DISTRIBUTIONS OF FLUX VACUA

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INTRODUCTION, OUTLINE, CONFESSIONS ...

BIG QUESTION HOW MANY CONSISTENT STRING COMPACTIFICATIONS ARE THERE? [d = 4]

TYPE IIB STRING THEORY ON A CALABI-YAU WITH FLUXES

- LOW ENERGY EFFECTIVE DESCRIPTION AS $\mathcal{N}=1$ SUPERGRAVITY IN $d=4$.
- FOCUS ON PROPERTIES OF VACUUM SOLUTIONS [SUPERSYMMETRIC / NON-SUPERSYMMETRIC]

QUESTIONS :

- NUMBER DENSITY OF VACUA OVER CONFIGURATION SPACE
- DISTRIBUTION OF $\left\{ \begin{array}{l} \text{COSMOLOGICAL CONSTANTS} \\ \text{SUPERSYMMETRY BREAKING SCALES} \\ \ldots \end{array} \right.$

$$N_{vac}(L \leq L_*, M_{susy} \leq M_*, \Lambda \leq \Lambda_*, \ldots)$$

OUTLINE : GENERAL PROPERTIES OF $\mathcal{N}=1$ $d=4$ SUGRA.

- BASICS OF FLUX COMPACTIFICATIONS
- GLOBAL CONSTRAINTS
- COUNTING : A FIRST EXAMPLE : SUPERSYMMETRIC VACUA
- VARIOUS LIMITS $\left\{ \begin{array}{l} \to \text{CONIFOLD} \\ \to \text{LARGE CPLX. STRUCTURE} \end{array} \right.$
- DISTRIBUTION OF COSMOLOGICAL CONSTANTS
- DISTRIBUTION OF NON SUPERSYMMETRIC VACUA
- ALTERNATE METHODS (MORE MODULI)
- CONCLUDING REMARKS

  STATISTICAL SELECTION & PREDICTIVITY

$\mathcal{N}=1$ $d=4$ LOW ENERGY EFFECTIVE THEORY :

TRIPLE $(\mathcal{M}, \mathcal{K}, W)$

$\mathcal{M}$: CONFIGURATION SPACE.
COMPLEX KÄHLER MANIFOLD.
LOCAL COORDINATES (z^a)

$\mathcal{K}$: KÄHLER POTENTIAL.
DETERMINES KÄHLER METRIC ON $\mathcal{M}$

$$g_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K}$$

W : SUPERPOTENTIAL
LOCAL ANALYTIC FUNCTION ON $\mathcal{M}$

$$V(z^a) = e^{K(z^a, \bar{z}^{\bar{a}})} \left[g^{a\bar{b}} D_a W \bar{D}_{\bar{b}} \bar{W} - 3 W \bar{W} \right]$$

$$= |DW|^2 - 3|W|^2$$

VACUA : $\partial V = (DDW)\bar{D}\bar{W} - 2(DW)\bar{W} = 0$

SUPERSYMMETRIC : $DW = 0$ (AdS, Minkowski)
NON-SUPERSYMMETRIC : $DW \neq 0$

FLUX COMPACTIFICATION :

F-THEORY ON A CALABI-YAU 4-FOLD. X

$\mathcal{M}$

$$K = -\ln\left[\int_X \Omega \wedge \bar{\Omega}\right] \qquad \Omega \in H^{(4,0)}(X) \hookrightarrow \Omega(z)$$

WHAT ABOUT W?

FLUXES : $G_4 \in H^4(X, \mathbb{Z}) \ni \{\Sigma_\alpha\}$ [SYMPLECTIC BASIS]

$$G_4 = N^\alpha \Sigma_\alpha = \eta^{\alpha\beta} N_\beta \Sigma_\alpha$$

$$\eta_{\alpha\beta} = \int_X \Sigma_\alpha \wedge \Sigma_\beta$$

CLAIM : $W = \int_X G_4 \wedge \Omega(z) = N^\alpha \int_X \Sigma_\alpha \wedge \Omega(z) = N^\alpha \Pi_\alpha(z)$

ORIENTIFOLD LIMIT : $X \longrightarrow (T^2 \times CY_3)/\mathbb{Z}_2$: $\boxed{\mathcal{M} \longrightarrow \mathcal{M}_\tau \times \mathcal{M}_{\text{CPLX OF } CY_3}}$

- SCALARS : $\{z^0, z^i\}$ AXIO-DILATON & CPLX. STRUCTURE MODULI OF CY_3

- $G_4 \rightarrow -\alpha \wedge F_{(3)} + \beta \wedge H_{(3)}$ $\qquad \alpha, \beta \in H^1(T_2, \mathbb{Z})$

  $F_{(3)}$: RR 3-FORM OF IIB

- $\Omega \rightarrow \Omega_{(1)}(z^0) \wedge \Omega_{(3)}(z^i)$ $\qquad H_{(3)}$: NS-NS 3-FORM ...

- $L \equiv \frac{1}{2}\int_X G_4 \wedge G_4 = \frac{1}{2} N \eta N \longrightarrow \int_{CY_3} F_{(3)} \wedge H_{(3)}$

CONSTRAINTS

- TADPOLE CANCELLATION (GAUSS' LAW)

Eg: F-theory ON A 4-FOLD:

$$L \equiv \frac{1}{2}\int_X G_4 \wedge G_4 = \frac{\chi(X)}{24} + N_{D3}$$

$$\boxed{L \leq L_*}$$

ENSURES FINITENESS FOR SUSY VACUA

- ABSENCE OF TACHYONS IN PERTURBATIVE SPECTRUM (V'' HAS NON-NEGATIVE EIGENVALUES)

- TO ENSURE FINITENESS OF NON-SUPERSYMMETRIC VACUA,

$$L \leq L_* \quad \text{AND} \quad |DW| \leq F_*$$

BASIC STRATEGY AND AN EXAMPLE

NUMBER OF SUPERSYMMETRIC VACUA $N_{vac}(L \leq L_*)$

$$\text{VACUUM}: \left\{ \{z^a, \vec{N}\} \mid DW = 0 \right\}$$

STRATEGY

- PICK A POINT (z_0^a) IN CONFIGURATION SPACE
- COUNT ALL FLUX VECTORS SUCH THAT (z_0^a) IS A SOLUTION TO $DW = 0$
- IMPOSE TADPOLE CANCELLATION CONSTRAINT AT THIS POINT, $d\mu_{vac}(z^a, L_*)$
- INTEGRATE THIS OVER ANY REGION $\mathcal{R}$ IN CONFIGURATION SPACE ; THIS GIVES CONTRIBUTION OF $\mathcal{R}$ TO THE NUMBER OF SUPERSYMMETRIC VACUA.

APPROXIMATION : $\displaystyle\sum_{\{N^a\}} \xrightarrow{?} \int d^k N$

N_{susy} ONLY TADPOLE CONSTRAINT.

$$N_{vac}(L \leq L_*) = \sum_{\substack{susy\\vac}} \Theta(L_* - L) = \frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha} \sum_{\substack{susy\\vac}} e^{\alpha(L_* - L)}$$

$$= \frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha} e^{\alpha L_*} N_{vac}(\alpha)$$ → LAPLACE TRANSFORM

$$N_{vac}(\alpha) = \sum_{\substack{susy\\vac}} e^{-\frac{1}{2}\alpha N\eta N}$$

$$\sum_{\substack{susy\\vac}} = \sum_{\substack{(\text{zeroes of}\\ DW)\cup\{\text{fluxes}\}}} = \int_M d^{2m}z \, . \sum_{\{N\}} . \, \delta^{(2m)}(DW) . |\det D^2W|$$

$$N_{vac}(\alpha) \cong \int_M d^2z \int d^KN \; \delta^{(2m)}(DW) . |\det D^2W| . \, e^{-\frac{1}{2}\alpha N\eta N}$$

$\tau, z^i \quad i=1\dots n$

$n+1 = m$

$4.m = K$

SCALING : $N \to \frac{N}{\sqrt{\alpha}}$

$$\Rightarrow N_{vac}(\alpha) = \alpha^{-K/2} N_{vac}(1)$$

α - INTEGRAL EASILY DONE AND L-DEPENDENCE OF $N_{vac}(L \leq L_*)$ FACTORS OUT.

$$\boxed{N_{vac}(L \leq L_*) = \frac{(L_*)^{2m}}{(2m)!} . \, N_{vac}(1)}$$

→ PURELY DEPENDENT ON GEOMETRY OF MODULI SPACE

$$N_{vac}(1) = \int d^KN \int_M d^{2m}z \; \delta^{(2m)}(DW) \; |\det D^2W| . \, e^{-\frac{1}{2}N\eta N}$$

$$\det D^2W = \det \left[\begin{array}{c|c} \bar{D}DW & DDW \\ \hline \bar{D}\bar{D}\bar{W} & D\bar{D}\bar{W} \end{array}\right]$$

$$N_{vac}(1) = \int d^k N \; e^{-\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta} \; \delta^{(2n)}(DW) \cdot |\det D^2 W| .$$

GAUSSIAN 'ENSEMBLE' WITH COVARIANCE $\eta^{\alpha\beta}$

$$\langle N^\alpha N^\beta \rangle = \eta^{\alpha\beta}$$

$$N_{vac}(1) = \left\langle \delta^{(2n)}(DW) \cdot |\det D^2 W| \right\rangle_{\eta^{\alpha\beta}}$$

$$W = N^\alpha \Pi_\alpha$$

EXACT SOLUTION POSSIBLE FOR INDEX DENSITY.

$$N_{ind}(1) = \left\langle \delta^{(2n)}(DW) \cdot (\det D^2 W) \right\rangle$$

$$= \det (R + \omega \mathbb{1})$$

$$\det_{c,d} \left[(R_{ab})^c{}_d \, dz^a \wedge dz^b + \omega_{ab} \, dz^a \wedge dz^b \, \delta^c{}_d \right]$$

$$N_{\substack{vac \\ (ind)}}(L \leq L_*) = \frac{(L_*)^{K/2}}{(K/2)!} \underbrace{\int_{\mathcal{M}} \det (R + \omega \cdot \mathbb{1})}_{\text{ROUGHLY, VOLUME OF CONFIGURATION SPACE}}$$

ROUGHLY, VOLUME OF ~~MODULI~~ CONFIGURATION SPACE

Eg: <u>ABELIAN VARIETY</u> :

$$\mathcal{M} : \left\{ Z_{ij} \;\middle|\; \begin{matrix} \operatorname{Im} Z > 0 \\ Z_{ij} = Z_{ji} \end{matrix} \right\}$$

$K = 40$; $L_* = 32$; $n = \frac{3 \times 4}{2} = 6 \ldots$ $\boxed{N_{vac} = 4 \times 10^{21}}$

• QUALITATIVE MESSAGE } : VACUA CONCENTRATED NEAR REGIONS OF HIGH <u>CURVATURE</u>. (Eg: CONIFOLD POINT)

NEW NOTATION & A CHANGE OF BASIS :

$$\langle A, B \rangle = \int_X A \wedge B \qquad A, B \in H^4(X, \mathbb{Z})$$

RECALL : OLD BASIS $\{\Sigma_\alpha\} \in H^4(X, \mathbb{Z})$

$$G_4 = N^\alpha \Sigma_\alpha = \eta^{\alpha\beta} N_\beta \Sigma_\alpha$$

WHERE $N_\alpha = \langle G_4, \Sigma_\alpha \rangle$

NEW BASIS : IN THE ORIENTIFOLD LIMIT,

$$\mathcal{M} = M_\tau \times M_{\substack{\text{CPLX of} \\ CY_3}}$$

$$\{\Omega, D_A\Omega, D_0 D_I \Omega\} \cup \{c.c.\}$$
$$1 + (n+1) + n + \ldots = 4(n+1) = 4m = K$$

$$D_A \Omega = e_A{}^a D_a \Omega = \boxed{e_A^a} \left(\frac{\partial \Omega}{\partial z^a} + \frac{\partial K}{\partial z^a} \Omega \right)$$

$$\langle B, \bar{B} \rangle = \mathrm{Diag}\left(1, -\mathbb{1}_{n+1}, \mathbb{1}_n, 1, -\mathbb{1}_{n+1}, \mathbb{1}_n\right)$$

$$G_4 = \bar{X}\Omega - \bar{Y}^A D_A \Omega + \bar{Z}^I D_0 D_I \Omega + c.c.$$

CHANGE OF BASIS $\quad \{N^\alpha\} \longleftrightarrow \{X, Y_A, Z_I, c.c.\}$

~~AS BEFORE~~

NATURAL COÖRDINATES:

$$X = \langle G_4, \Omega \rangle = W$$

$$Y_A = \langle G_4, D_A \Omega \rangle = D_A W$$

$$Z_I = \langle G_4, D_0 D_I \Omega \rangle = D_0 D_I W$$

DEFINING $\mathcal{F}_{IJK} = \langle \Omega, D_I D_J D_K \Omega \rangle$,

$$D_I D_J W = \mathcal{F}_{IJK} \bar{Z}^K$$

SIMILARLY, FOR ALL HIGHER DERIVATIVE TERMS...

WHY: INTEGRAND OF $N_{vac}(1)$ EASILY DESCRIBED IN TERMS OF $\{X, Y, Z\}$

Eg: $V = |DW|^2 - 3|W|^2 = |Y|^2 - 3|X|^2$

$$L = \frac{1}{2}\langle G_4, G_4 \rangle = |X|^2 - |Y|^2 + |Z|^2$$

SIMILARLY, $\det D^2 W \ldots$

$$N_{vac}(L \leq L_*) = \frac{(2\pi L_*)^{2m}}{(2m)!} \frac{1}{\sqrt{\eta}} \int_M d^{2m} z \, (\det g) \cdot \boxed{\rho(z)}$$

$$\rho(z) = \frac{1}{\pi^{2m}} \cdot \int d^2X \, d^{2n}Z \, e^{-|X|^2 - |Z|^2} \cdot |\det D^2 W|$$

$\delta(DW) \sim \delta(Y_A)$ ENFORCES $Y_A = 0$ FOR SUPERSYMMETRY.

$$\det D^2 W = |X|^2 \det \begin{bmatrix} \delta_{IJ}\bar{X} - \dfrac{z_I \bar{z}_J}{X} & \mathcal{F}_{IJK}\bar{z}^K \\ \bar{\mathcal{F}}_{IJK} z^K & \delta_{IJ} X - \dfrac{\bar{z}_I z_J}{\bar{X}} \end{bmatrix}$$

ONE CAN REPRODUCE THE $\det(R + \omega \mathbb{1})$ FORMULA FOR THE INDEX DENSITY BY USING

$$R_{I\bar{J}K\bar{L}} = \mathcal{F}_{IK}{}^{\bar{M}} \bar{\mathcal{F}}_{\bar{M}\bar{J}\bar{L}} - g_{I\bar{J}} g_{K\bar{L}} - g_{I\bar{L}} g_{\bar{J}K}$$

NOTE: $\rho(z)$, [THE DENSITY OF SUPERSYMMETRIC VACUA PER UNIT VOLUME OF MODULI SPACE] IS COMPLETELY DETERMINED IN TERMS OF SPECIAL GEOMETRY DATA $\mathcal{F}_{IJK}$.

CONIFOLD LIMIT: $|\mathcal{F}| \to \infty$

$$\det D^2 W \approx (-1)^n |X|^2 \left| \det \mathcal{F}_{IJK} \bar{z}^K \right|^2$$

(IN PARTICULAR, $|\rho_{vac}| = |\rho_{index}|$)

PLAN : RESTRICT TO ONE VARIABLE MODELS $(n=1)$

: STUDY CONIFOLD LIMITS

: OBTAIN NUMBER DENSITY OF SUSY VACUA WITH/WITHOUT THE $V'' > 0$ CONDITION

: REPEAT... FOR DISTRIBUTION OF COSMOLOGICAL CONSTANTS.

$\underline{n=1}$

$$\rho_{ind} = \frac{1}{\pi^4}\int d^2X\, d^2Z\, e^{-|X|^2-|Z|^2}\left[|X|^4+|Z|^4-|X|^2|Z|^2\left(2+|\mathcal{F}|^2\right)\right]$$

$$= \frac{2-|\mathcal{F}|^2}{\pi^2}$$

CONIFOLD LIMIT: $\quad K(z,\bar z) = c\, z\bar z\left[\log\left(\frac{\mu^2}{z\bar z}\right)+1\right] \quad (z\to 0)$

$$g_{z\bar z} = c\log\left(\frac{\mu^2}{z\bar z}\right) \quad ; \quad \mathcal{F} = g_{z\bar z}^{-3/2}\cdot\left(\frac{c}{z}\right) \to \infty \text{ as } z\to 0.$$

$$\int_{M|_R} d^2z\,(\det g)\,\rho_{ind} = -\int_{M|_R} d^2z\; c\log\left(\frac{\mu^2}{z\bar z}\right)\cdot\frac{1}{\pi^2}|\mathcal{F}|^2 = \frac{1}{\pi}\cdot\frac{1}{\log\left(\frac{\mu^2}{R^2}\right)}$$

Eg: MIRROR QUINTIC : c and μ^2 ARE KNOWN.

$$N_{vac}(L\leq L_*, |z|\leq R) = \frac{\pi^4 L_*^4}{18\,\ln\left(\frac{\mu^2}{R^2}\right)}$$

- VACUA STABILIZED AROUND CONIFOLD $\rightsquigarrow$ GENERATING HIERARCHIES
- GROWTH IN NUMBER OF VACUA NEAR HIGH CURVATURE POINTS SEEM GENERIC.

WHAT ABOUT POSITIVITY OF V''?

$$d^2V = \begin{bmatrix} |z|^2 - 2|x|^2 & \bar{\mathcal{F}} z^2 & 0 & -x\bar{z} \\ \mathcal{F}\bar{z}^2 & (1+|\mathcal{F}|^2)|z|^2 - 2|x|^2 & -x\bar{z} & -\bar{\mathcal{F}}\bar{x}\bar{z} \\ 0 & -x\bar{z} & |z|^2 - 2|x|^2 & \mathcal{F}\bar{z}^2 \\ -x\bar{z} & -\bar{\mathcal{F}} x z & \bar{\mathcal{F}} z^2 & (1+|\mathcal{F}|^2)|z|^2 - 2|x|^2 \end{bmatrix}$$

POSITIVITY $\Rightarrow$ CONSTRAINTS. eg:- $|z|^2 - 2|x|^2 > 0$

$\vdots$

RESTRICTS INTEGRATION REGIONS IN $|z|^2, |x|^2$ PLANE

FINAL RESULT IN THE CONIFOLD LIMIT : $\rho_{V''>0} = \dfrac{21}{16\pi^2} \cdot \dfrac{1}{|\mathcal{F}|^2}$

$$\left[|\rho| = +\frac{1}{\pi^2} |\mathcal{F}|^2 \right]$$

ALMOST NO VACUA WITH $V''>0$ NEAR CONIFOLD!

BUT THIS SUPPRESSION IS PROBABLY AN ARTIFACT OF THE 1-MODULUS CASE. $V''>0$ NOT TOO CONSTRAINING IN HIGHER MODULI EXAMPLES...

DISTRIBUTION OF COSMOLOGICAL CONSTANTS :

BASIC EXPRESSION:

$$N_{vac}(L \leq L_*, \lambda \leq \lambda_*) = \int_{\mathcal{M}} d^2 z \int_{C} \frac{d\alpha}{\alpha}\, e^{\alpha L_*} \int_0^{\lambda_*} d\lambda \;\; \nu(z,\lambda,\alpha)$$

$$\nu(z,\alpha,\lambda) = \int d^k N \; e^{-\alpha N \eta N}\, \delta^{(2m)}(DW)\, |\det D^2 W|\; \delta(|W|^2 - \lambda)$$

- CHANGE VARIABLES TO $\{X, Y_A, Z_I\}$
- IMPOSE DELTA FUNCTIONS
- SCALE OUT α-DEPENDENCE : $Y_A = 0$; $X = \alpha\lambda$

PHYSICS INTEREST : PUT $\lambda \to 0$ IN INTEGRAND TO OBTAIN NUMBER OF VACUA WITH $\Lambda \sim 0$. WE FIND UNIFORM DISTN.

$$N_{susy}(L \leq L_*, |W|^2 \leq \lambda_* \sim 0) = \lambda_* \left[\frac{(2\pi L_*)^{2m}}{(2m)!} \frac{1}{\sqrt{\eta}} \cdot \int_{\mathcal{M}} (\det g) \cdot \rho(z) \right]$$

$$\rho = \rho(\lambda)\big|_{\lambda=0} = \frac{2\pi m}{\pi^{2m} L_*} \int d^{2n} Z \; e^{-|Z|^2} \left| \det \begin{pmatrix} 0 & Z_J \\ Z_I & F_{IJk} \bar{Z}^k \end{pmatrix} \right|^2$$

ρ FIXED BY SPECIAL GEOMETRY DATA F_{IJK}

Eg: MIRROR QUINTIC : ρ INDEPENDENT OF F_{IJK}.

$m = 2$.

$$N_{susy}(L \leq L_*, |W|^2 \leq \lambda_* \sim 0) = \frac{4\pi^4}{45} L_*^3 \cdot \lambda_*$$

WHAT ABOUT POSITIVITY ?

POSITIVITY LEADS TO CONSTRAINTS. IN THE CONIFOLD LIMIT,

$$|z|^2 > 4|f|^2|x|^2$$

OR $$|W|^2 = |x|^2 < \frac{|z|^2}{4|f|^2} \to 0 \quad \text{as } |f| \to \infty$$

∴ VACUA NEAR CONIFOLD AUTOMATICALLY HAVE COSMOLOGICAL CONSTANTS ~ 0.

SOME NUMBERS:

EG: MIRROR QUINTIC: TAKE $L_* = 100$

USE $V'' > 0$ FORMULA FOR N_{vac};

$N_{vac} = 1 \Rightarrow |z| \sim 0.02$

THIS LEADS TO $|f| \sim 20$.

- CONSTRAINT BECOMES $\lambda_* \lesssim 10^{-3} L_* \sim 0.1$
- $N_{vac}(L \leq L_*, |W|^2 < \lambda_*) \sim 10^{+7} \lambda_* \lesssim 10^6$

$\Rightarrow |\Lambda| \gtrsim 10^{-6}$ (in string units)

METASTABLE VACUA NOT CLOSE ENOUGH TO THE CONIFOLD POINT.

NON-SUPERSYMMETRIC VACUA

RECALL : $L = \frac{1}{2}\langle G_4, G_4 \rangle = |X|^2 - |Y|^2 + |Z|^2$

SINCE $D_A W = Y_A \neq 0$, NEED TO IMPOSE $|Y| \lesssim F_*$ FOR FINITENESS

MANY BRANCHES

- "ANTI-SUPERSYMMETRIC" : $X = Z = 0$, $Y \neq 0$

  IT TURNS OUT: $\mathcal{N}_{X=Z=0}(|Y| \lesssim F_*) = N_{SUSY}(L \lesssim F_*^2)$

  $$\Lambda = |DW|^2 = (-L)$$

  SINCE $L \in \mathbb{Z}$, $\Lambda \sim$ STRING SCALE.

  EFFECTIVE FIELD THEORY BREAKS DOWN!?

- $X, Y, Z \neq 0$.

  ONE CAN REPEAT ANALYSIS AS BEFORE.

  $n=1$: NON-SUSY CONIFOLD VACUA ARE SPARSER THAN SUSY VACUA

  METASTABLE dS DO NOT EXIST AROUND LARGE CPLX STRUCTURE POINT.

  ALL OF THE NON-SUSY CONIFOLD VACUA HAVE $\Lambda < 0$. $\Lambda > 0$ KILLED BY $V'' > 0$.

  (ARTIFACT OF $n=1$)

NATURALNESS

- EXISTENCE OF SUCH LARGE NUMBERS OF VACUA LEAD TO A STATISTICAL SELECTION PRINCIPLE.

Eg: (DOUGLAS, Strings 2004 TALK)

SUPPOSE:

- LOW SCALE ~~SUSY~~ IN 10^{160}
  HIGH SCALE ~~SUSY~~ IN 10^{100}

[VACUA THAT REPRODUCE ALL OF SM, BUT NOT THE COSM. CONSTANT]

- ONE CAN SHOW THAT IN ALL THESE VACUA, COSMOLOGICAL CONSTANT IS UNIFORMLY DISTRIBUTED

$\Rightarrow$ $\sim 10^{40}$ VACUA WITH LOW SCALE ~~SUSY~~

$\sim 10^{-20}$ VACUA WITH HIGH SCALE ~~SUSY~~

- DEFINITE PREDICTION { STRING THEORY PREDICTS ~~HIGH~~ LOW SCALE ~~SUSY~~

IF HIGH SCALE ~~SUSY~~ IS OBSERVED, STRING THEORY IS FALSIFIED.

VALIDITY OF APPROXIMATION :

RECALL : ONLY APPROX. WAS TO IGNORE FLUX QUANTIZATION. HOW GOOD IS THIS APPROX.?

SPACE OF FLUXES : $\mathbb{R}^K$

GIVEN (z^a, τ) : $2n+2 = \frac{K}{2}$ CONDITIONS $N^\alpha D_i \Pi_\alpha = 0$

$$S_{(z,\tau)} \cong \mathbb{R}^{K/2}$$

$$\boxed{S = \bigcup_{(z,\tau)\in R} S_{(z,\tau)}} \quad (K\text{-dimensional})$$

$$\begin{array}{c}\#\text{ of}\\ \text{susy}\\ \text{vacua}\end{array} = \mathbb{Z}_K \cap S_L = \left\{ S \cap \mathbb{Z}_K \,\middle|\, \tfrac{1}{2} N \eta N \leq L \right\}$$

$$N(L) = L^{K/2} V(S_{L=1}) + L^{\frac{K-1}{2}} A(S_{L=1}) + \cdots$$

Approximation good when

$$L^{K/2} V(S_{L=1}) \gg L^{\frac{K-1}{2}} A(S_{L=1}) \quad \text{or} \quad \sqrt{L} \gg \frac{A(S_1)}{V(S_1)}$$

$$\boxed{L \gg \frac{K}{\gamma^2}}$$