Assignment-3: Classical Mechanics

1. Consider Rutherford scattering in 4 space and 1 time dimensions. That is, consider a particle of mass m incident on a fixed potential α/r^2, where $\alpha > 0$.
(a) Compute the differential scattering cross section as a function of the scattering angle.
(b) Now consider the case of two particle scattering in the same potential where the masses of both the particles are equal ($m_1 = m_2 = m$ say) when one of the particles is at rest and the other is approaching with a velocity v. Calculate the scattering cross section of the incident particle as a function of the scattering angle.
(c) Also compute the scattering cross section as a function of the final energy of both the particles.

2. Suppose a particle in p space and 1 time dimensions dissociates into two daughter particles of masses m_1 and m_2. In the rest frame of the original particle, the velocities of the daughter particles are $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. The velocity of the original particle in the lab frame was $\mathbf{V}$.
(a) Determine $\mathbf{v}_2$ in terms on $\mathbf{v}_1$.
(b) Find a formula for the kinetic energy of the particle with mass m_1 in the lab frame.
(c) Assuming that the particles were emitted isotropically in the rest frame of the original particle, find a formula for the probability distribution of the kinetic energy of the particle with mass m_1 in the lab frame. What are the smallest and largest values this kinetic energy can take?

3. Consider the system described by the following Lagrangian:

$$L = \frac{m}{2}\left(\dot{x}^2 - \omega_1^2 x^2 + \dot{y}^2 - \omega_2^2 y^2\right) - \lambda(x+y)^4$$

When $\lambda = 0$, this system admits an oscillatory solution of the form

$$a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t)$$

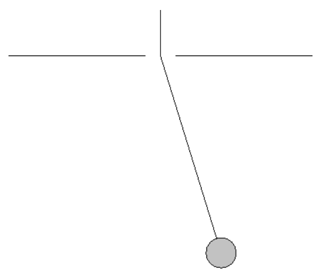
Assuming a_1 and a_2 are small but of the same order, compute the corrections to this solution at $\mathcal{O}(a_1^3)$ and $\mathcal{O}(a_2^3)$ when $\lambda \neq 0$. What can you say about the frequencies of the resultant oscillations?

4. Consider the equation

$$\ddot{x} + \omega_0^2 \left(1 + h\cos[(2\omega_0 + \epsilon)t]\right) x = 0$$

Assuming that h and $\frac{\epsilon}{\omega_0}$ are small but of the same order, compute the two linearly independent solutions to this differential equation to $\mathcal{O}(h)$. Also calculate the rate constant that governs exponential behaviour to $\mathcal{O}(h^2)$. Under what conditions on h and ϵ does parametric resonance happen?

5. Consider the system shown in the figure below



The figure shows a pendulum suspended from a hole on the roof such that the length l of the string can be periodically varied as

$$l = L\left(1 + h\cos\left[\left(2\sqrt{\frac{g}{L}} + \epsilon\right)t\right]\right)$$

where L is the length from the hole to the bob of the pendulum. If h and ϵ are small and are of the same order, then

(a) Compute under what conditions on h and ϵ will this pendulum undergo simple, bounded small harmonic oscillations and under what conditions will these become exponentially unstable?

(b) Suppose air resistance provides a frictional force term equal to $2m\lambda\dot{x}$, how does this modify your answer?

6. Consider an oscillator given by the Lagrangian

$$L = \frac{m}{2}\left(\dot{x}^2 - \omega^2 x^2\right)$$

at rest. At $t = 0$, it is forced as

$$f(t) = \begin{cases} f_0 e^{\nu t} & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$$

where $\nu > 0$ and f_0 is a positive constant. Find the harmonic solution for $t > T$.