Large N Thermodynamics from Lattice

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SU(N) Gauge Theory at Large N

No natural small expansion parameter in SU(N) gauge theory
'tHooft: Use $1/N$ as an expansion parameter
Perturbation theory requires taking $g^2 \to 0$ as $N \to \infty$ such that $\lambda_{tH} = g^2 N$ is finite.
Some simplifications and insights, though the $N \to \infty$ theory not solvable

Lattice study : more expensive. Glueball spectrum, chiral symmetry etc. studied.

Lucini & Teper; Narayanan & Neuberger; etc.

Study of SU(N) theory at finite T

Interesting questions at finite temperature, which may help our understanding of SU(3)

- Symmetry of the Polyakov loop $\longrightarrow$ strong first order transition
- For $N > 3$ the latent heat $\approx N^2$
- Does one reach the asymptotic state soon after the transition?
- Deviation from conformality in the plasma phase?

Deconfinement transition for SU(4)

Gavai; Ohta & Wingate

Deconfinement transition and pressure of the high temperature phase

Lucini & Teper; Lucini, Teper & Wenger; Bringholtz & Teper

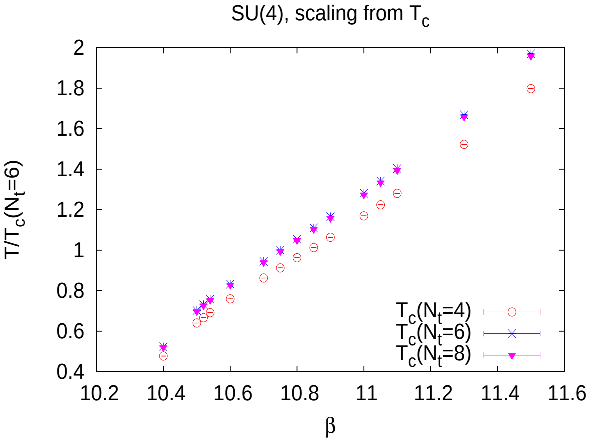
Need to be careful about discretization errors

Equilibrium Thermodynamics on Lattice

- $Z(T) = \int dU \ \exp(-\beta \int_{0,pbc}^{1/T} d\tau \int d^3x \mathcal{L}(U))$
  As lattice spacing $a \to 0$, $\beta\mathcal{L} \to \frac{1}{g^2}\mathrm{Tr} F_{\mu\nu}^2$ with
  $\beta = 2N/g^2 = 2N^2/\lambda_{tH}$
- Asymmetric $N_s^3 x N_\tau$ lattice
- Order parameter for deconfinement transition:
  $L = \frac{1}{N}\mathrm{Tr} \prod_{x_0=1}^{N_\tau} U_{(x_0,\mathbf{x}),\hat{0}}$
  Aperiodic gauge transformation $U_\mu(\vec{x}, 1/T) = e^{2\pi i/N} U_\mu(\vec{x}, 0)$
  $Z_N : L = \frac{1}{V}\sum_x L(\vec{x}) \to e^{2\pi i/N} L$
- Need $N_s \to \infty$ to study phase transition. Check effect of changing $N_s/N_t = L/T$. We take ratios between 2.5 and 4.
- To reach continuum limit, need to go to higher N_τ and check scaling

Scaling

Set the scale with $T_c(N_\tau)$ for different N_τ



V-scheme used. Coarse lattices behave worse than SU(3).

S. Gupta, PRD 64, 034507

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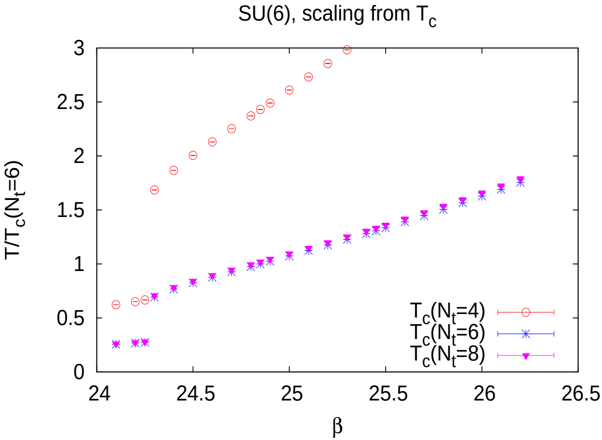

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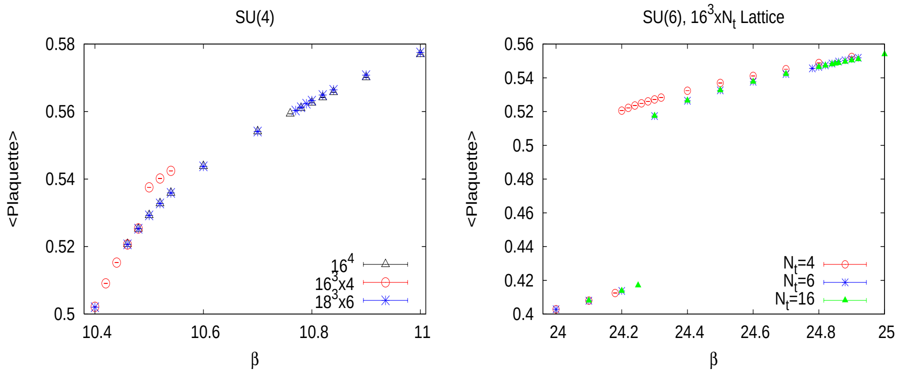
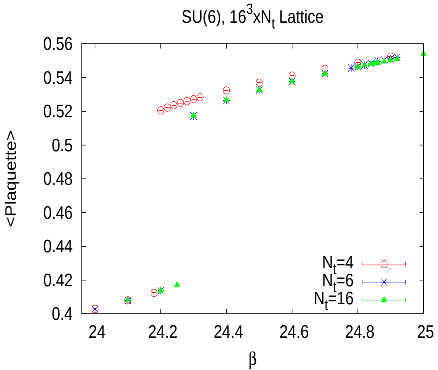
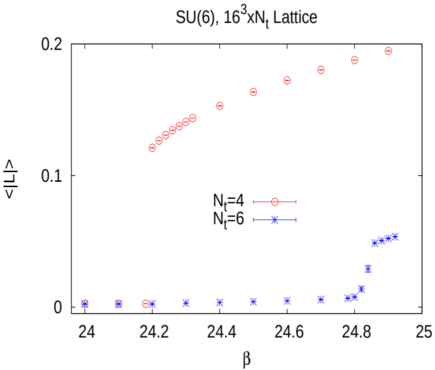
Bulk Transition for SU(6)

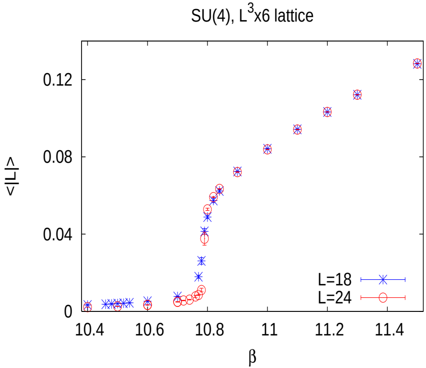


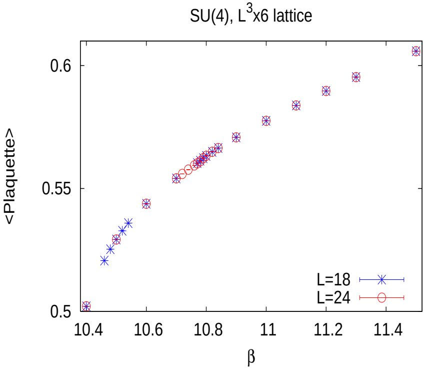
Bulk vs. Deconfinement Transition for SU(6)





Deconfinement Transition for SU(4)





$\langle P_t - P_s \rangle$ associated with deconfinement:

$\frac{\epsilon}{T^4} = 6N^2 N_\tau^4 \frac{P_t - P_s}{\lambda_{tH}} + corrections$

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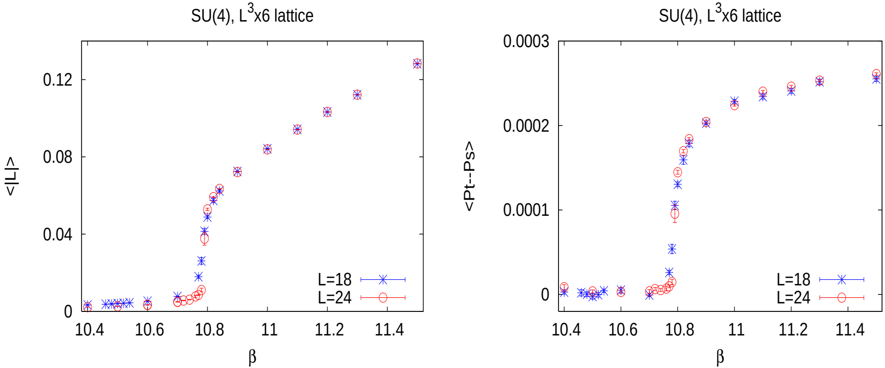

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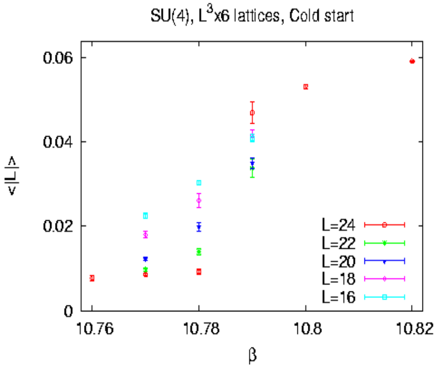
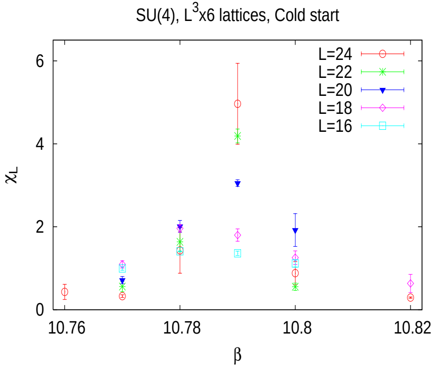
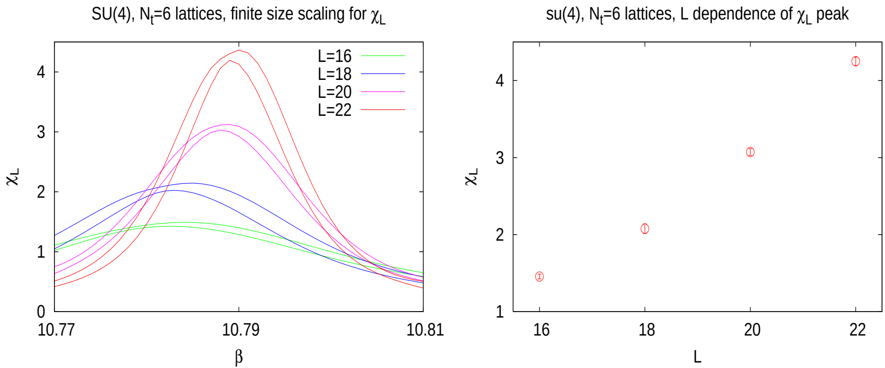
Finite Size Analysis





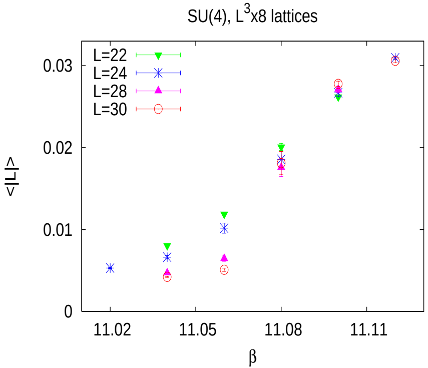
Susceptibility $\chi_L = V(\langle|L|^2\rangle - \langle|L|\rangle^2)$ $\sim V$ for 1st order transition.

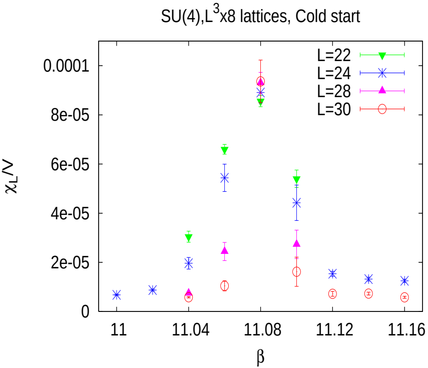
Finite Size Analysis[Contd.]



A naive fit to aL^b gives $b = 3.39 \pm 0.07$

β_c for $N_\tau = 8$





Equation of State from Lattice

Define thermodynamic quantities

$F(T,V) = T\,\ln Z(T,V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V}\frac{\partial F(T,V)/T}{\partial(1/T)}$

$p = \frac{\partial F}{\partial V} \qquad = F/V$ for homogeneous

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Trace of energy momentum tensor $\theta^{\mu\mu} = \Delta = \frac{\epsilon - 3p}{T^4}$

measure of breaking of conformal invariance

$$\boxed{\Delta/T^4 = T\frac{\partial}{\partial T}(p/T^4)}$$

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Convenient to calculate this on lattice

$T\frac{\partial}{\partial T} \rightarrow -a\frac{\partial}{\partial a}$

$p/T^4 \rightarrow \frac{N_t^3}{N_s^3} \ln Z$

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$p/T^4 \to \frac{N_t^3}{N_s^3}\ln Z$

$$\boxed{\Delta/T^4 = \frac{\partial}{\partial a}\beta\,\frac{N_t^3}{N_s^3}\langle\frac{dS}{d\beta}\rangle}$$

Eqn. of State (Contd.)

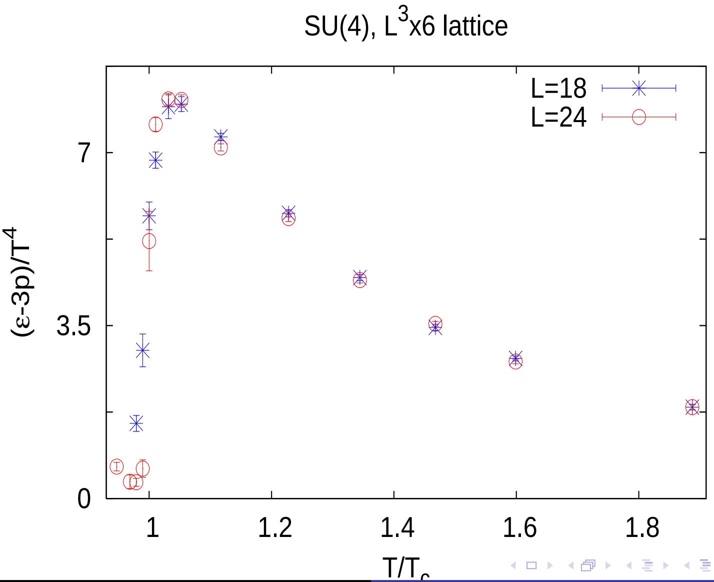
$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Delta(T')}{T'^5}$$

$$\frac{\epsilon}{T^4} = 3\frac{p}{T^4} + \frac{\Delta}{T^4} \qquad \frac{s}{T^3} = \frac{\epsilon}{T^4} + \frac{p}{T^4}$$

For free gas, Stefan-Boltzmann limit
$\frac{\epsilon}{T^4} = 3\frac{p}{T^4} = (N^2 - 1)\frac{\pi^2}{15} R(N_\tau)$
Here $R(N_\tau)$ discretization error
$= 1 + \frac{10}{21}(\frac{\pi}{N_\tau})^2 + \ldots$

Boyd et al., Nucl.Phys. B 469('96) 419;
Engels et al., Nucl.Phys. B 205('82) 545.

Interaction measure $\epsilon - 3p$



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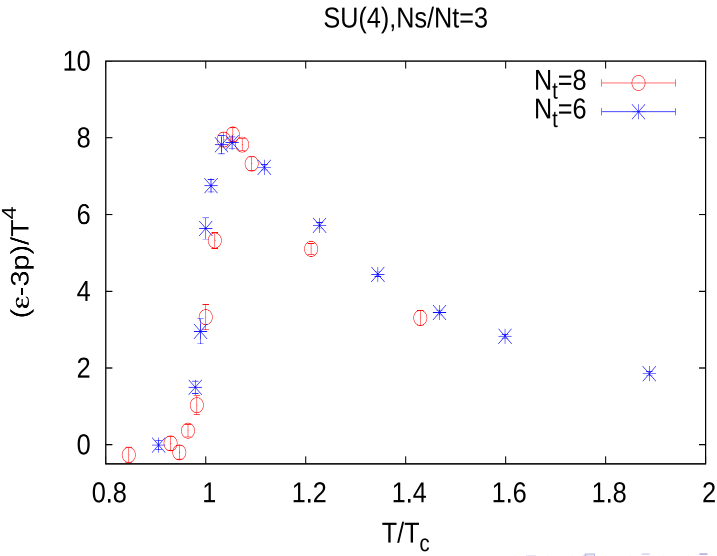
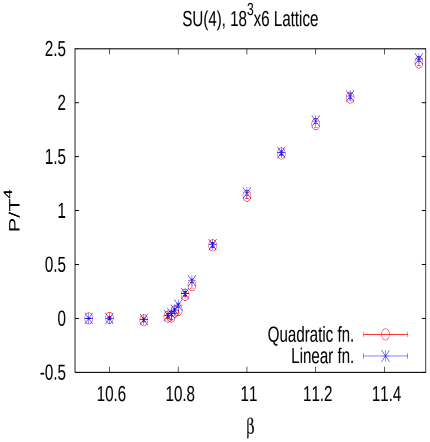
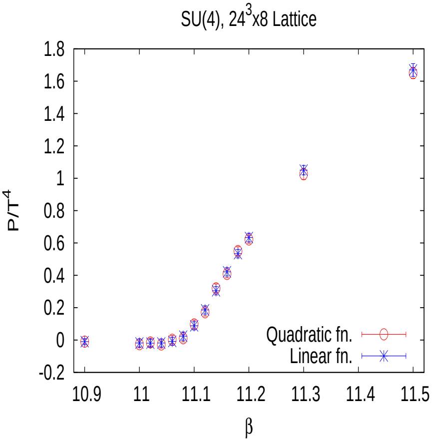


Pressure

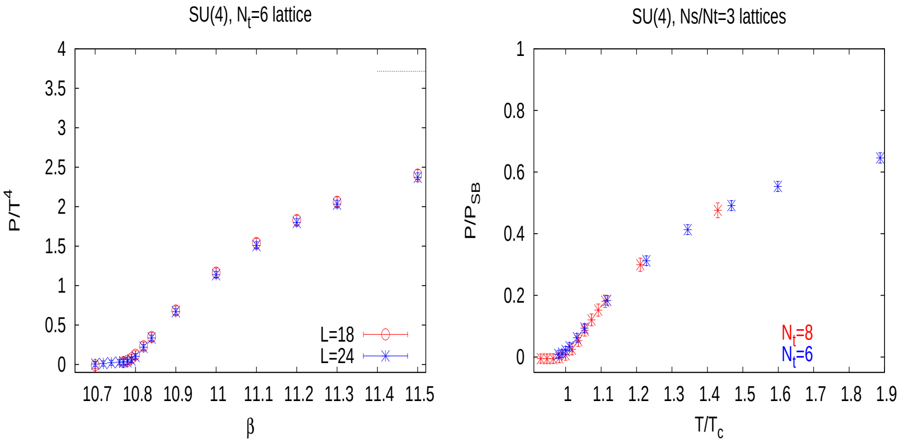


SU(4), 18^3x6 Lattice
P/T^4
2.5
2
1.5
1
0.5
0
-0.5
10.6
10.8
11
11.2
11.4
β
Quadratic fn.
Linear fn.



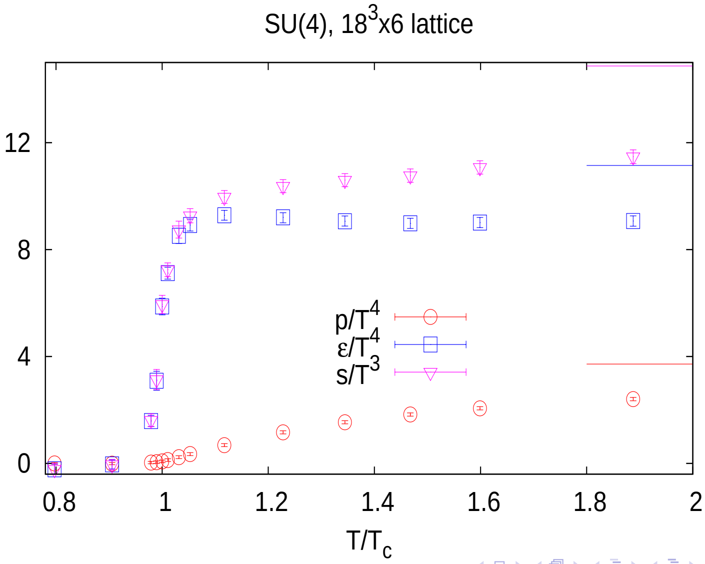
SU(4), 24^3x8 Lattice
P/T^4
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
10.9
11
11.1
11.2
11.3
11.4
11.5
β
Quadratic fn.
Linear fn.

Pressure

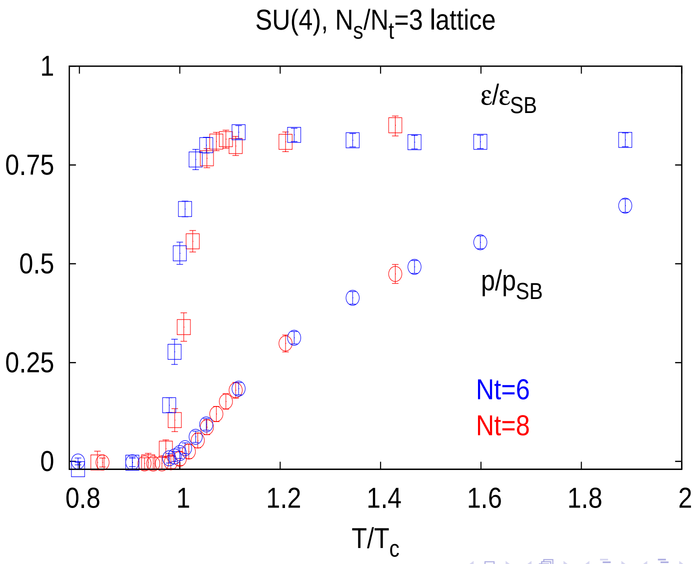


SU(4), N_t=6 lattice
P/T^4
β
L=18
L=24
SU(4), Ns/Nt=3 lattices
P/P_SB
T/T_c
N_t=8
N_t=6

Summary of Basic Thermodynamic Quantities



Cutoff Effects



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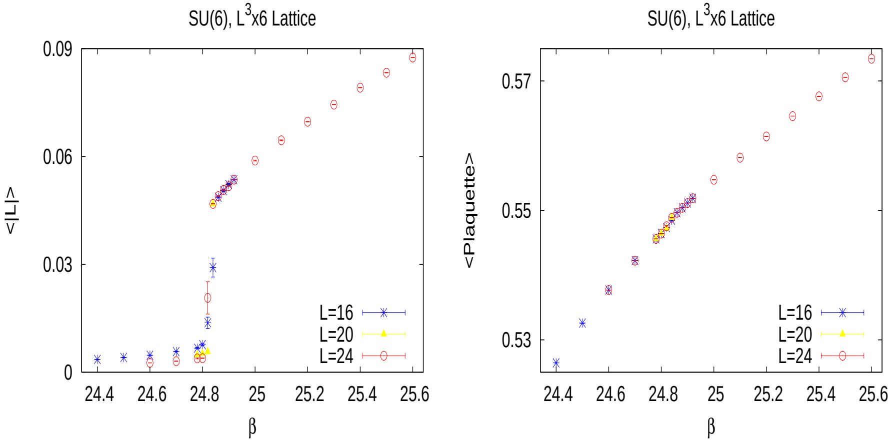

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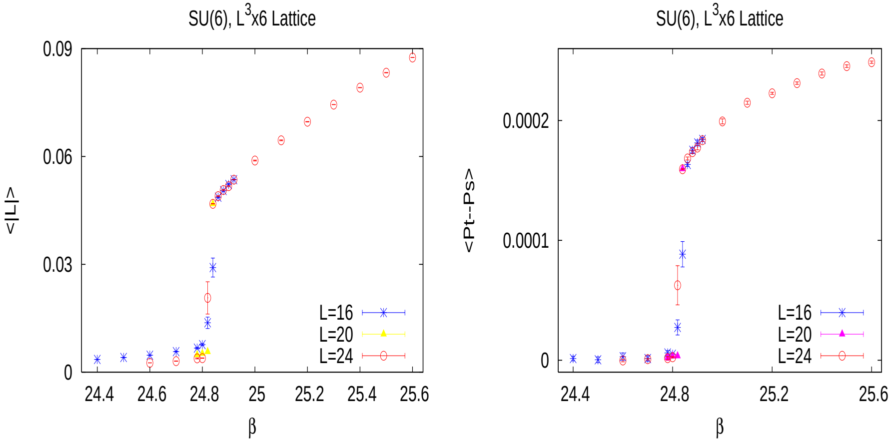
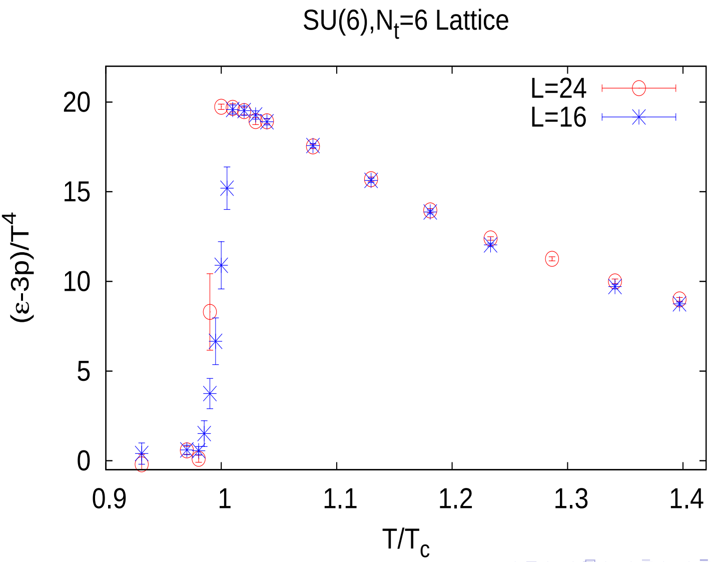


Interaction measure $\epsilon - 3p$



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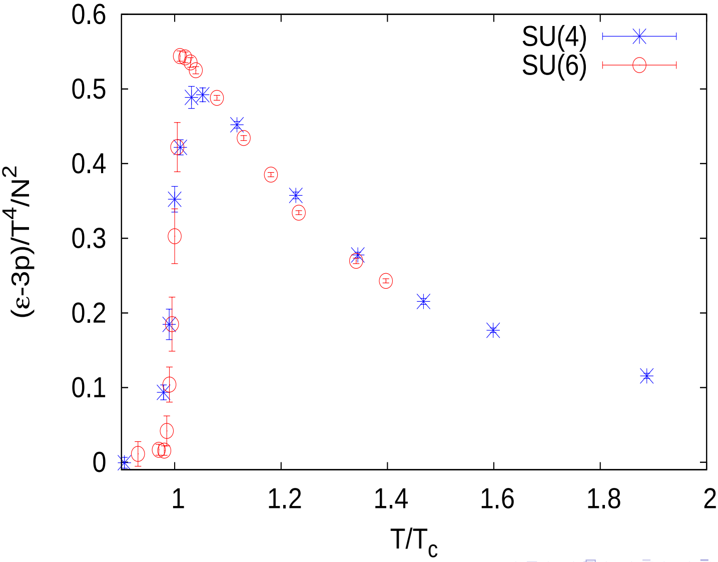

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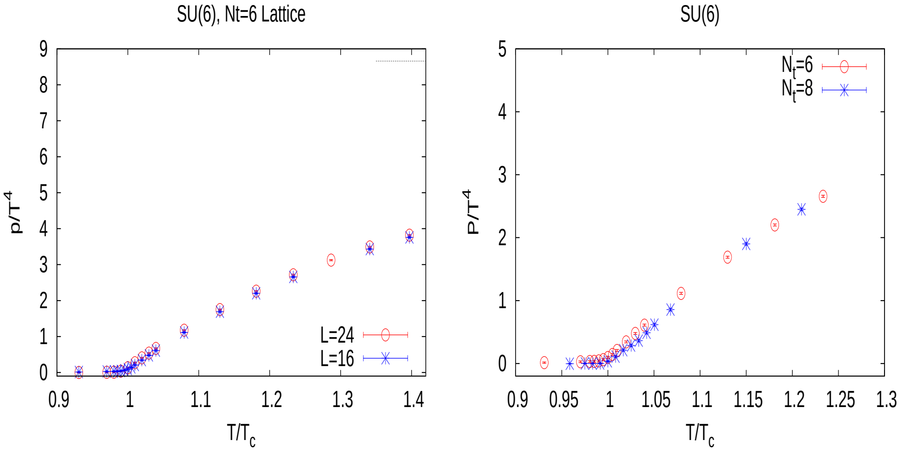
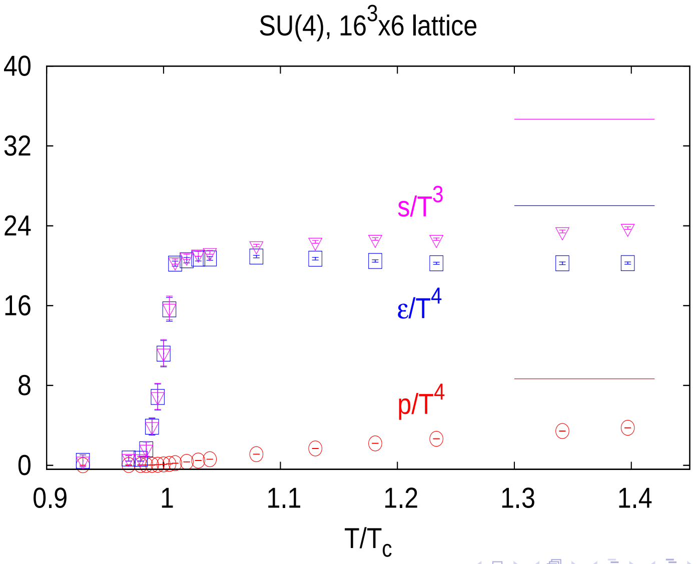
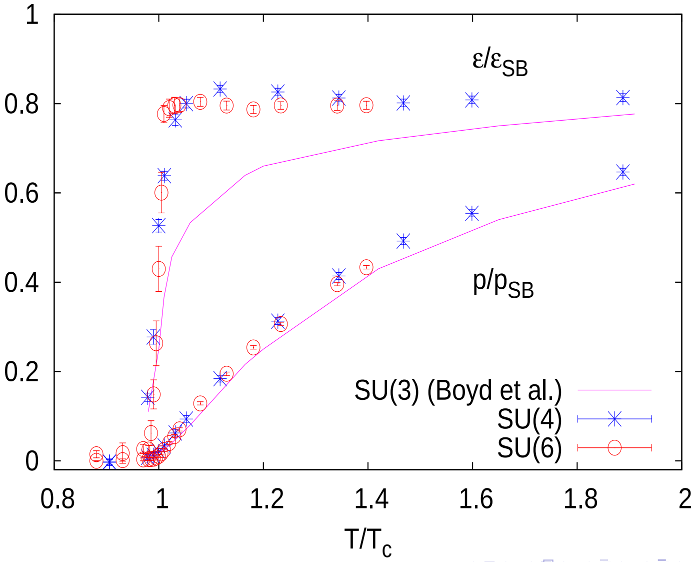


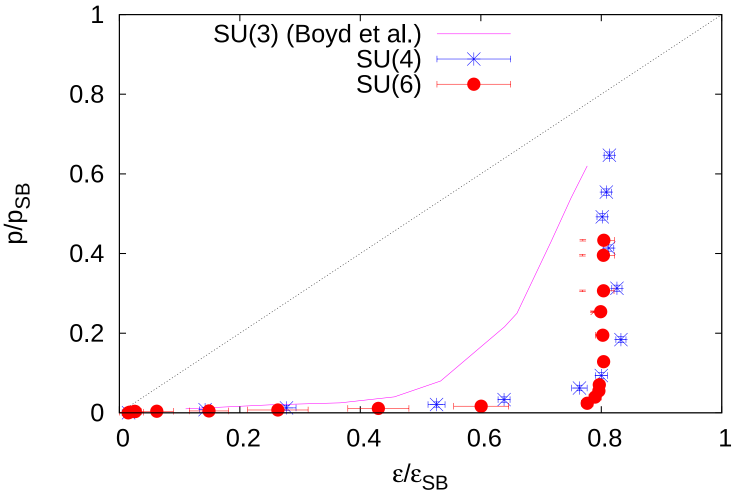
Thermodynamic Quantities for SU(6)



Thermodynamic Quantities for SU(N)



Approach to Conformal Symmetry



Gavai, Gupta & Mukherjee, Pramana(2008)

Summary

- Important to check for discretization errors.
  Very large discretization effect for coarse lattices as N increases.
  However, under control for $N_\tau \gtrsim 6$.
- $(\epsilon - 3p)/T^4$: large deviation from conformality above T_c.
  Possible slow movement of peak towards T_c with increasing N.
  At SU(4), peak position very close to SU(3).
- Pressure continues to deviate significantly from weak coupling till high temperatures.
  Would tend to disfavor certain models. [Teper.]
  Energy density almost immediately stabilizes to a value about 80% of Stefan-Boltzmann limit for $N > 3$.
- Approach to conformal limit similar for $N \gtrsim 3$.