Assignment-4: Classical Mechanics

1. Consider a symmetric top on a flat surface. The tip (point of contact) can slide on surface without friction. Solve for the motion of the top. The moment of inertia of the top is I, mass m and the distance of the center of mass from the point of contact is l.

2. While a professor at Cornell, Feynman at a restaurant noticed that a waiter spun a plate and threw it into the air. The plate both spun and precessed about the vertical. Feynman noticed that there was a simple relationship between the angular velocity of the plate and the angular velocity of precession. Work out this relationship. For the purposes of this problem, assume the plate to be a uniform planar disk symmetrical about its center. Also assume that the angle between the angular momentum vector and the symmetry axis of the plate to be small.

3. A ball is thrown up from the surface of the earth by an athlete located at latitude θ. The ball is thrown at speed v at an angle α to the vertical. The projection of the initial velocity vector of the ball onto the surface of the earth makes an angle β with respect to the direction to the north pole.
(a) Ignoring the earth's rotation, where will the ball land? You may assume that the distance traversed by the ball is small compared to the radius of the earth, and accordingly take the acceleration due to gravity g to be constant.
(b) Find the correction to your answer after including the effects of rotation of the earth. Do this in two ways:
(i) First in the rotating frame of reference attached to the earth;
(ii) Second in the inertial frame of the "fixed stars".

4. Determine the free motion of an asymmetrical top for which $M^2 = 2EI_2$, where M is the angular momentum of the top, E is the energy of the top and the three moments of inertia about the principal axes of the top are I_1,

I_2 and I_3 and where $I_1 > I_2 > I_3$. Explicitly determine the Euler angles as a function of time for this motion.

5. Consider the motion of a symmetric top with the point of support fixed on the surface of a table. Let the conserved angular momentum of the top about the vertical be given by M_z and the conserved angular momentum about the axis be given by M_3. Specialize to the case $M_3 = M_z$.
(a) Determine the point of stable equilibrium of the effective potential (see problem 2 on pg. 113 of Landau-Lifshitz).
(b) Determine the frequency of small oscillations about this equilibrium point. Also determine the angular motion of the top as it undergoes these oscillations.
(c) Which of the figures (a), (b) or (c) in Fig. 49 of Landau-Lifshitz qualitatively describes the motion?