Classical Mechanics: Mid Term Test

1. Consider a spherical pendulum of mass m and angular momentum M. (A spherical pendulum is a particle of mass m moving on the surface of a sphere of radius l in a gravitational field.)
(i) Determine the effective potential as a function of the polar angle θ for fixed angular momentum M.
(ii) Determine the minimum of the above effectice potential at fixed angular momentum M.
(iii) Determine the frequency of small oscillations around this minimum.

2. A particle of mass m moves in a potential $V(x)$ and has time period T. Determine the time period of a particle of mass m' moving in the potential $\alpha V(x)$.

3. Find the time dependence of the co-ordinates of a particle with energy $E = 0$ moving in a parabola in a field $U = -\alpha/r$.

4. Determine the effective cross-section for particles of mass m_1 to strike a sphere of mass m_2 and radius R to which they are attracted in accordance with Newton's law.

5. Determine the final amplitude for the oscillations of a harmonic oscillator

$$\ddot{x} + \omega_0^2 x = f(t)$$

under a force $f(t)$ which is as

$$f(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ F_0 t/T & \text{for} \quad 0 \leq t \leq T \\ 0 & \text{for} \quad t > T \end{cases}$$

6. Obtain expressions for the Cartesian components and the magnitude of the angular momentum of a particle in cylindrical co-ordinates r, ϕ, z.