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Non-trivial 2d space-times from matrices

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Abstract

Solutions of matrix quantum mechanics have been shown to describe time dependent backgrounds in the holographically dual two dimensional closed string theory. We review some recent work dealing with non-trivial space-times which arise in this fashion and discuss aspects of physical phenomena in them.

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1 Introduction

One of the most profound lessons of recent research in String Theory is that gravitational physics typically has a holographic description [1] in terms of lower dimensional theories which do not contain gravity. This is a manifestation of the duality between closed strings and open strings and the most celebrated example is AdS/CFT correspondence [2] where the presence of horizons result in a truncation of the open string theory to its low energy field theory limit.

The earliest example of such a holographic correspondence is two dimensional noncritical string theory [3] . Here the holographic model is matrix quantum mechanics which has no space. The closed string theory lives in one space and one time dimension - the space dimension arising from the space of eigenvalues of the matrix in the the large-N limit [4]. Recently it has been realized that this is also an example of open-closed duality [5]. Two dimensional string theory is a toy model of string theory - but a useful toy model because of its solvability. In particular, the emergence of space can be understood in an explicit fashion using the techniques of collective field theory [6, 4].

As is well known, the ground state of the quantum mechanical system is dual to the static linear dilaton vacuum of the closed string theory with a liouville wall provided by an additional tachyon background. In addition to the ground state, time-dependent backgrounds have been known for a while [8]. Recently such solutions have been proposed as models of cosmology [9]. Using standard techniques, the dual space-times may be constructed which provide interesting models for posing and answering questions about time-dependent backgrounds in string theory. In this talk I will review some results in this area.

Some of these models can be used as useful tools to understand issues of particle production in time dependent backgrounds [10, 11] and display characteristic stringy effects [11]. Another class of solutions naturally lead to space-like boundaries. Normally these space-times would be regarded as geodesically incomplete. However, the underlying fundamental description of dynamics in terms of the matrix model precludes any extension of the space-times beyond these boundaries. The results show that the construction of extra dimensions from the degrees of freedom of a holographic model can be rather non-trivial.

2 Space-time around the ground state

Let us first recapitulate how space-time emerges around the ground state of the model.

The dynamical variable of the model is a single $N \times N$ matrix $M_{ij}(t)$ and there is a constraint which restricts the states to be singlets. In the singlet sector and in the double scaling limit [7] matrix quantum mechanics reduces to a theory of an infinite number of fermions with the single particle hamiltonian given by

$$H = \frac{1}{2}[p^2 - x^2] \tag{1}$$

where we have adopted conventions in which the string scale $\alpha' = 1$ for the bosonic theory and $\alpha' = \frac{1}{2}$ in the fermionic theory. The fermi energy in this rescaled problem will be denoted by $-\mu$.

In the classical limit, the system is equivalent to an incompressible fermi fluid in phase space. The ground state is the static fermi profile

$$(x-p)(x+p) = 2\mu \tag{2}$$

The dual closed string theory is best obtained by rewriting the theory in terms of the collective field $\phi(x,t)$ which is defined as the density of eigenvalues of the original matrix.

$$\partial_x \phi(x,t) = \frac{1}{N}\mathrm{Tr}\delta(M(t) - x\cdot I) \tag{3}$$

At the classical level the action of the collective field is given by

$$S = N^2 \int dxdt\ [\frac{1}{2}\frac{(\partial_t\phi)^2}{(\partial_x\phi)} - \frac{\pi^2}{6}(\partial_x\phi)^3 - (\mu - \frac{1}{2}x^2)\partial_x\phi] \tag{4}$$

This is of course a theory in $1+1$ dimension, the spatial dimension arising out of the space of eigenvalues.

All solutions represented by Fermi seas do not necessarily appear as *classical* solutions to collective field theory[12] . However fermi surfaces with *quadratic profiles* do. Nonquadratic profiles can be still represented as states of the quantum collective theory, albeit with large fluctuations which survive the classical limit [13]. The ground state profile (2) is such a quadratic profile and the classsical solution is

$$\partial_x\phi_0 = \frac{1}{\pi}\sqrt{x^2 - 2\mu} \qquad \partial_t\phi_0 = 0 \tag{5}$$

The space-time which is generated may be obtained by looking at the dynamics of fluctuations of the collective field around the classical solution. For later use, we will in fact expand around an *arbitrary* classical solution $\phi_0(x,t)$

$$\phi(x,t) = \phi_0(x,t) + \frac{1}{N}\eta(x,t) \tag{6}$$

The action for these fluctuations at the quadratic level may be written as

$$S_\eta^{(2)} = \frac{1}{2}\int dtdx\ \sqrt{g}g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta \tag{7}$$

where $\mu,\nu = t,x$. The line element determined by $g_{\mu\nu}$ is conformal to

$$ds^2 = -dt^2 + \frac{(dx + \frac{\partial_t\phi_0}{\partial_x\phi_0}dt)^2}{(\pi\partial_x\phi_0)^2} \tag{8}$$

Therefore, *regardless of the classical solution* the spectrum is always a single massless scalar in one space dimension given by x. The metric can be determined only upto a conformal factor. However, as we will see below, the global properties of the space-time can be determined from the nature of the classical solution.

The classical interaction hamiltonian is purely cubic when expressed in terms of the fluctuation field η and its canonically conjugate momentum Π_η,

$$H_\eta^{(3)} = \int dx[\frac{1}{2}\Pi_\eta^2\partial_x\eta + \frac{\pi^2}{6}(\partial_x\eta)^3] \tag{9}$$

Around the ground state, the metric (8) is given by

$$ds^2 = -dt^2 + \frac{dx^2}{x^2 - 2\mu} \tag{10}$$

The perturbative fluctuations live in the region $|x| > \sqrt{2\mu}$ and the field η satisfies Dirichlet boundary condition at the "mirrors" given by $x = \pm\sqrt{2\mu}$. The fields on the "left" and "right" side are decoupled. The physics of these fields is made transparent by choosing Minkowskian coordinates (σ, τ) which in this case are

$$t = \tau \qquad x = \pm\sqrt{2\mu}\cosh\sigma \tag{11}$$

In these coordinates

$$ds^2 = -d\tau^2 + d\sigma^2 \tag{12}$$

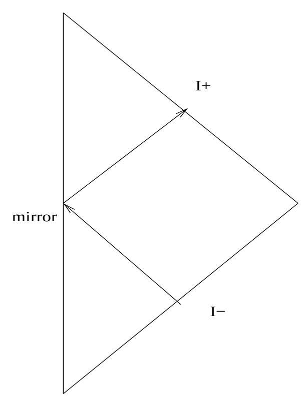

Figure 1: Penrose diagram of space-time produced by ground state solution showing an incoming ray getting reflected at the mirror

The field η may be now thought of being made of *two* fields, $\eta_{S,A}(x,t)$ each of which live in the region $x > 0$

$$\eta_{S,A}(x,t) = \frac{1}{2}[\eta(x,t) \pm \eta(-x,t)] \tag{13}$$

In terms of the Minkowskian coordinates, solutions to the linearized equations are plane waves $\eta_{S,A} \sim e^{-i\omega(t\pm\sigma)}\eta_{S,A}(\omega)$ and these fourier components are related to the two spacetime fields - the tachyon T and the axion C - which appear in the standard formulation of Type 0B string theory [14] :

$$\begin{aligned} T(\omega) &= (\pi/2)^{-i\omega/8}\,\frac{\Gamma(i\omega/2)}{\Gamma(-i\omega/2)}\,\eta_S(\omega) \\ C(\omega) &= (\pi/2)^{-i\omega/8}\,\frac{\Gamma((1+i\omega)/2)}{\Gamma((1-i\omega)/2)}\,\eta_A(\omega) \end{aligned} \tag{14}$$

Recall that we are working in string units. These transforms therefore imply that the position space fields are related by a transform which is non-local at the string scale. Therefore points on the Penrose diagram should be thought of as smeared over the string scale. But then, this should be true of any Penrose diagram drawn in a string theory.

In any case, the space-time generated is quite simple. The Penrose diagram is that of two dimensional Minkowski space with a mirror at $\sigma = 0$, as shown in Figure (1). The fluctuations are massless particles which come in from $\mathcal{I}^-_{L,R}$, get reflected at the mirror and arrive at $\mathcal{I}^+_{L,R}$. In terms of the Minkowskian coordinates the interaction hamiltonian becomes

$$H_3 = \int d\sigma \frac{1}{2\sinh^2\sigma}[\frac{1}{2}\tilde{\Pi}_\eta^{\ 2}\partial_\sigma\eta + \frac{\pi^2}{6}(\partial_\sigma\eta)^3] \tag{15}$$

The interactions therefore vanish at $\sigma = \infty$ and are strong at $\sigma = 0$ - this gives rise to a non-trivial wall S-matrix.

In the Type 0B string theory interpretation, the Penrose diagram has to be folded across the center and a point which is localised in σ space is smeared out in the string theory space over string length.

As emphasized above, the metric is determined only upto a conformal transformation. So long as the conformal transformation is non-singular, this is sufficient to draw Penrose diagrams. A conformal transformation would, however, mix up the space σ and time τ and it would appear that this leads to an ambiguity. The special property of the space and time coordinates defined above is that the *interaction Hamiltonian is time-independent* with this choice and a conformal transformation would destroy this property. This makes the physics transparent and easy to compare with string theory results. Of course a different coordinatization with a time dependent Hamiltonian is physically equivalent and should be compared with the string theory results in an appropriately chosen gauge.

3 Moving Fermi Seas

One class of time-dependent solutions are generated from (2) by the action of W_∞ symmetries in phase space [11]. In the classical limit, these are represented by moving fermi seas :

$$x^2 - p^2 + \lambda_- \ e^{-rt}(x+p)^r + \lambda_+ \ e^{rt}(x-p)^r + \lambda_+\lambda_-(x^2-p^2)^{r-1} = 2\mu \ . \tag{16}$$

where r is a non-negative integer and

$\lambda_\pm$ are finite parameters. The $r = 1$ solution was considered in [9] and proposed as a model of cosmology. Formally, the state of the fermion system is related to the ground state $|\mu\rangle$ by

$$|\lambda\rangle = \exp[i\lambda Q]|\mu\rangle \ , \tag{17}$$

where Q denotes the W_∞ charge which generates this solution. However this state is not normalizable and therefore not contained in the Hilbert space of the model. Rather, this corresponds to a deformation of the hamiltonian of the theory to

$$H' = e^{-i\lambda Q} H e^{i\lambda Q} \ . \tag{18}$$

In the following we will study the dual spacetimes which arise from the solutions with $r = 1$ and $r = 2$. For these solutions the fermi surfaces are quadratic, so that

these correspond to classical solutions of collective field theory.

For $r = 1$ and $\lambda_- = 0, \lambda_- > 0$ we can choose the origin of time to choose $\lambda_+ = 2$. Then the classical collective field is

$$\partial_x\phi_0 = \frac{1}{\pi}\sqrt{(x+e^t)^2 - 2\mu} \qquad\qquad \partial_t\phi_0 = -e^t \ \partial_x\phi_0 \tag{19}$$

We will call this the "draining/flooding fermi sea" solution. The fermi surface is displayed in (2)

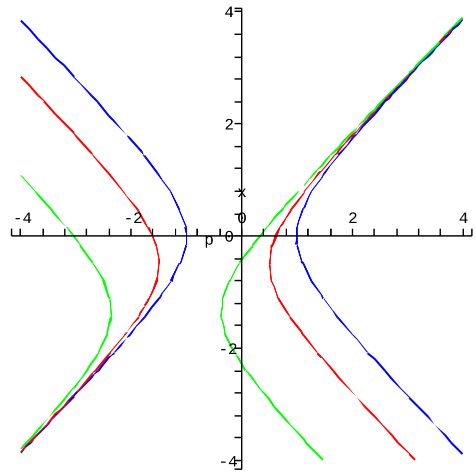

Figure 2: The draining/flooding sea solution. The fermi surface moves to the left and downwards. Fermions which are in the $x < 0$ region at $t = -\infty$ are drained out while fermions originally in the $x > 0$ flood in, eventually crossing over to the other side.

The physics of the $r = 2$ solutions was considered in [15]. For $\lambda_- = 0$ with $\lambda_+ < 0$ which can be chosen to be -2 we have

$$\partial_x \phi_0 = \frac{1}{\pi(1+e^{2t})}\sqrt{x^2 - (1+e^{2t})} \qquad \partial_t \phi_0 = -\frac{xe^{2t}}{1+e^{2t}}\,\partial_x \phi_0 \tag{20}$$

Here we have rescaled x and t to set $2\mu = 1$. This will be called the "closing hyperbola" solution shown and explained in Figure (3).

Finally, for $r = 2$ and $\lambda_- = 0$ with $\lambda_+ > 0$ which can be chosen to be 2 we have

$$\partial_x \phi_0 = \frac{1}{\pi(1-e^{2t})}\sqrt{x^2 - (1-e^{2t})} \qquad \partial_t \phi_0 = \frac{xe^{2t}}{1-e^{2t}}\,\partial_x \phi_0 \tag{21}$$

where we have again set $2\mu = 1$. This will be called the "opening hyperbola" solution. The phase space fermi surfaces are shown in (4)

What do these solutions mean in the worldsheet string theory ? This question has not been settled yet, but it is clear that this corresponds to a closed string tachyon condensation. Proposals for the worldsheet perturbations corresponding to these solutions have been made in [9, 10, 15]. This issue will not be discussed here further.

4 Space-time structures

We will study the nature of space-times generated by these solutions by looking at fluctuations around them. The strategy will be to pass to Minkowskian coordinates in each case so that the

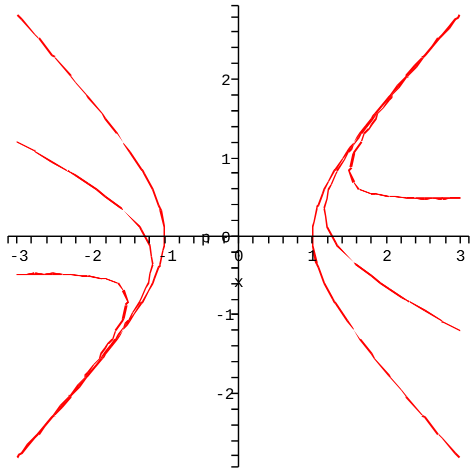

Figure 3: The closing hyperbola solution. At late times the hyperbola closes on into itself drianing all the fermions

Penrose diagrams may be drawn easily. As argued above, points on these diagrams should be considered fuzzy at the string scale.

4.1 The draining/flooding sea

The draining/flooding sea solution is a simple modification of the ground state, but encodes non-trivial physics.

Let us first consider the fluctuations around the left branch of the fermi sea. It is trivial to see that the Minkowskian coordinates (σ, τ) are given by

$$t = \tau \qquad x = -\cosh\sigma - e^{\tau} \tag{22}$$

The mirror is always at $\sigma = 0$. However in the original "space" x, the mirror is *moving* towards the asymptotic region.

The moving mirror problem has served as an excellent toy model for various properties of quantum fields in time dependent geometres, e.g. Hawking radiation from a collapsing black hole. Here, the moving mirror arises from within a string theory. In fact the mirror is "soft" since there can be tunelling across the barrier of the inverted oscillator potential. In the following, however, we will restrict our attention to perturbative processes which ignore tunneling.

At early times, the solution is identical to the static solution so that it is natural to define a space coordinate q appropriate for incoming modes by

$$x = -\cosh q \tag{23}$$

The Penrose diagram in (τ, σ) space is the same as in the ground state. On $\mathcal{I}^-$, defined as $\sigma_- = (\sigma - \tau) \to \infty$ with $\sigma_+ = \sigma + \tau$ finite, the two coordinates q and σ coincide. However, on

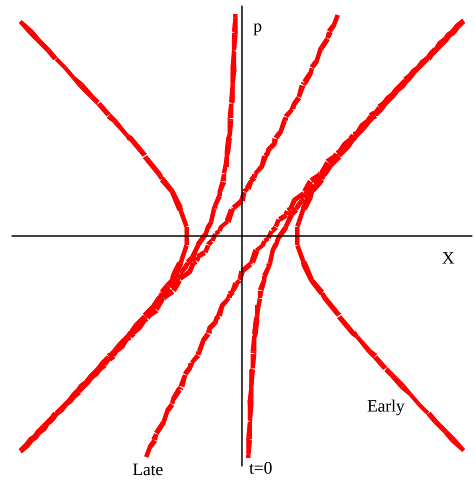

Figure 4: The opening hyperbola solution. At $t = 0$ the surface becomes vertical and tips over for $t > 0$ eventually asymptotic to $x = p$ line

$\mathcal{I}^+$

$$\sigma \pm \tau = (q \pm t) + \log(1 - 2e^{-(q-t)}) \tag{24}$$

which shows that the mirror hits $\mathcal{I}^+$ at a point $q_- = q - t = \log(2)$. The interaction hamiltonian is once again given by (15) and therefore vanish on $\mathcal{I}^\pm$.

Equation (24) implies a nontrivial relation between the in modes starting on $\mathcal{I}^-$ and the out modes which end on $\mathcal{I}^+$ after reflection from the mirror and results in particle creation [10],[11]. However, as discussed below, the fact that our moving mirror is embedded in a *string theory* has nontrivial consequences.

The fluctuations of the right branch of the fermi surface have a slightly different physics. Now the mirror is *receding* away from the asymptotic region. As a result modes coming in from $\mathcal{I}^-$ will reflect back to $x = \infty$ if they start off at early times, but will not be able to catch up with the mirror if they start off at late times. These modes then cross over to the other side of the potential. Therefore $\mathcal{I}^+$ has two pieces : $\mathcal{I}^+ = \mathcal{I}_1^+ + \mathcal{I}_2^+$ where $\mathcal{I}_1^+$ is located at large positive values of x at late times while $\mathcal{I}_2^+$ is located at large negative values of x at late times. In the Type 0B string theory interpretation, however, a crossing of the barrier means a transformation of the relative strengths of the tachyon and axion fields.

4.2 The closing hyperbola solution

For both the ground state and the moving mirror solutions, the original matrix model time t is identical to the time τ in terms of which the metric is Minkowskian. A direct consequence of this is that the Penrose diagrams are quite similar. In contrast, the opening and closing hyperbola solutions involve a non-trivial relationship between τ and t. The necessary coordinate transformations may be obtained using the techniques of [16]. For the closing hyperbola solution one has, for the right side

$$x = \cosh\sigma\sqrt{1+e^{2t}} \qquad e^{\tau} = \frac{e^t}{\sqrt{1+e^{2t}}} \tag{25}$$

This immediately shows that as $-\infty < t < \infty$ the time τ has the range $-\infty < \tau < 0$. Since the dynamics of the matrix model ends at $t = \infty$ the resulting space-time appears to be geodesically incomplete with a space-like boundary at $\tau = 0$.

There is always a mirror. Fluctuations coming in from $\mathcal{I}^-$ along $\sigma_+ = \sigma + \tau = \tau_0$ will get reflected by the mirror at $\sigma = 0$ so long as $\tau_0 < 0$. For $\tau_0 > 0$ this ray cannot reach the mirror before time ends - rather it directly hits the space-like boundary at $\tau = 0$. The Penrose diagram with these two classes of rays is shown in Figure (5).

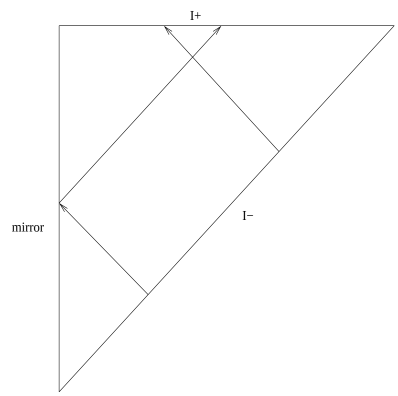

Figure 5: Penrose diagram for the closing hyperbola solution showing two classes of null rays

In the original space-time (t, x) however, small fluctuations of the Fermi surface are *always* reflected back to large x region. The exact trajectory of a point on the fermi surface is given by

$$x(t) = e^t \cosh\tau_0 + \frac{1}{2}e^{-t}e^{\tau_0} \tag{26}$$

It may be easily verified that $dx/dt = 0$ always has a solution, and the point turns back in x space *before it reaches the mirror* at $\sigma = 0$. In fact the whole of $\tau = 0$ surface is at $x = \infty$.

The interaction hamiltonian which evolves in time τ may be obtained by starting from the expression (9). The result is

$$H_3 = \frac{\partial}{\partial\tau} = (\frac{dt}{d\tau})\frac{\partial}{\partial t} = (\frac{dt}{d\tau})\int d\sigma \frac{1}{(\partial x/\partial\sigma)^2}[\frac{1}{2}\tilde{\Pi_\eta}^2\partial_\sigma\eta + \frac{\pi^2}{6}(\partial_\sigma\eta)^3] \tag{27}$$

where we have used the fact that $\tau = \tau(t)$ only. In this case this reduces exactly to equation (15). Thus the coupling on the space-like boundary is exactly the same as on any other space-like slice - strong at $\sigma = 0$ and small in the asymptotic region. In particular this implies that the coupling is strong even when x is large. Our conclusions should be trustworthy in the region of large σ.

As in the case of the ground state, we have chosen the space and time such that the *interaction Hamiltonian is time-independent.* This property will not be maintained under a conformal transformation.

4.3 The opening hyperbola space-time

The space-time generated by the closing hyperbola solution is even more non-trivial. It is clear from the profile of the fermi surface that the "mirror" disappears at the time $t = 0$. In fact we now require two patches of Minkowskian space-time to describe time evolution. Let us first discuss the space-time perceived by fluctuations of the right branch of the fermi surface. For $t < 0$ the Minkowskian coordinates (τ, σ) are given by

$$x = \cosh\sigma\sqrt{1-e^{2t}} \qquad e^{\tau} = \frac{e^t}{\sqrt{1-e^{2t}}} \tag{28}$$

The range $-\infty < t < 0$ maps into the complete range in Minkowski time $-\infty < \tau < \infty$ and the line $\sigma + \tau = \infty$ with finite σ_- marks a horizon of this coordinate system. For $t > 0$ we have another patch with

$$x = \sinh\sigma\sqrt{e^{2t}-1} \qquad e^{-\tau} = \frac{e^t}{\sqrt{e^{2t}-1}} \tag{29}$$

Note that for $t > 0$ the entire x space is covered by this coordinate patch whereas for $t < 0$ one has to have an additional patch which is relevant to the fluctuations of the left branch of the fermi surface.

Once again, space-time ends on a space-like surface at $\tau = 0$ in the second patch. The exact trajectory of a point on the fermi surface is given by

$$x(t) = -e^t \sinh\tau_0 + \frac{1}{2}e^{-t}e^{\tau_0} \tag{30}$$

In the (τ, σ) space *all* the trajectories turn around at the mirror at $\sigma = 0$. Upon reflection, they go across the horizon at $\sigma_+ = \infty$ of the first patch into the second patch and end up on the $\tau = 0$ surface of the second patch. For $\tau_0 < 0$ these end up in the $x \to \infty$ region while for $\tau_0 > 0$ they end up in the $x \to -\infty$ region.

The trajectories look rather different in the original (t, x) space-time. It is straightforward to see from (30) that (i) for $-\infty < \tau_0 < -\frac{1}{2}\log\ 2$ they reflect back in x space at a negative value of t. (ii) For $-\frac{1}{2}\log\ 2 < \tau_0 < 0$ they reflect back in x space for positive t and (iii) For $\tau_0 > 0$ the rays never reflect back in x space but proceed to the other side.

The story for fluctuations originating on the left branch of the hyperbola is exactly similar. Putting these together we can draw a Penrose diagram as in Figure (6).

Finally, on $\mathcal{I}^+$ the interaction hamiltonian (27) is again time-independent :

$$H_3 = \frac{\partial}{\partial\tau} = \int d\sigma \frac{1}{2\cosh^2\sigma}[\frac{1}{2}\tilde{\Pi}_\eta{}^2\partial_\sigma\eta + \frac{\pi^2}{6}(\partial_\sigma\eta)^3] \tag{31}$$

While the whole of $\tau = 0$ surface has $x = \pm\infty$ the interactions are nontrivial. However they again vanish in the asymptotic region of large σ and this is where our results are trustworthy.

4.4 The nature of boundaries

For the two classes of closing and opening hyperbola solutions we concluded that the space-time have space-like boundaries. This is based on the fact that the boundary is a $\tau = 0$ surface in terms of the Minkowskian time. Normally one would extend the space-time beyond this surface

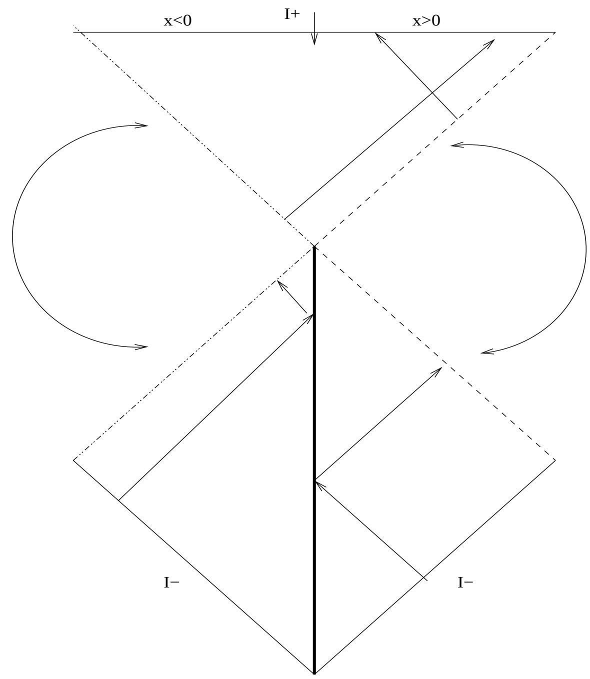

Figure 6: Penrose diagram for the opening hyperbola solution showing two classes of null rays. The identifications are indicated by curved arrows

and complete it. However, this extension does not make sense in this theory. Recall that space-time is a derived concept in this model and the true dynamics is that of infinite number of eigenvalues of the matrix evolving in time t. This dynamics is interpreted as a dynamics of a field in one more dimension. Extension of the space-time beyond the surfaces $\tau = 0$ would mean that we extend the fundamental time evolution beyond $t = \infty$ and this does not make any sense.

In two dimensions, any metric is conformally flat. Since our model contains only massless scalars, the free part of the Hamiltonian is not sensitive to the conformal factor. One could possibly choose a conformal factor which is singular at the boundary and makes the space-time geodesically complete. This should lead to an equivalent description of the physics. The problem with this is that the necessary conformal transformation would make the interaction Hamiltonian time dependent and possibly singular at *all points in space* at late times. In our choice the interaction Hamiltonian is time-independent. While it does not vanish everywhere on the space-like boundary, it does vanish in the asymptotic region at all times. This makes the physics more transparent. For fluctuations around the ground state, this is a useful choice since this facilitates comparison with worldsheet results. In the case of non-trivial solutions, such a comparison is not yet possible.

The presence of space-like boundaries in the solutions above should be interpreted in view of these comments. The unambiguous statement is that the boundaries are space-like in a choice of coordinates in which the interaction Hamiltonian is time-independent.

5 Aspects of particle production and stringy effects

In the moving mirror background, modes receive large blue(red) shifts as they get reflected and this results in particle production. A standard way to study this is to consider the components of the "out" energy momentum tensor in the "in" vacuum, using the anomalous transformation properties in two dimensions. In this case $\mathcal{I}^-$ is parametrized by $\sigma_+ = \sigma + \tau$ (as defined in equation (22) which is identical to $q_+ = q + t$ in this region. The "in" vacuum is defined in terms of the modes $u_{in} = e^{-i\omega(\tau \pm \sigma)}$. On the other hand $\mathcal{I}^+$ is defined by $\sigma_+ \to \infty$ and parametrized by either σ_- or q_- which are related by (24). This means that one measure of the outgoing energy-momentum flux is the quantity $< T_{q_-q_-} >_{in}$ where the quantity $T_{q_-q_-}$ is defined in terms of the derivatives of the field with respect to q_- with constant large σ_+ (which is not the same as constant q_+). This quantity can be calculated in terms of $< T_{\sigma_-\sigma_-} >_{in}$ by performing a coordinate transformation from $\sigma_\pm$ to (q_-, σ_+). The relation (24) shows that this is a conformal transformation, so that the anomaly relation gives

$$< T_{q_-q_-} >_{in} = (\frac{\partial \sigma_-}{\partial q_-})^2 < T_{\sigma_-\sigma_-} >_{in} + \frac{1}{24\pi}\{\sigma_-, q_-\}_S \tag{32}$$

where $\{A, B\}_S$ denotes the Schwarzian derivative. In our case

$$\frac{1}{24\pi}\{\sigma_-, q_-\}_S = \frac{1}{48\pi}\frac{4e^{-q_-}(1 - e^{-q_-})}{(1 - 2e^{-q_-})^2} \tag{33}$$

In usual quantum field theory one would set $< T_{\sigma_-\sigma_-} >_{in} = 0$ - by definition the quantity is normal ordered in the in vacuum. Therefore the energy-momentum flux of particles is simply given by (33)

However, here we are dealing with a string theory and the quantity under consideration is the torus worldsheet partition function. Unlike usual quantum field theory, this quantity has a well defined meaning since string theory does not have ultraviolet divergences in physical quantities - and this quantity is nonvanishing and finite in theories without target space supersymmetry. This important fact seems to be at odds with the corresponding calculation in collective field theory.

The answer to this apparent puzzle is well known. The action of collective field theory given in (4) is only the *classical* action. At the loop level there are additional terms in the action [6], which are calculated concretely by carefully making the change of variables from the matrix to collective fields. The explicit term which is relevant for our purposes is given by the singular expression

$$\Delta S_{(1)} = -\frac{1}{2\pi}\int dx\ (\partial_x \phi)\ [\partial_x \partial_{x'}\ \log|x - x'|]_{x=x'} \tag{34}$$

This is clearly subdominant in the large-N and hence $\hbar$ expansion of the theory. At one loop one can substitute $\phi \to \phi_0$. For fluctuations around the ground state this term leads to an infinite contribution to the one loop ground state energy. The singular term cancels the infinite contribution from integrating out fluctuations (the usual one loop diagram in the field theory of η) leaving a finite result which is in exact agreement with worldsheet calculations [4]. This result may be expressed in terms of a ground state energy momentum tensor

$$< T_{++} >_{gs} = < T_{--} >_{gs} = -\frac{1}{48\pi} \tag{35}$$

This finite one loop ground state energy must be clearly taken into account in the calculation of the energy-momentum flux from the moving mirror [11]. In this problem $\sigma_\pm$ are Minkowskian coordinates - this means that the quantity $< T_{\sigma_-\sigma_-} >_{in}$ which appears in the anomaly relation (32) is precisely the ground state quantity $< T_{--} >_{gs}$ given by (35). Substituting this in (32) one finds the q_- dependent contribution from the Schwarzian derivative in the second term is cancelled by the first term, leaving with *exactly the ground state answer*

$$< T_{q_-q_-} >_{in} = -\frac{1}{48\pi} \tag{36}$$

Since we continue to use σ_+ as the relevant coordinate at late times the value of $< T_{\sigma_+\sigma_+} >$ is unchanged and is also $-1/48\pi$ so that in this sense there is no energy flux, even though there is particle production. What has happened is that the constant one loop energy density in the moving coordinate system σ, τ translates into an energy flux in the coordinates (q_-, σ_+) which cancels the energy flux due to particle production. This is a very stringy effect - since the additional one loop term in the collective field action is a reflection of the underlying string theory.

Instead of computing the quantities considered above one could also calculate the expectation values of canonical quantities in the original (t, q) coordinate system leading to $< T_{q_-q_-} >_{in}$ which are defined in terms of derivatives $\frac{\partial}{\partial q_-}|_{q_+}$. Since the transformation between q_+ and σ_+ is not a canonical transformation for large q_+ one cannot use the anomaly relation as above. Rather the calculation has to be done ab initio with careful regularization. This has been done in [17]. The result is that while divergent terms cancel, the finite result has a nontrivial q_- dependence.

The questions of physical effects like particle production in the other geometries considered in this talk are more complicated and have not been fully addressed yet.

An issue which is closely related to particle production is the question of thermality. This requires a time-symmetric model. In [18] a time symmetric version of $r = 1$ solution was shown to lead to Hartle-Hawking type states, despite nontrivial time dependence.

6 Other solutions

There are several other classes of time-dependent solutions of the $c = 1$ which have been studied using methods similar to those discussed above. One interesting class of such solutions involve finite size droplets in phase space [19]. In fact there has been recent progress in understanding the question of black hole (non) formation in this theory and Hawking radiation [20]. In a slightly different direction, [21] have found that a *time-like* linear dilaton background in two dimensional string theory can be also described in terms of a matrix model, which is in fact quite similar to the standard one. Some non-perturbative aspects of time-dependence have been considered in [22]. The outstanding problem is to discuss these backgrounds in the worldsheet formulation. It is likely that recent work on minimal string theory [23] can shed some light on this problem.

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