Finite-temperature Field Theory

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Initial Conditions in Heavy Ion Collisions
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Outline

Phase diagram of QCD

Tools for finite-temperature QCD
The phase diagram
The phases of QCD

The deconfinement transition

Preliminaries
Pure Yang-Mills theory
Dynamical quarks

Perturbative thermal QCD

Basic thermal field theory
IR Problems
Effective Theory Approach

Summary

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Summary

Setting the stage

- Today, specialize to equilibrium thermodynamics of QCD, keeping N_c and quark masses unfixed

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\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_f (\not{D} + i m_f) \psi_f, \\
F^a_{\mu\nu} &\equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \\
D_\mu &\equiv \partial_\mu - i g A^a_\mu T^a, \quad \mathrm{Tr}\, T^a T^b = \frac{1}{2} \delta^{ab}
\end{aligned}
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- Want to know (among other things):
  - Structure of QCD phase diagram: Location of transition lines, critical points,...
  - Properties of deconfinement transition
  - Equation of state

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Tools available

- Lattice QCD
  - Only non-perturbative first principles tool: Most trustworthy whenever available
  - Limitations: $T \sim T_c$, small μ (sign problem!), realistic quark masses,...
  - Limited use with real time phenomena
- Weak coupling methods
  - Asymptotic freedom $\Rightarrow$ At asymptotically high T or μ, perturbation theory guaranteed to "converge"
  - Can probe entire parameter space of theory (μ, N_c, N_f, quark masses); even real time no problem
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- Effective theories
  - Ideology: Identify relevant degrees of freedom, and study simpler theory
  - Chiral lagrangian, EQCD, Polyakov loop models, Z_NQCD,...
  - Seldom obtain quantitatively new insight independent of above two methods
- Gauge-gravity duality
  - Conjectured duality between SU(N_c) $\mathcal{N}=4$ Super Yang-Mills in 4d and type IIB string theory in AdS$_5\times$S$_5$
  - Enables analytic calculations in strongly coupled large N_c non-Abelian gauge theory — including real time observables
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Phase diagram

- Most fundamental question to answer when studying equilibrium thermodynamics: What are the phases of your theory as a function of its parameters?
  - Phase transitions characterized by jumps (continuous or discontinuous) in thermodynamic quantities
- Universality arguments (based on symmetries, etc.) useful, but ultimately a numerical problem
  - Lattice QCD invaluable at small μ — compare to situation at high μ, small T
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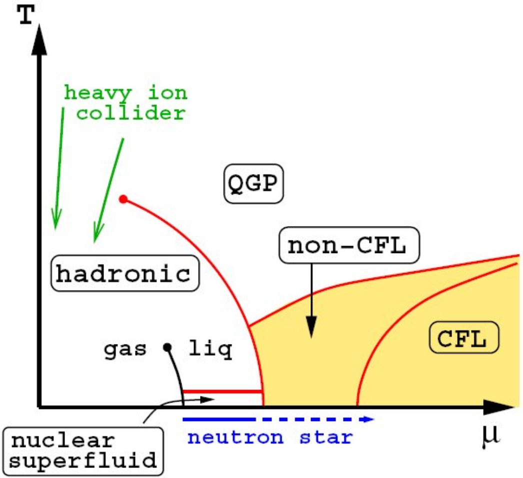
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collider
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neutron star
μ

The phases of QCD

1. Small T and μ_B: Hadron gas
   - Dilute gas of color neutral bound states
   - Thermodynamic description in terms of resonance gas model
2. $T \lesssim 10$ MeV, 308 MeV $< \mu_B \lesssim 1$ GeV: Hadron liquid (including nuclear superfluid)
   - Beyond nuclear ground state, hadron gas becomes dense, liquid-like
   - Separated from hadron gas by first order (water-vapor-like) transition line, ending at 2nd order critical point

3. $T \gtrsim 180$ MeV or $\mu_B \gtrsim 1$ GeV, $T \gtrsim 100$ MeV: Quark gluon plasma (QGP)
   - Description in terms of deconfined quarks, gluons; much more to follow
4. Asymptotically high μ_B, $T \lesssim 100$ MeV: Color-flavor locked (CFL) phase
   - Single gluon exchange provides attractive coupling between quarks on Fermi surface $\Rightarrow$ BCS pairing
   - Condensate invariant under simultaneous transformation in color and flavor space, hence CFL
   - Description via effective theories (NJL models,...)
5. 1 GeV $\lesssim \mu_B \lesssim$?, $T \lesssim 100$ MeV : Non-CFL superconductor
   - Precise nature still undetermined (kaon condensation?)
   - Problem: Not many first principles method available

In the rest of the talk, look at the high T, small μ region in more detail: Quark gluon plasma

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Elementary considerations

- Old qualitative argument: When density of nuclear matter exceeds hadron density, nucleons start overlapping
  - Asymptotic freedom ⇒ Description of system in terms of quarks, gluons
- Hard to study experimentally — weak coupling methods only available in asymptopia
- Lattice QCD most important quantitative tool
  - Reveals strong dependence on N_c, N_f, m_q
- Let's fix $N_c = 3$ and start from the cleanest case...

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Pure Yang-Mills theory: The center symmetry

- Full gauge symmetry of SU(3) Yang-Mills theory

$$A_\mu(x) \to s(x)\,(A_\mu(x) + i\,\partial_\mu)\,s(x)^\dagger,\;\; s(x) \in SU(3)$$
$$s(x + \beta\,\hat{e}_t) = z\,s(x)\,,\;\; z \in Z(3)$$

- The Wilson line transforms as a Z(3) fundamental

$$\Omega(\mathbf{x}) \equiv \mathcal{P}\exp\left[i\int_0^\beta \mathrm{d}\tau\, A_0(\tau,\mathbf{x})\right]$$
$$\mathrm{Tr}\,\Omega(\mathbf{x}) \to z\,\mathrm{Tr}\,\Omega(\mathbf{x})$$

- Ω order parameter for deconfinement transition
  - $|\langle \mathrm{Tr}\,\Omega(\mathbf{x})\rangle| = e^{-\beta\,\Delta F_q(\mathbf{x})}$
  - Non-zero value signals existence of free color charges

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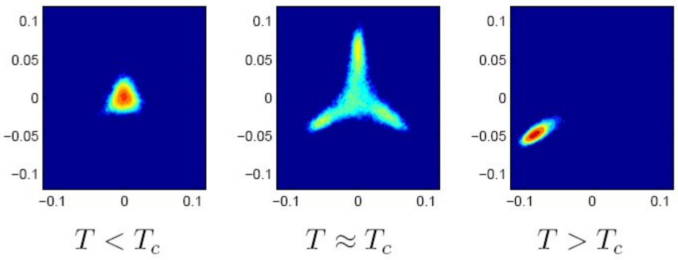
- In deconfined phase, effective potential for Ω has degenerate minima $\Omega_{\rm min} \sim e^{i2\pi n/3}\mathbf{1}$, $n \in \{0,1,2\}$
  - Tunnelings between different vacua important near T_c
  - At (1st order) phase transition quadruple point with phase coexistence with the confining one



Inclusion of dynamical quarks

- Dynamical ($m_q < \infty$) quarks break Z(3) symmetry explicitly
  - Wilson line no longer a strict order parameter for transition
  - Jump (rapid change) in $|\langle \mathrm{Tr}\,\Omega(\mathbf{x})\rangle|$ nevertheless still visible in phase transition region
- With N_f flavors of (nearly) massless quarks, chiral symmetry explicit at high T
  - At smaller temperatures, spontaneously broken via appearance of quark condensates
  - Chiral and deconfinement transitions closely related
- Huge lattice effort in determining phase diagram as function of quark masses
  - Current understanding: Physical transition cross-over at $\mu = 0$ — first order line starts from critical point at $(T, \mu) \approx$ (170,290) MeV

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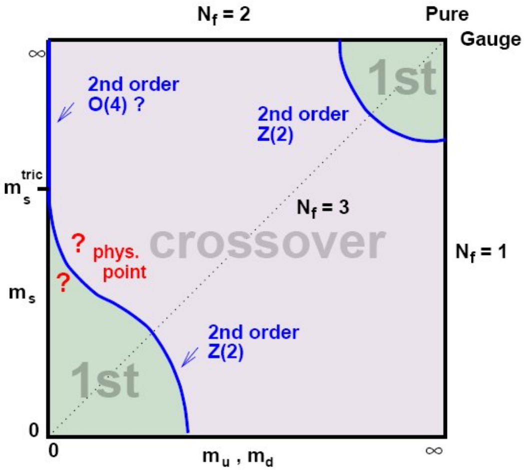
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Recap of thermal field theory

- For equilibrium thermodynamics, want to compute most importantly the partition function

$$p = \lim_{V\to\infty} \frac{T}{V} \ln Z,$$
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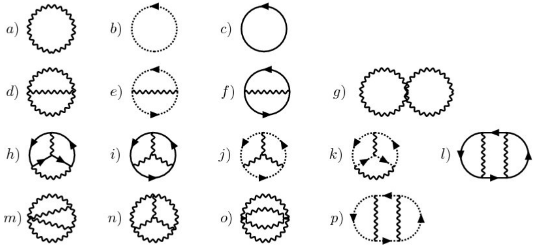
IR Sector of QCD



- Immediate problem: Strict loop expansions of thermodynamic quantities IR divergent at three-loop order
  - To obtain finite result, need to resum infinite classes of diagrams $\Rightarrow \mathcal{O}(g^3)$ term for pressure
- Need detailed understanding of energy scales contributing to the problem

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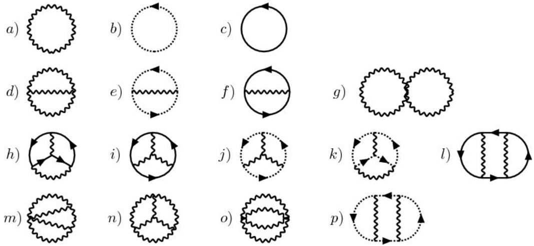

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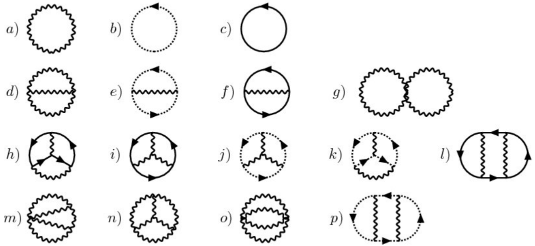

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Energy Scales in Hot QCD

At asymptotically high T, with $g \ll 1$, clear separation of three length scales:

- $\lambda \sim 1/(\pi T)$: Wavelength of thermal fluctuations, inverse effective mass of non-static field modes ($p_0 \neq 0$)
  - $n(E)g^2(T) \sim g^2(T) \Rightarrow$ Contributes perturbatively at high T
- $\lambda \sim 1/(gT)$: Screening length of static color electric fluctuations, inverse thermal mass of A_0
  - $n_b(E)g^2(T) \sim g(T) \Rightarrow$ Physics barely perturbative at high T
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- Strict loop expansions of thermodynamic quantities IR divergent
  - Solution: Resum contributions of scales gT and g^2T
  - Get terms non-analytic in g^2 in expansions
- Two competing (and completing) approaches: Direct 4d resummations and effective 3d theories
- Resummed perturbation theory systematized with hard thermal/dense loops (HTL/HDL) (Braaten, Pisarski)
  - Reorganize perturbation expansions by treating hard and soft scales on separate footing
  - Not limited to static quantities: Classical result gluon/quark damping rates (Braaten, Pisarski)
  - In equilibrium QCD, notice improved convergence of expansions; extensive work by Andersen, Braaten & Strickland and Blaizot, Iancu & Rebhan

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Effective Theories and Hot QCD

- Scale hierarchy $\Rightarrow$ Natural to integrate out massive (non-static) modes (Appelquist, Pisarski)
  - Effective description accurate for $\lambda \gtrsim 1/(gT)$
- Integrate out heavy modes to obtain 3d effective theory EQCD for static bosonic dof's (Braaten, Nieto)

$$\mathcal{L}_{\rm EQCD} = g_{\rm E}^{-2}\left\{\frac{1}{2}\,\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}\left[(D_i A_0)^2\right] + m_{\rm E}^2\,\mathrm{Tr}(A_0^2) + \lambda_{\rm E}\,\mathrm{Tr}(A_0^4)\right\} + \delta\mathcal{L}_{\rm E},$$

$$g_{\rm E} \equiv \sqrt{T}g,\ m_{\rm E} \sim gT,\ \lambda_{\rm E} \sim g^2$$

- Parameters available through comparison of long distance correlators in EQCD and full QCD

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- Parameters available through comparison of long distance correlators in EQCD and full QCD

Effective Theories and Hot QCD

- Scale hierarchy $\Rightarrow$ Natural to integrate out massive (non-static) modes (Appelquist, Pisarski)
  - Effective description accurate for $\lambda \gtrsim 1/(gT)$
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Effective Theories and Hot QCD

- EQCD valuable in reorganizing perturbation theory
  - No need for resummations in full theory
- IR sensitive sector described by EQCD: Non-perturbative contributions available through simulations in a 3d theory
- Near T_c theory unphysical due to loss of $Z(3)$ symmetry
  - Can be cured by integrating in some heavy dof's (AV, Yaffe), resulting in a physical phase diagram (Kurkela)
  - Breaking of the symmetry automatic in perturbative expansions; no effect at high enough T
- Finite μ has only minor effects as long as $m_D \ll T$ (Hart, Laine, Philipsen; Ipp, Kajantie, Rebhan, AV)
  - One new operator generated to Lagrangian, parameter matching affected

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Recent Applications of Dim. Red. Approach

- Equation of state (Kajantie, Laine, Rummukainen, Schröder)
- μ-dependence of p & quark number susceptibilities (AV)
- Spatial 't Hooft loop (Giovannangeli, Korthals Altes)
- Two-loop gauge coupling at high T (Laine, Schröder)
- Correlation lengths (Hart, Laine, Philipsen; Laine, Vepsäläinen)
- Spatial string tension (Laine, Schröder)
- Standard model pressure (Gynther, Vepsäläinen)
- Four-loop pressure of ϕ^4 theory (Gynther, Laine, Schröder, Torrero, AV)

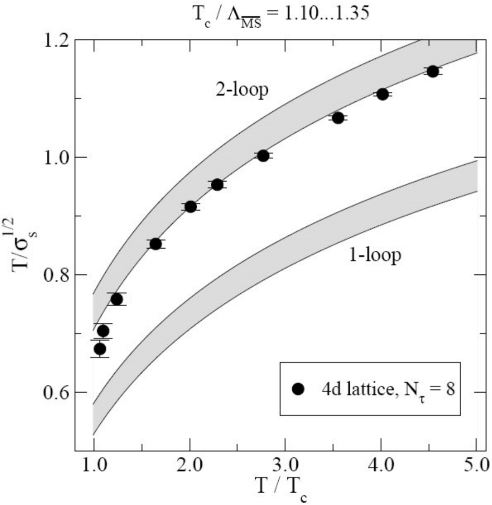
Example: Spatial String Tension

Laine, Schröder (2005): Compute in EQCD

$$\sigma_s \equiv -\lim_{R_1\to\infty}\lim_{R_2\to\infty}\frac{1}{R_1 R_2}\ln W_S(R_1,R_2)$$

to 2-loop order and compare to full theory lattice data.

Example: Spatial String Tension



Outline

Phase diagram of QCD
- Tools for finite-temperature QCD
- The phase diagram
- The phases of QCD

The deconfinement transition
- Preliminaries
- Pure Yang-Mills theory
- Dynamical quarks

Perturbative thermal QCD
- Basic thermal field theory
- IR Problems
- Effective Theory Approach

Summary

Summary

- At present, limited tools available for studying equilibrium thermodynamics of QCD
  - Lattice QCD most fundamental, weak coupling methods most versatile
- QCD has rich phase structure, now largely determined through lattice studies
  - Open questions: Nature/location of critical endpoint, non-CFL superconducting phases, value of T_c for deconfinement / chiral transitions,...
  - Deconfinement transition believed to be cross-over, critical point at $(T, \mu) \approx (170, 290)$ MeV
- At high T, perturbative QCD suffers from IR problems
  - Long distance properties of QCD describable through dimensionally reduced effective theory