LANDAU-GINZBURG MODEL FOR A CRITICAL TOPOLOGICAL STRING

Debashis Ghoshal
Mehta Research Institute
of Mathematics & Mathematical Physics
10, Kasturba Gandhi Marg, Allahabad 211 002, India

and

Sunil Mukhi
Tata Institute of Fundamental Research
Homi Bhabha Road, Bombay 400 005, India

ABSTRACT

We argue that the one-superfield topological Landau-Ginzburg model with superpotential $W(X) = X^{-1}$ defines a topological string background with critical central charge $\hat{c} = 3$. This model is shown to be equivalent to $c = 1$ string theory compactified at the self-dual radius. We also comment on a possible Calabi-Yau phase of this model.

1. Introduction

There has been a lot of progress in our understanding of the exactly solvable string theories. Among these are the non-critical string theory in the background of matter with central charge $c_M \leq 1$, which were first studied to all orders in perturbation theory using a discretization of the world-sheet via random matrix models[12]. A topological field theory description was subsequently developed[23], where it was shown that perturbations of pure topological gravity can reproduce an infinite subclass of $c_M < 1$ non-critical string models. The remaining models are obtained by coupling specific topological matter systems to topological gravity[17]. These topological string theories have their characteristic topological central charges $\hat{c}$.

Physically $c_M = 1$ is perhaps the most interesting. This theory has a propagating degree of freedom, the massless 'tachyon', and an infinite number of discrete states, the remnant of the graviton and other higher tensor modes in a two-dimensional target space. One topological formulation for this two-dimensional string theory is given by the twisted Kazama-Suzuki[15] coset model $\mathrm{SU}(2)_k/\mathrm{U}(1)$ with the level $k = -3$ defined by continuation[21]; (the twisted cosets with $k \in \mathbf{Z}^+$ describe the unitary $c_M < 1$ series[25]). While the topological central charge of the unitary series is $\hat{c}_k = \frac{k}{k+2} < 1$, the two-dimensional string is distinguished by the 'critical' value $\hat{c} = 3$.

Topological string models also appear in the compactification of critical string theories and are closely related to the extremely intriguing mirror symmetry[24]. The Calabi-Yau compactification, in particular, has $\hat{c}=3$. This symmetry has been exploited to obtain a set of recursion relations between some correlators of the theory. Simplification occurs for $\hat{c}=3$, leading to an exact solution[2].

In what follows, we describe a Landau-Ginzburg-type topological formulation of the $c_M=1$ string theory compactified at the self-dual radius. This is equivalent to the topological Kazama-Suzuki model of Ref.(21), but has the advantage that everything is explicitly calculable from purely topological arguments and consistency of the operator algebra.

2. Symmetry algebra of a string background

The following "modern" version of the axioms for consistent string backgrounds has emerged from these recent developments[4]. In an (abstract) string background there should exist

- a stress-energy tensor $T(z)$, a spin-2 bosonic current satisfying a Virasoro algebra with vanishing central charge $c_{\rm vir}=0$, (and its hermitian conjugate),
- a spin-2 fermionic current $G^-(z)$ (and its hermitian conjugate),
- a nilpotent fermionic charge Q_B ($Q_B^2=0$),
- a bosonic U(1) charge J

satisfying the (anti-)commutation relations

$$\{Q_B, G^-(z)\} = T(z) \tag{1}$$

$$[J,T(z)]=0 \qquad [J,G^-(z)]=-G^-(z) \qquad [J,Q_B]=+Q_B \tag{2}$$

From Eq.(1) we see that G^- is the fermionic partner of T, while Eq.(2) gives the U(1) charges of T, G^- and Q_B to be $0, \mp 1$ respectively. For the usual bosonic string in a flat background, T is the total (matter-ghost) stress-energy tensor; the antighost b plays the role of the fermionic current G^-; Q_B is the BRS charge corresponding to the gauge fixing of diffeomorphism and J is the ghost number operator.

While it would be interesting to look for realizations of this most general 'topological string algebra', one can look for special cases where this basic structure enhances to a larger symmetry algebra. To this end, let us note that the above is reminiscent of a part of the (twisted) $N=2$ superconformal algebra. For it to extend to the full $N=2$ superconformal algebra, we must require that the charges Q_B and J come from local spin-1 currents $G^+(z)$ and $j(z)$ respectively, and the OPE between all the currents close to yield the twisted $N=2$ SCA (also called a topological conformal algebra). In this case, the theory is characterized by a topological central charge $\hat{c}$ that appears in the jj OPE.

Under what condition does the former algebra extend to the latter? The only answer existing so far is a technical one: the existence of a (possibly anomalous)

U(1) current $\partial\eta(z)$ in the string background[9]. If this current has a background charge Q_η, then the topological central charge turns out to be[1]

$$\hat{c} = 3 + \frac{Q_\eta}{4}(-Q_\eta + \sqrt{Q_\eta^2 - 8}) \tag{3}$$

The 26-dimensional bosonic string is an obvious example which meets the criterion. There are 26 U(1) currents ∂X^μ with no background charge. Critical bosonic string background therefore has a topological symmetry algebra with $\hat{c} = 3$. (The full symmetry in this background should be much larger and involve, in particular, the target space Poincaré algebra.)

Two-dimensional string theory is another, perhaps more tractable, example. In this case, there are two U(1) currents, ∂X and $\partial\varphi$, corresponding to the matter and Liouville fields. The matter current is non-anomalous, but translational invariance is broken along Liouville which has a background charge $Q_\varphi = 2\sqrt{2}$. Choosing the scalar field η to be the Liouville field[1] φ, we get a topological algebra with $\hat{c} = 1$, while the choosing η to be the matter variable X[21] leads to the critical value $\hat{c} = 3$. The latter choice is natural for the theory at a non-zero cosmological constant,[21] where the Liouville current no longer splits into holomorphic and anti-holomorphic parts.

We therefore conclude that any translation invariant direction in a string background gives rise to a topological symmetry algebra with the critical value of the topological central charge $\hat{c} = 3$. To our knowledge, this fact has not yet been fully exploited.

One can now look at the known topological models and try to interpret them as string backgrounds. A simple class of theories are the so called topological Landau-Ginzburg models[6,22]. They are defined by the superspace action

$$S = \int d^2z\, d^2\theta\, d^2\bar{\theta} D(X, \bar{X}) + \int d^2z\, d^2\theta W(X) + \int d^2z\, d^2\bar{\theta}\bar{W}(\bar{X}) \tag{4}$$

obtained by twisting from the $N = 2$ supersymmetric models. For (quasi-) homogeneous superpotential $W(X)$, the theory defined by the Eq.(4) is believed to flow to a CFT in the infrared limit[16]. The superpotential $W(X)$, being an invariant of the flow, determines the properties of the fixed-point theory. For a single chiral superfield X with superpotential $W(X) = \frac{1}{k+2}X^{k+2}$, one finds $\hat{c} = \frac{k}{k+2}$. These are well-known to describe the unitary $c_M < 1$ minimal matter coupled to gravity[6,22].

If we solve for the critical value $\hat{c} = 3$ in this class, we get $k = -3$ leading to a Landau-Ginzburg theory with singular superpotential $W(X) = -X^{-1}$. Because of the singularity at the origin, we must take X to be valued on the punctured plane $\mathbf{C}\backslash\{0\}$, which has the topology of a cylinder. The conventional LG theories, on the other hand, are valued on $\mathbf{C}$. So which string theory, if any, is this? In Ref.(10), it was shown to be the topological model of the $c_M = 1$ string theory with the matter field compactified at the self-dual radius. In the following, we will summarize some of the arguments of Ref.(10) referring the reader to the original paper for further details. The results of Ref.(10) were also subsequently, but independently, obtained in Ref.(14).

3. Landau-Ginzburg tachyons and their genus-0 correlators

The physical operators of a topological string theory are defined by the BRS cohomology. In conventional LG theories with polynomial superpotential, these arise from the chiral primaries of the untwisted theory, and the topological algebra given by the factorization of the zero-form operators is derived from the isomorphism to the chiral ring $\mathbf{C}[x]/W'(x)$. This leaves us with a finite number of primaries $\{\phi_i\}$ $0 \leq i \leq k$, and their infinite number of gravitational descendants $\{\sigma_m \cdot \phi_i\}$, $m \geq 0$, where $\sigma_0 \cdot \phi_i \equiv \phi_i$.

An arbitrary correlation function of the physical operators involving primaries and descendants can be reduced, by using the gravitational recursion relations[23,7], to those of the primaries, which in turn can be evaluated in the LG[6] or lagrangian[25] framework. Yet another approach is the LG gravity of Losev[18], who proves that, after coupling to gravity, the equation of motion $W'(x) \sim 0$ no longer implies the decoupling of arbitrary polynomials $f(x)W'(x)$. In this picture, the descendant $\sigma_m \cdot x^i$ is represented by $x^{i+m(k+2)}$.

The LG theory with superpotential $W(X) = X^{-1}$ requires extra work to define it precisely. Due to the non-polynomial nature of the superpotential many of the properties of the conventional LG, including the existence of a nilpotent chiral primary ring, do not strictly hold.* Nevertheless, it turns out that consistency requirements are powerful constraints and these suffice to find the spectrum of physical operators and determine the correlation functions.

To begin with let us consider all powers x^i, $i \in \mathbf{Z}$. Thus we have the ring $\mathbf{C}[x, x^{-1}]$. Notice that the equation of motion $W'(x) = x^{-2} \sim 0$ does not give any useful information. This is because the BRS variation of the fermion $\psi^- \equiv \frac{1}{2}(\psi - \psi^*)$ of the anti-chiral multiplet $\bar{X} = \{\bar{x}, \psi, \bar{\psi}, \bar{F}\}$ is $\delta_B \psi^- = W'(x)$. Naively therefore

$$x^i = \delta_B(\psi^- x^{i+2}) \qquad \text{for all } i \in \mathbf{Z} \tag{5}$$

all fields are BRS exact. We are, however, dealing with the cohomology relative to $G_0^- = \oint zG^-(z) - (\text{h.c})$. Using the fact that $G^- = \rho\partial\bar{x}$, (where $X = \{x, \rho, \bar{\rho}, F\}$ is the chiral multiplet), we see that

$$G_0^-(\psi^- x^{i+2}) = \oint zG^-(z)(\psi^- x^{i+2}) - (\text{h.c}) = x^{i+1} \neq 0 \tag{6}$$

Therefore x^{i+2} is not exact in the equivariant cohomology[8].

---

*The difficulties involved here are analogous to those encountered in defining an $\mathrm{SL}(2)_k$ conformal field theory, where (unlike the case of $\mathrm{SU}(2)_k$) it is not possible to rigorously determine the integrable representations and other basic properties.

Now, for a generic topological field theory coupled to topological gravity, an analysis of the fermion zero modes gives the conservation law[6,17,25]

$$\sum_{i=1}^{N}(m_i + q_i - 1) = (g-1)(3-\hat{c}) \tag{7}$$

for an arbitrary N-point correlator involving $\sigma_m \cdot \phi$, where ϕ is a primary of U(1) charge q. In the LG theory with $W(X) = X^{k+2}$, the chiral field X has charge $q[X] = 1/(k+2)$. So the monomial x^i has U(1) charge $q_i = i/(k+2)$. In our non-polynomial LG theory the U(1) charge of X is $q[X] = -1$, and therefore x^i has $q_i = -i$. Moreover, since the topological central charge is $\hat{c} = 3$, the conservation law

$$\sum_{i=1}^{N}(m_i - i - 1) = 0 \tag{8}$$

is independent of the genus.

Notice the curious degeneracy in this formula: for a fixed $(m-i)$, $\sigma_m \cdot \phi_i$ satisfies the same conservation law for all $m, i \in \mathbf{Z}$. This suggests a 'collapse' of many states of the polynomial LG theory into a singe set labelled by $i \in \mathbf{Z}$. In particular, the correlator of the operators x^{k-1} satisfies

$$\sum_{i=1}^{N} k_i = 0 \tag{9}$$

We propose that Eq.(9) is the momentum conservation law for the tachyons of the two-dimensional string theory with the $c_M = 1$ matter field compactified at the self-dual radius. The operators of the two theories are related as

$$T_k = x^{k-1}, \qquad k \in \mathbf{Z} \tag{10}$$

Recall, that the tachyons T_k's have integer momenta in appropriate normalization (self-dual radius is unity).

The cosmological operator $T_0 = x^{-1}$ coincides with the superpotential of the theory. The LG model therefore describes the two-dimensional string theory at non-zero cosmological constant. As expected, the cosmological operator satisfies charge conservation, Eq.(9), for any number of insertions and in any genus.

It turns out[10] that the positive momentum tachyons, that is, the monomials $\{x^{-1}, 1, x, x^2, \cdots\}$ can be viewed as primary fields. The negative momentum tachyons T_k, $k < 0$ on the other hand, can be thought of as the gravitational descendants of the cosmological operator T_0. (Actually, the negative momentum tachyons play a somewhat subtle role, appearing in various different "pictures", as described in Ref.(11). This point will not be essential here.)

We can now calculate the 3-point function on the sphere by the standard residue formula

$$c_{k_1 k_2 k_3} = \langle T_{k_1} T_{k_2} T_{k_3} \rangle$$

$$
\begin{aligned}
&= \operatorname{Res}\left(\frac{x^{k_1-1}x^{k_2-1}x^{k_3-1}}{x^{-2}}\right) \\
&= \delta_{k_1+k_2+k_3,0} \qquad (11)
\end{aligned}
$$

where the residue is taken around the circle at infinity. The result, as expected, is the momentum conservation condition.

Next we turn to the computation of the four-point function which involves an integration over the moduli space $\overline{\mathcal{M}}_{0,4}$. The projective symmetry fixes the position of any three tachyons, which are taken as local operators and the fourth tachyon insertion is integrated over the sphere. One can add this integrated tachyon as a perturbation to the superpotential — differentiating the 3-point function, with respect to the coupling of the perturbation, would then give the 4-point function. As was shown by Losev[18], this has be corrected by adding contact terms which arise when the fourth field collides with the other three. The latter correspond to the contribution from the boundary of $\overline{\mathcal{M}}_{0,4}$.

Explicitly, if $C_W(T_{k_i}, T_{k_j})$ is the contact term between the fields T_{k_i} and T_{k_j}, the 4-point correlator is given by

$$
\begin{aligned}
\langle T_{k_1}T_{k_2}T_{k_3}T_{k_4}\rangle_W &= \frac{\partial}{\partial t_4}\langle T_{k_1}T_{k_2}T_{k_3}\rangle_{W+t_4T_{k_4}}\Big|_{t_4=0} + \langle C_W(T_{k_4},T_{k_1})T_{k_2}T_{k_3}\rangle_W \\
&+ \langle T_{k_1}C_W(T_{k_4},T_{k_2})T_{k_3}\rangle_W + \langle T_{k_1}T_{k_2}C_W(T_{k_4},T_{k_3})\rangle_W \qquad (12)
\end{aligned}
$$

Total symmetry of the 4-point function is obtained if we require that

$$
\begin{aligned}
C_W(T_{k_i},T_{k_j}) &= \frac{d}{dx}\left(\frac{T_{k_i}(x)T_{k_j}(x)}{W'(x)}\right)_- \\
&= (k_i+k_j)T_{k_i+k_j}\theta(-k_i-k_j) \qquad (13)
\end{aligned}
$$

where $\theta(k)$ is the step function. Using this, we get the answer

$$
\begin{aligned}
\langle T_{k_1}T_{k_2}T_{k_3}T_{k_4}\rangle &= \delta\,(\sum_{i=1}^{4}k_i)\ (1-\max|k_i|) \\
&= \delta\,(\sum_{i=1}^{4}k_i)\ [-\frac{1}{2}|k_1+k_2| - \frac{1}{2}|k_1+k_3| - \frac{1}{2}|k_2+k_3| + 1] \qquad (14)
\end{aligned}
$$

which is precisely the tree-level correlation function of four tachyons computed in the matrix model[20,5] evaluated at the cosmological constant $\mu=-1$.

Similarly, the correlator of N tachyons can be computed[10]. We get answers in agreement with those obtained by other means[3,5]. The correct μ-dependence at an arbitrary non-zero μ can be restored by replacing the superpotential by $-X^{-1} \to \mu X^{-1}$. In the present simplified picture of the problem, one can perturb the superpotential only by primaries. Going to the "big phase space", in which perturbations by secondaries are also allowed, requires the addition of extra multi-point contact terms. An elegant way to accomplish this is described in Ref.(11).

In particular, without carrying out this procedure, we can only compute N-point functions with a maximum of 3 negative tachyons.

4. Flow equations

It is useful to study the LG theory perturbed by adding (integrated) primaries to the superpotential[6]. The superpotential, as well as the fields, in the perturbed theory acquire non-trivial dependence on the couplings of the perturbation t — they are said to *flow*.

One can define the formal generating function of the t-dependent multipoint correlator

$$\langle T_{k_1} \cdots T_{k_N} \rangle_W(t) \equiv \langle T_{k_1} \cdots T_{k_N} e^{\sum_{k_i>0} t_i T_{k_i}} \rangle_W \tag{15}$$

This is equal to the multipoint correlator $\langle T_{k_1}(t) \cdots T_{k_N}(t) \rangle_{W(t)}$ in the perturbed theory.

The self-consistent flows of the operators and the superpotential are obtained from the solutions of the differential equations[18,19]

$$\begin{aligned} \frac{\partial}{\partial t_i} T_{k_j}(t) &= C_{W(t)}(T_{k_i}(t), T_{k_j}(t)), \\ \frac{\partial}{\partial t_i} W(t) &= T_{k_i}(t) \end{aligned} \tag{16}$$

where, the index i is restricted to tachyons with positive k_i (primaries) only. The space of couplings of the primaries $\{t_i\}$ is called the *small phase space*. For most theories, a knowledge of correlators as a function of the coordinates of the small phase space is sufficient to determine them completely[7].

It is easy to explicitly integrate Eqs.(16). Simplification occurs because of the fact that the contact term Eq.(13) between two tachyons with positive momenta vanishes. Thus we find that the primaries do not flow

$$T_k(t) = T_k = x^{k-1} \quad \text{for } k > 0, \tag{17}$$

and there is no higher order correction to the superpotential

$$W(t) = -x^{-1} + \sum_{i=1}^{\infty} t_i x^{i-1} \tag{18}$$

which is a linear function of the couplings t_i.

Only the tachyons with negative momenta have non-trivial flow, which can be determined order by order in t, and expressed in the compact form

$$T_{-k}(t) = \left(\frac{(-W(t))^k}{-k} \right)'_- \tag{19}$$

This is analogous to the solution in the polynomial LG theory[6,8].

Consider the t-dependent 3-point correlator with one negative momentum tachyon T_{-n} and two positive momentum tachyons T_{k_1}, T_{k_2}. Using, Eqs.(11),(17) and (19), we get

$$\begin{aligned} \langle\langle T_{-n} T_{k_1} T_{k_2} \rangle\rangle &= \mathrm{Res}\left[\frac{1}{W'(t)} \left(\frac{(-W(t))^n}{-n} \right)'_- \frac{\partial W(t)}{\partial t_{k_1}} \frac{\partial W(t)}{\partial t_{k_2}} \right] \\ &= \mathrm{Res}\left[(-W(t))^{n-1} \partial_{t_{k_1}} W(t) \partial_{t_{k_2}} W(t) \right] \end{aligned} \tag{20}$$

To get the second step above, first notice that in taking the residue we can remove the minus-subscript in the expression of the negative momentum tachyon. Explicit differentiation then gives the desired result. Using Eq.(18), we can integrate Eq.(20) to obtain the generating function for the so called $1 \to N$ amplitudes

$$\langle\langle T_{-n} \rangle\rangle = \mathrm{Res}\left[\frac{(-W)^{n+1}}{n(n+1)} \right] \tag{21}$$

The above is in perfect agreement with the spherical limit of the W_∞ ward-identity derived from the matrix model[5]

$$\frac{\partial}{\partial \bar{t}_n} Z = \mathrm{Res}\left[\frac{(-W)^{n+1}}{n(n+1)} \right] Z \tag{22}$$

where the $\bar{t}_n$ denote the coupling corresponding to the negative momentum tachyon T_{-n}. In this comparison, the LG superpotential W is identified to $-\partial\phi$, where $\partial\phi$ is the bosonized current corresponding to the free fermions of the matrix model.

5. Calabi-Yau/Landau-Ginzburg correspondence at $k = -3$

LG models are closely related to the Calabi-Yau (CY) hypersurfaces in some appropriate weighted projective space[13]. The equation defining the hypersurface turns out to be the (quasi-homogeneous) superpotential of the LG theory. These heuristic arguments have recently been made more precise[26] by coupling a U(1) gauge field to the original $N = 2$ theory. Roughly speaking, one gets a class of theories characterized by one parameter r. One recovers the LG theory in the limit $r \to -\infty$, while for $r \to \infty$, one gets a sigma model with the CY hypersurface as the target space. In this sense, LG and CY σ-model are two phases of the same theory.

What, if any, is the CY phase of the LG theory with superpotential X^{-1}? A naive analysis, (assuming that the results derived for the polynomial LG theories[26] with $k \in \mathbf{Z}$ can be continued to $k = -3$), suggests some interesting generalizations of the CY/LG correspondence.

Notice that the LG theory with $k = -3$ already has the critical topological central charge $\hat{c} = 3$, which corresponds to a 3-dimensional CY hypersurface. Let us modify our superpotential in a standard way by adding extra quadratic terms, (in the following, X_0 refers to our original LG superfield X):

$$G(X) = -X_0^{-1} + X_1^2 + X_2^2 + X_3^2 + X_4^2 \tag{23}$$

which do not affect the central charge. $G(X)$ is quasi-homogeneous if we assign the weights $q_0 = -2$, $q_i = 1$, for $i \neq 0$, to the chiral superfields. In coupling to the gauge fields, one introduces a sixth chiral superfield P with charge $q = -2$ so as to ensure that one gets an anomaly free theory.

Let $\mathbf{C}^6$ be the space whose coordinates are the lowest component of the superfields $x_0, \cdots, x_4$ and p. Actually the superpotential (23) is analytic in $Y = \mathbf{C}^6 - \{x_0 = 0\}$, which is the domain relevant for us. There is a natural $\mathbf{C}^*$ action in Y: for $\lambda \in \mathbf{C}^*$, $p \to \lambda^{-2} p$, $x_0 \to \lambda^{-2} x_0$ and $x_i \to \lambda x_i$ for $i = 1, \cdots, 4$.

The superpotential of the gauge theory is $P \cdot G(X)$; and the potential energy, (after eliminating the gauge field), is

$$U = \frac{1}{2e^2} D^2 + |G|^2 + |p|^2 \sum_{i=0}^{4} \left| \frac{\partial G}{\partial x_i} \right|^2 , \tag{24}$$

where, e is the gauge coupling and D is

$$D = -e^2 \left\{ -2(|p|^2 + |x_0|^2) + (|x_1|^2 + |x_2|^2 + |x_3|^2 + |x_4|^2) - r \right\} . \tag{25}$$

Let us now look at the ground state and low energy excitations of this theory. This depends on the value of r. For $r < 0$, take $r \to -\infty$ and $|x_0| \to \infty$ such that $|p| \neq 0$ is finite. To get the minimum energy, we must now demand that G as well as all its partial derivatives $\partial G / \partial x_i$ vanish. This fixes $x_i = 0$ for $i \neq 0$. Expanding around this classical vacuum, one recovers (a $\mathbf{Z}_2$ orbifold of) the LG theory with superpotential $G(X)$.

For $r \gg 0$, the condition that D vanishes together with the transversality of G (in any finite region), requires that $p = 0$. The equation $D = 0$ gives $\sum_1^4 |x_i|^2 - 2|x_0|^2 = r$. Dividing the solutions of this by the U(1) action of the gauge group (this is the residual $\mathbf{C}^*$ action for $|\lambda| = 1$), we get the weighted projective space $\mathbf{WCP}^4_{-2,1,1,1,1}$. This is isomorphic to a properly defined $\mathbf{C}^*$ quotient of Y. To get the ground state, we must also set $G = 0$. The classical vacuum is therefore the hypersurface S defined by the vanishing of the quasi-homogeneous polynomial $G(x)$ in $\mathbf{WCP}^4$. Following standard arguments[13,26], S defines a *non-compact* generalization of Calabi-Yau manifold, and the low energy theory is a sigma model based on this target space.

6. Discussion

In the two-dimensional string theory there is a $\mathbf{Z}_2$-invariance that simply sends the $c_M = 1$ free scalar field to minus itself — this is variously interpreted as time-reversal or parity. In the topological LG model discussed here, such a symmetry would have to take x^{k-1} to x^{-k-1} which is clearly not obtainable by any transformation on the superfield. The LG model therefore fails to show a basic invariance of string theory.

The origin of this problem seems to be in the shift of 1 unit between the tachyon momentum and the topological U(1) charge. Consider, however, a generic perturbation of the LG theory,

$$\Psi(X) = \sum_k t_k X^{k-1} \tag{26}$$

which can be thought of as the string field $\Psi(X)$. Interpreting X as a complex coordinate, this is precisely the mode expansion of a spin-1 current in a conformal field theory. If $\Psi(X)$ were really treated as a spin-1 conformal field, then the inversion $X \to X^{-1}$ would effect the change $k \to -k$ thus recovering the $\mathbf{Z}_2$ invariance. Interestingly, an analysis of the target space symmetry algebra W_∞, both in continuum[27] and in matrix model[5] approaches, suggests a target-space conformal dimension of 1 for the string field corresponding to the tachyons. Transforming the variable to the cylinder by $X = e^z$ would eliminate the 'shift' by 1 unit, and we would have perfect $\mathbf{Z}_2$ symmetry on the cylinder. Recall that X is valued on $\mathbf{C}\backslash\{0\}$ — this suggests that we need to understand better the 'target-space' properties of our theory, and the right variables in which to describe it.

References

1. M. Bershadsky, W. Lerche, D. Nemeschansky and N. Warner, Nucl. Phys. **B401** (1993) 304.
2. M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, Nucl. Phys. **B** (1994) ; K.S. Narain's talk in this volume and references therein.
3. P. di Francesco and D. Kutasov, Phys. Lett. **261B** (1991) 385.
4. This point of view was emphasized by R. Dijkgraaf in his lectures in the Trieste Spring School, 1994.
5. R. Dijkgraaf, G. Moore and R. Plesser, Nucl. Phys. **B394** (1993) 356.
6. R. Dijkgraaf, H. Verlinde and E. Verlinde, Nucl. Phys. **B352** (1991) 59.
7. R. Dijkgraaf and E. Witten, Nucl. Phys. **B342** (1991) 486.
8. T. Eguchi, H. Kanno, Y. Yamada and S.-K. Yang, Phys. Lett. **298B** (1993) 73.
9. B. Gato-Rivera and A. Semikhatov, Phys. Lett. **288B** (1992) 38.
10. D. Ghoshal and S. Mukhi, hep-th/9312189 (to appear in Nucl. Phys. **B**).
11. D. Ghoshal, C. Imbimbo and S. Mukhi, hep-th/9410034.
12. For a review and references, see P. Ginsparg and G. Moore, *Lectures on 2-D Gravity and 2-D String Theory*, Los Alamos and Yale Preprint LA-UR-92-3479, YCTP-P23-92; hep-th/9304011.
13. B. Greene, C. Vafa and N. Warner, Nucl. Phys. **B324** (1989) 371.
14. A. Hanany, Y. Oz and R. Plesser, hep-th/9401030.
15. Y. Kazama and H. Suzuki, Nucl. Phys. **B321** (1989) 232.

16. W. Lerche, C. Vafa and N. Warner, Nucl. Phys. **B324** (1989) 427;
E. Martinec, in "Physics and Mathematics of Strings", eds. L. Brink et.al.
17. K. Li, Nucl. Phys. **B354** (1991) 711,725.
18. A. Losev, ITEP Preprint PRINT-92-0563 (Jan 1993), hep-th/9211089.
19. A. Losev and I. Polyubin, ITEP Preprint PRINT-93-0412 (May 1993), hep-th/9305079.
20. G. Moore, Nucl. Phys. **368** (1992) 557;
G. Mandal, A. Sengupta and S. Wadia, Mod. Phys. Lett. **A6** (1991) 1465;
K. Demeterfi, A. Jevicki and A. Rodrigues, Nucl. Phys. **B362** (1991) 173;
J. Polchinski, Nucl. Phys. **B362** (1991) 125.
21. S. Mukhi and C. Vafa, Nucl. Phys. **B407** (1993) 667.
22. C. Vafa, Mod. Phys. Lett. **A6** (1991) 337.
23. E. Witten, Nucl. Phys. **B340** (1990) 281.
24. E. Witten in *Essays on Mirror Manifold*, Ed. S.T. Yau, International Press, (1992).
25. E. Witten, Nucl. Phys. **B371** (1992) 191.
26. E. Witten, Nucl. Phys. **B403** (1993) 159.
27. E. Witten and B. Zwiebach, Nucl. Phys. **B377** (1992) 55.