# Classical Mechanics: Mid Term Test

1. (a) Write down the Lagrangian for a simple pendulum of mass $m$ whose point of support
(i) moves uniformly on a vertical circle with constant frequency $\gamma$,
(ii) oscillates horizontally in the plane of motion of the pendulum according to the law $x = a\cos\gamma t$,
(iii) oscillates vertically according to the law $y = a\cos\gamma t$.
(b) For the case (ii), determine the positions of the stable equilibrium of the pendulum, and
(c) determine the frequencies of small oscillations about the equilibria determined in part (b).
(assume that the frequency $\gamma >> \sqrt{g/l}$)

2. A particle of mass $m$ moves in a potential $V(x)$ and has time period $T$. Determine the time period of a particle of mass $m'$ moving in the same potential.

3. When a small correction $\delta U(r)$ is added to the potential energy $U = -\alpha/r$, the paths of finite motion are no longer closed, and at each revolution the perihelion is displaced through a small angle $\delta\phi$. Find $\delta\phi$ when $\delta U = \gamma/r^3$.

4. Determine the effective cross-section for a particle to "fall" to the center of a field $U = -\alpha/r^2$.

5. Determine the forced oscillations due to an external force $f = f_0\exp(\alpha t)\cos\gamma t$ in the presence of friction. The equation of motion is

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = (f_0/m)\exp(\alpha t + i\gamma t)$$

6. Obtain expressions for the Cartesian components and the magniude of the angular momentum of a particle in spherical co-ordinates $r$, $\theta$, $\phi$.