Vacancy-induced local moment instability of RVB spin liquids

Kedar Damle (TIFR Mumbai) @ BU (Jan 2026)

unpublished:

Bhola, KD, arXiv:2311.05634v2 (2025); Ansari, Kundu, KD (in preparation)

recent:

Ansari, KD, PRL 132 226504 (2024)
Bhola, Biswas, Islam, KD, PRX 12, 021058 (2022)

background: KD, PRB 105 235118 (2022)

Sanyal, KD, Chalker, Moessner PRL 127 127201 (2021)

Sanyal, KD, Motrunich, PRL 117 116806 (2016)

Antiferromagnetism and Frustration

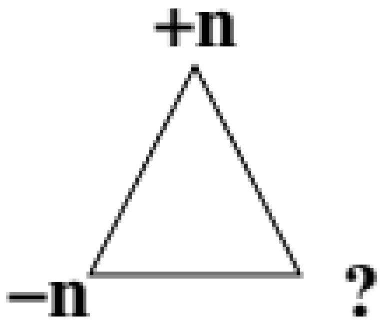
Bipartite lattices: A-sublattice spins point “up”, B-sublattice spins point “down”

up and down about what axis? Spontaneous symmetry breaking

Geometric frustration

Triangles in the nearest neigbhor connectivity

Collinear antiferromagnet frustrated

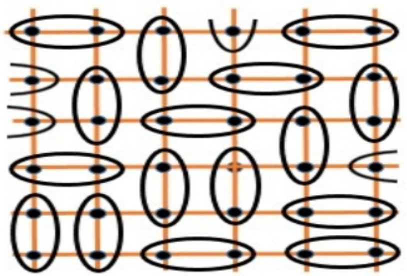


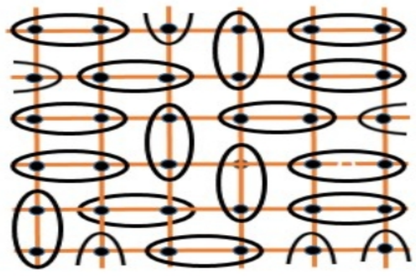
Higher orders in strong coupling expansion:

Four-spin couplings: Ring exchange around plaquettes

Simple antiferromagnetic state again frustrated.

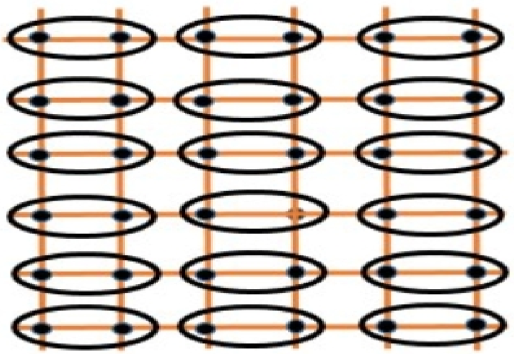
Frustration induced quantum disordered states





+ .....

Short-range resonating valence bond (sRVB) spin liquid



Valence bond solid (VBS)

(with spontaneous lattice symmetry breaking)

Our focus: Effect of quenched disorder aka dirt

Substitutional impurities, interstitial adatoms, structural defects...

Quenched (on electronic timescales).

Not to be confused with doping with mobile holes...

Our focus: Non-magnetic substitutional impurities--- e.g Zn for Cu, Ga for Cr

e.g. in Herbertsmithite, SCGO...

Variety of effects

Weak disorder: Can be irrelevant for low energy properties (not always).

Strong disorder: new phases of matter (e.g. spin glasses, Anderson insulator, MBL..)

When weak: Can probe correlations of underlying state (e.g. spin textures in frustrated magnets)

"Central dogma"

In large-size limit -

Strong version:

Self-averaging of properties: Sample-to-sample fluctuations small (average = typical)

Violations exist – e.g. Disordered quantum spin chains (infinite-disorder fixed points)

Weak version:

At a minimum, two samples prepared using some protocol must be in same phase.

Violations? May exist in infinite-range spin glass models (?)

Percolation

Sharp threshold behavior (in the limit of $L \to \infty$) of the end-to-end connectivity of a random medium as a function of the density n_v of blocked pathways or vacancies.

Simplest and entirely *geometric* example of a phase transition

Mathematical model: Regular lattice with random site or bond dilution n_v ("Bernoulli percolation")
(Broadbent and Hammersley Proc. Cam. Phil. Soc. 53, 629, 1957)

Boolean variable: Answer to YES/NO connectivity question.

$P(L, n_v)$: the probability of answering YES

Universal scaling at Bernoulli percolation transition

Scaling in the vicinity of the threshold: $P(L, n_v) = f((n_v - n_v^{crit})L^{1/\nu})$

$f(x) \to 1$ as $x \to -\infty$

$f(x) \to 0$ as $x \to +\infty$

Critical point exhibits scale invariant behavior

$n_v < n_v^{crit}$: diluted lattice has single infinite cluster with probability 1 (in limit $L \to \infty$)

In $L \to \infty$ limit: $P(L, n_v)$ can only be 0 or 1 at any dilution.

There is only one percolated phase and one unpercolated phase
(no reentrant transitions)

Our basic message

Maximum-density dimer packings of weakly-diluted lattices have unusual percolation transitions

Some of the percolated phases show violations of (even the weak form of the) "central dogma"

Root cause: Kinematic constraints that induce long-range correlations

Consequences for magnetism:

In short-range RVB spin liquids on triangular lattice

(vacancy disorder = nonmagnetic impurities)

Vacancy-induced local moment instability of sRVB state

At a minimum: Strong violations of thermodynamic self-averaging in the susceptibility at low vacancy density

Likely:

"R-type samples" have vacancy-induced spin-glass order but not "P-type" samples

Also: Chaotic (deterministic but unpredictable) response to changes in disorder configuration.

In contrast:

short-range RVB spin liquids on the kagome lattice are stable to weak vacancy disorder(!)

Quantum dimer model framework for RVB/VBS states

Rokhsar and Kivelson: Effective Hamiltonian living in subspace of singlets spanned by nn VB

$$H_{QDM} = -t(\,|\,=\,\rangle\langle\,\shortparallel\,|+|\,\shortparallel\,\rangle\langle\,=\,|\,) + \ldots$$

More generally: Ring-exchange kinetic terms on “flippable” plaquettes, and local interactions

Additional terms incorporate the effect of matrix elements to further-neighbor singlet states

Z2 spin liquid example: Triangular QDM

Triangular lattice: Moessner-Sondhi (within QDM framework):

Triangular lattice QDM has truly quantum disordered phase

Short-range spin correlations, valence bond correlations, genuine Z2 spin liquid

(also for kagome lattice)

Language primer: Fully-packed dimers (perfect matchings)

Fully-packed hard-core dimer models in stat-mech: Match **each** site to an adjacent site monogamously

In graph theory/computer science: The perfect matching problem

Easy to see (for regular lattices like square, triangular, honeycomb, kagome...):

Extensive entropy of fully-packed dimer covers (perfect matchings)

(exact computation of entropy on planar graphs: Classic papers by Kasteleyn & Fisher)

(also exact results on special non-planar graphs: Chandra & Dhar)

Extending QDM framework to disordered lattices

Basic question arises: Can a diluted lattice with even number of vertices be perfectly matched? (CS language)

If bipartite, need $|A| = |B|$

But: generally not possible (even with $|A|=|B|$)

Then have *maximum matching* but not *perfect matching*

Maximum matchings have unmatched sites that host *monomers*

Generally, nonzero vacancy density implies nonzero density of monomers (effect of irregularity and local structure)

Monomers correspond to “emergent” local moments in spin system

Each monomer corresponds to a disorder-induced “emergent” local moment
(purely kinematic effect, independent of VBS vs RVB nature of ground state)

Signature: Large intermediate temperature range with Curie tail in susceptibility

Quenched below scale set by residual interactions

$$\chi_{\mathrm{imp}} \sim \frac{\mathcal{C}}{T} \text{ for } J_{\mathrm{eff}} \ll T \ll J$$
$$\mathcal{C} \propto n_{\mathrm{monomer}}$$

But wait: This conclusion seems to rely too much on having only nearest-neighbor singlets?
Does it hold for more generic short-range RVB liquid?

To answer: large-N route to quantum dimer model

$$H = J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \dots$$

$$= -J \sum_{\langle rr' \rangle} \left(\mathcal{P}_{rr'} - \frac{1}{4} \right) + \dots$$

Enlarge symmetry group:

$$H = -\frac{J_m}{N} \sum_{\langle r_1 r_2 \rangle} \sum_{\alpha,\beta=1}^{N} |\alpha\rangle_{r_1} |\alpha\rangle_{r_2} \langle\beta|_{r_1} \langle\beta|_{r_2} + \cdots ,$$

Affleck, Read, Sachdev, Auerbach, Penc, Mila, Coleman, Sandvik, Alet, Kawashima, Beach, Kaul...(1988 - now)

What's the enlarged symmetry?

$$\begin{aligned}
\mathcal{A}_{\alpha\beta}(r) &= -i(|\alpha\rangle_r\langle\beta|_r - |\beta\rangle_r\langle\alpha|_r) \;\forall\; \text{pairs} \;\; \alpha < \beta \\
\mathcal{S}_{\alpha\beta}(r) &= (|\alpha\rangle_r\langle\beta|_r + |\beta\rangle_r\langle\alpha|_r) \;\forall\; \text{pairs} \;\; \alpha < \beta \\
\mathcal{Q}_{\alpha\alpha}(r) &= (|\alpha\rangle_r\langle\alpha|_r - 1/N) \;\forall\; \alpha = 1 \ldots N-1
\end{aligned}$$

$$\mathcal{A}^{\text{tot}}_{\alpha\beta} = \sum_r \mathcal{A}_{\alpha\beta}(r)$$

SO(N) symmetry on any arbitrary lattice

$$\mathcal{S}^{\text{tot}}_{\alpha\beta} = \sum_r (-1)^r \mathcal{S}_{\alpha\beta}(r)$$

$$\mathcal{Q}^{\text{tot}}_{\alpha\beta} = \sum_r (-1)^r \mathcal{Q}_{\alpha\beta}(r)$$

Bipartite case: Enhanced “staggered” SU(N) symmetry

Large N limit in pure case

Any perfect (fully packed) dimer cover is a ground state (each dimer interpreted as singlet state)

Leading 1/N corrections: Captured precisely by QDM Hamiltonian with ring-exchange

Higher orders in 1/N: Additional local terms in QDM Hamiltonian

(Affleck, Read, Sachdev, Kaul...)

Recover the same QDM framework---without nearest-neighbor singlet assumption.

Disordered case: Large N limit

Any maximum matching now gives a large-N ground state.

Monomers correspond to free moments (additional degeneracy)

Leading 1/N corrections: QDM Hamiltonian with ring-exchange + monomer kinetic energy terms

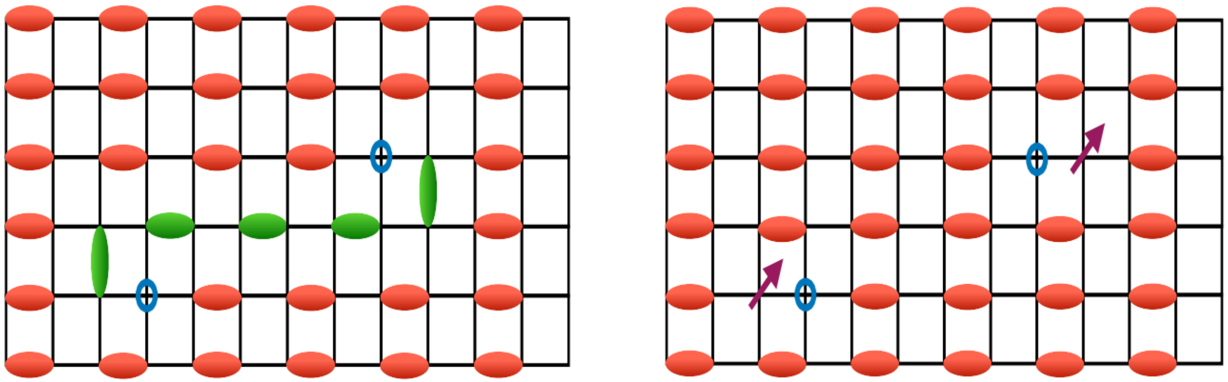
Higher orders in 1/N: Additional local terms in QDM Hamiltonian

Correspond to residual interactions between local moments...(?)

These control fate of system at lowest energies

So: Large N also gives maximally-packed QDM description of disorder effects in short-range RVB liquid

Contrast with VBS state



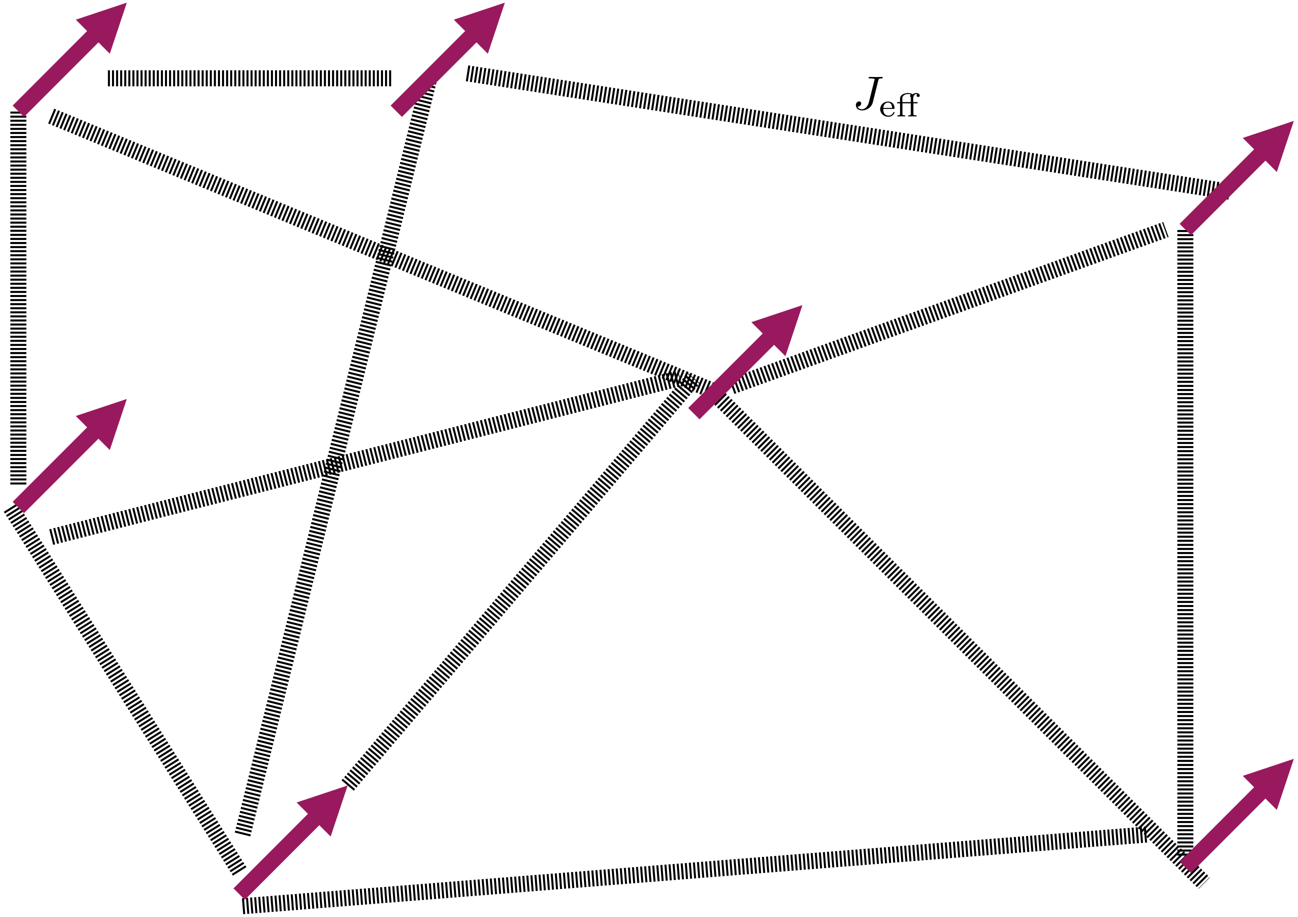
Each vacancy, even if isolated from other vacancies, seeds a local moment in a VBS state

(even when perfect matchings are possible, i.e even when there are no monomers)

In contrast, for sRVB case: Monomers of maximum matchings are sole mechanism

Ansari, KD, PRL 132 226504 (2024)

Summary: Distinct vacancy-induced local moment instabilities of RVB and VBS states



In RVB case, only if

$$w \neq 0$$

In VBS case, even when

$$w = 0 \text{ but } n_v \neq 0$$

Ansari, KD, PRL 132 226504 (2024)

Striking implication: Stability of the kagome RVB liquid

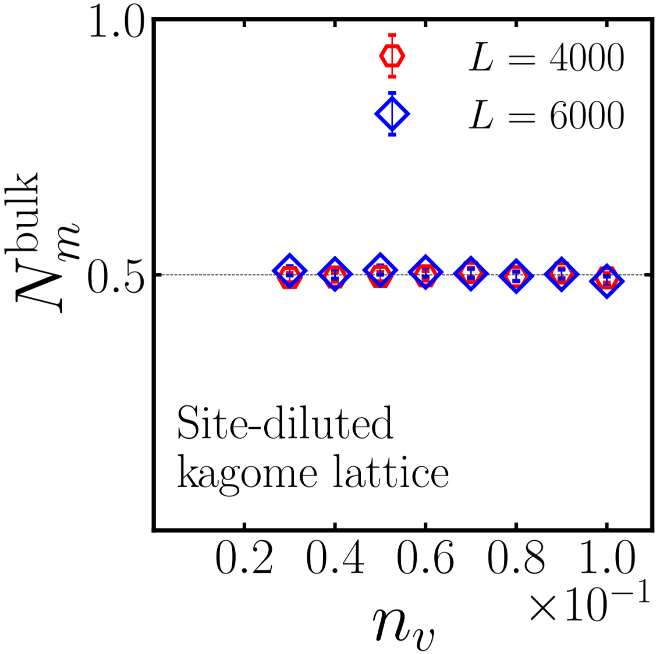
w=0 in the thermodynamic limit of the diluted kagome lattice with nonzero vacancy density

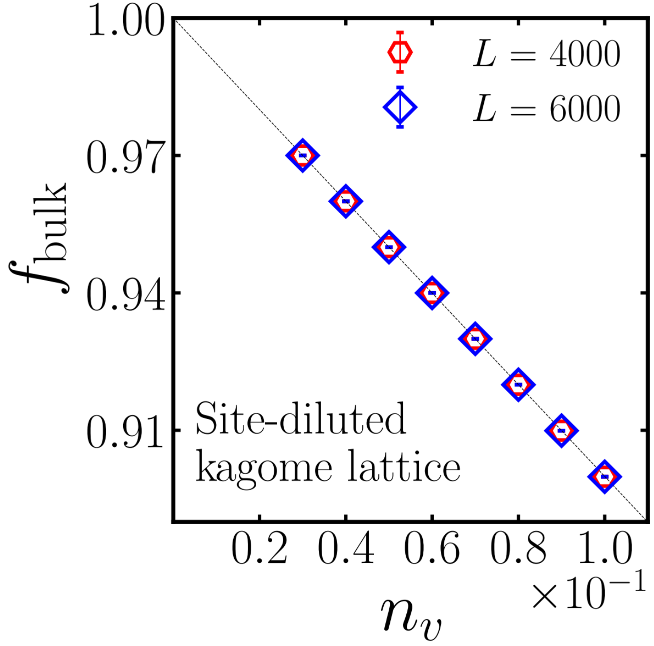
Short-range RVB state stable to vacancy disorder on kagome lattice (!)

Generally true on all claw-free lattices (pyrochlore lattice, star lattice etc)

Ansari, KD, PRL 132 226504 (2024)
Bhola, KD, arXiv:2512.23639

Basis for claim: Explicit check followed by proof





Any maximum matching has at most 1 monomer in each connected component of lattice(!)

Ansari, KD, PRL 132 226504 (2024)

proof: Bhola, KD, arXiv:2512.23639

Story so far:

VBS states always have vacancy-induced local moment instability (single-vacancy effect)

sRVB states have such an instability if maximum matchings have nonzero bulk monomer density.
(multi-vacancy effect)

Key implication: Kagome sRVB liquid is stable

When there's an instability:
Nature of the actual many-body ground state controlled by random geometry of monomer-carrying regions

Motivates study of this random geometry

key claims need computational test

Isolated vacancies do not seed local moments in sRVB states, but do so in VBS states.

Monomer-carrying regions of lattice correspond to local moments in both kinds of states

Primer: Computational tests

O(N) models on non-bipartite lattices, SU(N) models on bipartite lattices

Ideal unified test: $\chi^{\mathcal{A}}$ (runs into computational difficulties)

For SU(N) systems, equivalent to checking: $\chi^{\mathcal{Q}}$

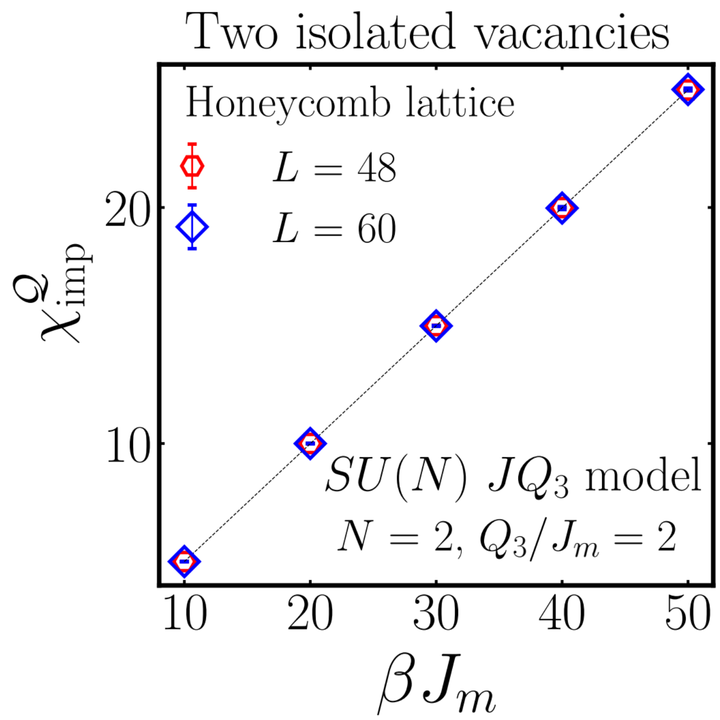
This is not defined for nonbipartite O(N) models

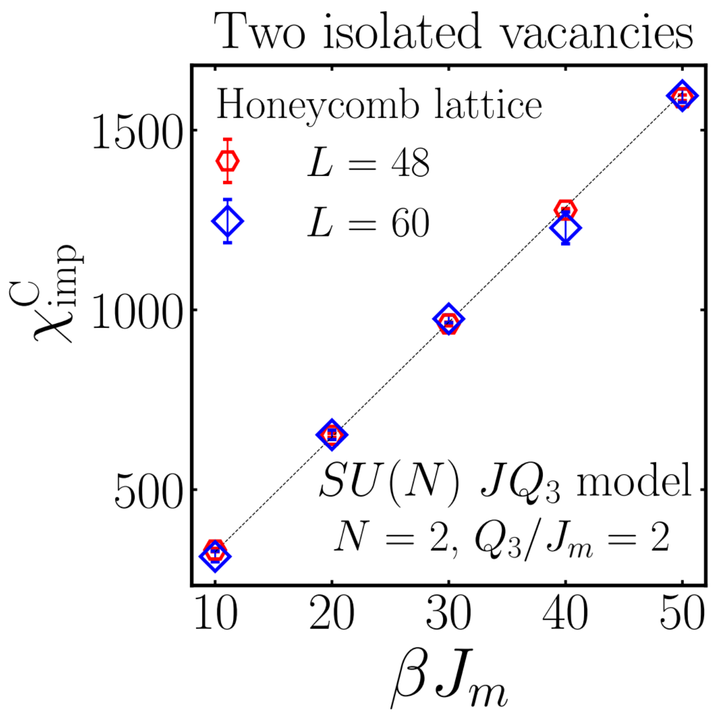
For O(N) systems, can instead check: χ^{C}

$$C^{\mathrm{tot}}_{\alpha\alpha} = \sum_r \mathcal{Q}_{\alpha\alpha}(r)$$

expected to be equivalent for $J_m \gg T \gg J_{\mathrm{eff}}$

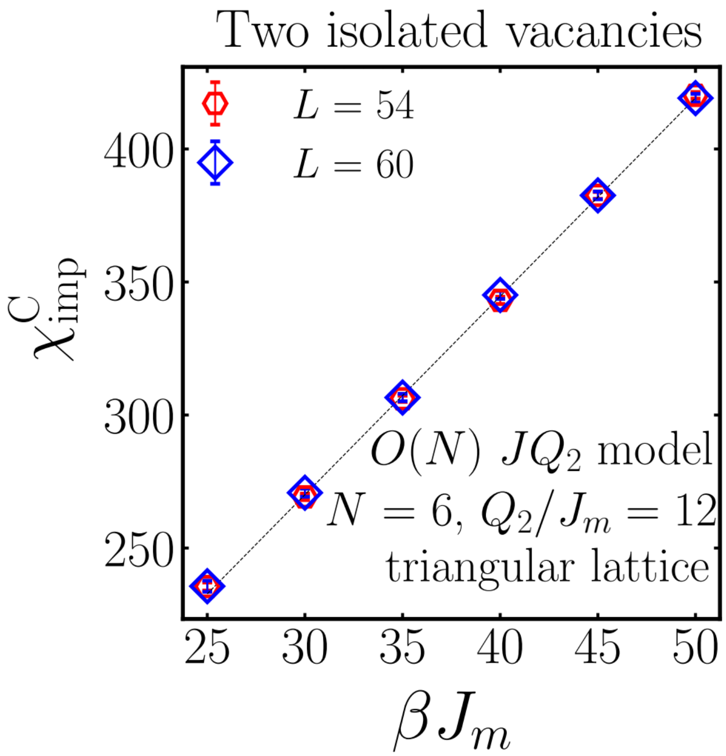
Isolated vacancies: VBS state (bipartite)





Ansari, Kundu, KD, in preparation

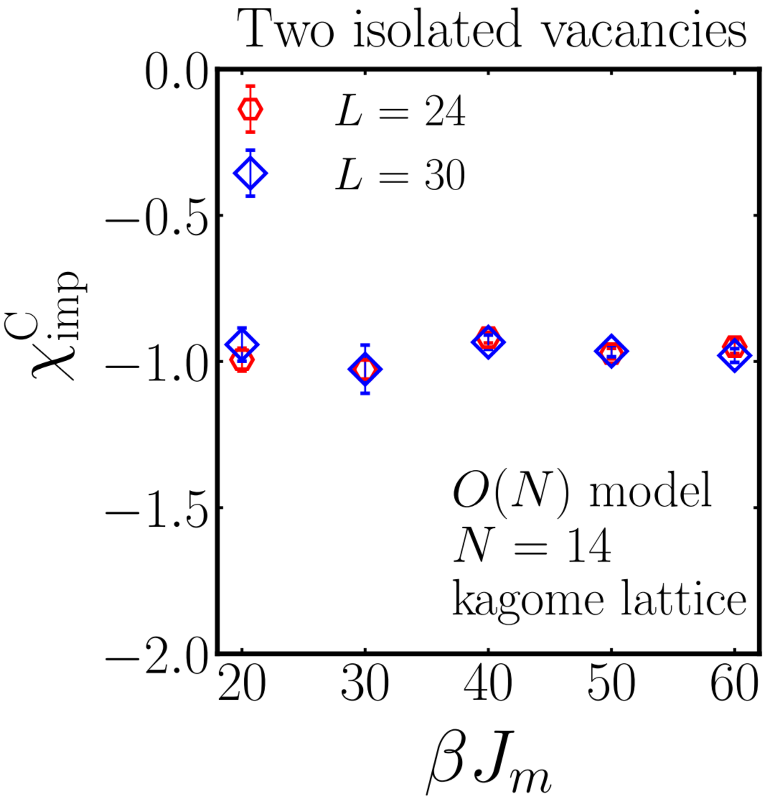
Isolated vacancies: VBS state (nonbipartite)



Ansari, KD, PRL 132 226504 (2024)

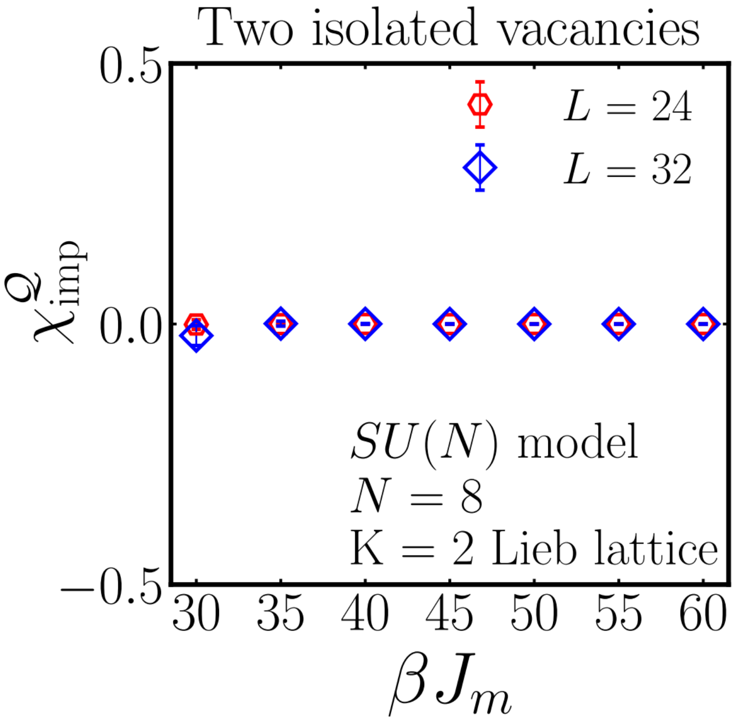
Isolated vacancies: kagome RVB state (non-bipartite)

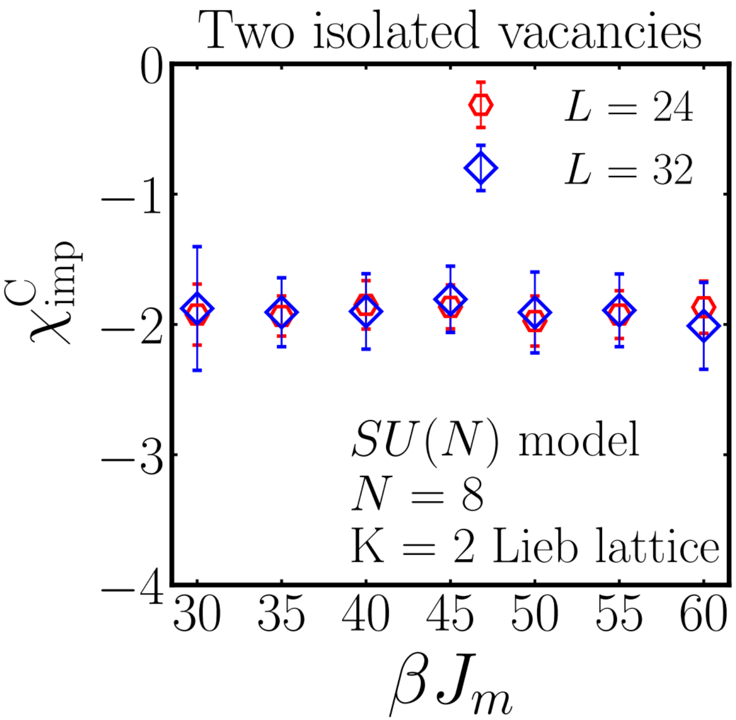
RVB state established in Block,D'Emidio, Kaul 2020



Ansari, KD, PRL 132 226504 (2024)

Isolated vacancies: sRVB regime (bipartite)





Ansari, Kundu, KD, in preparation

To summarize: Vacancy-induced local moments in sRVB liquids associated with monomers

Emergent local moments are a multi-vacancy effect, and confined to R-type regions of lattice

Dominant short-range interactions between these local moments also confined within R-type region

Geometry of $\mathcal{R}$-type regions expected to determine low-energy state and magnetic response

Very different from vacancy effects in VBS states:

Each vacancy individually nucleates a local moment bound to it

Ansari, KD, PRL 132 226504 (2024)

Focus on random geometry of maximum-density dimer packings

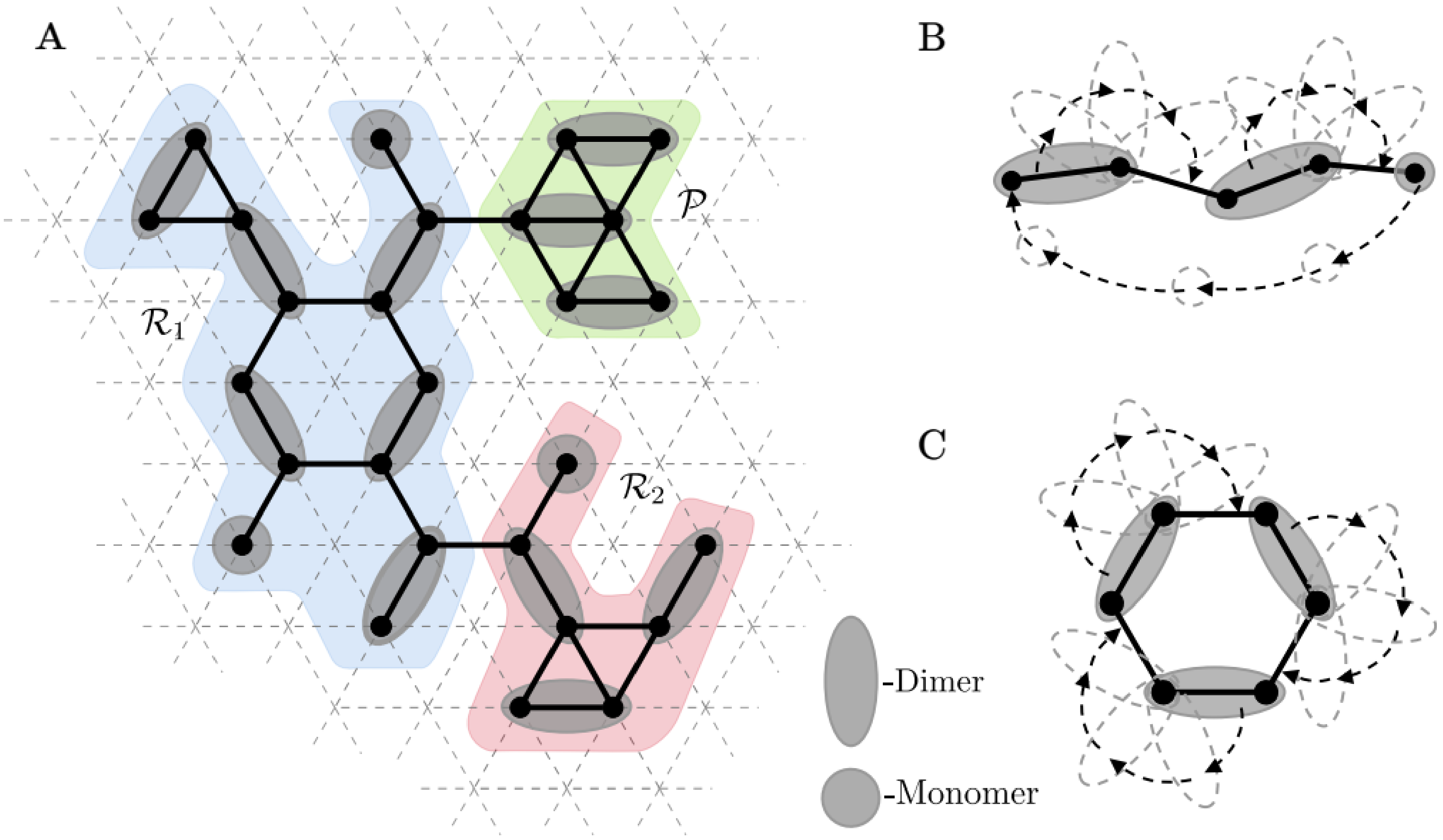
We ask: Where on the lattice do the monomers live? (in the ensemble of maximum matchings)

The answer should give us

partial information on vacancy-induced local moment instabilities

How does one implement this?

The setting: Maximum-density dimer packings of diluted lattices



Conclusions (from pictures):

Consequences of hard-core and maximum-density constraints:

Constrained kinematics: ring-exchange or monomer-hopping

Constraint on links of ring-exchange and monomer-hopping process paths:

Each such link must be occupied by a dimer in at least one such dimer packing

Constraint on monomer and dimer motion:

Monomers confined to well-defined regions of disordered lattice. Other regions fully-packed.

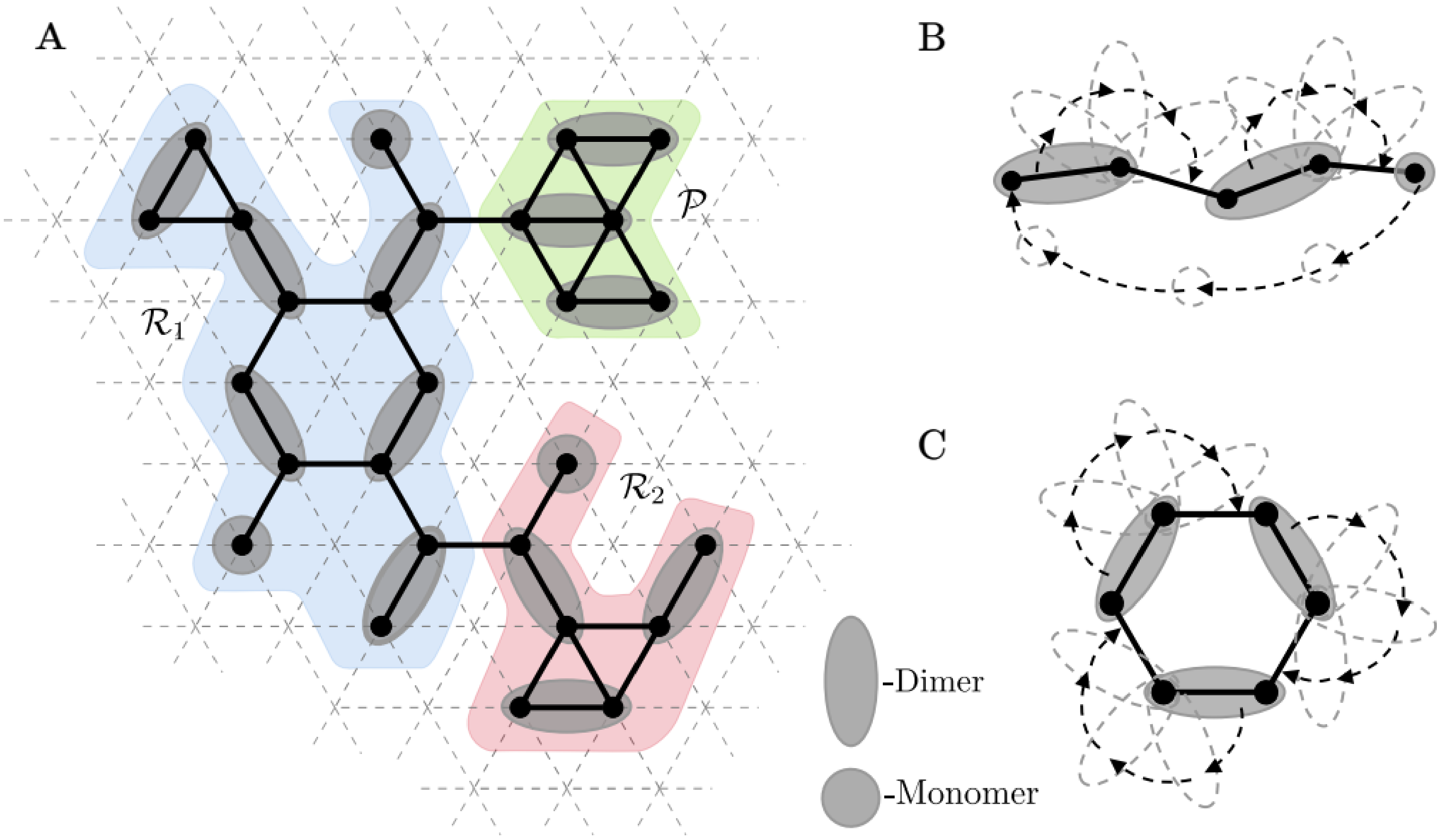
Defining monomer-carrying and perfectly-matched regions:

Boundaries of monomer-carrying $\mathcal{R}$-type, fully-packed $\mathcal{P}$-type regions:

Some "forbidden" links of disordered lattice can never be occupied by a dimer in any such packing

Boundaries of these regions demarcated by the "forbidden" links

Geometry of monomer-carrying and fully-packed regions



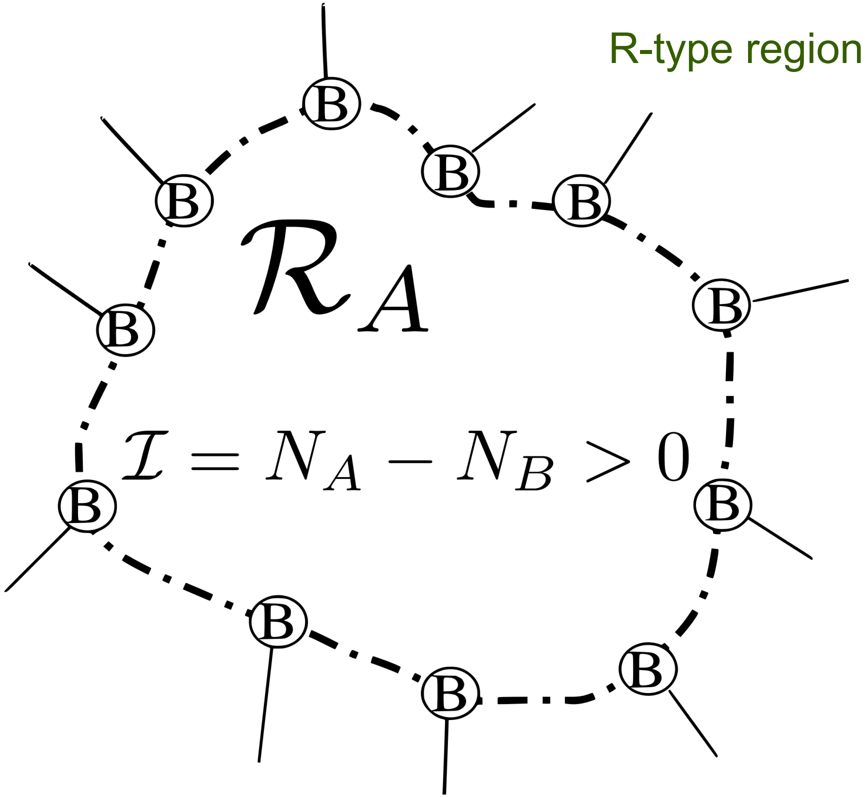
Identifying monomer-carrying and perfectly-matched regions:

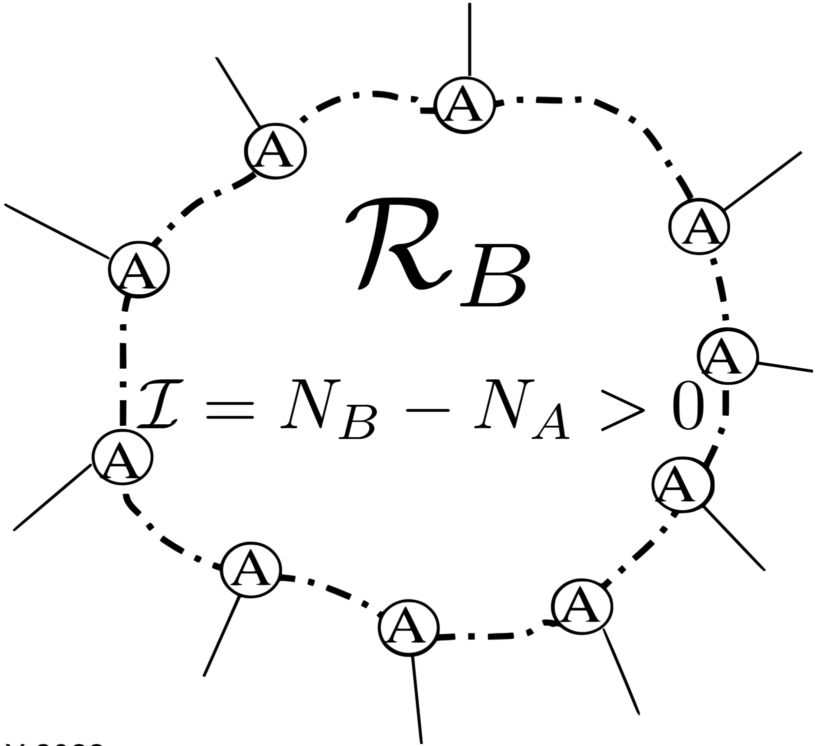
Can we identify the forbidden links in a systematic way given a disordered sample?

Some simple ad hoc answers possible:

Bipartite case: rare “R-type” regions with local sublattice imbalance

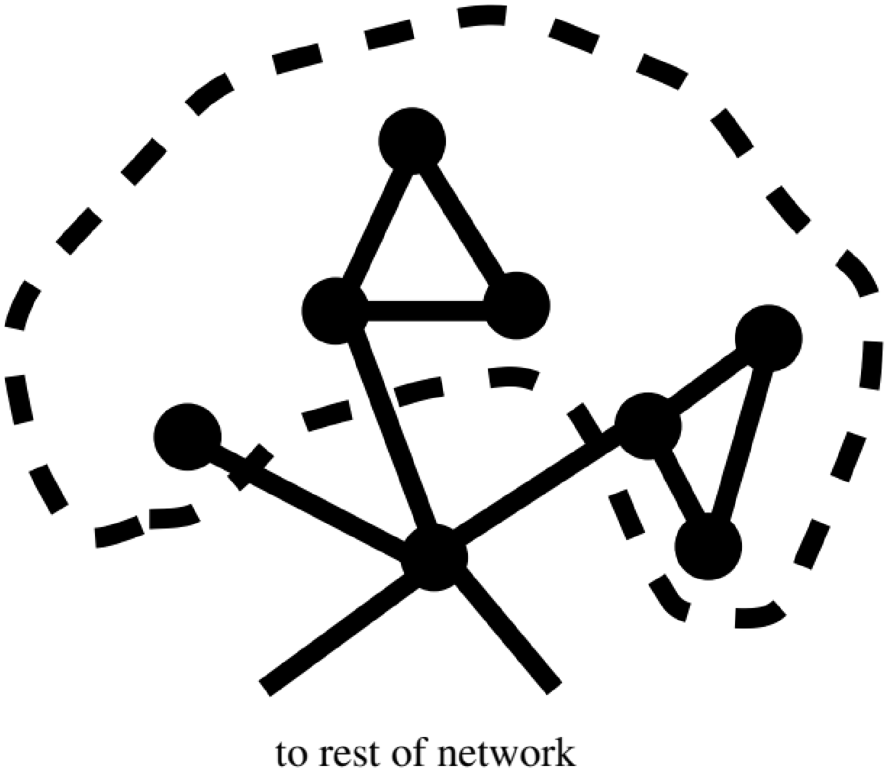
R-type regions each trap fixed number of monomers





Bhola, Biswas, Islam, KD, PRX 2022

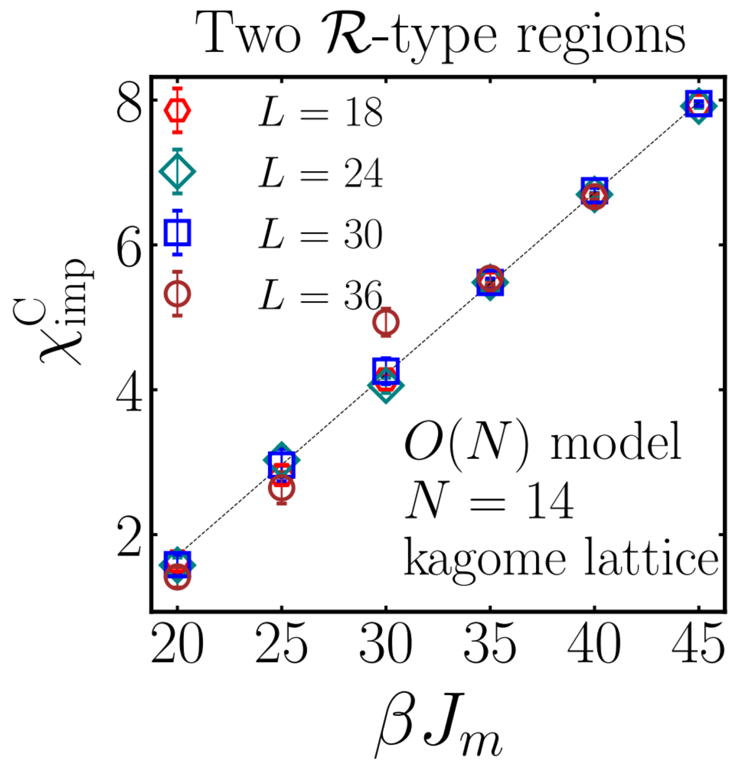
Non-bipartite case is trickier

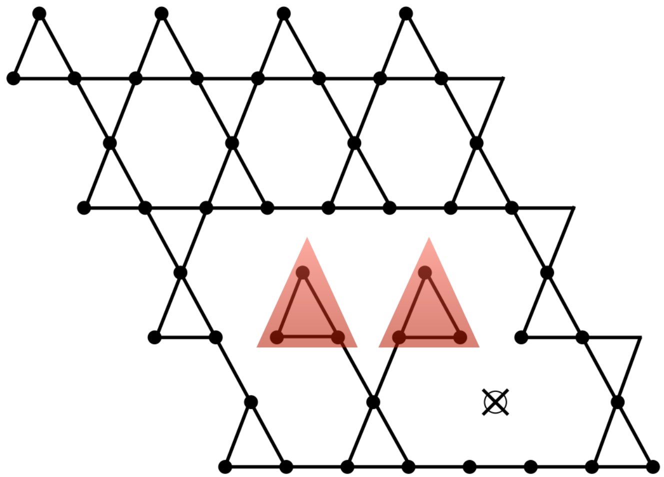


e.g: This R-type region traps two monomers

KD, PRB 2022

Two R-type regions in RVB state (non-bipartite)

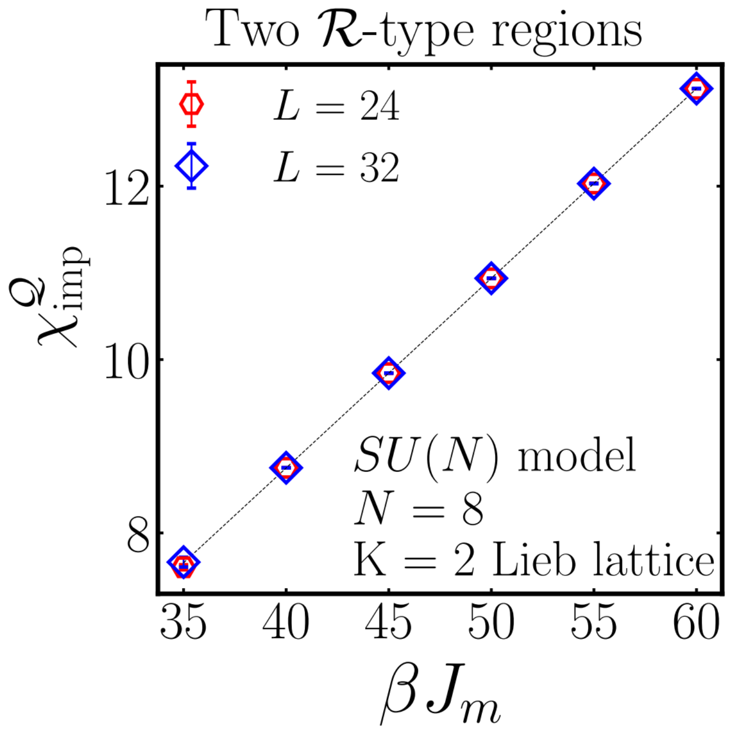


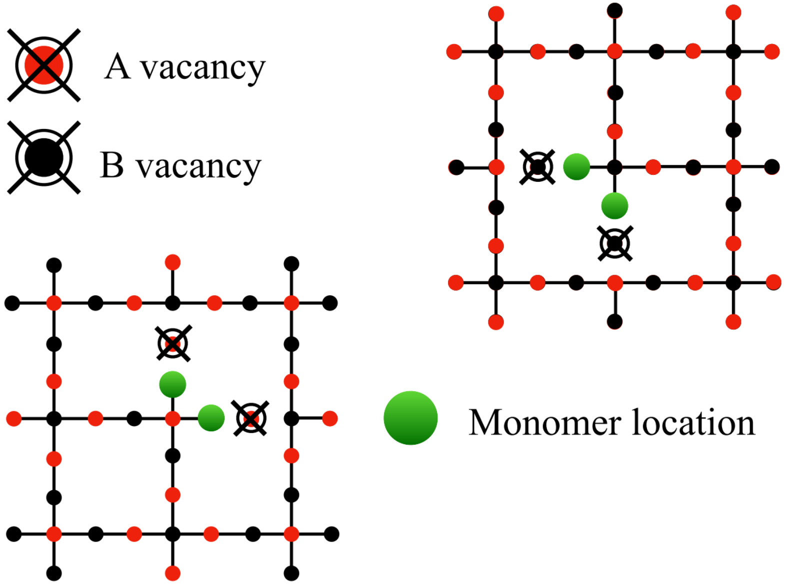


Note: deleted bonds, not sites

Ansari, KD, PRL 132 226504 (2024)

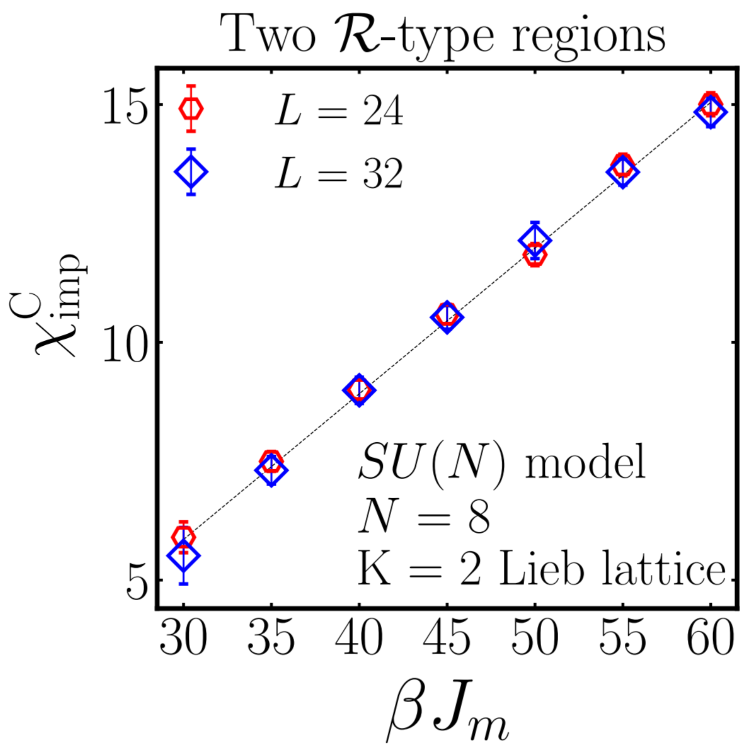
Two R-type regions Q-response: RVB state (bipartite)

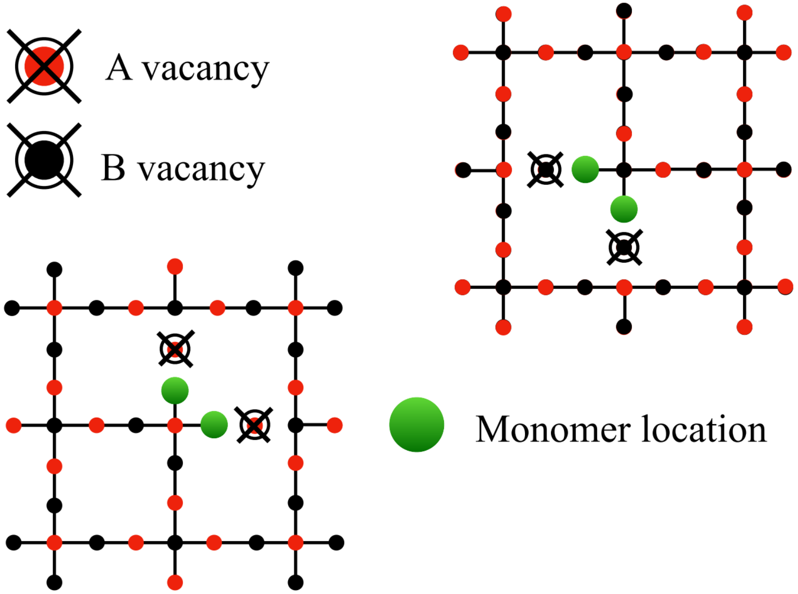




Ansari, Kundu, KD, in preparation

Back to tests: Two R-type regions C-response: RVB state (bipartite)



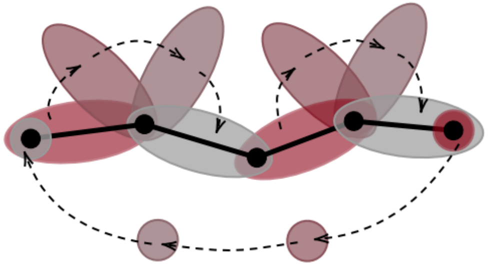


Ansari, Kundu, KD, in preparation

Systematize this?

What's the general procedure???

Math to the rescue: Gallai-Edmonds & Dulmage-Mendelsohn theory



Definitions:

Pick favorite maximum-density dimer packing

Explore forest of alternating paths starting from all monomers

Label vertices e (even) if they can be reached along an even-length path of this forest

Label vertices u (unreachable) if they cannot be reached along any paths of this forest

Label vertices o (odd) otherwise (i.e. can be reached by odd-length path but not even-length path)

Theorem of Gallai-Edmonds (general case) & Dulmage-Mendelsohn (bipartite case):

Labeling is property of disordered lattice, not of your favorite maximum-density dimer packing.

COVERINGS OF BIPARTITE GRAPHS

A. L. DULMAGE AND N. S. MENDELSOHN

Dulmage & Mendelsohn, Canadian J. Math 1958

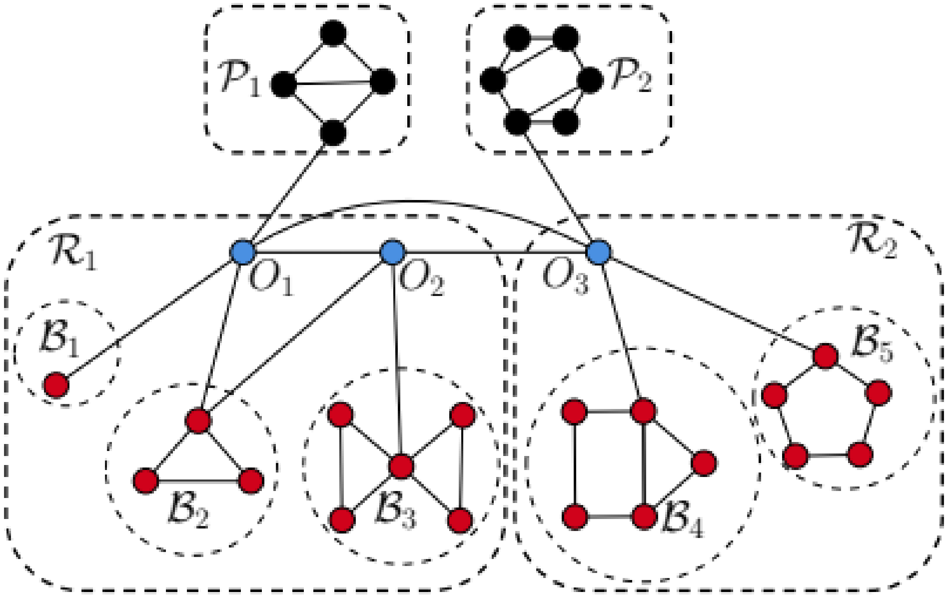
T. Gallai 1963,'64

PATHS, TREES, AND FLOWERS

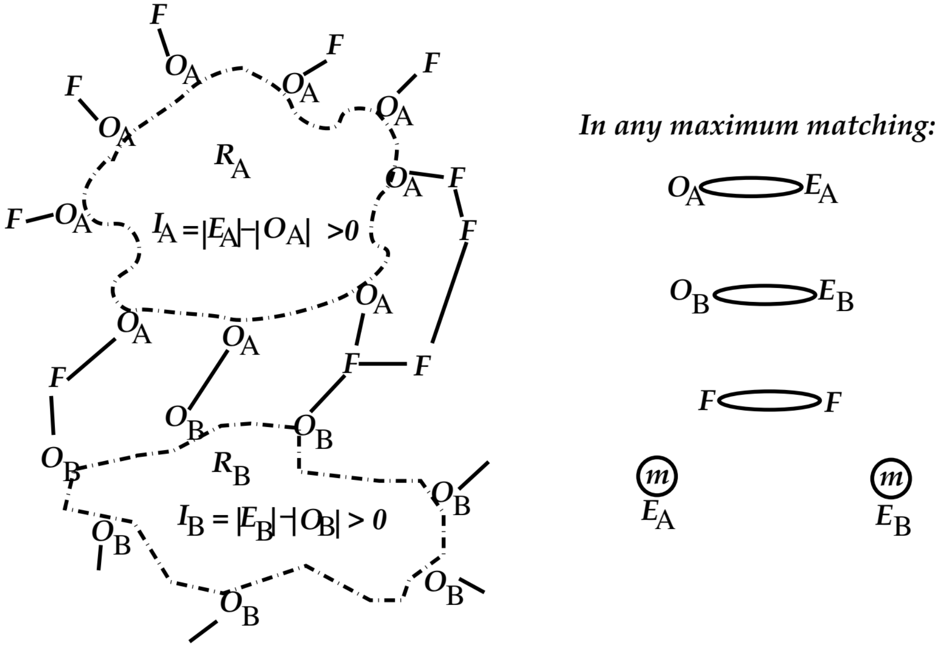
JACK EDMONDS

J. Edmonds, 1965

Our answer



KD, PRB 105 235118 (2022)



Bhola, Biswas, Islam, KD, PRX 2022

Key observation: o – o and o – u links are the "forbidden" links. Delete!

Gives us an alternate local proof of Longuet-Higgins & Lovasz results.

Number of monomers in an R-type region = Number of zero modes localized in same region.
(adding contributions from all regions gives older global statements of Longuet-Higgins and Lovasz)

Striking theorem on kagome lattice

$$n_{\mathrm{monomer}} = 0$$

in (infinite connected cluster of) the diluted kagome lattice with nonzero vacancy density

Short-range RVB state has no vacancy-induced local moment instability on kagome lattice (!)

Theorem generally true on all “claw-free lattices” (pyrochlore lattice, star lattice etc)

Ansari, KD, PRL 132 226504 (2024)
Bhola, KD, arXiv:2512.23639 (2025)

More generally: tractable computations(!)

Can obtain complete set of R-type and P-type regions from one maximum matching of diluted lattice

via BFS for augmenting alternating paths (Blossoms in non-bipartite case)

Opens door to detailed computational study of random geometry of $\mathcal{R}$-type and $\mathcal{P}$-type regions

(..and thence, (hopefully) to deductions about the physics...)

Bhola, KD, arXiv:2311.05634v2 (2025)

KD, PRB 105 235118 (2022)

Bhola, Biswas, Islam, KD, PRX 2022

Enter: Percolation...

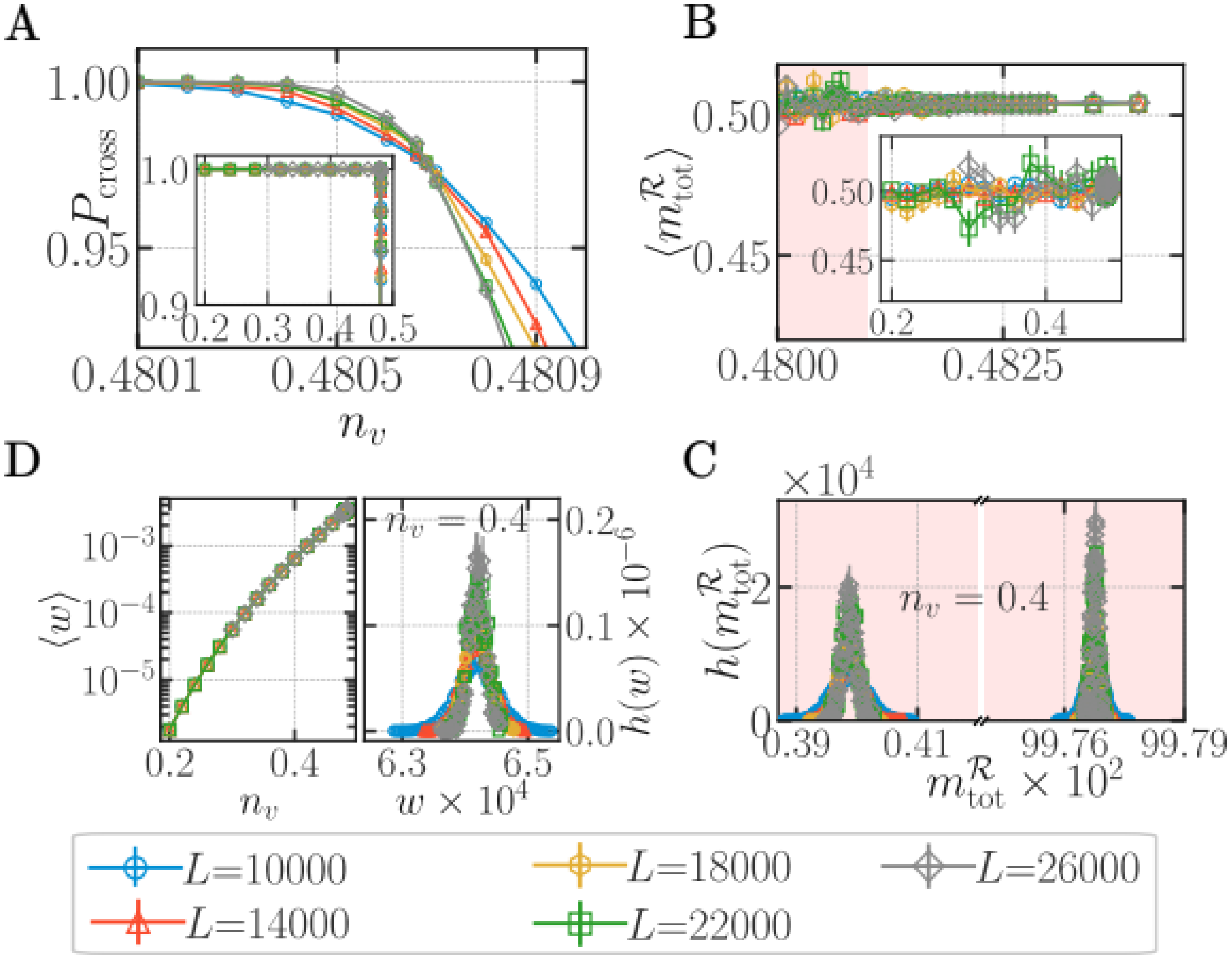
Typical regions are large at low dilution: Think in terms of percolation

The “right” yes/no question to ask: Can one walk from one end of a sample, staying within a single region?

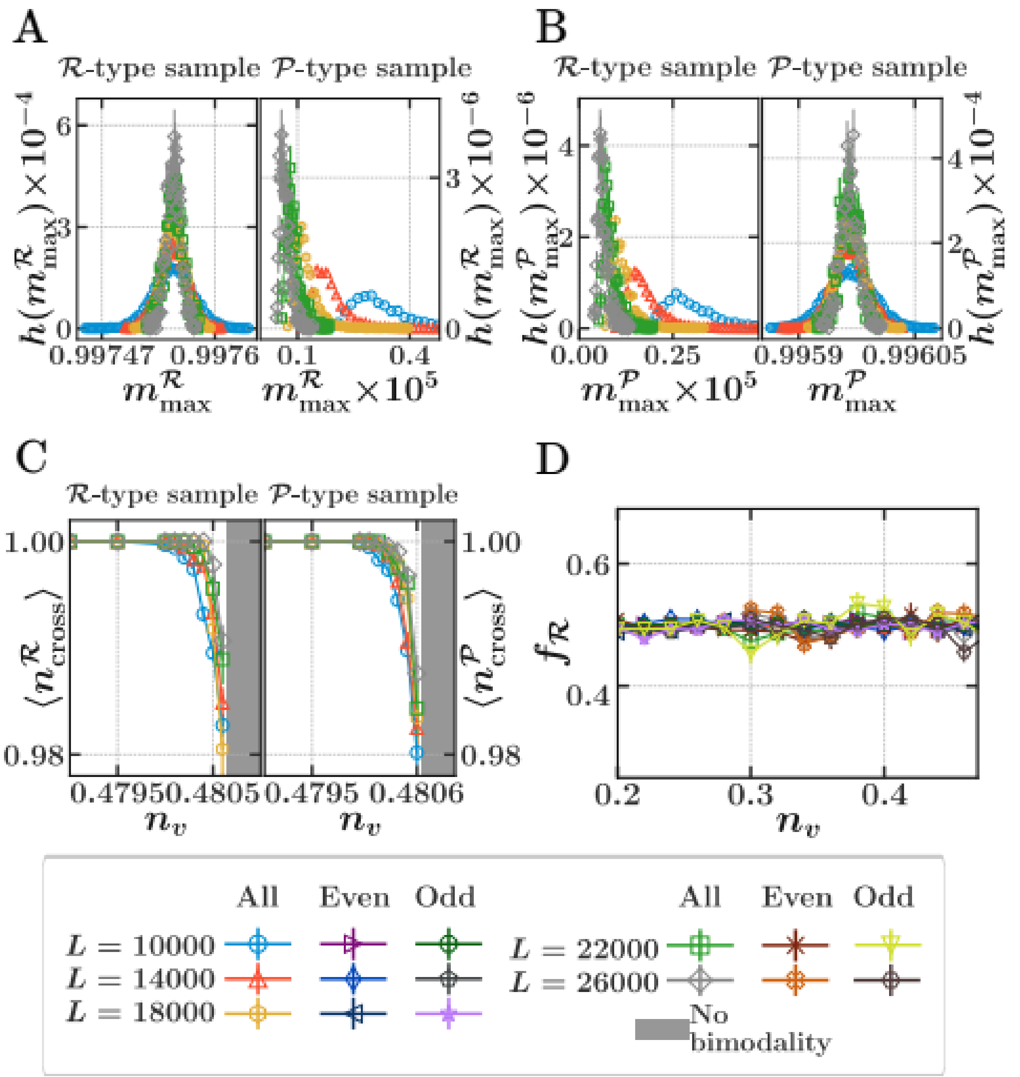
Nonbipartite case

Bhola, KD, arXiv:2311.05634v2 (2025)

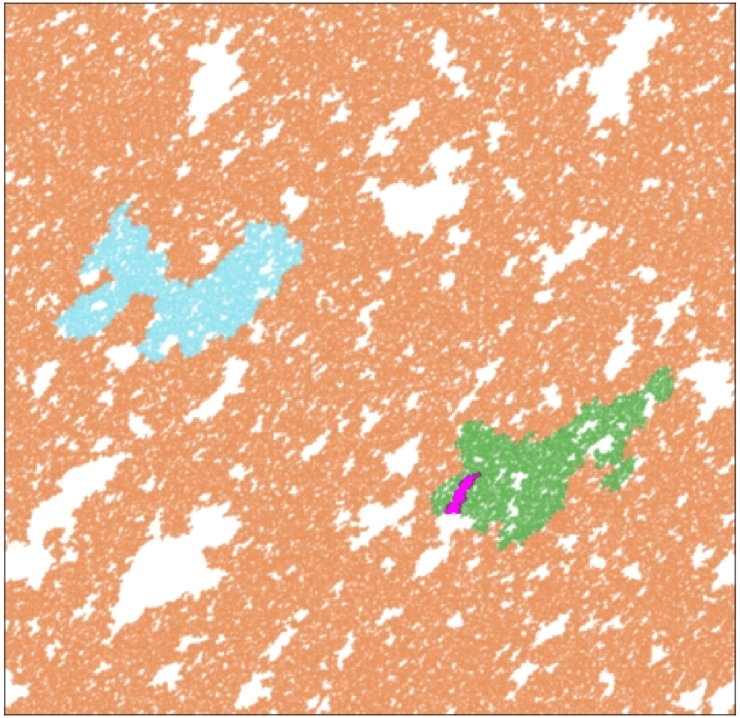
On the diluted triangular lattice



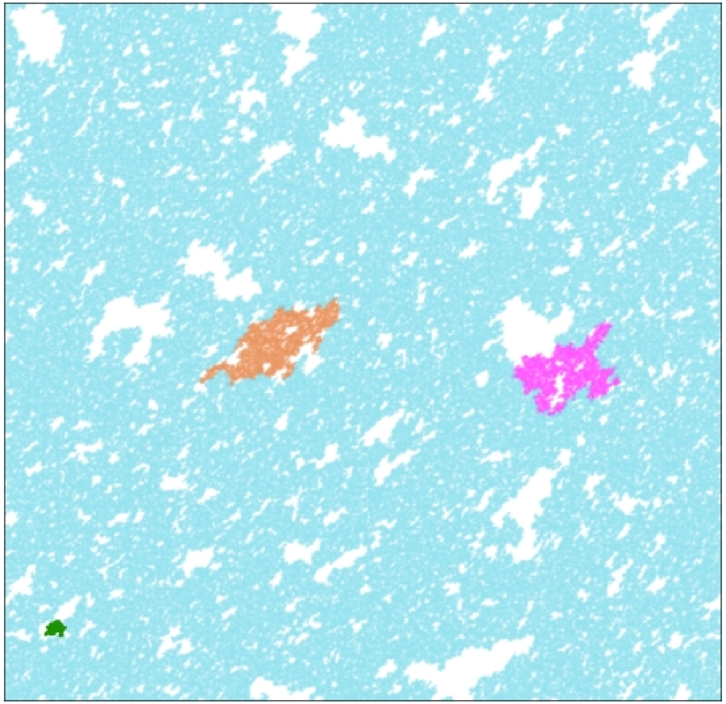
On the diluted triangular lattice



Pictorially on the diluted triangular lattice



R-type sample



P-type sample

On the diluted triangular lattice

$$f_{\mathcal{R}} = 1/2$$

Why???

Does this suggest some emergent symmetry between monomer-carrying and fully-packed regions

Again: Parity of largest geometric cluster plays no role!

On the diluted triangular lattice

Violation of even the weak form of “central dogma” at low vacancy concentration:

Monomers delocalized in half the samples, localized to O(1) regions in the other half!

All samples identically prepared, randomly diluted, with the exact same density of vacancies

On the diluted triangular lattice

Suggests extreme sensitivity of large-scale geometry to micro-scale details of disorder configuration

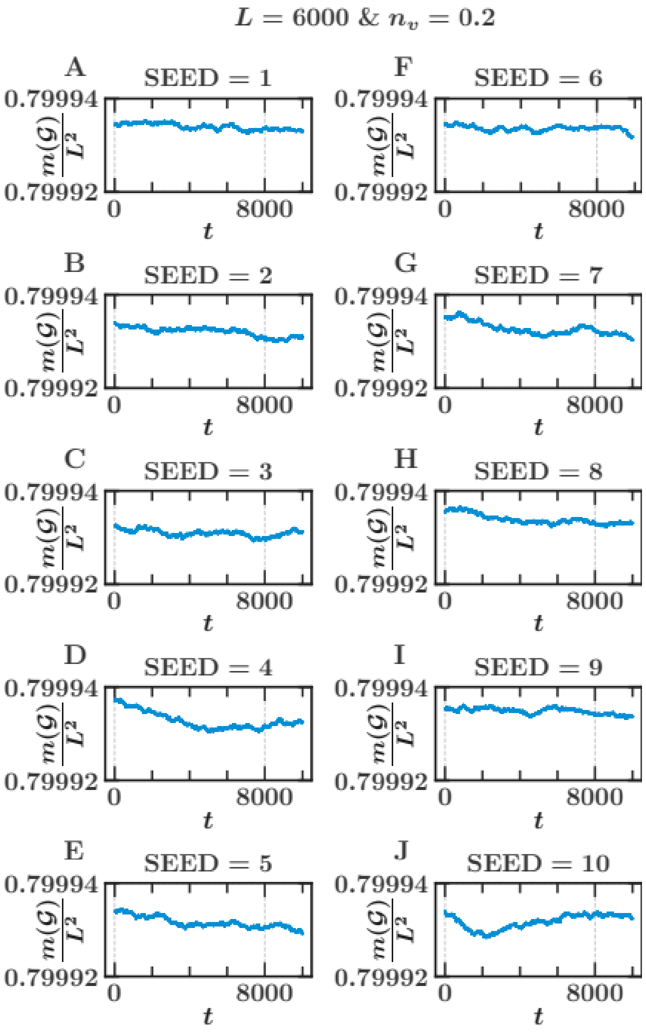
Can we quantify this?

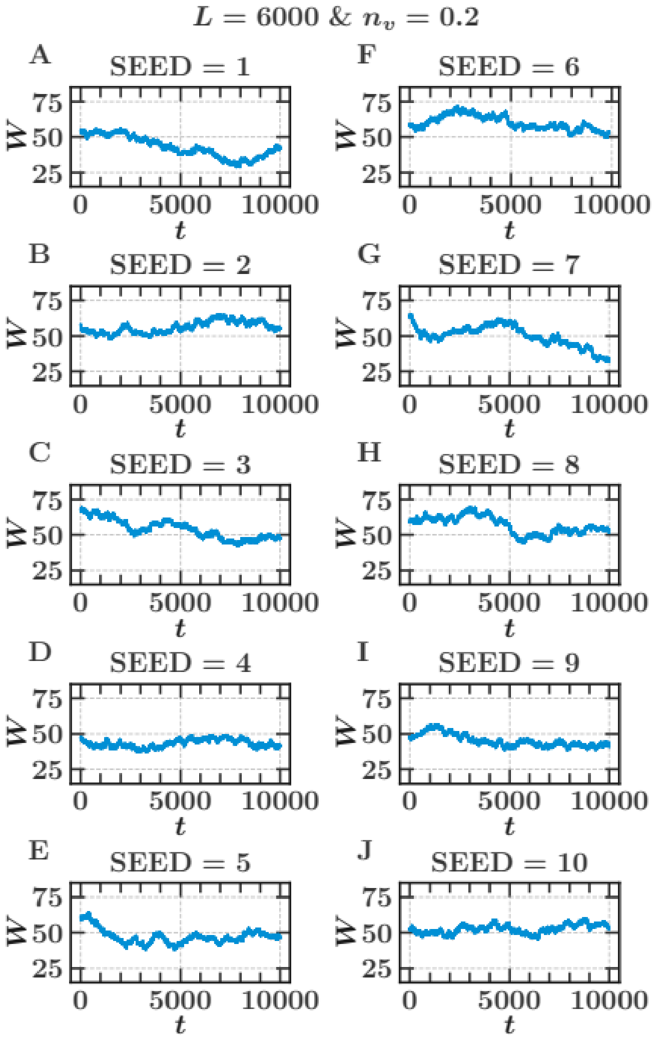
Model dynamics: Set vacancies in motion and watch what happens!

Small fraction of vacancies exchange position with neighboring surviving site at each time step

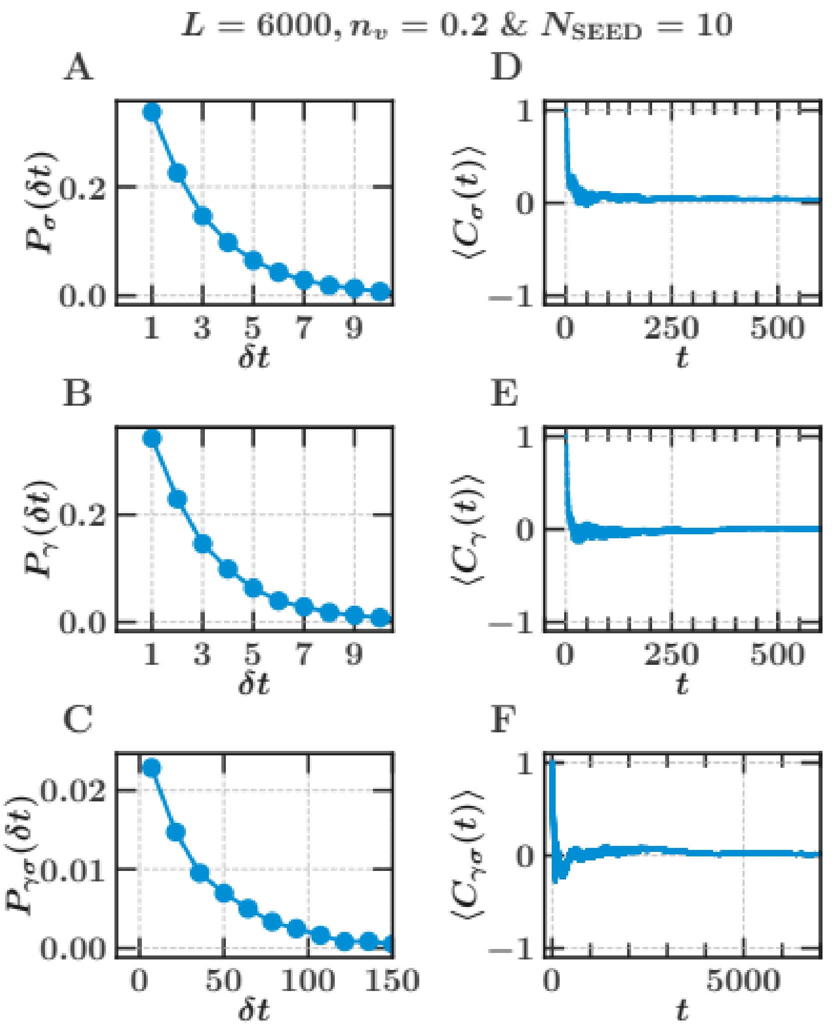
How does the large-scale geometry of these regions react?

Dynamics doesn’t disturb underlying lattice much





Yet: Large-scale geometry of monomer-carrying/fully-packed regions responds chaotically



And thus, consequences for magnetism

Effects of weak vacancy disorder (nonmagnetic impurities) in short-range RVB spin liquids on triangular lattice

At a minimum: Strong violations of thermodynamic self-averaging in the susceptibility

Likely: "R-type samples" have spin-glass order but not "P-type" samples

Also: Chaotic (deterministic but unpredictable) susceptibility response to changes in disorder configuration.

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