Monomer percolation

Implications for topologically protected Majorana modes & local moments

Kedar Damle, TIFR Mumbai

Ansari, KD, arXiv:xxxx:xxxxx

Bhola, KD, arXiv:2311.05634

KD, PRB 105 235118 (2022)

Bhola, Biswas, Islam, KD, PRX 12, 021058 (2022)

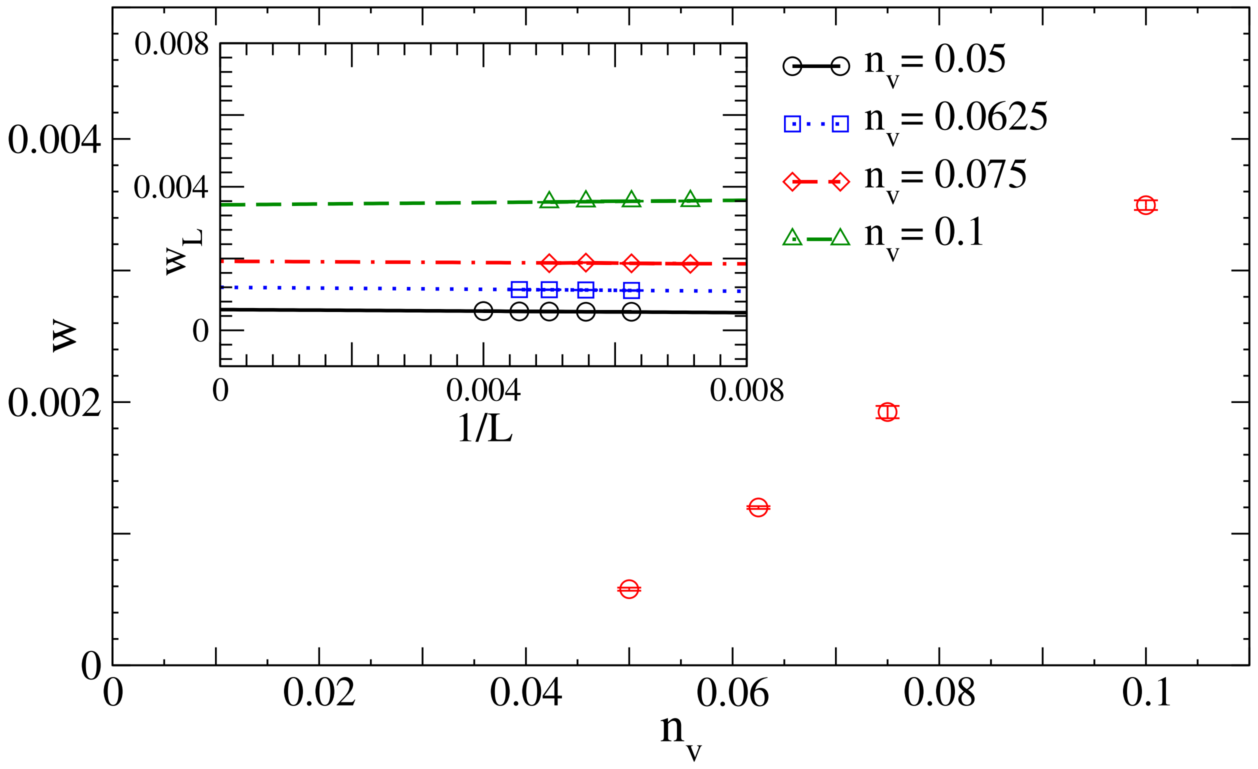
Motivation: Sanyal, KD, Motrunich, PRL 117 116806 (2016)





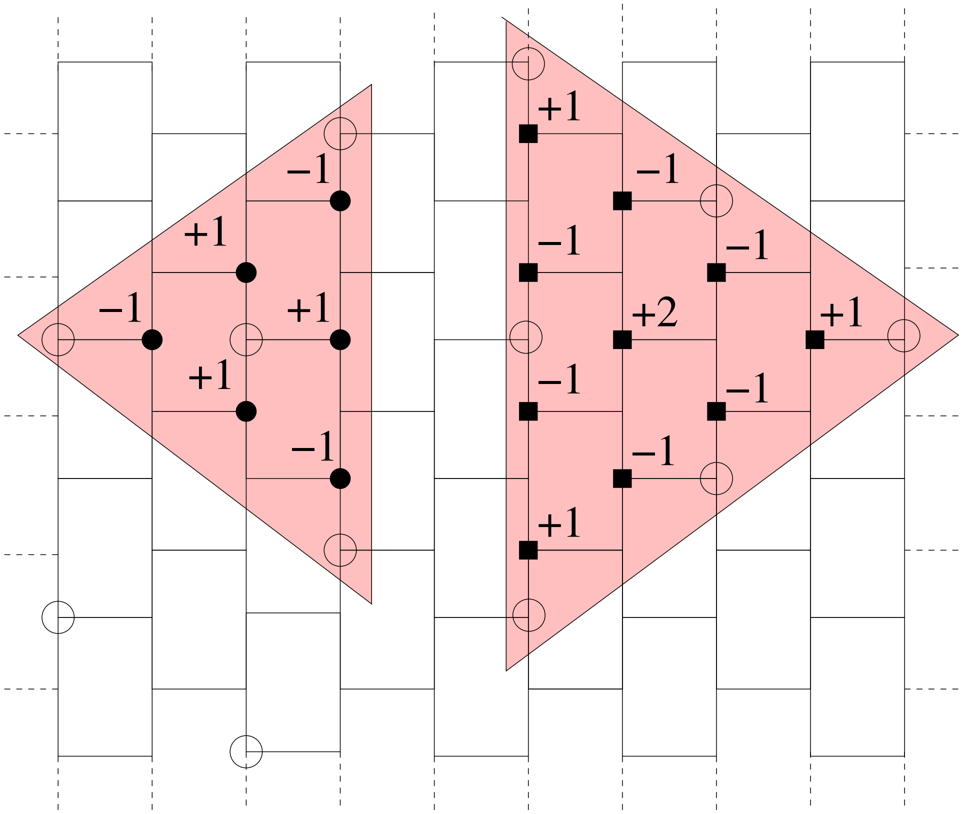

Generous funding: DAE, SERB, Infosys-Chandrasekharan Random Geometry Center

Graphene with vacancies: “surprising” E=0 states



Sanyal, KD, Motrunich PRL 2016

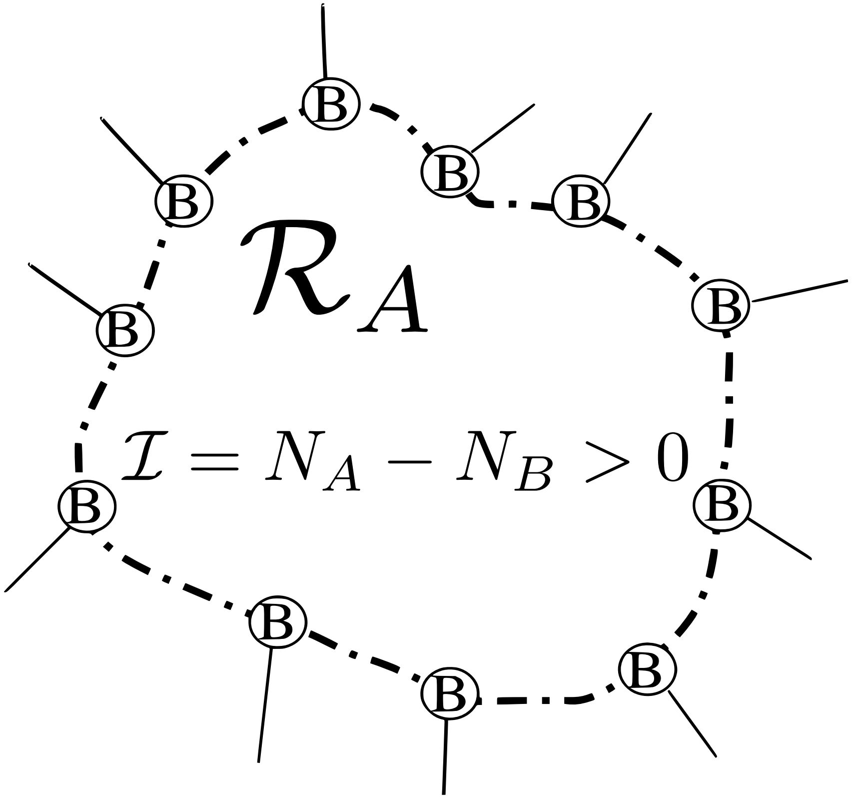
Naive guesses and an important distinction

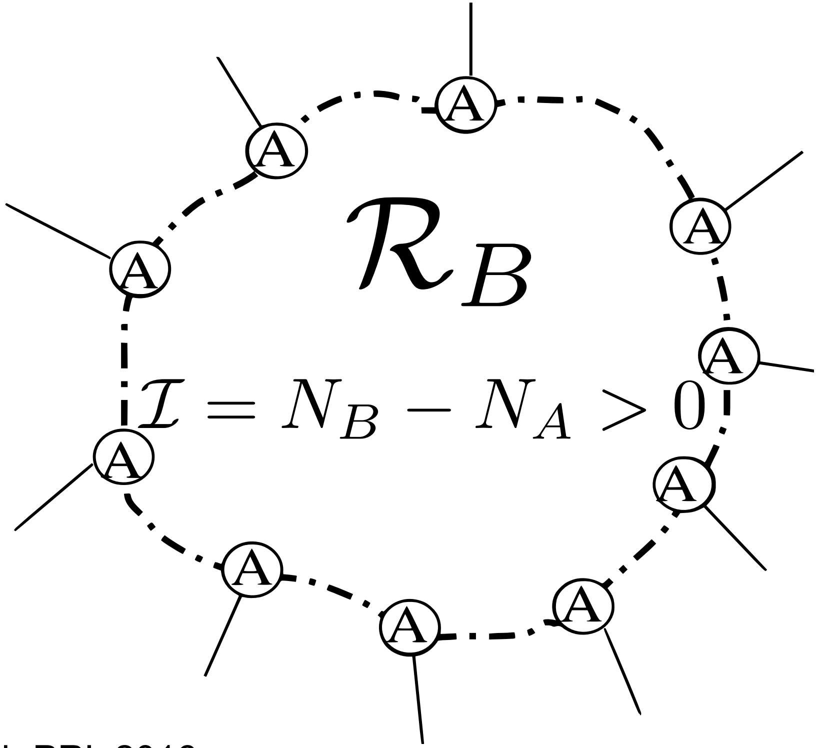


Sanyal, KD, Motrunich PRL 2016

Key distinction: R6 topologically protected

Topologically protected zero modes: A mechanism





Sanyal, KD, Motrunich PRL 2016

Key unanswered question

Hand-drawn examples provide lower bounds on DOS at E=0.

But computed value much greater.

What actually determines density of zero modes??

Local sublattice imbalance and dimers: A first clue

Disorder-robust zero modes only depend on connectivity, not hopping strengths.

R-type regions rely on local imbalance between A and B type site densities.

Suggests thinking in terms of *matchings* a.k.a *dimer covers*

Regions of lattice that cannot be covered perfectly by dimers host wavefunctions

Confirmed by: Longuet-Higgins on zero modes

AIP | The Journal of Chemical Physics

Some Studies in Molecular Orbital Theory I. Resonance Structures and Molecular Orbitals in Unsaturated Hydrocarbons

1950

H. C. Longuet-Higgins

E=0 molecular orbitals correspond to magnetic moments in MO theory of benzenoid molecules

Effectively studying a tight-binding model and asking about E=0 states.

Result: (transcribed to our language)

Number of monomers in maximum matching = number of topolgically protected zero modes

Language primer: Dimers and Matchings

Dimer model in statistical mechanics: Match each site to an adjacent site monogamously

In graph theory/computer science: The matching problem

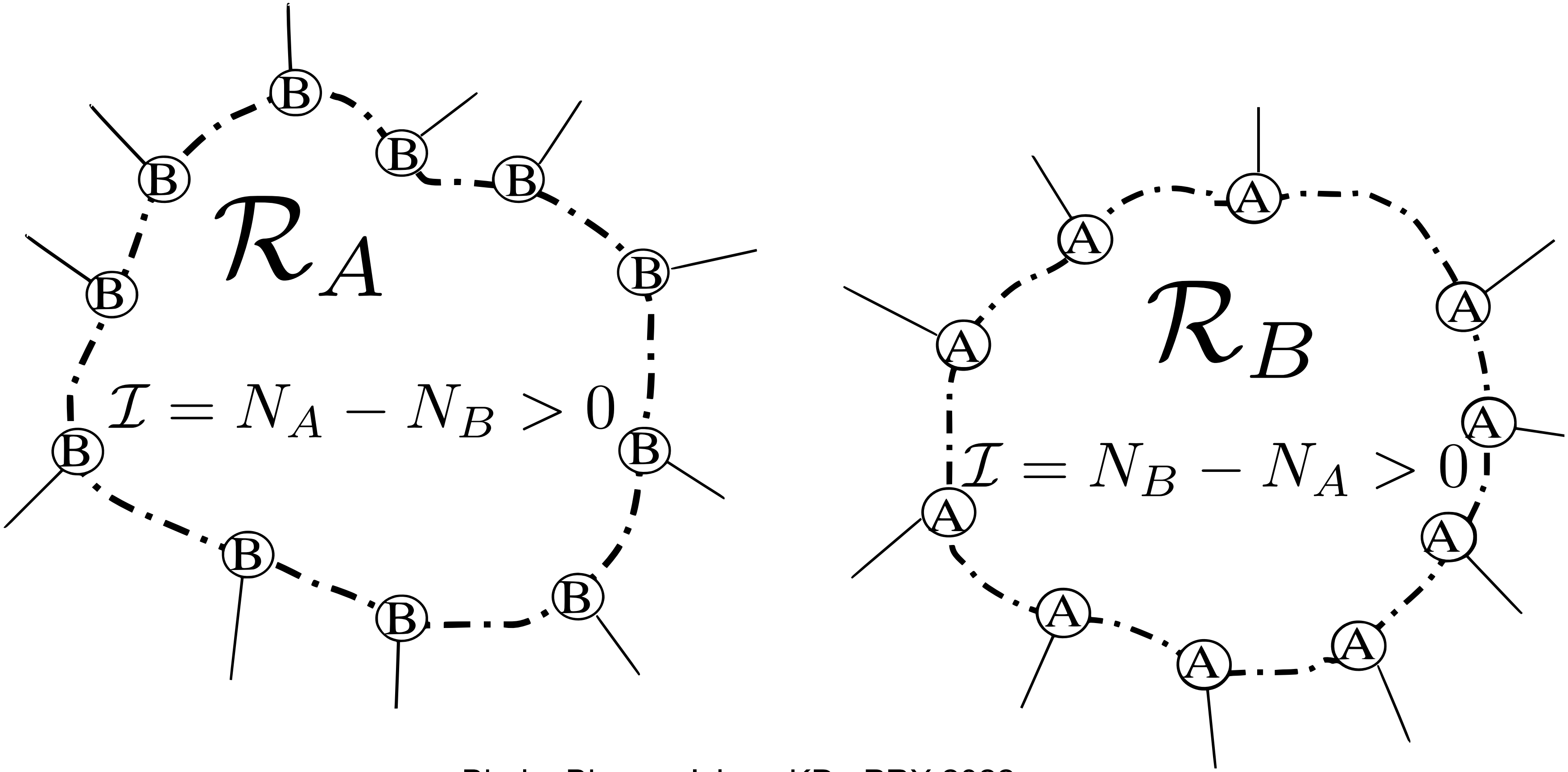
Question: Can a lattice with even number of vertices be perfectly matched?

Note: if bipartite, need $|A| = |B|$

Sometimes not possible: Then have *maximum matching* but not *perfect matching*

Maximum matchings have unmatched sites that host *monomers*

Key observation: R-type regions trap monomers



Bhola, Biswas, Islam, KD, PRX 2022

So “where” do the modes “live”?

How does one find a complete set of R-type regions?

What does this question even mean in algebraic terms??

A useful maximally-localized basis for topologically protected part of zero mode subspace

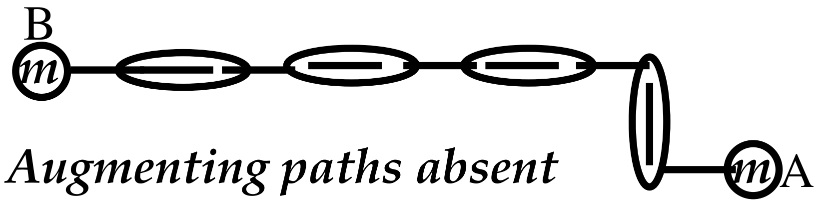
Structure theory of Dulmage-Mendelsohn

COVERINGS OF BIPARTITE GRAPHS

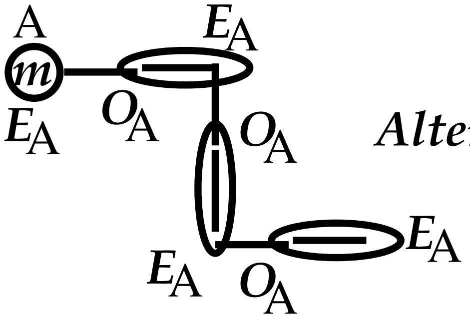
A. L. DULMAGE AND N. S. MENDELSOHN

Canadian J. Math 1958

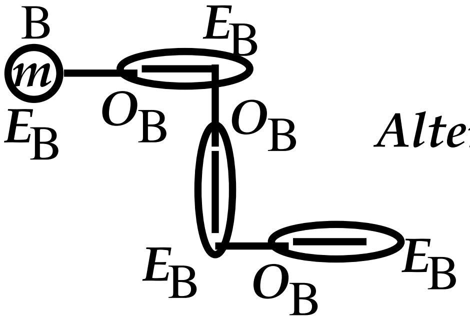
In any maximum matching M:



Augmenting paths absent

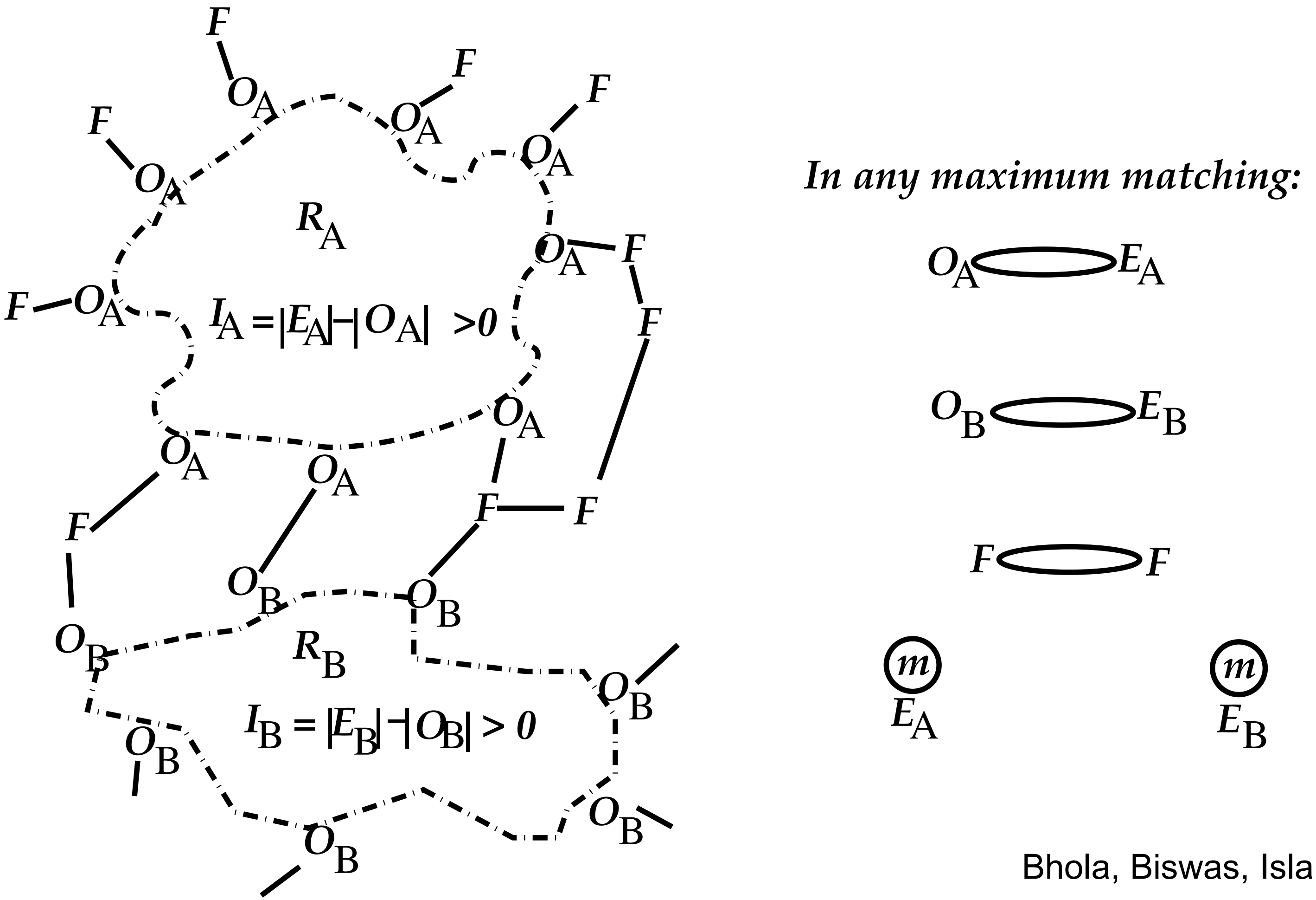


Alternating path



Alternating path

Maximally-localized basis for topologically protected zero modes



Bhola, Biswas, Islam, KD, PRX 2022

Switch gears: Topologically protected collective Majorana modes of Majorana networks?

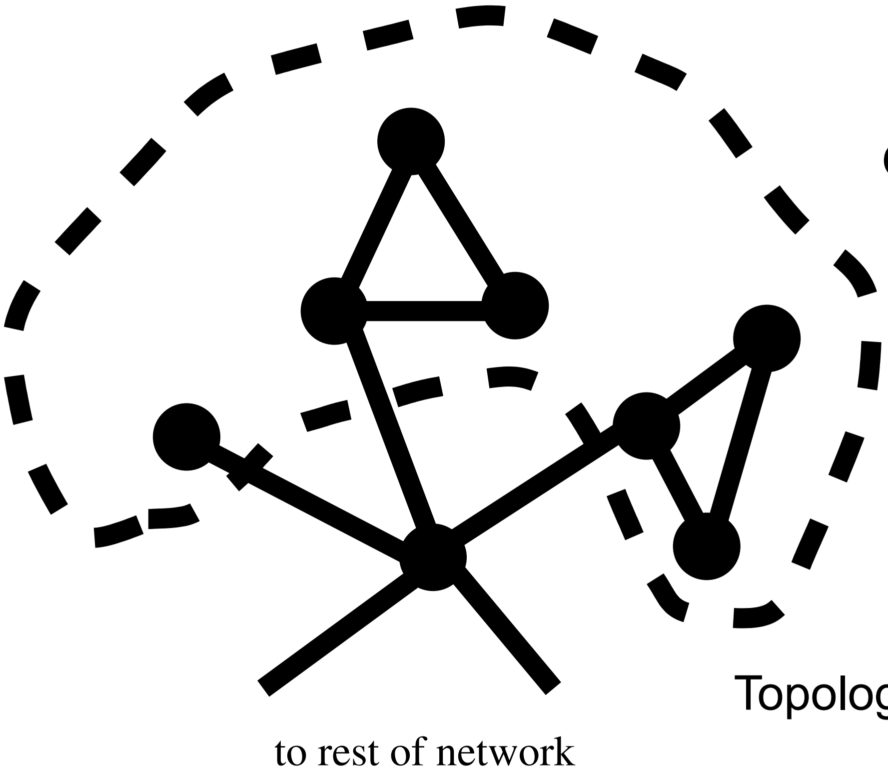
$$\mathcal{H}_{\text{network}} = \frac{i}{4} \sum_{rr'} \mathcal{A}_{rr'} \eta_r \eta_{r'}$$

$\mathcal{A}_{rr'}$ Antisymmetric matrix of quantum mechanical mixing amplitudes

η_r Majorana operators corresponding to localized Majorana modes (e.g. cores of p+ip vortices)

Note: Bipartite random hopping special case of this

Basic picture for collective Majorana modes



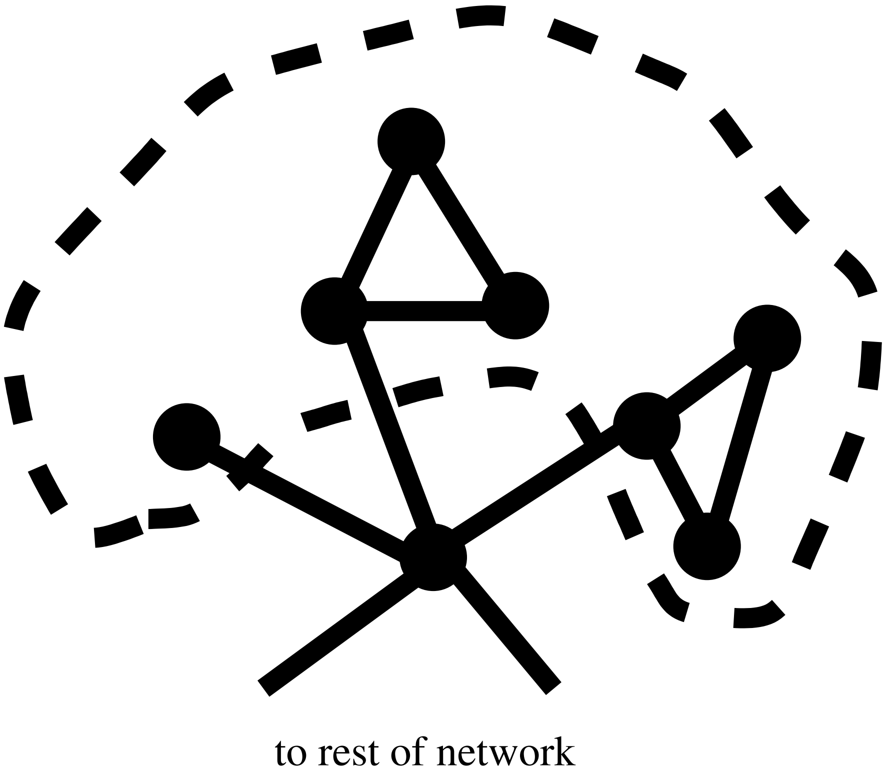
Odd cyles in isolation guaranteed to have zero mode.

Many such isolated modes mix inside “R-type” region

Topologically protected collective Majorana modes survive

KD, PRB 2022

Key observation: Such motifs trap monomers



This region also traps two monomers

KD, PRB 2022

Gels with theorem of Lovasz

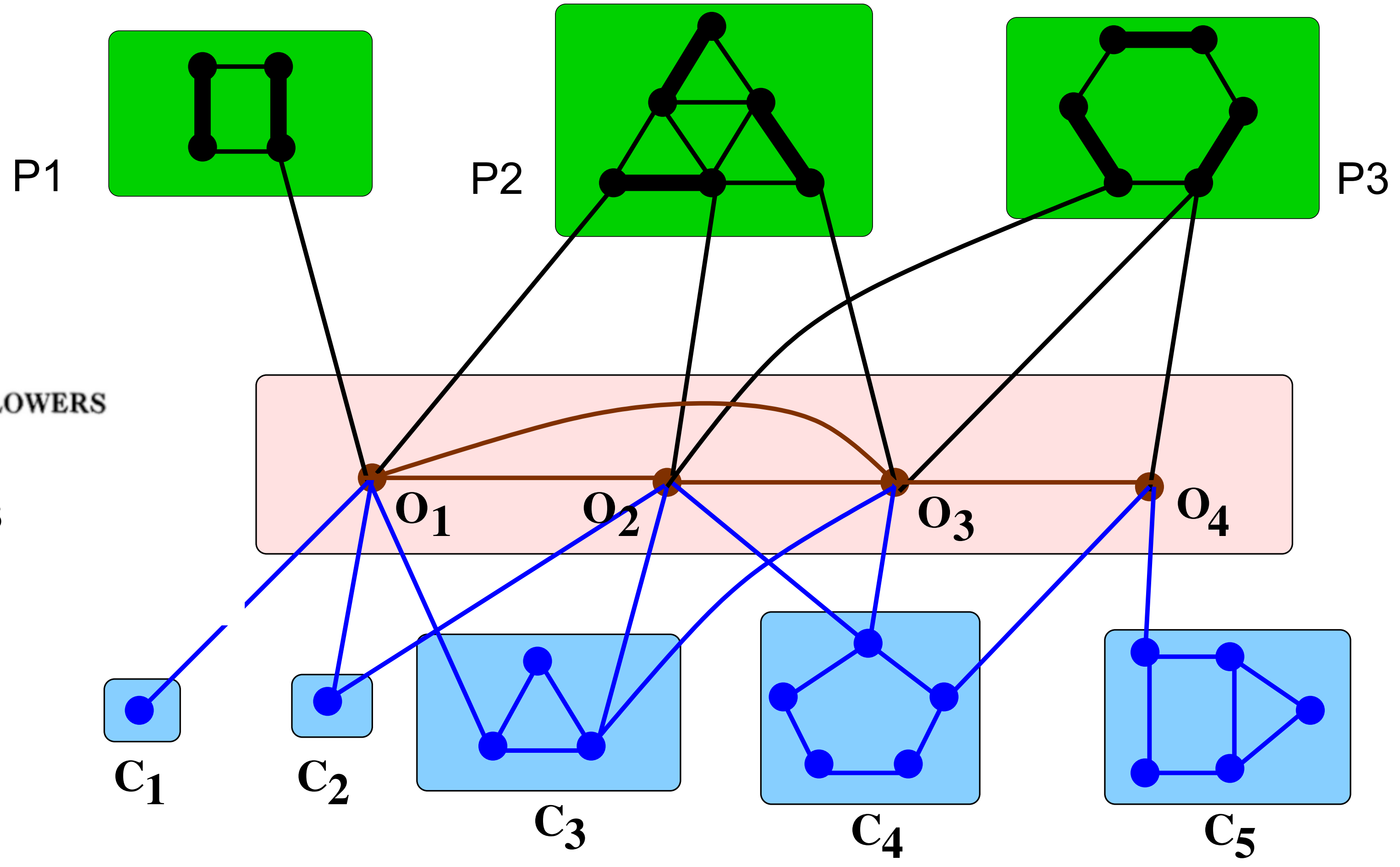
ON DETERMINANTS, MATCHINGS, AND RANDOM ALGORITHMS

by L. Lovász*

Fund. Comp. Th. 1979

Monomer number = number of topologically protected zero modes of $\mathcal{A}_{rr'}$

Structure theory of Gallai & Edmonds



PATHS, TREES, AND FLOWERS

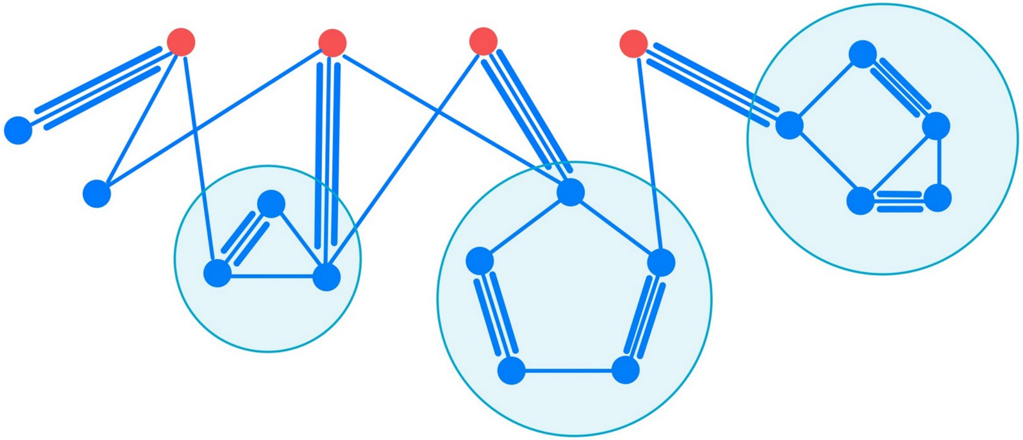
JACK EDMONDS

Canadian J. Math 1965

Constructing R-type regions and zero modes

Alternate “local” proof of Lovasz’s Thm:

- Each blossom hosts 1 (would-be) mode.
- Number of monomers in each R-type region of auxillary bipartite graph fixed, determines number of collective zero modes.



R-type region in bipartite auxiliary graph

KD, PRB 2022

Tractable computations(!)

Can obtain complete set of R-type regions from one maximum matching of diluted lattice

Opens door to detailed computational study of random geometry of R-type regions

Bhola, KD, arXiv:2311.05634

Bhola, Biswas, Islam, KD, PRX 2022

Random geometry of R-type and P-type regions

- 2D: Triangular, Shastry-Sutherland, square, honeycomb lattices.
- 3D: Cubic, stacked triangular, corner sharing octahedral lattices.
- Uncorrelated dilution, with global compensation (|A|=|B|) in bipartite case
- Random geometry characterized in multiple ways, e.g.:

Thermodynamic densities

- w
- $m^{\mathcal{R}}_{\mathrm{tot}}$ $n_{\mathcal{R}}$
- $m^{\mathcal{P}}_{\mathrm{tot}}$ $n_{\mathcal{P}}$

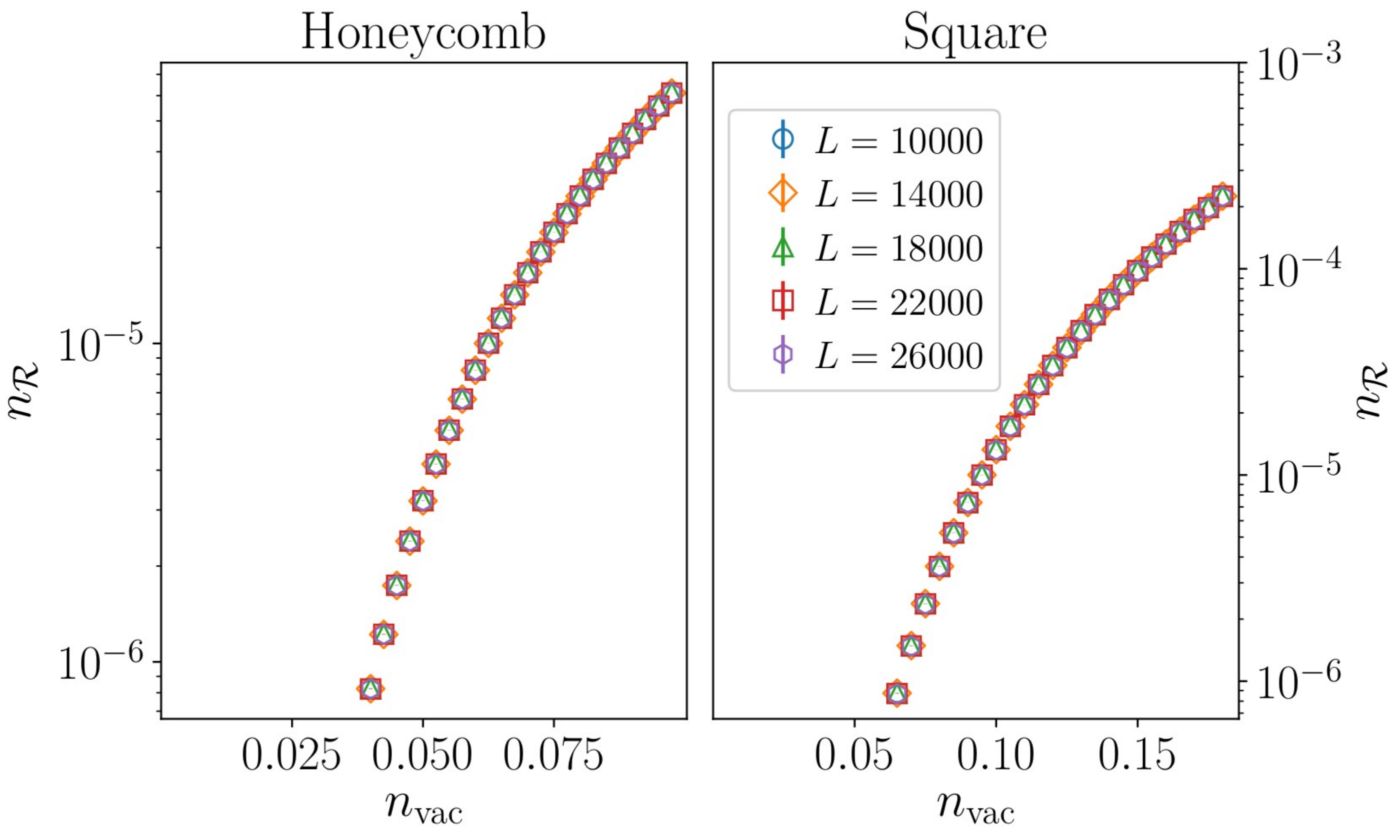
Percolation related measurements

- $m^{\mathcal{R}}_{\mathrm{max}}$
- $m^{\mathcal{P}}_{\mathrm{max}}$
- $P_{\mathrm{cross}}, P_{\mathrm{single}}$

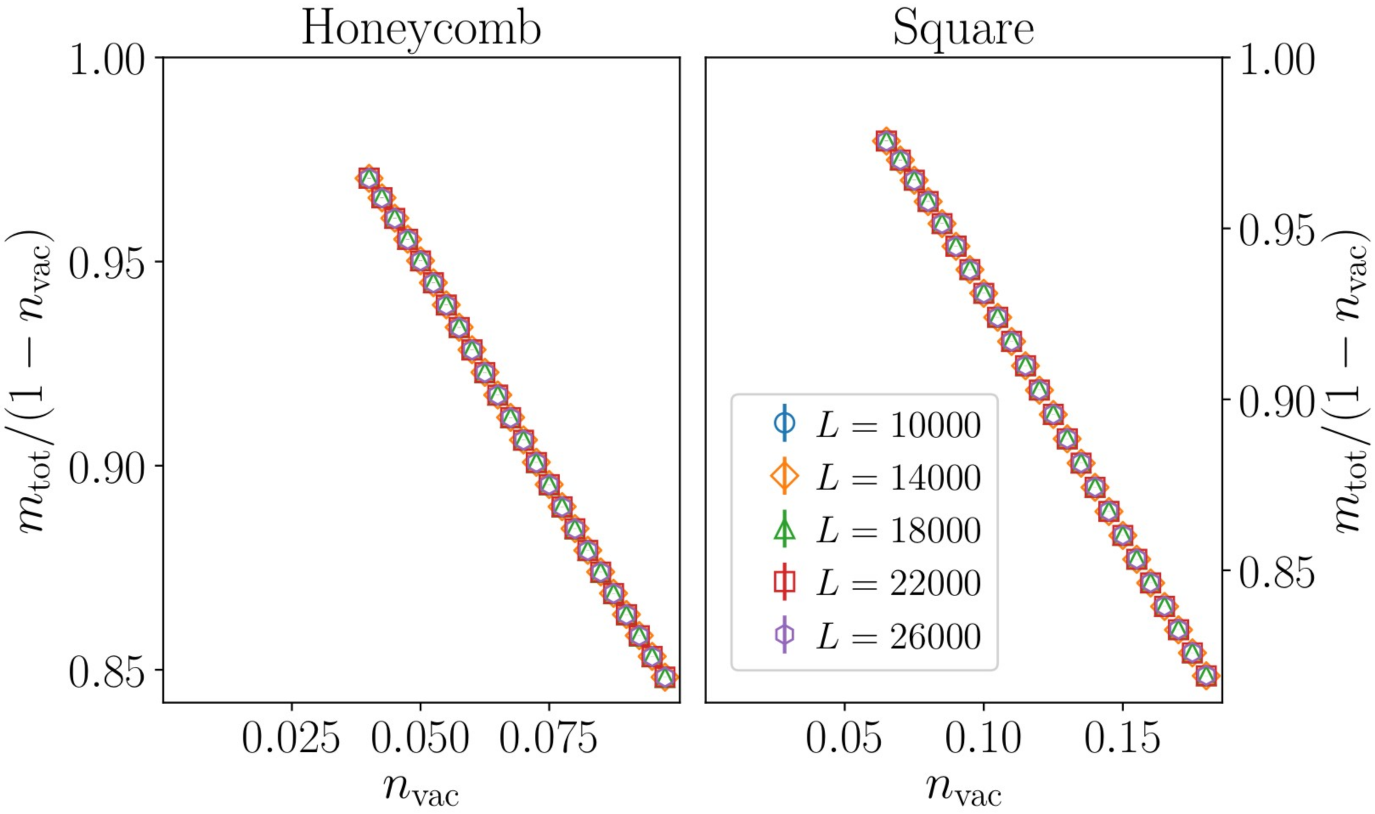
Bipartite case

Bhola, Biswas, Islam, KD, PRX 2022

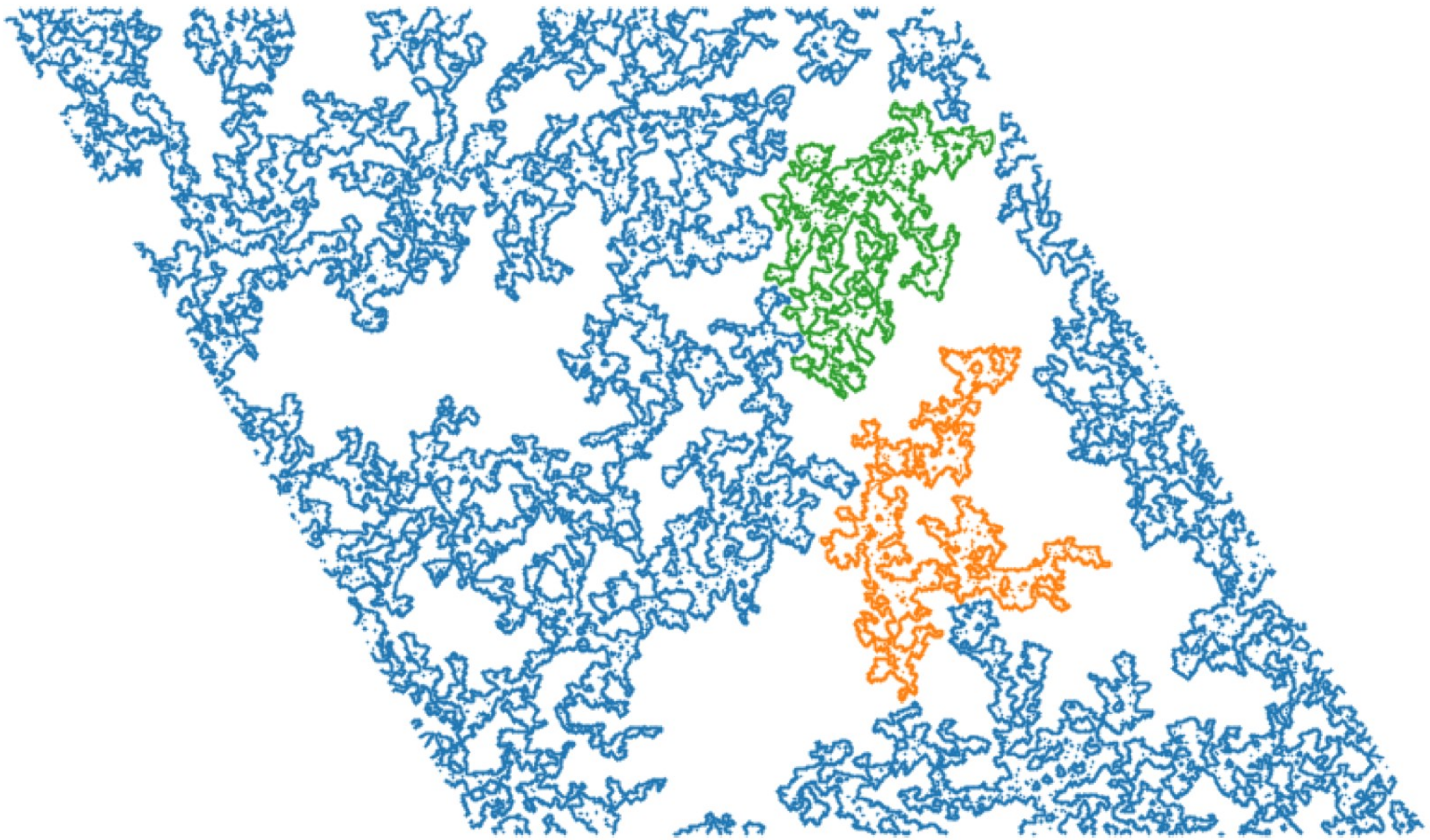
Number density of R-type regions



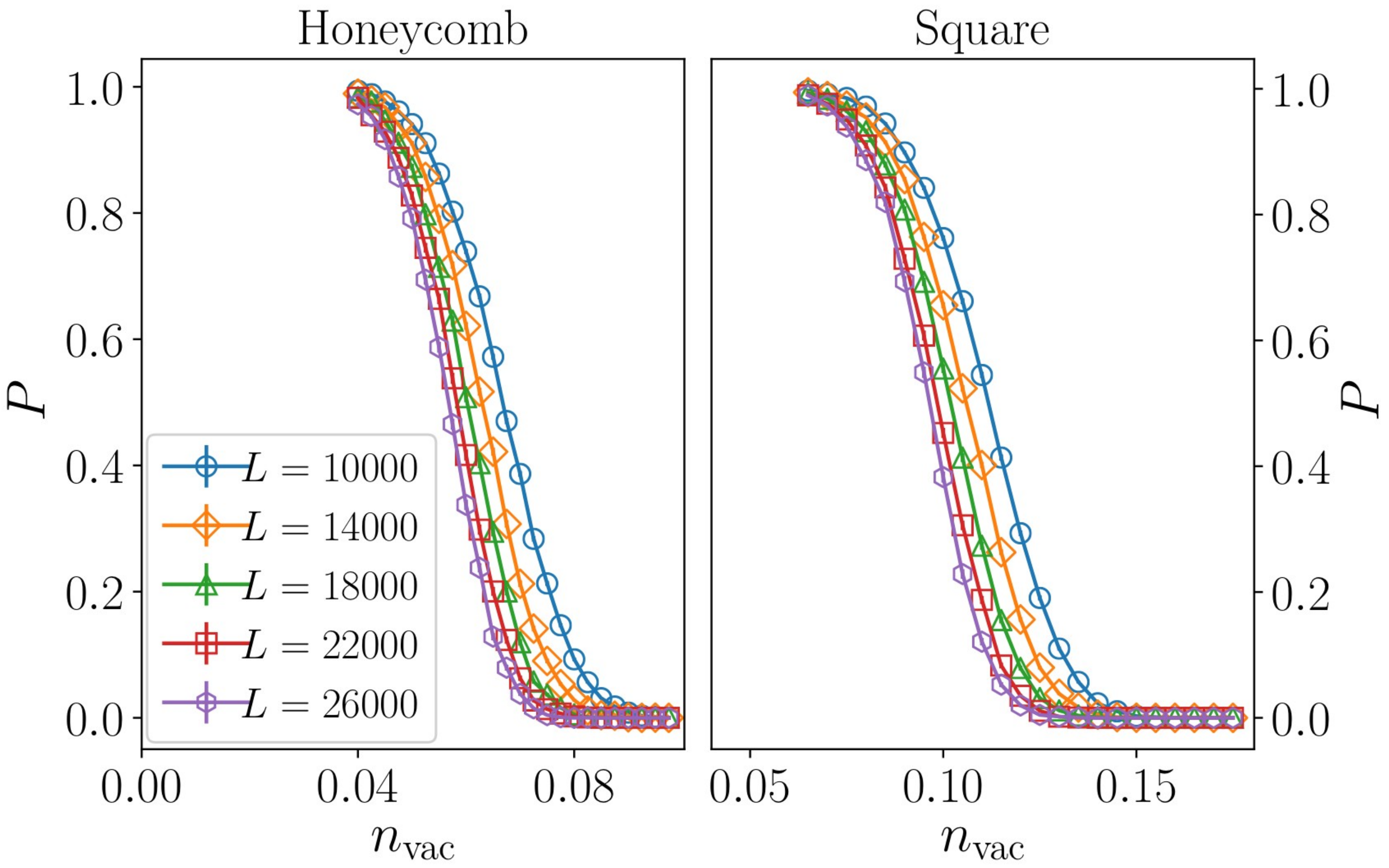
R-type regions take over lattice



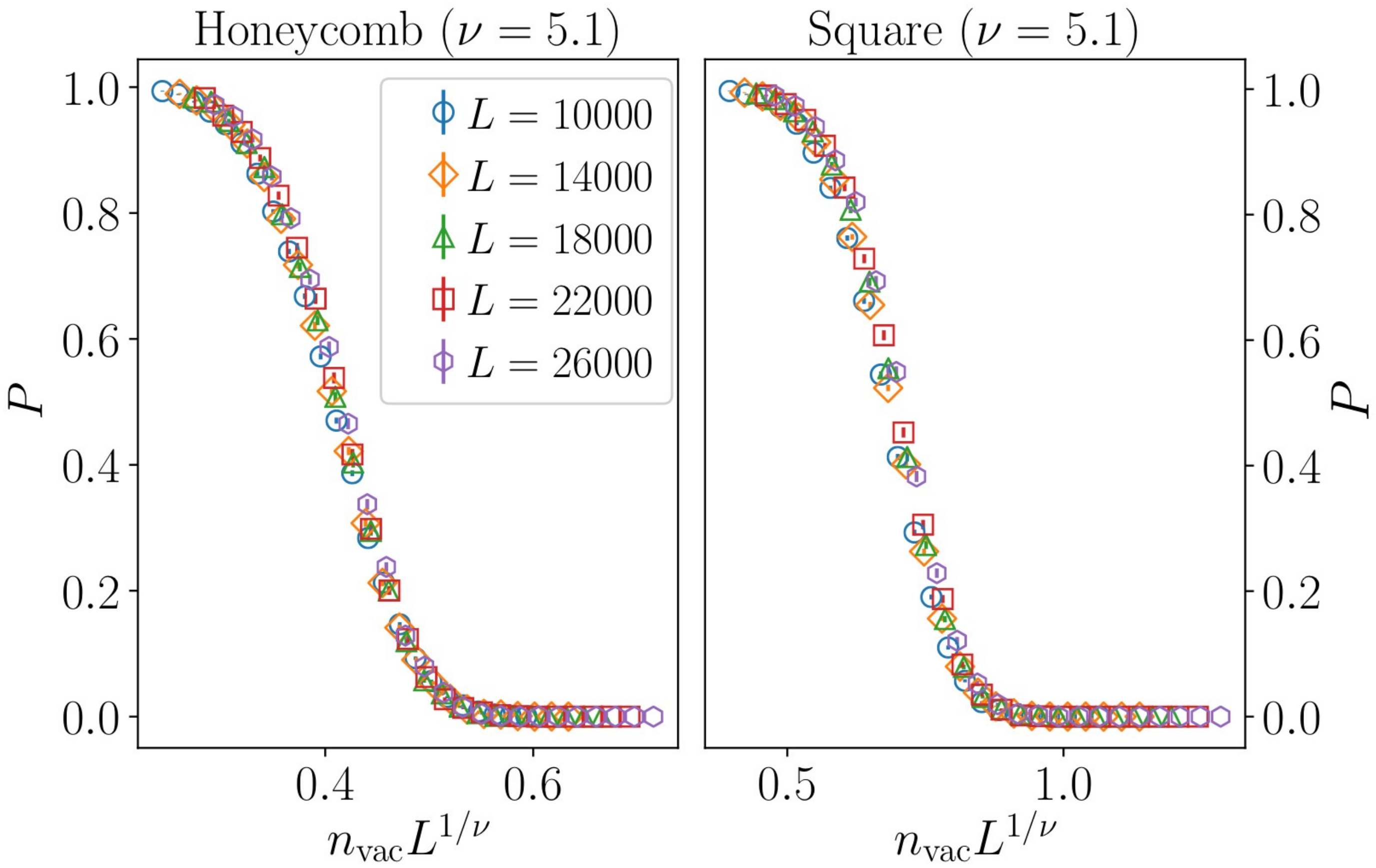
Percolation of R-type regions at low dilution?



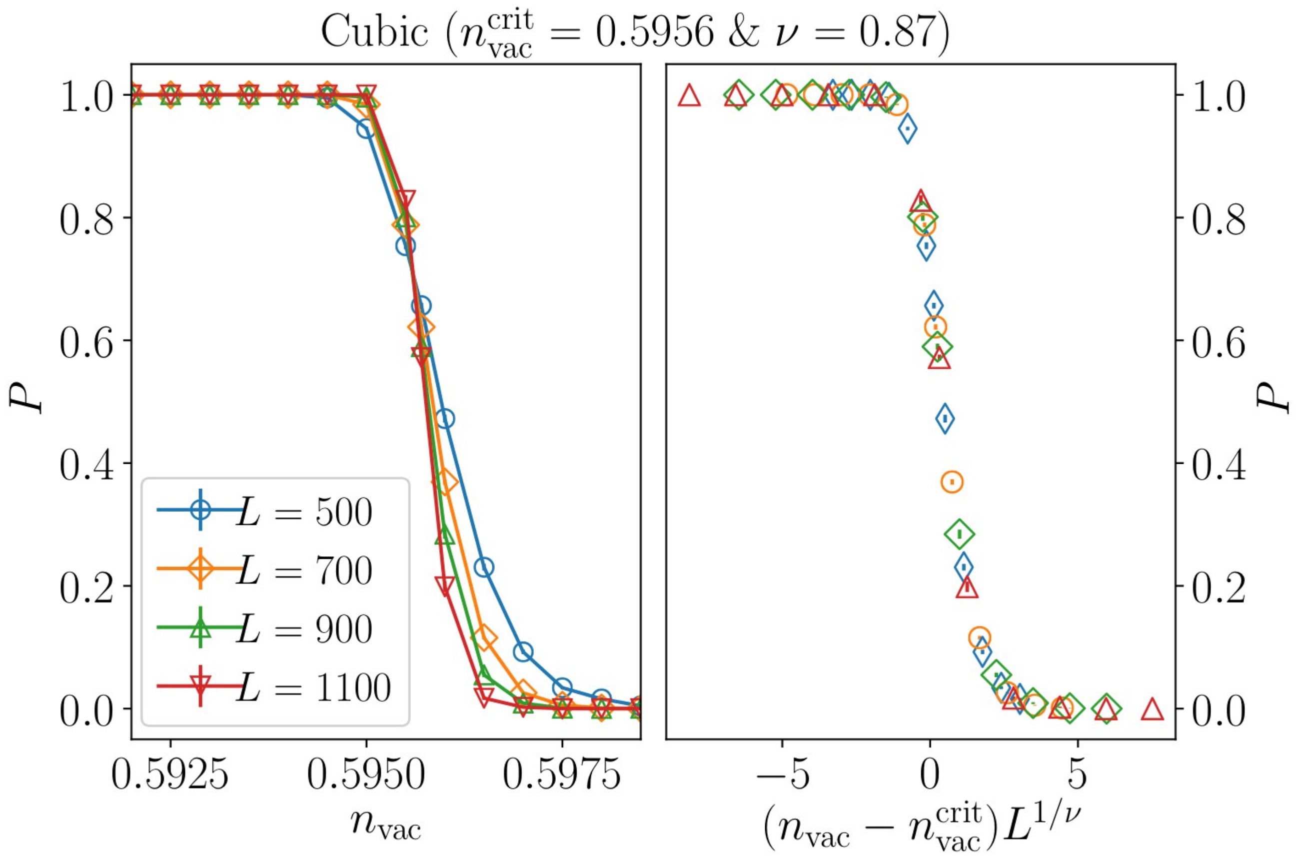
Incipient percolation at nv=0(?)



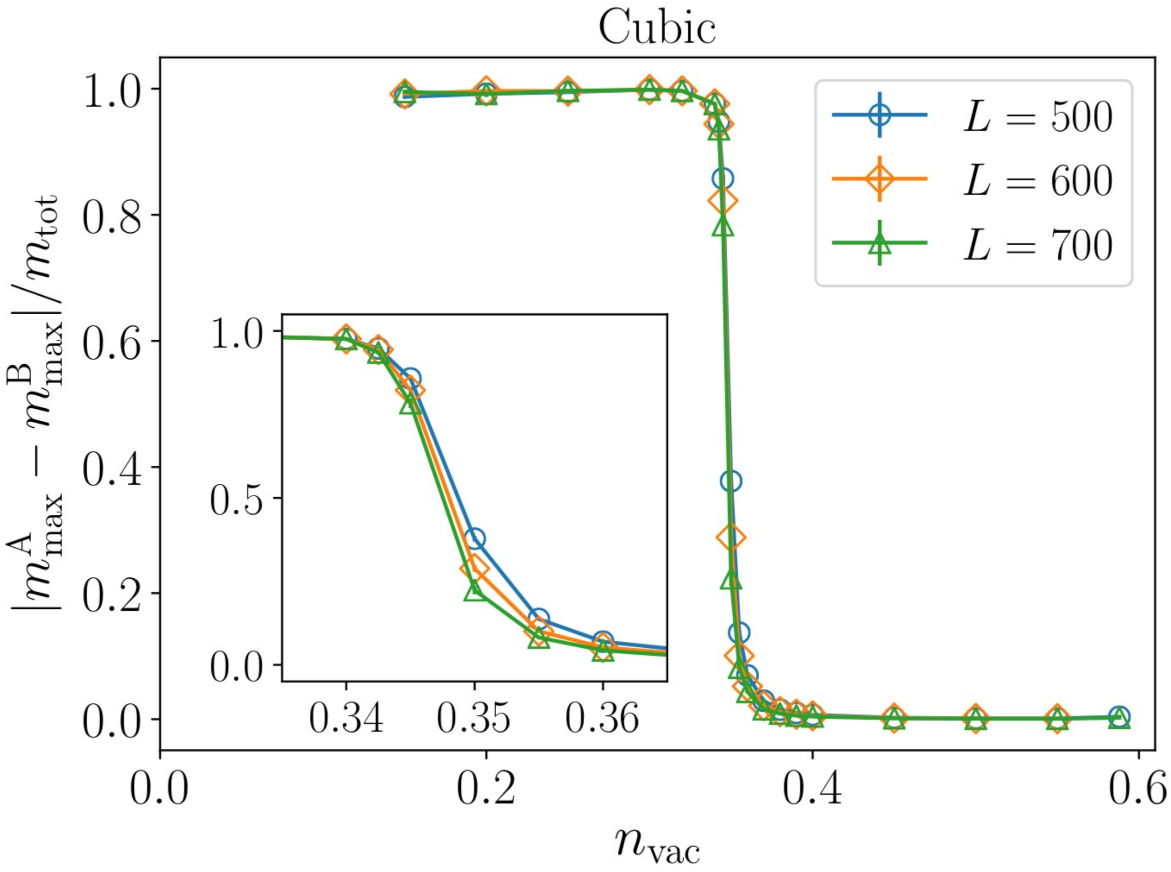
Universal scaling at nv=0 critical point(!)



Percolation transition on cubic lattice



Spontaneous sublattice symmetry breaking deep inside percolated phase

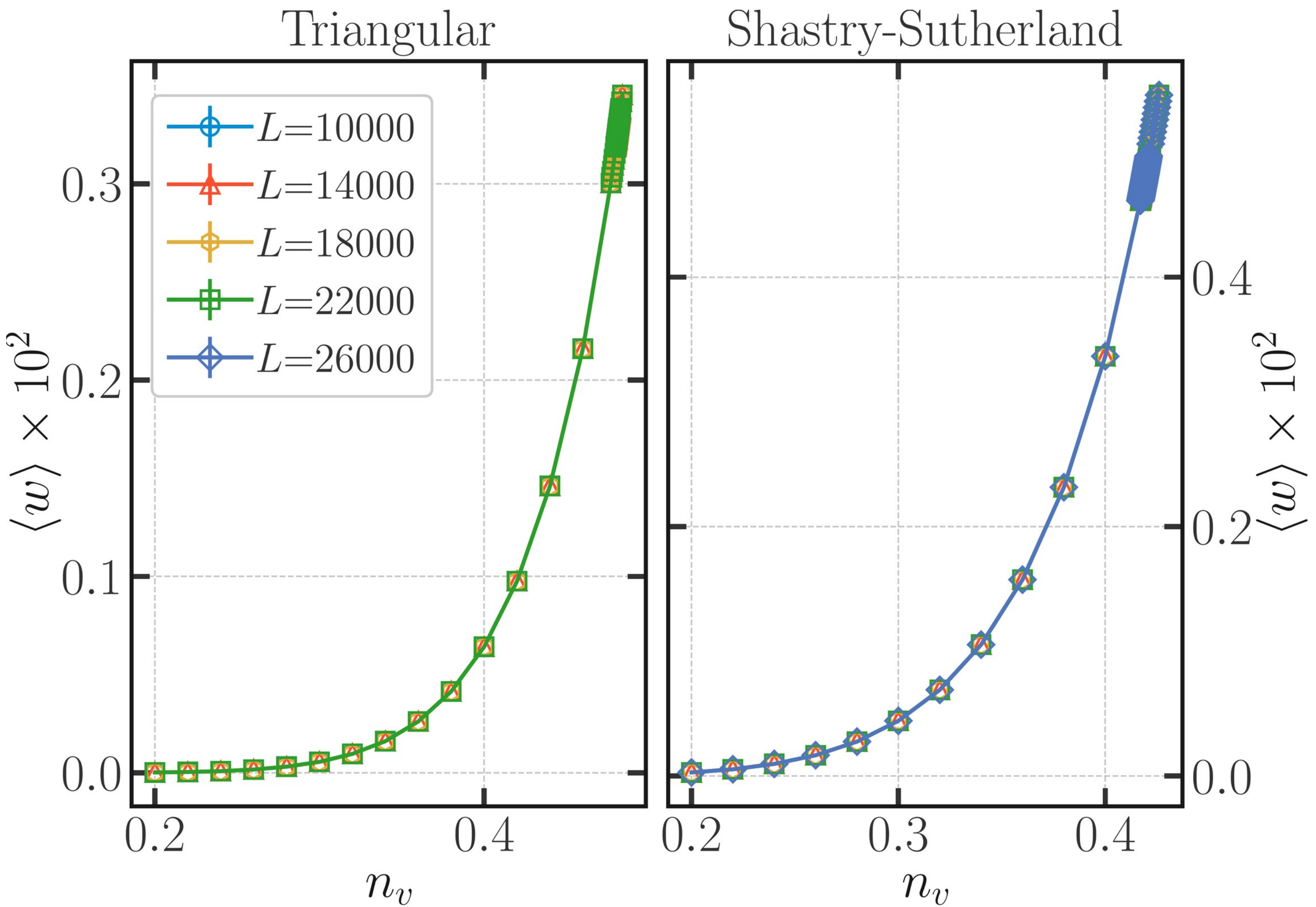


Nonbipartite case

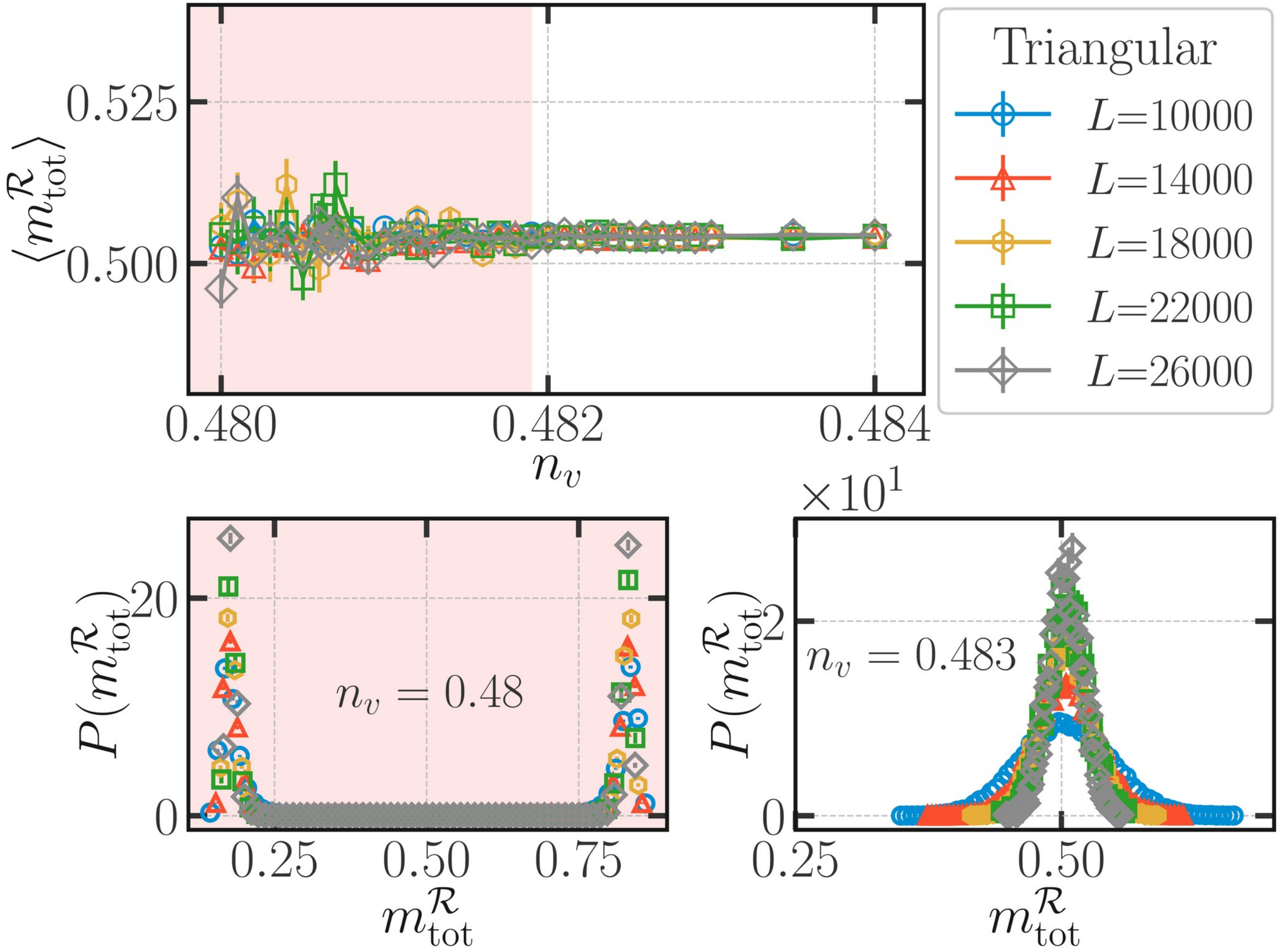
Bhola, KD, arXiv:2311.05634

Bhola, KD, arXiv:xxxx:xxxxx

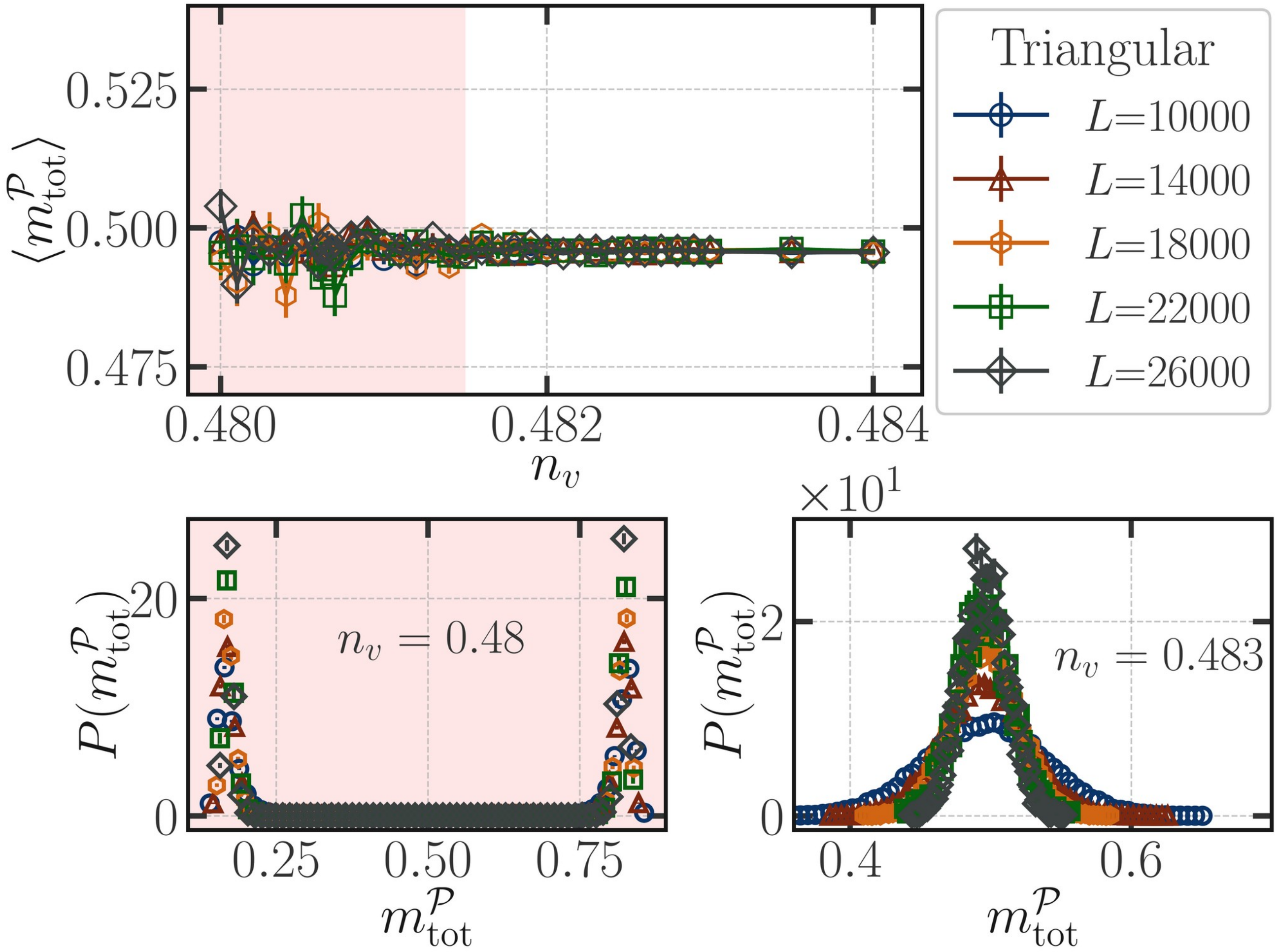
Monomer density



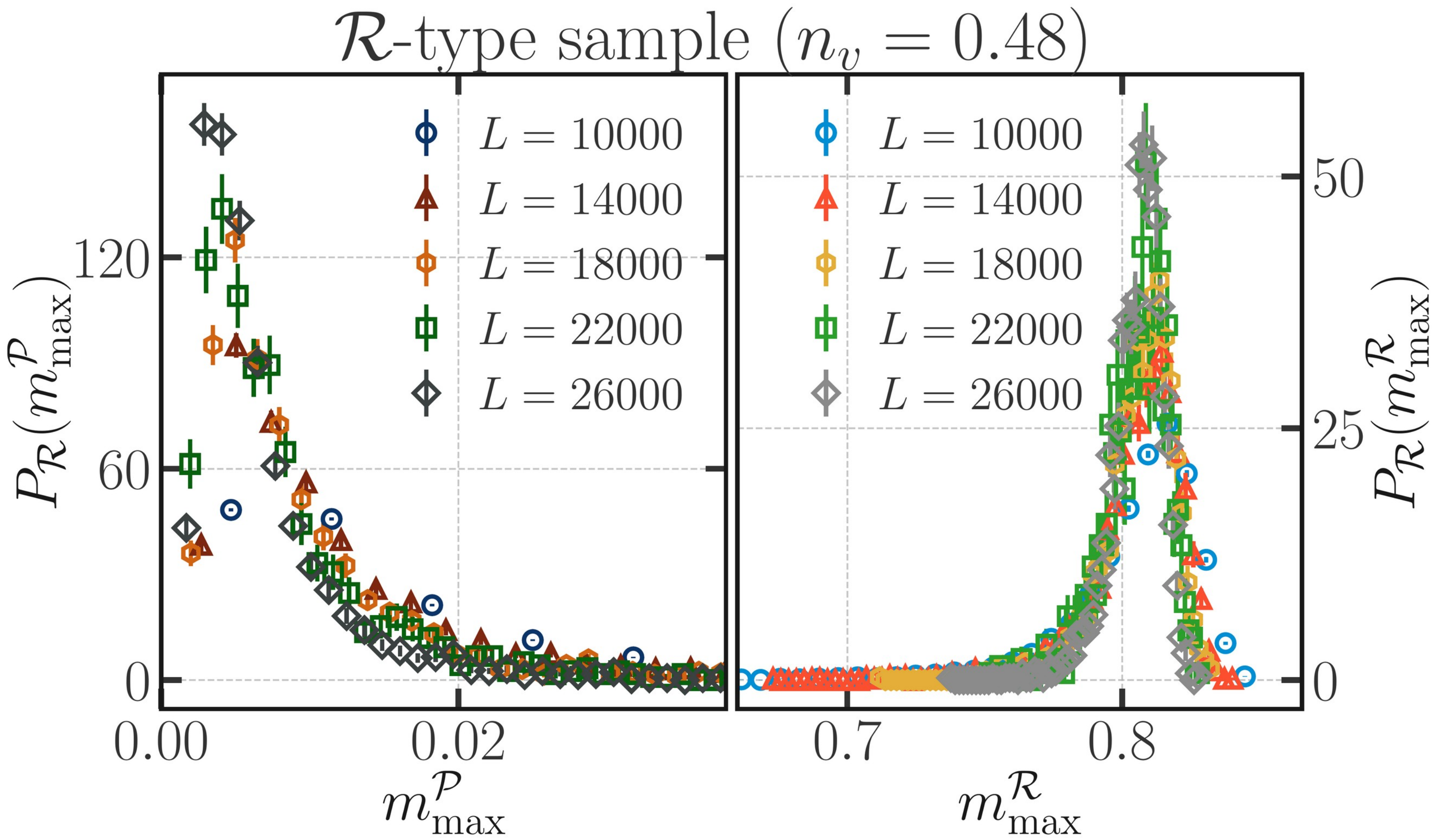
Fraction of sites in R-type regions



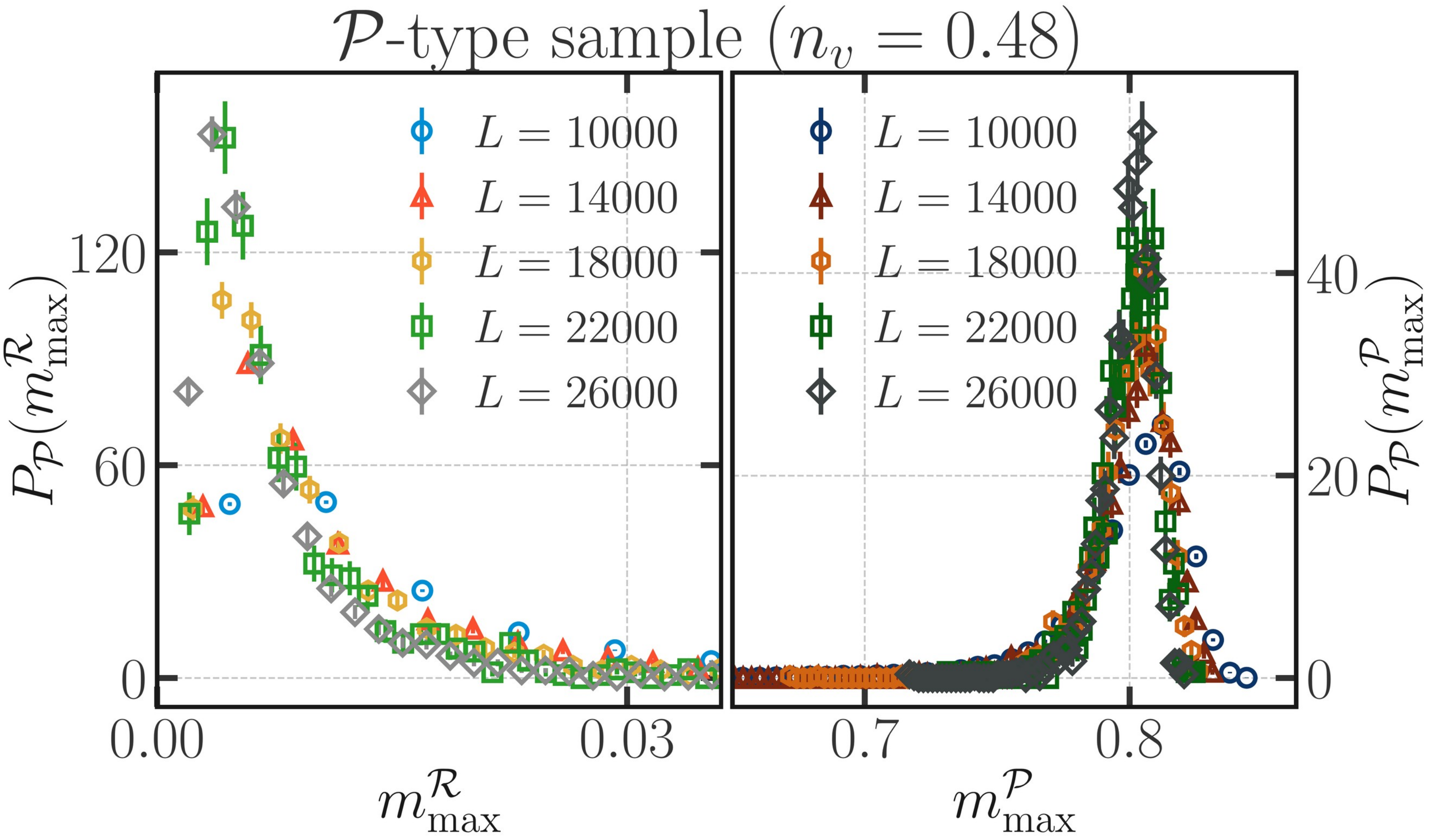
Fraction of sites in P-type regions



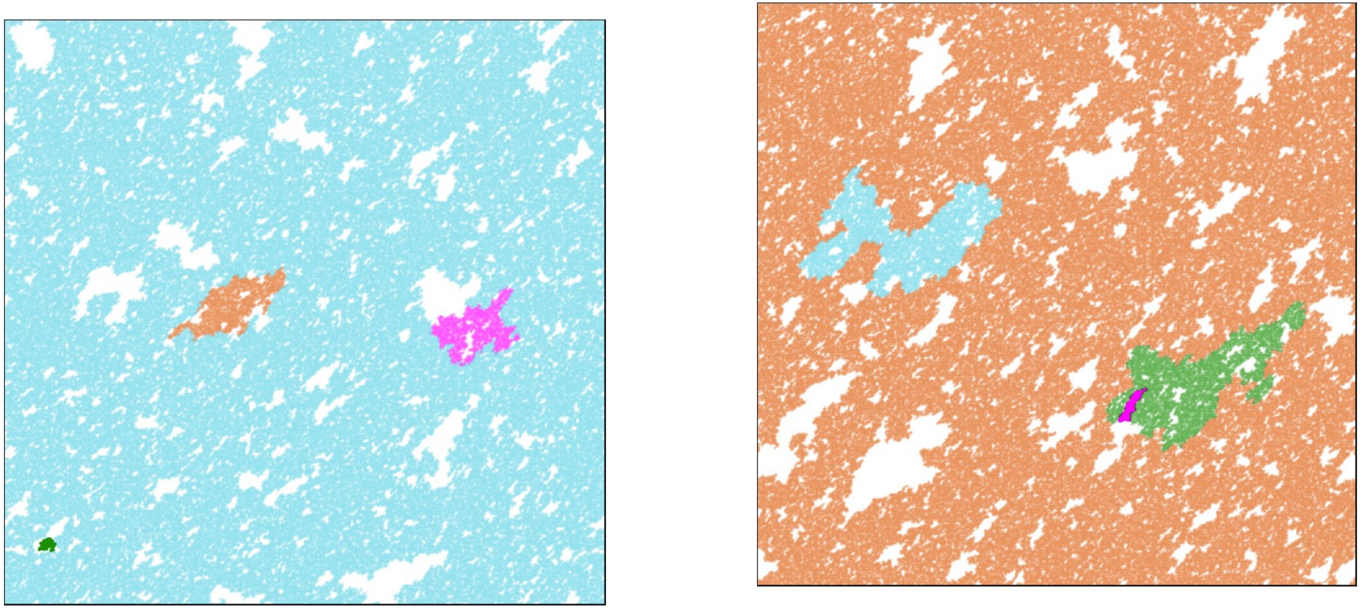
Cross-correlations



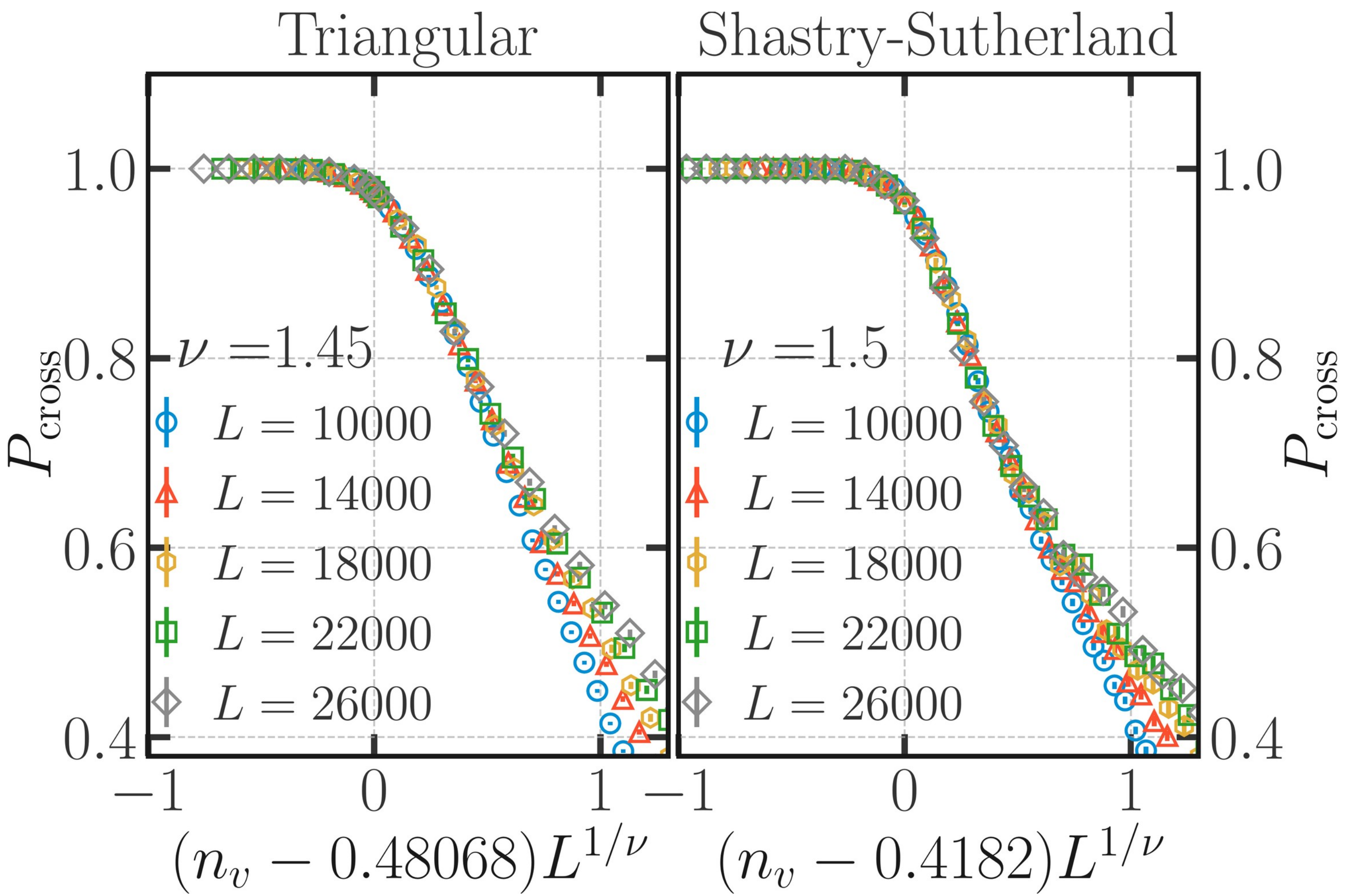
Cross-correlations



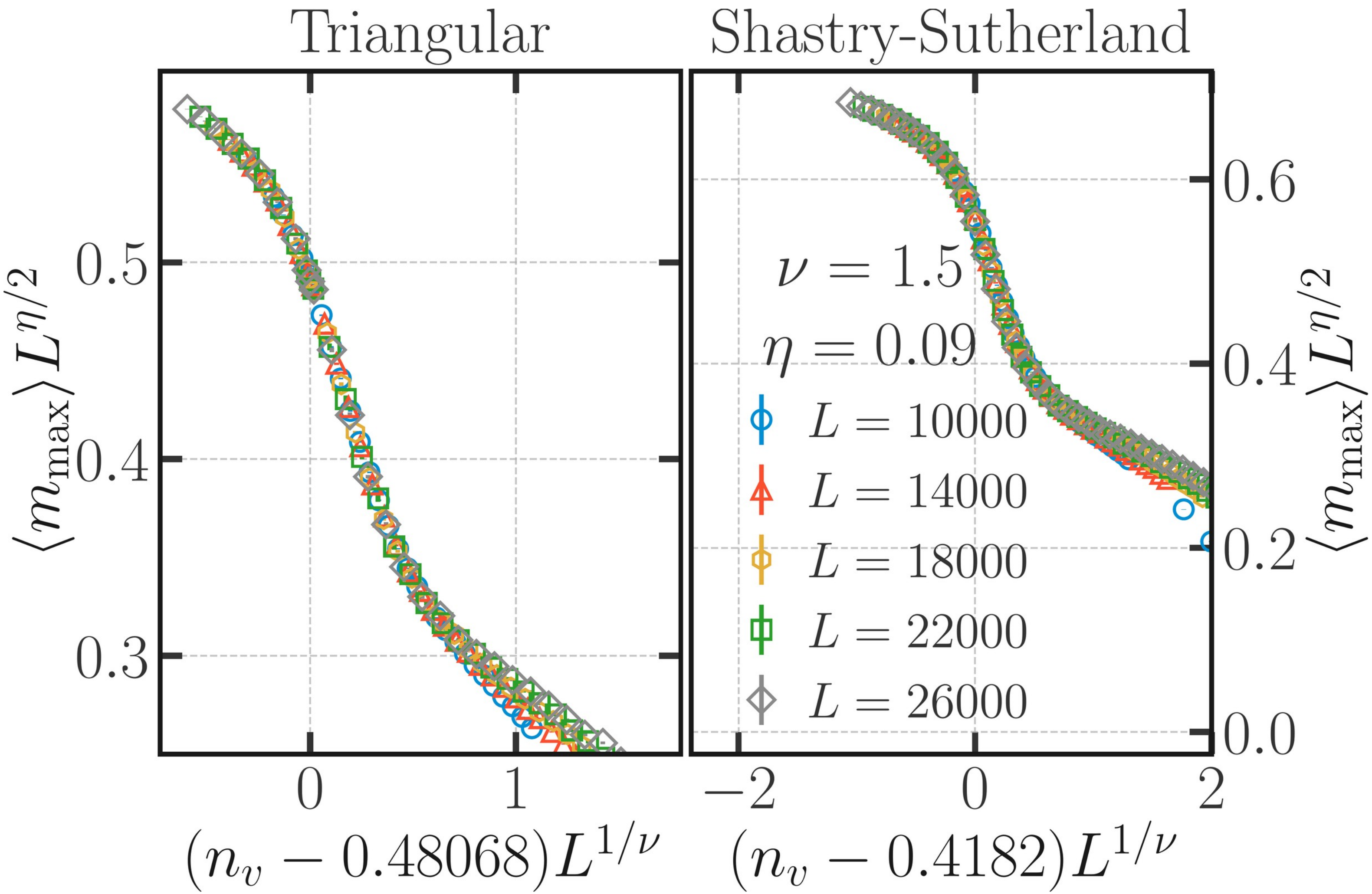
Viiolation of self-averaging in thermodynamic limit(!)



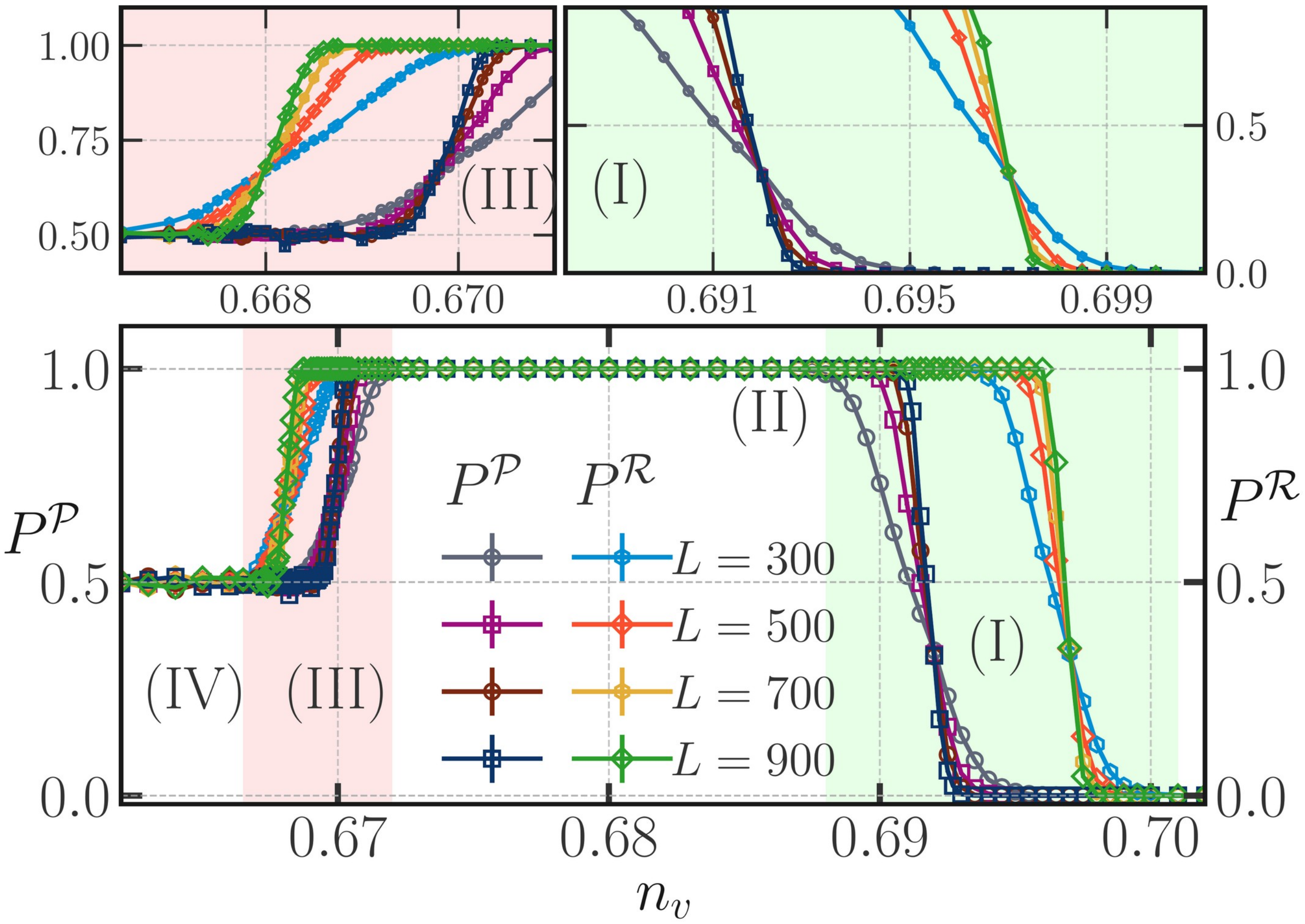
Percolation transition



Percolation transition



3D Phase diagram via wrapping probabilities



Thanks to

For crucial pointers to graph theory literature: T. Kavitha (TIFR CS) and A. Mondal (TIFR Math)

For useful discussions: A. Sandvik, S. Roy, Mahan Mj, S. Goswami, D. Dhar, M. Barma...

For crucial technical support: K. Ghadiali and A. Salve (DTP SysAds)

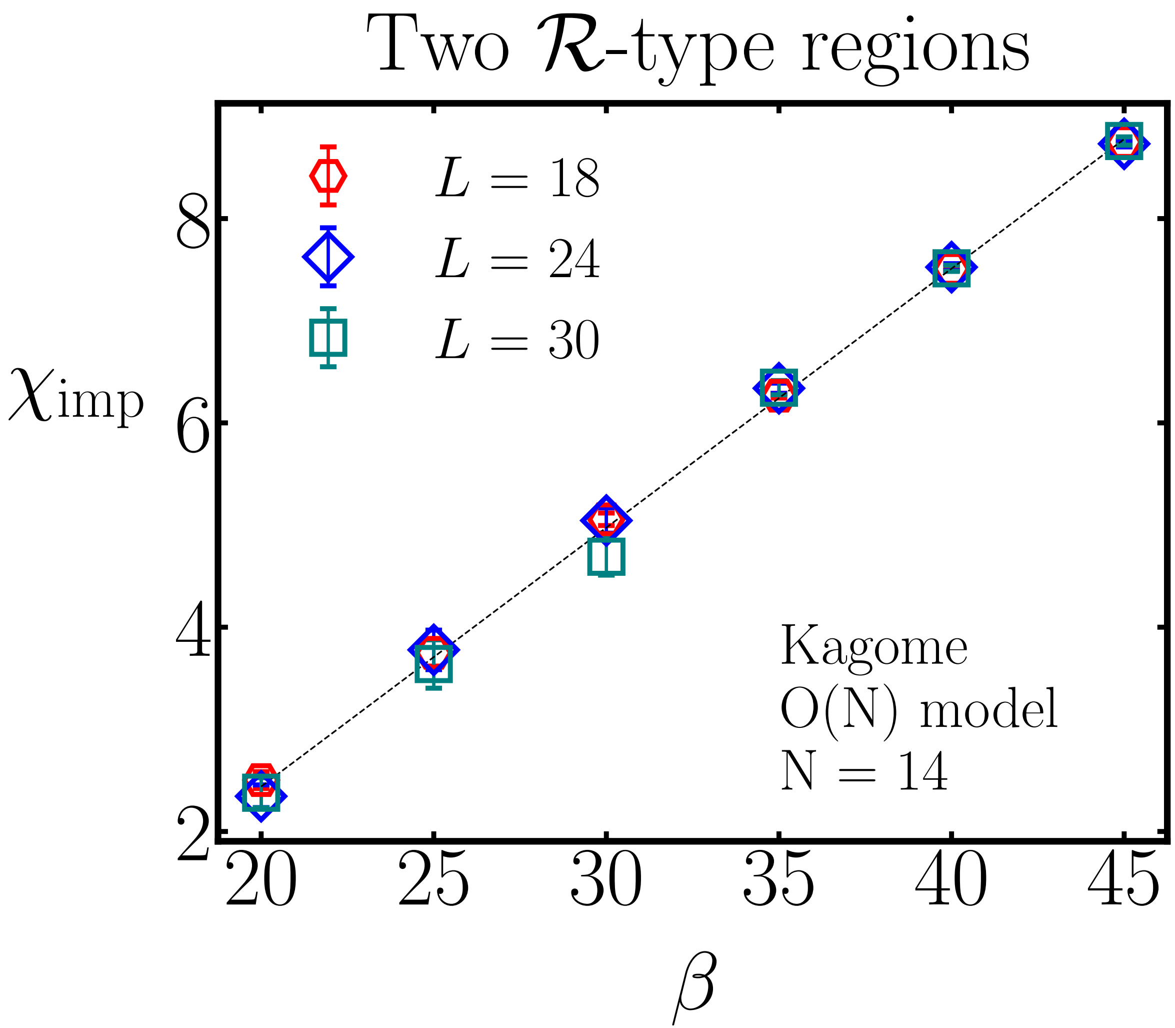
Aside: Implications for RVB spin liquids

Vacancy-induced emergent local moments live in R-type regions

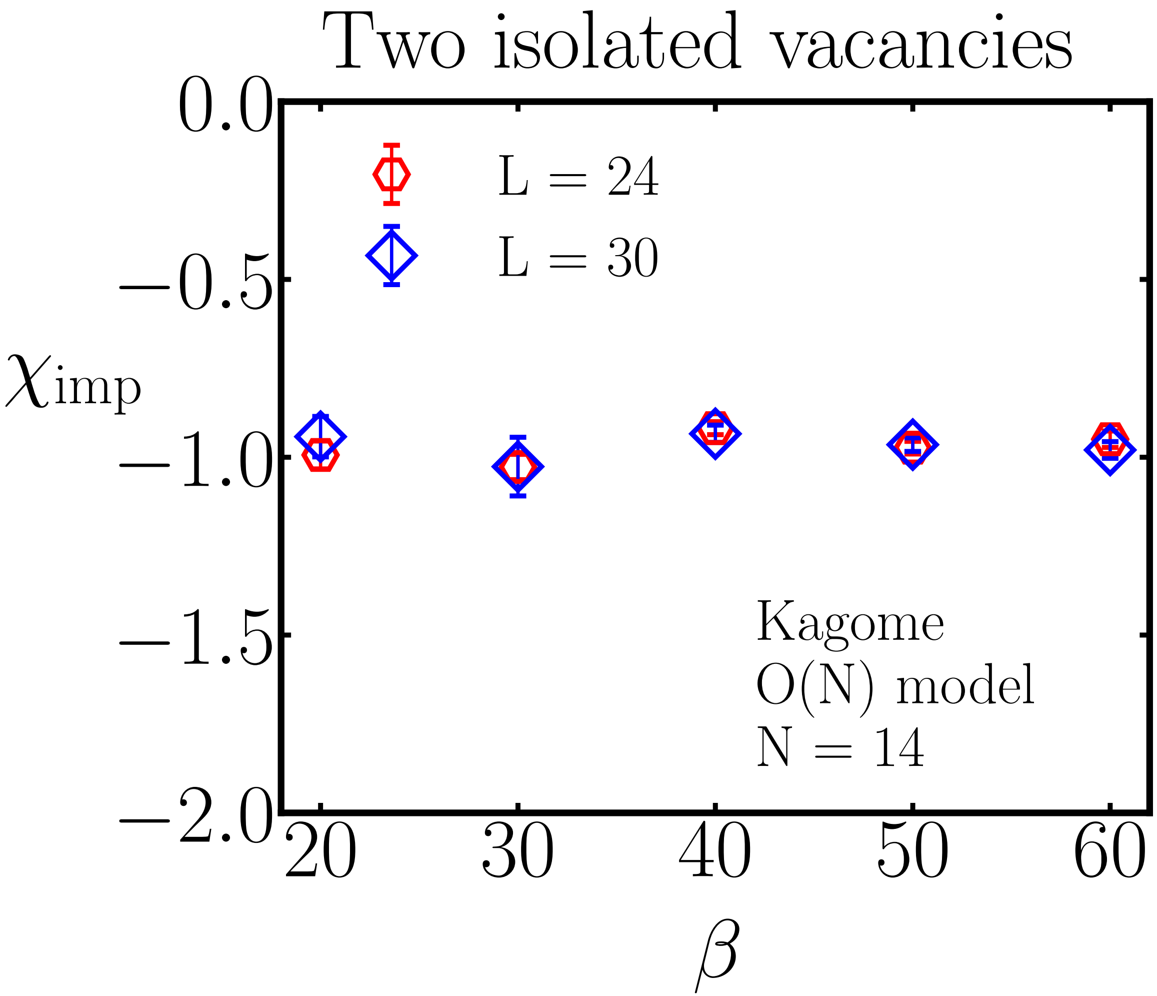
Random geometry of R-type regions controls local moment instability of such quantum magnets

Ansari & KD, arXiv:xxxx.xxxxx, in preparation

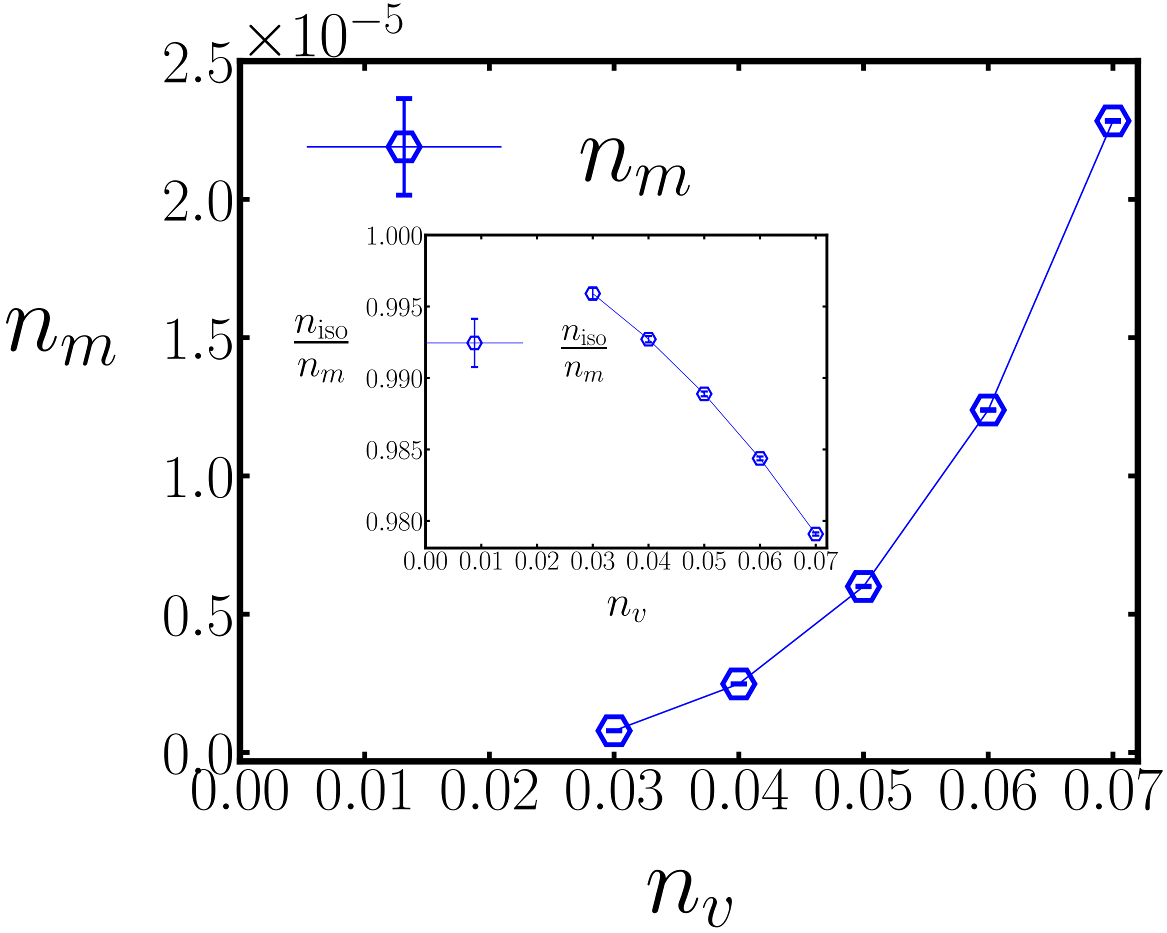
Check: Local moments associated with monomers



No local moments seeded by isolated vacancies



No bulk monomer density on site-diluted kagome



Gapped short-range
RVB liquids stable to vacancy disorder