Some aspects of open string tachyon dynamics

1. Classical tachyon dynamics in open string theory

2. Closed string emission

3. Completeness of open string description

4. Two dimensional string theory

During this talk I shall try to emphasize the open problems in this field.

Notations and conventions:

We shall set

$$\hbar = 1, \qquad c = 1, \qquad \alpha' = 1$$

g_s: closed string coupling constant

We shall be studying D-branes in bosonic string theory or unstable D-branes or brane-antibrane systems in type II string theory.

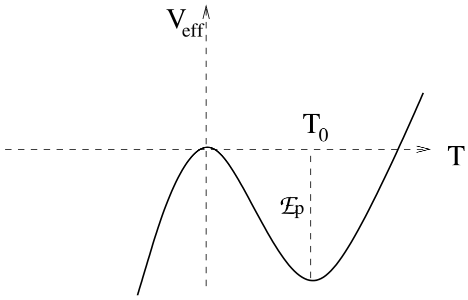
Each of these systems is characterized by the existence of tachyonic modes.

Define $\mathcal{E}_p$ to be the total energy / unit p-volume of the system.

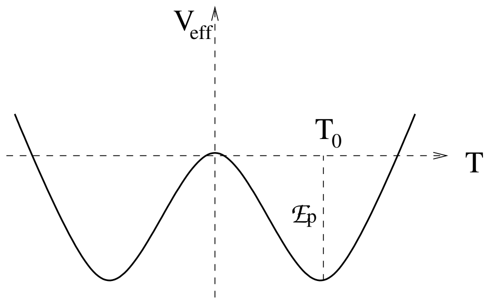
$\mathcal{E}_p$ = tension for a single unstable Dp brane in bosonic or type II string theory.

$\mathcal{E}_p$ = twice the tension of a BPS D-brane for Dp − $\bar{\text{D}}p$ system in type II string theory.

Shape of tree level tachyon effective potential $V_{eff}(T)$ for a D-p-brane in bosonic string theory:



Shape of $V_{eff}(T)$ for non-BPS D-p-brane in type II string theory:



For Dp-$\bar{\text{D}}p$ system we have to revolve this figure about the vertical axis to get the tachyon potential as a function of complex T.

1. At $T = T_0$ the total energy density vanishes identically.

$$V_{eff}(T_0) + \mathcal{E}_p = 0$$

$T = T_0$ configuration describes the vacuum of string theory without any D-brane.

2. Thus around this minimum there are no physical open string excitations.

3. Time independent but space-dependent classical solutions of the equations of motion of T represent lower dimensional D-branes.

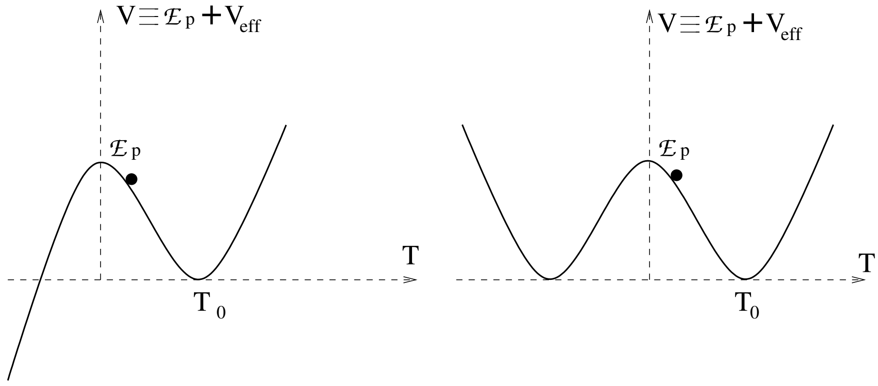
Example: On a non-BPS Dp-brane of type II string theory, a kink solution interpolating between $\pm T_0$ represents a BPS D-$(p-1)$-brane.

The indirect evidence for these results come from various studies based mainly on conformal field theory techniques, non-commutative solitons, boundary string field theory, etc.

The direct evidence comes from numerical results in Witten's cubic open bosonic string field theory and Berkovits' superstring field theory.

However despite many attempts there has not been any progress in finding analytical solutions in these string field theories representing either the tachyon vacuum or the classical solutions representing lower dimensional D-branes.

Time dependent solutions:



What happens if we displace the tachyon field away from the maximum of the potential and let it roll down according to its classical equations of motion?

Consider spatially homogeneous solution for simplicity.

Had T been an ordinary scalar field, there would be a two parameter family of solutions labelled by the initial values of T and its time derivative $\partial_0 T$.

Even in string theory we can construct a two parameter family of time dependent solutions describing the rolling of the tachyon away from its maximum.

Detailed computation shows that as $x^0 \to \infty$, the rolling tachyon approaches a configuration of

Energy density $\mathcal{E} =$ constant

Pressure $p = 0$

Dilaton charge density $Q = 0$

We shall outline the procedure for constructing the solution for D-branes in bosonic string theory.

Equations of motion of the tachyon near 0:

$$(\partial_0^2 + m^2)T = \mathcal{O}(T^2), \qquad m^2 = -1$$

Solution:

$$T(x^0) = \lambda \cosh(x^0 + a) + \mathcal{O}(\lambda^2)$$

λ, a: constants of integration

$T(x^0) = \lambda \cosh(x^0 + a) + \mathcal{O}(\lambda^2)$

To order λ this corresponds to deforming the world-sheet CFT by a boundary term:

$$\lambda \int dt \cosh\left(X^0(t) + a\right)$$

t: parameter labelling the boundary of the world-sheet.

Since T is on-shell, this is guaranteed to be a marginal deformation to order λ.

In this case it turns out to be a marginal deformation to all orders in λ.

$\rightarrow$ gives a two parameter family of CFT's

$\rightarrow$ two parameter family of time dependent solutions of classical equations of motion.

The same method works for superstring theory.

General case:

Suppose the theory has n tachyons $T_1, \ldots T_n$ with mass$^2 = -m_1^2, \ldots - m_n^2$.

Let $V_1, \ldots V_n$ be the zero momentum vertex operators for these tachyons.

Then at the linearized level we have a $2n$-parameter family of solution

$$T_k = \lambda_k \cosh(m_k x^0 + a_k)$$

for arbitrary constants λ_k, a_k.

This corresponds to deforming the world-sheet theory with the boundary perturbation:

$$\sum_k \lambda_k \int dt\, V_k(t)\ \cosh\left(m_k X^0(t) + a_k\right)$$

Since each tachyon is on-shell this is guranteed to be marginal to linear order in λ_k.

Beginning with this solution one can solve the β function $= 0$ equations iteratively.

Result: in the generic case it is possible to construct a $2n$ parameter family of boundary CFT's with boundary interaction of the form:

$$\sum_k \lambda_k \int dt\, V_k(t)\ \cosh\left(\omega_k(\vec{\lambda})\, X^0(t) + a_k\right)$$

with

$$\omega_k(\vec{\lambda}) = m_k + \mathcal{O}(\lambda^2)$$

$\omega_k(\vec{\lambda})$ can be calculated by systematically solving the β-function=0 equations.

A variant of this procedure can be used to construct a $2n$ parameter family of classical solutions in the open string field theory.

Open question: Does this $2n$ parameter family of solutions exhaust the time dependent solutions that one can construct involving the n-tachyons?

Had these tachyons been ordinary scalar fields with at most two derivative terms in the action, the answer would be yes.

However open string field theory has interaction terms with arbitrarily large number of derivatives.

Hence there could be other solutions.

On the other hand if there are really additional solutions, then we expect that upon quantization they will give rise to new states which are not seen in perturbative quantization of open string theory.

Note: The problem associated with higher derivative terms is a generic problem in all string theories and is not specific to open string theory.

Even for closed strings, the effective field theory computed from the β-function vanishing condition, as well as the full closed string field theory action, has higher derivative terms.

This apparently leads to additional solutions to the equations of motion, which, if present, will give rise to additional quantum states which are not present in the spectrum of string theory.

We come face to face with this problem when we try to construct supersymmetric black hole solutions in the presence of higher derivative terms.

The effective lagrangian density of closed string theories often contains higher derivative terms of the form:

$$\sqrt{-\det g}\, R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma}$$

Supersymmetrization $\to$ many more terms

We can try to construct BPS black hole solutions taking these additional terms into account.

Boundary condition on various fields: Smoothness at the horizon.

Evolving the fields in the radial direction one finds that in the asymptotic region various fields start oscillating about the expected asymptotic values.

This is possible because these various fields have additional oscillatory solutions of the linearized equations of motion due to the presence of higher derivative terms.

Example: Consider a scalar field ϕ with action

$$\frac{1}{2}\int d^{p+1}x\,\phi\,\partial_\mu\partial^\mu\left(1+M^{-2}\partial_\mu\partial^\mu\right)\phi$$

Solutions of the equations of motion:

$$\phi = A\,e^{ik.x}$$

with $k^2 = 0$ or $k^2 = M^2$.

Similarly, due to the presence of higher derivative terms, metric and various other fields have additional oscillatory solutions and these are responsible for oscillatory behaviour of the metric of the black hole in the asymptotic region.

However these additional oscillatory solutions must be unphysical, since otherwise by quantizing these oscillatory solutions we shall get additional states which are not present in the spectrum of string theory.

Resolution: We must redefine fields to remove these solutions.

Define $\psi = \left(1 + M^{-2}\,\partial^\mu \partial_\mu\right)^{1/2}\,\phi$

Then the action is

$$\frac{1}{2}\int d^4x\,\psi\,\partial^\mu\partial_\mu\,\psi$$

Under this field redefinition the solutions $\phi = A\,e^{ik.x}$ with $k^2 = M^2$ gets mapped to $\psi = 0$.

Similar field redefinitions need to be carried out for the metric and other fields.

When the black hole solution is expressed in terms of these new field variables, the oscillations must disappear.

How does the solution near the horizon look in terms of these new variables?

Coming back to open string theory one might contemplate three possibilities:

1. Cubic open string field theory automatically uses the right choice of variables and hence the only continuous parameters labelling a classical solution correspond to boundary condition on the physical fields and their first derivatives.

2. Cubic open string field theory contains additional parameters labelling its classical solutions, but these parameters are spurious and disappear from the solution once we make the 'right' choice of field variables.

3. Cubic open string field theory contains additional parameters labelling its classical solutions, and these parameters cannot be removed by field redefinitions.

Which of these possibilities is correct???

So far we have considered tree level open string field theory ignoring possible backreaction due to closed string emission from the brane.

The rolling tachyon solution on the D-brane acts as a classical source for all the closed string fields.

$\longrightarrow$ produces a classical closed string field configuration.

$$\updownarrow$$

a state $|\Psi_c\rangle$ of ghost number two in the two dimensional CFT

Equation of motion of $|\Psi_c\rangle$

$$(Q_B + \bar{Q}_B)|\Psi_c\rangle = |\mathcal{B}\rangle$$

$(Q_B + \bar{Q}_B)$: BRST charge

$|\mathcal{B}\rangle$: Boundary state describing the rolling tachyon solution

Solution in Siegel gauge:

$$|\Psi_c\rangle = (L_0 + \bar{L}_0)^{-1} (b_0 + \bar{b}_0) |\mathcal{B}\rangle$$

$L_n, \bar{L}_n$: total Virasoro generators

$b_n, \bar{b}_n, c_n, \bar{c}_n$: ghost oscillators

Solution:

$$|\Psi_c\rangle = (L_0 + \bar{L}_0)^{-1}\,(b_0 + \bar{b}_0)\,|\mathcal{B}\rangle$$

In Minkowski space $(L_0 + \bar{L}_0)$ has zero eigenvalues.

$\longrightarrow$ the solution is ambiguous.

Hartle-Hawking prescription: Solve this in Euclidean space and then replace $x \to ix^0$.

We can divide $|\mathcal{B}\rangle$ into two parts:

$$|\mathcal{B}\rangle = |\mathcal{B}_1\rangle + |\mathcal{B}_2\rangle$$

1. $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$ are separately BRST invariant.

2. $|\mathcal{B}_1\rangle$ approaches a finite limit as $x^0 \to \pm\infty$.

$|\mathcal{B}_2\rangle$ contains exponentially growing terms in these limits.

3. The time dependence of various terms in $|\mathcal{B}_2\rangle$ is determined from the requirement of BRST invariance.

Time dependence of $|\mathcal{B}_1\rangle$ is not determined from BRST invariance.

4. In the limit where the initial D-brane energy density goes to zero, only $|\mathcal{B}_1\rangle$ contributes to $|\Psi_c\rangle$.

Results for Dp-brane with all tangential directions compactified on a torus (either in bosonic or superstring theory)

Closed string field produced by $|\mathcal{B}_1\rangle$

$$|\Psi_c^{(1)}\rangle = (L_0 + \bar{L}_0)^{-1} (b_0 + \bar{b}_0) |\mathcal{B}_1\rangle$$

has the following features:

1. The field configuration is essentially independent of the energy of the brane (up to a shift of the time coordinate x^0).

2. The total amount of energy in all the closed string modes is infinite.

3. The contribution to the energy of a closed string mode of mass m_N comes predominantly from modes with momentum $|\vec{k}_\perp| \sim (m_N)^{1/2}$ along directions transverse to the brane and of winding $|w_\parallel| \sim (m_N)^{1/2}$ along directions tangential to the brane.

Since the Dp-brane has a finite energy, the total energy carried by the closed string fields cannot really be infinite.

There should be an upper cut-off on the energy of the emitted closed string $\sim$ the Dp-brane mass $\sim 1/g_s$

Using the upper cut-off the results are modified as follows:

1. All the energy of the wrapped Dp-brane is radiated away into closed strings.

2. Most of the emitted energy is carried by closed strings of mass $\sim \frac{1}{g_s}$.

3. Typical momentum of an emitted closed string along directions transverse to the brane is of order $(g_s)^{-1/2}$.

4. Typical winding of an emitted closed string along directions tangential to the brane is of order $(g_s)^{-1/2}$.

From this analysis it would seem that the effect of closed string emission invalidates the tree level open string analysis.

However comparison of the properties of the final state closed strings with those inferred from the tree level open string analysis points to an alternative interpretation.

1) Tree level open string analysis tells us that the final system has:

$$Q/T_{00} = 0$$

Q: Dilaton charge density

On the other hand we know that the closed string world-sheet does not couple to the zero momentum dilaton.

$$s_{world-sheet} = \int d^2z(G_{\mu\nu}(X)+B_{\mu\nu}(X))\partial_z X^\mu \partial_{\bar{z}} X^\nu$$

Thus the final state closed strings carry zero total dilaton charge.

This shows that the dilaton charge of the final state closed strings agrees with that computed in the open string description.

2) Tree level open string analysis tells us that the final system has:

$$p/T_{00} = 0$$

On the other hand, closed string analysis tells us that the final closed strings have momentum/mass and winding/mass $\sim \sqrt{g_s}$

For such a system

$$p/T_{00} \sim g_s \to 0 \qquad \text{as} \quad g_s \to 0$$

Thus the pressure of the final state closed strings match the result computed in the open string description.

Conclusion: Properties of the final state calculated from tree level open string analysis agree with the properties of the final state closed strings into which the D-brane decays.

Such agreements also hold for more general cases.

Consider the 'decay' of a Dp-brane along $x^1, \ldots x^p$ plane, with an electric field e along the x^1 axis.

The final state is characterized by its energy-momentum tensor $T_{\mu\nu}$, source $S_{\mu\nu}$ for anti-symmetric tensor field $B_{\mu\nu}$ and the dilaton charge Q.

Non-zero charges in $x^0 \to \infty$ limit

$$T^{00} = \Pi\, e^{-1}, \quad T^{11} = -\Pi\, e, \quad S^{01} = \Pi$$

Π: a parameter labeling the solution

These tree level open string results again agree exactly with the properties of the final state closed strings into which the D-brane decays.

This leads to the following conjecture:

Open string theory provides a description of the rolling tachyon system which is dual to the description in terms of closed string emission.

We could give a completely consistent description of the dynamics of the D-brane without ever having to talk about closed strings.

Note: This conjecture does not require that quantum open string theory on a given system of unstable D-branes should be able to describe an arbitrary closed string state in the full string theory.

However the open string theory describes all the quantum states which are produced in the decay of this D-brane.

In this sense, the open string theory on each D-brane system is a consistent quantum mechanical system that encodes, in a highly non-local manner, part of the information about the full string theory.

A given system of D-branes is like a part of the hologram describing the full string theory.

What about the field produced by $|\mathcal{B}_2\rangle$?

1. In the $\mathcal{E} \to 0$ limit the fields produced by $|\mathcal{B}_2\rangle$ vanish.

$\longrightarrow$ the only contribution comes from $|\mathcal{B}_1\rangle$ and our previous results hold.

2. For $\mathcal{E} > 0$, the fields produced by $|\mathcal{B}_2\rangle$ grow exponentially in the past and future.

$\longrightarrow$ another possible source of large backreaction of closed string fields on open string dynamics.

However if the earlier conjecture is correct then the open string dynamics should be consistent without taking into account the backreaction due to closed strings.

According to this conjecture the growth of the closed string fields should indicate a breakdown of the closed string description rather than of the open string description.

Analogy: For many D-branes the closed string fields produced by the brane become singular near the origin.

$\rightarrow$ breakdown of the description of the D-brane in terms of weakly coupled closed string fields.

But the description in terms of weakly coupled open string theory does not get affected by this divergence.

Can we find a precise test of the conjecture that despite the apparently large backreaction due to closed string fields, the open string field theory describes the complete dynamics of the D-brane?

Two dimensional string theory

Continuum description: based on the world-sheet action:

$$s_{X^0} + s_L + s_{ghost}$$

s_{X^0}: Time-like scalar field X^0 with $c = 1$

s_L: A Liouville field theory of a scalar field φ with a potential $e^{2\varphi}$ and a background charge $Q = 2$ so that $c = 1 + 6Q^2 = 25$

For large negative φ the potential is small and φ behaves like a free field.

Also for large negative φ the dilaton $= Q\varphi$ is large and negative and hence the string coupling is small.

s_{ghost}: Usual action for ghost fields b, c, $\bar{b}$, $\bar{c}$

Physical closed string spectrum: A single massless scalar field Ψ in (1+1) dimensions labelled by X^0 and φ.

For large negative φ the vertex operator for Ψ carrying φ momentum P and energy E is:

$$e^{(1+iP)\varphi}\, e^{iEX^0}$$

Besides this there are discrete states carrying discrete momentum and energy.

These are labelled by SU(2) quantum numbers (j, m) with $-(j-1) \le m \le (j-1)$ and have vertex operators (for large -ve φ):

$$e^{(1+j)\varphi}\, V_{j,m}$$

$V_{j,m}$: Vertex operator of a higher level primary state in the $c = 1$ CFT of a scalar field X^0.

$$V_{j,m} \sim e^{2mX^0} \times \text{world-sheet derivatives of } X^0$$

This theory admits a D0-brane.

1. Neumann boundary condition on X^0 and ghosts

2. 'Dirichlet' boundary condition on φ.

Given this, we can now deform the conformal field theory by boundary interaction:

$$\lambda \int dt \cosh X^0(t)$$

to construct a one parameter family of rolling tachyon solutions.

Boundary state $|\mathcal{B}\rangle$ of this theory can be constructed explicitly.

As before we can divide $|\mathcal{B}\rangle$ into two parts $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$, each of which are separately BRST invariant.

We can calculate the closed string fields produced by $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_2\rangle$ following the same procedure as before.

Results:

1. $|\mathcal{B}_1\rangle$ produces a classical Ψ field background.

2. This field configuration is independent of the D0-brane energy $\mathcal{E}$ up to a shift in the time coordinate.

3. The total energy carried by this field configuration is infinite.

4. $|\mathcal{B}_2\rangle$ produces closed string fields associated with discrete states.

5. Since vertex operators for discrete states $\propto e^{2mX^0}$, these fields grow exponentially in the far future or the far past.

6. Fields produced by $|\mathcal{B}_2\rangle$ vanish at $\mathcal{E} = 0$.

These results are very similar to the corresponding results in the critical string theory.

Does the backreaction due to closed strings invalidate the classical open string results?

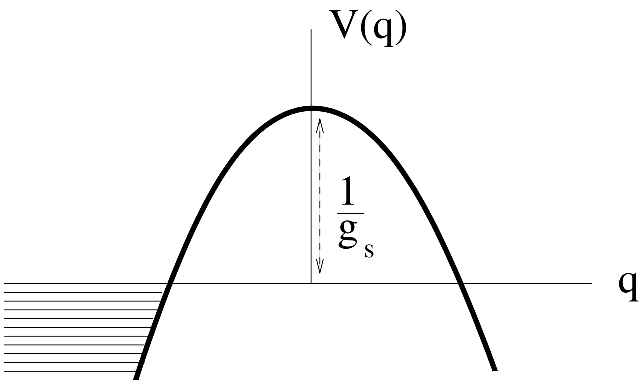
Fortunately for this two dimensional string theory we have an alternative formulation based on matrix model.

This allows us to analyse the various questions we raised earlier not only at the string tree level, but to all orders in string perturbation theory.

The matrix model description is equivalent to a theory of free fermions moving in an inverted harmonic oscillator potential.

Hamiltonian

$$h = \frac{1}{2}p^2 - \frac{1}{2}q^2 + \frac{1}{g_s}$$



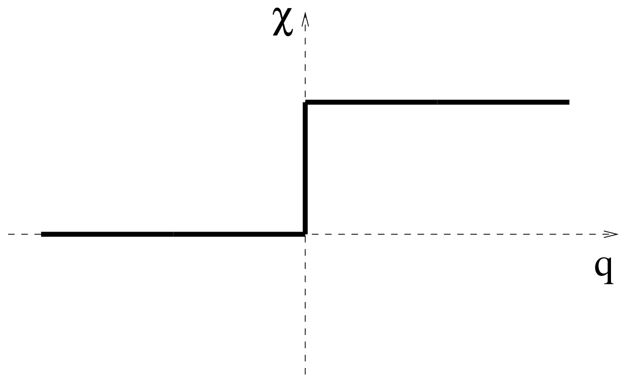
The energy levels below zero are filled, and above zero are empty.

Since to all orders in perturbation theory we can ignore tunneling between the two sides, we shall concentrate on the negative q side.

For large negative q the fermion behaves like a relativistic fermion.

Close to the fermi level, we can bosonize the second quantized fermion field to a scalar field $\chi(q, x^0)$.

A single fermion excited from the fermi level corresponds to a step function in the scalar field χ.



$\chi(q, x^0)$ is related to the scalar field $\Psi(\varphi, x^0)$ in the continuum description.

$$\int dq\, e^{-iPq}\chi(q, x^0)$$

$$= \frac{\Gamma(iP)}{\Gamma(-iP)} \int d\varphi\, e^{-iP\varphi}\Psi(\varphi, x^0)$$

Using this we can convert the $\Psi(\varphi, x^0)$ background produced by $|\mathcal{B}_1\rangle$ to $\chi(q, x^0)$.

$\longrightarrow$ precisely gives a step function describing a single excited fermion!

Conclusion: A single D0-brane in the two dimensional string theory corresponds to a single fermion excited from the fermi level to an excited level.

This also explains the origin of the infinite energy in the Ψ field produced by $|\mathcal{B}_1\rangle$

This is due to the step function form of χ.

A step function costs an infinite energy due to infinite spatial derivative.

In the fermionic description this infinite energy is due to the exact localization of the fermion at a spatial point.

$\rightarrow$ causes infinite uncertainty in momentum.

Thus the classical energy of the closed string field configuration is infinite because it includes the effect of infinite momentum uncertainty of the D0-brane that is strictly localized in position space.

Note: Our analysis establishing the correspondence between the D0-brane and single fermion excitations has been done for fermion energy $\mathcal{E}$ close to the Fermi level.

From the continuum viewpoint this requirement comes from the fact that the effect of $|\mathcal{B}_2\rangle$, which has been ignored so far, can be ignored only in the $\mathcal{E} \to 0$ limit.

From the matrix model side this requirement comes from the fact that the bosonization of the fermion system in terms of a single scalar holds only close to the fermi level.

However given the identification of the D0-brane with a single fermion state for $\mathcal{E} \simeq 0$, it is natural to assume that this identification holds for all energy (i.e. all $\mathcal{E}$)

From this analysis we conclude that the quantum dynamics of a single D0-brane is described by the inverted harmonic oscillator Hamiltonian

$$\hat{h} = \frac{1}{2}\hat{p}^2 - \frac{1}{2}\hat{q}^2 + \frac{1}{g_s}$$

with a sharp cut-off on the energy levels:

$$\mathcal{E} > 0$$

This cutoff is due to the Pauli exclusion principle.

The variables (p, q) are like the open string degrees of freedom living on the D0-brane.

Thus there must be a map from this inverted harmonic oscillator system (IHO) to the open string field theory (OSFT) describing the dynamics of the D0-brane.

At present this map between IHO and OSFT is not known.

In the IHO description the various classical solutions are explicitly known.

Thus if we can find this map then using it we should be able to construct classical solutions in OSFT.

e.g. in IHO the tachyon vacuum corresponds to a trajectory at the Fermi level.

Its image will give the tachyon vacuum in OSFT.

We also see that in the IHO description a general time dependent solution is labelled by precisely two parameters.

Thus once we understand the map between IHO and OSFT, we should be able to address the role of higher derivative terms in OSFT.

The analysis also provides support for the conjecture that the dynamics of a D0-brane can be described completely using open string field theory without any need to couple it to closed strings.

The IHO with an energy cut-off is a consistent quantum mechanical system in its own right.

It describes only the single fermion excitations in the theory.

Thus it encodes, in a highly non-local manner, partial information about the full two dimensional string theory.

Given this correspondence between D0-branes and single fermion excitations, we can now seek an interpretation of the exponentially growing terms in $|\mathcal{B}_2\rangle$.

It turns out that a particle moving in an inverted harmonic oscillator potential has infinite number of conserved charges.

$$e^{(l-k)x^0}\,(q-p)^l\,(q+p)^k, \qquad l,k\text{: integers}$$

These conserved charges are in one to one correspondence with the coefficients of the exponentially growing terms appearing in $|\mathcal{B}_2\rangle$ in the continuum theory.

Thus the natural interpretation of the exponentially growing terms in $|\mathcal{B}_2\rangle$ is that they encode information about conseved charges of the D0-brane.

Clearly from the point of view of matrix model these exponentially growing charges do not invalidate the 'open string description' in which we describe the system as a single particle moving in an inverted harmonic oscillator potential.

This is again consistent with the conjecture that open string field theory gives a complete description of a D-brane system without any need to take into account backreaction due to closed strings.

The matrix model description of two dimensional string theory, while resolving some of the earlier questions, raises a new puzzle.

What is the continuum description of hole states?

Proposal 1. Take the boundary state of the D0-brane, analytically continue the energy parameter $\mathcal{E}$ to negative value, and change the sign of the boundary state.

The new state carries conserved charges appropriate for a hole.

However the closed string field configuration produced by this boundary state, instead of producing an anti-kink configuration as appropriate for a hole, produces a kink.

This seems to be inconsistent with the idea that this boundary state describes a hole state.

Also if this proposal is correct, then a similar construction can be carried out for the boundary states of ordinary D-branes in critical string theory.

This will imply existence of new type of D-branes in critical string theory.

$\longrightarrow$ seems unlikely.

Proposal 2. In the presence of linear dilaton background, an ordinary D-brane experiences a force that pushes it towards the strong coupling region.

We can have rolling D0-brane solutions which travel from the region of finite φ towards large negative φ, reaches a turning point whose position depends on the energy of the brane, and turns back towards the finite φ region.

We could identify these states as hole states.

A summary of some of the open problems

1. Construct analytically classical solutions in open string field theory describing tachyon vacuum and other solutions.

2. Understand the role of higher derivative terms in open string field theory.

Do they give rise to more solutions than what is expected from counting of physical states in the theory?

3. Find a precise test for the conjecture that open string field theory is a completely consistent quantum theory describing the dynamics of a system of D-branes.

4. Find the relation between IHO and OSFT for two dimensional string theory.

5. Find the correct description of hole states in continuum description of two dimensional string theory.