Assignment-1: Classical Mechanics

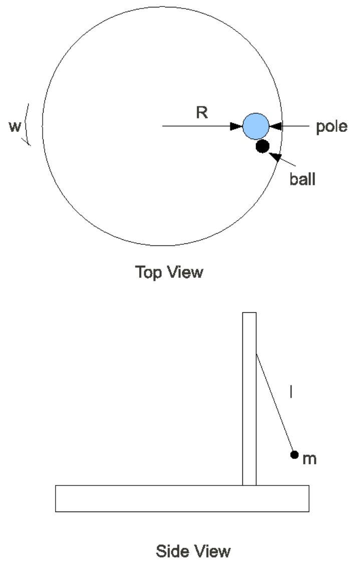
1. Derive the Noether charge for Galilean boosts. Show that this charge is automatically conserved if the total momentum of the system is conserved.

2. Consider the following action for a single particle in one dimension:

$$S = \int \left(\tfrac{1}{2}m\dot{x}^2 - kx^{10}\right) dt$$

where k is a constant. The particle undergoes periodic motion with period T.
(a) Compute the equations of motion
(b) Given that $x(t)$ is a solution, find β such that $\lambda x(\lambda^\beta t)$ is also a solution (λ is a scaling parameter)
(c) Given that $T = \alpha A^k$ is the time period of the motion where α is a constant and A is the amplitude of the oscillation, find k
(d) Compute the ratio of average kinetic energy to the average potential energy in the periodic motion of this problem

3. Consider the following mechanical systems:
(a) The earth is spinning around its axis with angular velocity ω. Write down the Lagrangian of a particle moving around near the surface of the earth. As coordinates, use the latitude, longitude and the height of the particle above the surface of the earth. Assume that the acceleration due to gravity g is a constant and the height of the particle above the surface of the earth is much smaller in comparison with the radius of the earth.
(b) The figure on the adjoining page shows a turntable (a merry-go-round) rotating with angular velocity ω around its axis and a pole attached to it at a distance R from the center. A ball of mass m is hanging from a massless rigid rod of length l from the pole as shown.
(i) How many degrees of freedom (i.e., the number of positions and velocities) does the ball have?



(ii) Set up the Lagrangian for the ball

4. Consider the following Lagrangian for a two particle system:

$$L = \tfrac{1}{2}m_1 v_1^4 + \tfrac{1}{2}m_2 v_2^4 - |\mathbf{r}_1 - \mathbf{r}_2|^2$$

where $\mathbf{v}_i = \dot{\mathbf{r}}_i$.

(a) Demonstrate that this action is invariant under translations, rotations and time translations, but neither the Lagrangian nor the equations of motion following from it are invariant under Galilean boosts.
(b) Compute the conserved charges that are consequences of spatial invariance, time translational invariance and rotational invariance of the above Lagrangian.