Assignment-2: Classical Mechanics

1. A particle of mass m in one dimension moves symetrically about a point. The time period of oscillation as a function of the energy is measured to be

$$T(E_0) = \frac{\pi}{\alpha}\sqrt{\frac{2m}{E_0+U}}$$

where E_0 is the energy and α and U are constants. Determine the potential in which it moves.

2. As you know, the gravitational potential between two objects is

$$V(r) = -\frac{a}{r}$$

in 3 space and 1 time dimensions.
(a) Use the 'Gauss law' to determine the form of the potential in p space and 1 time dimensions.
(b) Consider the special case $p = 4$ and a particle moving with an angular momentum M.
(i) Reduce the motion to an effective one dimensional problem.
(ii) Plot the effective potential in this 1-d problem. This potential is qualitatively different for $M > M_c$ and $M < M_c$ for a critical value of the angular momentum M_c. What is the value of M_c and what is the qualitative difference?
(iii) Qualitatively describe the nature of the orbits of the particle in the above two cases. Assume that the motion ends when the two objects collide. In the case $M < M_c$, compute the time taken for the planet from the farthest point to the point of collision.
(iv)Find an explicit expression for $\phi(r)$ by integrating the orbit equation and describe (in words) the shape of the orbit you obtain.

3. Consider the following potential

$$V = a(x^4 + y^4)$$

(a) Solve for the motion of a particle moving in two dimensions in the above potential. (An answer in terms of an integral is acceptable)
(b) Qualitaitvely describe the nature of the orbits the particle undergoes.
(c) Does the particle orbit repeat in general? If so, under what conditions?

4. Consider a planet moving around a star originally in a circular orbit of radius R. Suppose at $t = 0$ the mass of the star were to suddenly change to
(i) $2M$
(ii) $\frac{M}{2}$
(iii) $\frac{M}{4}$
(a)In each case, use the virial theorem to determine whether the subsequent motion would be elliptic, parabolic or hyperbolic.
(b) Find the equation of the orbit for the subsequent motion.

5. Consider the following potential

$$V(x) = \begin{cases} k(\frac{x^2}{2} - \frac{a^2}{2}) & \text{for} \quad |x| < a \\ 0 & \text{for} \quad |x| > a \end{cases}$$

where k is a constant. A particle of mass m is moving with a velocity v with the following initial conditions

$$x = vt \quad \text{for} \quad t < -a/v$$

(a) Determine the particle trajectory at late times for:
(i) $k > 0$
(ii)$k < 0$
(b) In the $k < 0$ case, the particle trajectories are qualitatively different for $v > v_c$ and $v < v_c$ for a critical value of the velocity v_c. What is the difference and determine v_c.
(c) In the $k > 0$ case, at what time does the particle reach the point $x = L$, where $L > a$? What is the time difference between the arrival of this particle at $x = L$ and a free particle (particle with no potential) with the same initial conditions?