_n/c_{n+2}}$$

$$\rho = \lim_{n\to\infty} \rho_n$$

(Ch. Schmidt FAIR Lattice QCD Days, Nov 23-24, 2009.)

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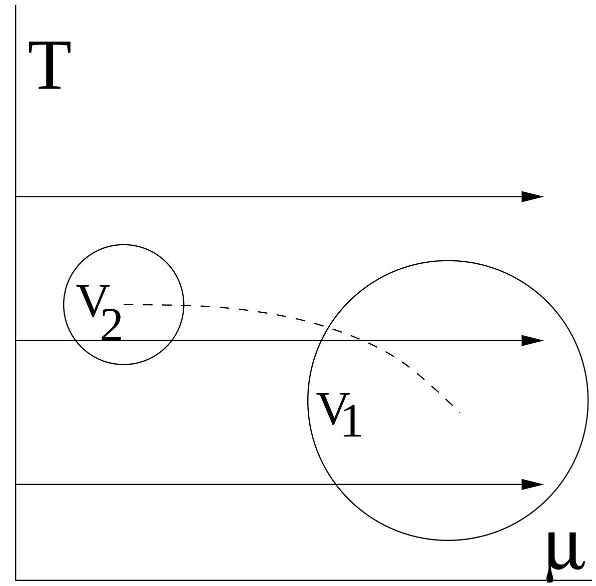
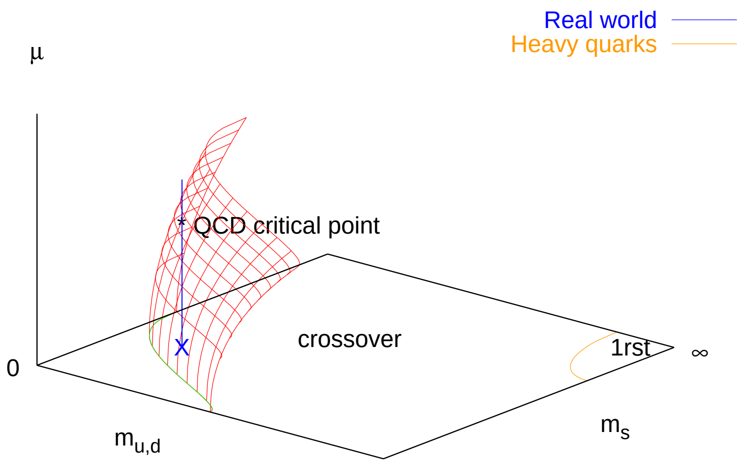
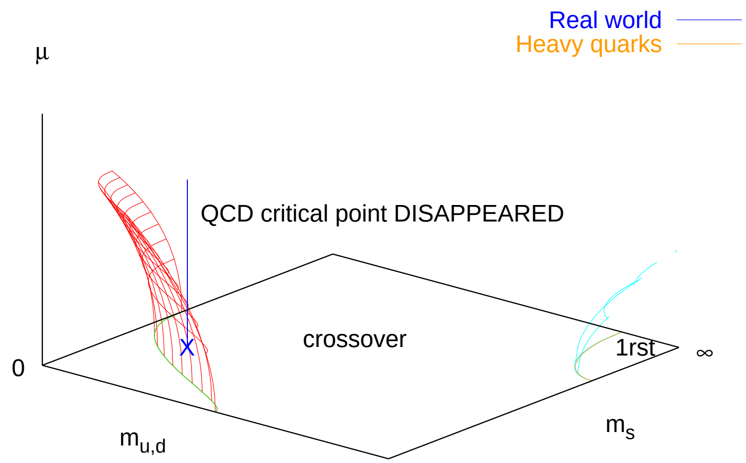


We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811





For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T_c}\right)^2 - 47(20)\left(\frac{\mu}{\pi T_c}\right)^4$, i.e., m_c shrinks with μ.

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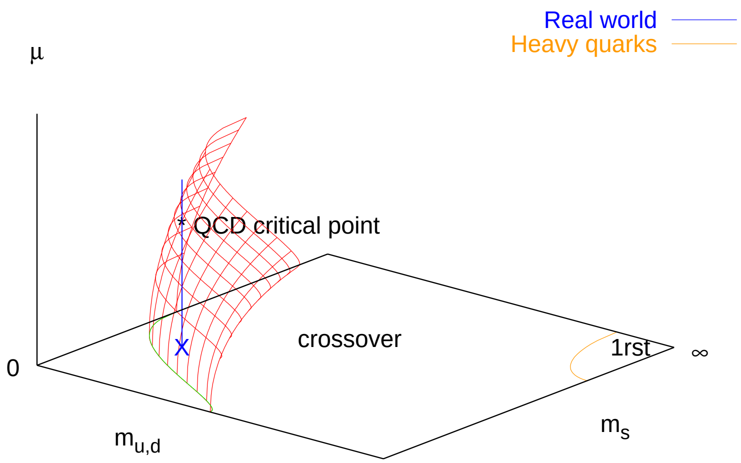

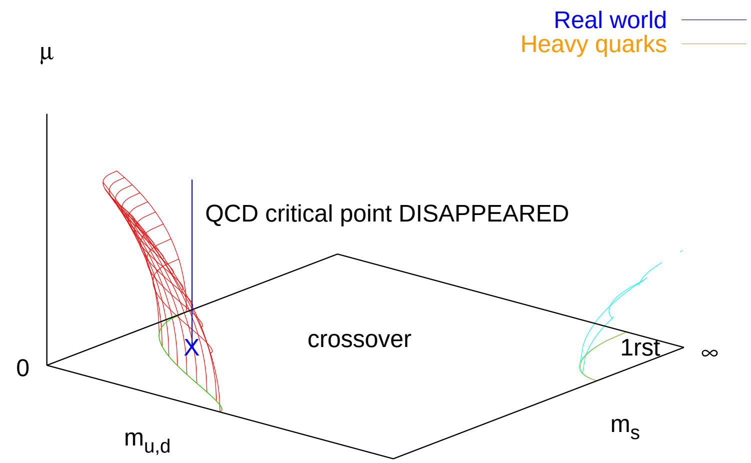

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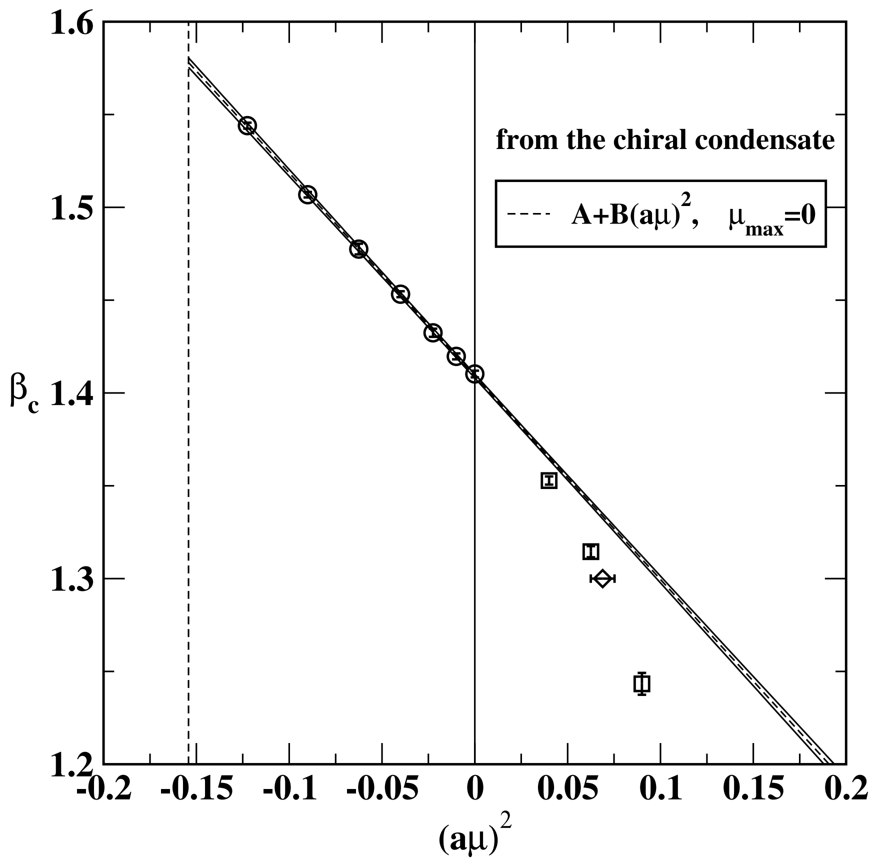
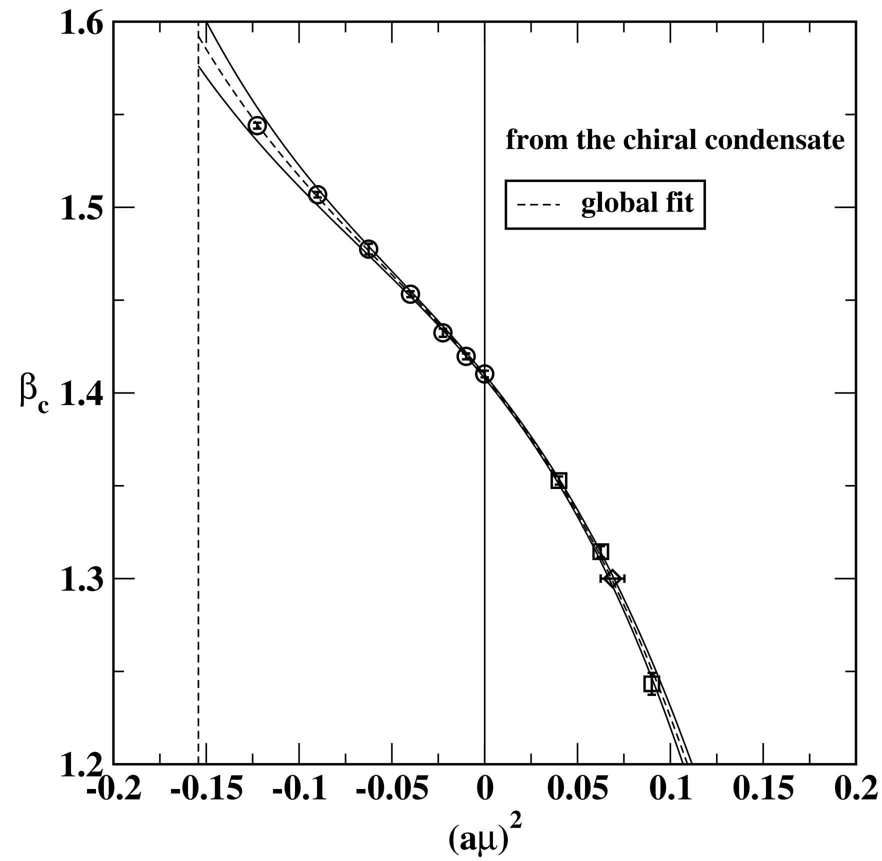
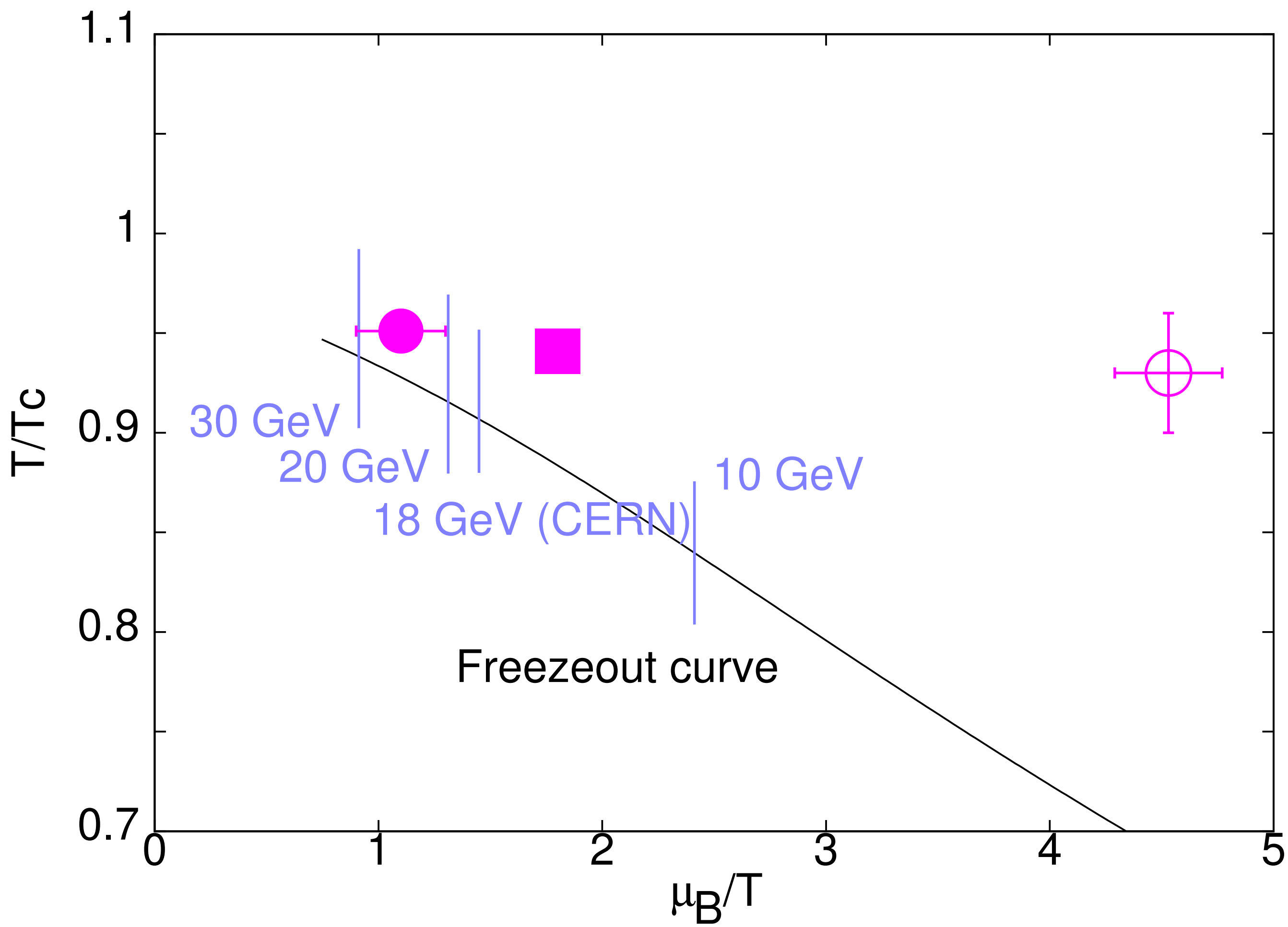
Problems : i) Positive coefficient for finer lattice (Philipsen, CPOD 2009), ii) Known examples where shapes are different in real/imaginary μ, “The Critical line from imaginary to real baryonic chemical potentials in two-color QCD”, P. Cea, L. Cosmai, M. D’Elia, A. Papa, PR D77, 2008






1.1
1
0.9
0.8
0.7
T/Tc
0
1
2
3
4
5
μ_B/T
30 GeV
20 GeV
18 GeV (CERN)
10 GeV
Freezeout curve